### **HOMEWORK 6**

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### **PROBLEM 0: Homework checklist**

I worked alone.

### PROBLEM 1:

Using the codes from class (or your own implementations in another language) illustrate the behavior of the simplex method on the LP from problem 13.9 in Nocedal and Wright:

minimize 
$$-5x_1 - x_2$$
  
subject to 
$$x_1 + x_2 \le 5$$
  
$$2x_1 + \frac{1}{2}x_2 \le 8$$