Comparison of standard k-ε and k-ε RNG calculations of the flow around a square cylinder

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Abstract

Among the possible studies of the flow around blunt bodies, the square cylinder case is one of the simplest and more commonly analyzed in order to assess the quality of a turbulence model. This paper presents numerical solutions for the physical problem of a square cylinder immersed in a flow with an uniform velocity profile and $Re = 22000$. The calculations were performed on the same grid but varying the turbulence model (standard k-ε or k-ε RNG); a discussion follows to examine the adherence of each simulation to experimental results.

1. Introduction

The k-ε has been for many years the most popular two-equation turbulence model. In its standard formulation, defined by Launder and Sharma (1974), it is as follows:

$$\frac{\partial k}{\partial t} + \frac{\partial}{\partial x_j}(\overline{u}_j k) = P_k - \varepsilon + \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_T}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] \tag{1}$$

$$\frac{\partial \varepsilon}{\partial t} + \frac{\partial}{\partial x_j}(\overline{u}_j \varepsilon) = C_{\varepsilon 1}\frac{\varepsilon}{k}P_k - C_{\varepsilon 2}\frac{\varepsilon^2}{k} + \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_T}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right] \tag{2}$$

$$\nu_T = C_\mu \frac{k^2}{\varepsilon} \tag{3}$$

With constant coefficients: $C_\mu = 0.09$, $C_{\varepsilon 1} = 1.44$, $C_{\varepsilon 2} = 1.92$, $\sigma_k = 1.0$ and $\sigma_\varepsilon = 1.3$.

The ε equation in the standard formulation is known [5] to have a singularity where $k \to 0$ (see, for example, the stagnation region in front of an obstacle). Indeed, one can observe that $\frac{\partial \varepsilon}{\partial t} \to -\infty$ for fixed ε as $k \to 0$; since the dissipation rate quantifies how rapidly turbulent kinetic energy is converted into thermal energy, a hugely negative time derivative of ε implies an unrestrained growth of k.

The problem arises since ν_T is only determined from a single turbulence length scale, whereas in reality all scales of motion contribute to the turbulent diffusion. A more recent formulation has been developed by Yakhot and Orszag (1986) using techniques from renormalization group theory, the RNG k-ε model, in which they introduce the adimensional quantity:

$$\eta = \left(\left| \frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i} \right| \right) \frac{k}{\varepsilon} \tag{4}$$

Which is a function of both the characteristic time scale of the strain rate and the turbulence time scale. Therefore it is possible to avoid the singularity by conveniently rewriting the coefficients as functions of η. As an example, one could rewrite $C^*_{\varepsilon 2}$ as follows:

$$C^*_{\varepsilon 2} = C_{\varepsilon 2} + \frac{C_\mu \eta^3 (1 - \eta/\eta_0)}{1 + \beta \eta^3} \quad \text{where} \begin{cases} \eta_0 = 4.38 \\ \beta = 0.012 \end{cases} \tag{5}$$

In order to have:

$$\begin{cases} C^*_{\varepsilon 2} \approx C_{\varepsilon 2} & \text{if } \eta \leq \eta_0 \\ C^*_{\varepsilon 2} \ll C_{\varepsilon 2} & \text{if } \eta > \eta_0 \end{cases} \tag{6}$$

2. Computational setup

2.1. Problem domain and mesh

Given a square section $B = 1$, it was found that a rectangular domain of $30B$ and $45B$ was sufficient. The obstacle was put $15B$ away from the inlet and $30B$ away from the outlet. In order to obtain more accurate results in the area of major interest, an hybrid mesh was designed: a triangular unstructured grid was applied around the cylinder, whereas on the farther area the grid was quadrangular and structured.

2.2. Boundary conditions and numerical schemes

The boundary conditions have been defined as follows: a periodic condition on lateral borders, a Dirichlet condition on the inlet (2% turbulence intensity and $0.5B$ characteristic turbulence lenght) and an homogeneus Dirichlet condition for the pressure on the outlet. Non-equilibrium wall functions have been used.

The integration in space was made with QUICK scheme, while in time a second order implicit Euler method was used (with a timestep of 0.02).

3. Numerical results and analysis

3.1. Flow parameters

For both models 20 sets of data were acquired over a shedding period, following the partition in 20 phases identified by Lyn and Rodi [1]. One can see from table 1 that the std formulation predicts oscillations of C_L with a much lower amplitude, and that it poorly predicts the Strouhal number measured by Lyn in [1]. These results are in accordance with previous works in [2].

	C'_L	C'_D	$\overline{C}_D$	St
k-ε std [2]	0.050	0.0003	1.618	0.126
k-ε std [present]	0.054	0.00013	1.497	0.109
k-ε RNG [present]	1.459	0.044	1.900	0.138
Exp. [6]	-	-	$1.9 - 2.2$	0.132

Table 1: Comparison of parameters of fully developed flows.

3.2. Contour lines of vorticity, ν_T and k

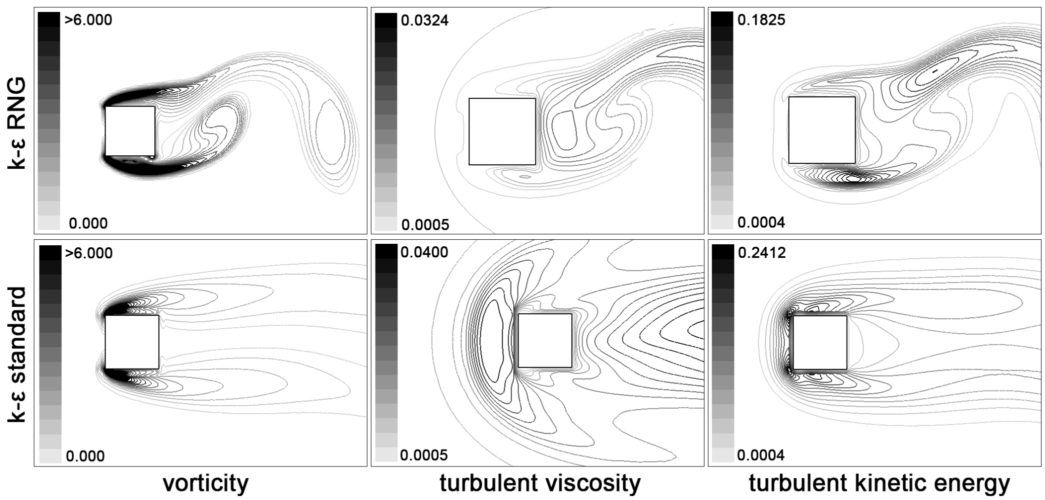

Figure 1: Comparison for phase 1 of contour lines of vorticity, ν_T and k.

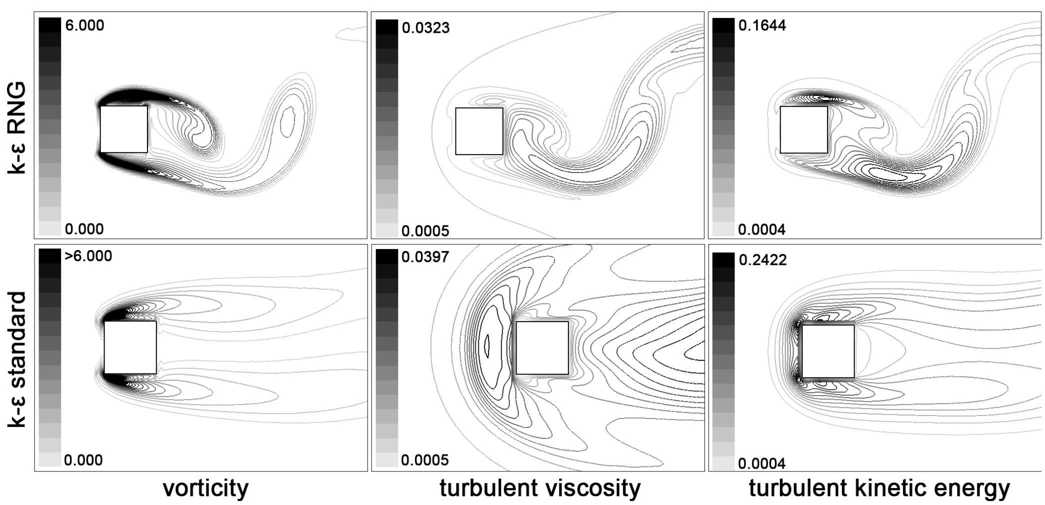

Figure 2: Comparison for phase 9 of contour lines of vorticity, ν_T and k.

From figure 1 and 2 and considering the experimental results in [3] it is clear that, while the RNG model realistically predicted the periodic vortex shedding in the wake, the standard model failed to do so (this explains the very low lift fluctuations already observed in table 1). Also, contour lines of ν_T obtained with the std model show a large amount of viscosity in front of the obstacle, where the flow is supposed to be nearly inviscid.

3.3. Turbulent kinetic energy

In accordance with the contour of ν_T (using eq. 3) and as expected from the analysis of the ε equation in section 1, there is an anomalous peak of $\langle k \rangle$ in front of the obstacle when the std model is used. It is useful to compare figure 3 with experimental results presented in [4]: both models underestimate $\langle k \rangle$ in the near wake, but the standard prediction over the stagnation region is clearly unphysical.

Indeed, the viscosity contour and the plot of $\langle k \rangle$ suggest that the standard model is simulating a flow with a Reynolds number much lower than 22000, since $Re \propto \frac{1}{\nu}$.

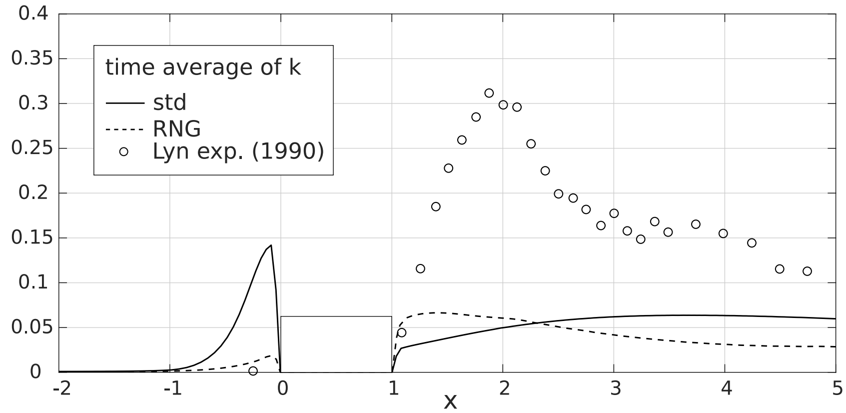

Figure 3: Time average of turbulent kinetic energy along domain centerline.

3.4. Velocity components

Figures 4, 5 and 6 show that the standard model results are again quite far from experimental ones, while RNG predicts nearly all the characteristics of the velocity profiles. This is no surprise: since the standard model is overlooking turbulent shedding, velocities along the x axis are more homogeneous (figure 4 and 6) and the u_x predicted by the standard model (figure 5) justifies the non-flapping shear layers observed in the vorticity contours.

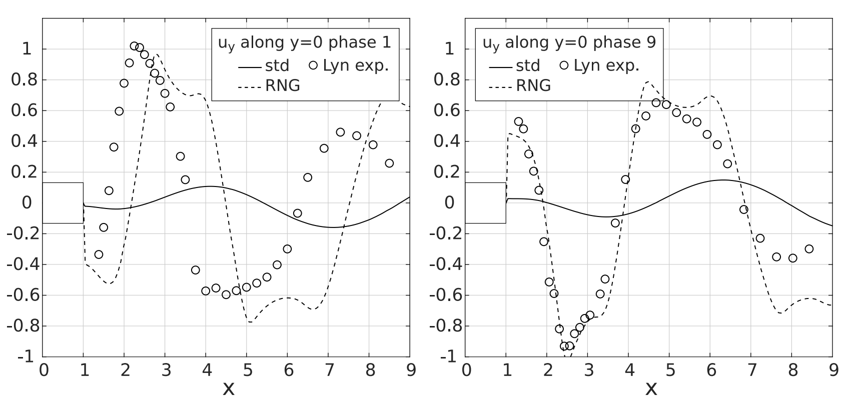

Figure 4: u_y profile along the line $y = 0$ for phases 1 and 9.

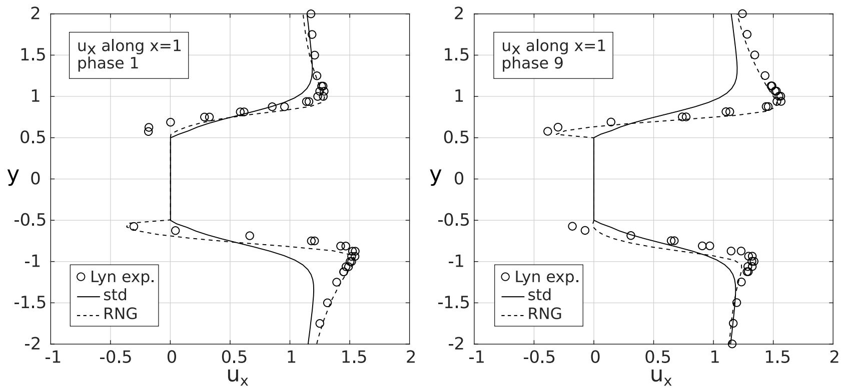

Figure 5: u_x profile along the line $x = 1$ (the square rear) for phases 1 and 9.

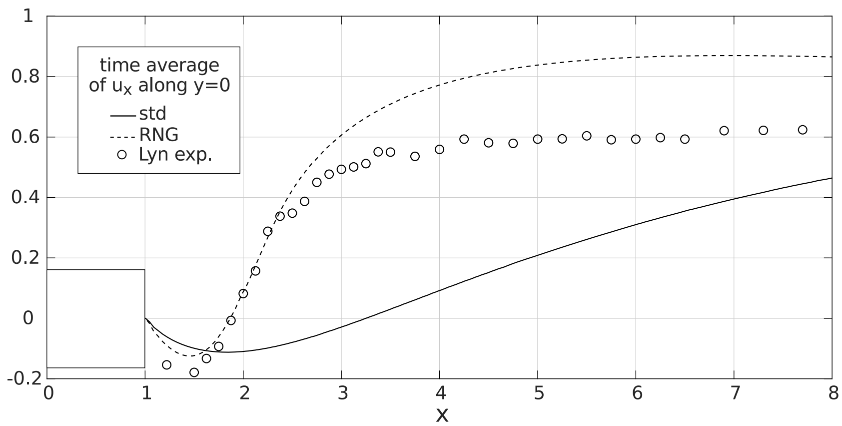

Figure 6: Time average of u_x along the line $y = 0$.

3.5. Pressure coefficient

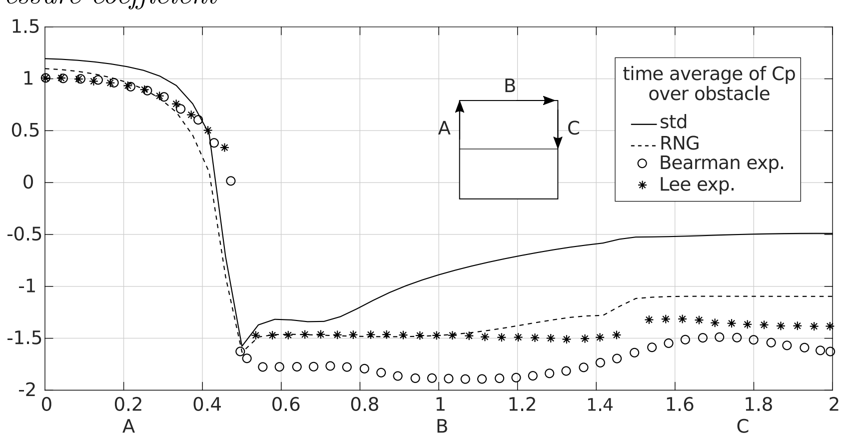

Figure 7: Time average of the pressure coefficient over the obstacle.

Figure 7 shows that the standard model overpredicts the time averaged pressure coefficient on the square rear. This is likely due to the fact that the rear pressure drops every time a vortex is shed; thus, if vortex shedding is non-existent, then the time averaged C_p would increase - since the instantaneous values of pressure are no longer oscillating.

4. Conclusions

The results highlight the already known deficiencies of the standard k-ε model, which appear to be conveniently corrected by the RNG formulation.

It may be observed that a badly formulated ε equation is to be blamed for the main discrepancies of the standard simulation with experimental results. Indeed, as the equation becomes singular in front of the obstacle, an unphysical stack of turbulent kinetic energy is predicted in this area; this prevents the transport of turbulence downstream and the subsequent release of vortices, thus affecting flow parameters such as drag, lift and pressure coefficients.

References

[1] D. A. Lyn and W. Rodi (1993) *The flapping shear layer formed by flow separation from the forward corner of a square cylinder.* Journal of Fluid Mechanics, 267, 353-376

[2] G. Bosch and W. Rodi (1998) *Simulation of vortex shedding past a square cylinder with different turbulence models.* International Journal for Numerical Methods in Fluids, 28, 601-616

[3] D. A. Lyn, S. Einav, W. Rodi and J.-H. Park (1995) *A laser-Doppler velocimetry study of ensemble-averaged characteristics of the turbulent near wake of a square cylinder.* Journal of Fluid Mechanics, 304, 285-319

[4] R. Franke and W. Rodi (1991) *Calculation of vortex shedding past a square cylinder with various turbulence models.* Proc. 8th Symp. Turbulent Shear Flows, 189–204

[5] Warren C. Strahle (1985) *Stagnation Point Flows with Freestream Turbulence — The Matching Condition.* AIAA Journal, 23-11, 1822-1824

[6] W. Rodi (1997) *Comparison of LES and RANS calculations of the flow around bluff bodies.* Journal of Wind Engineering and Industrial Aerodynamics, 69-71, 55-75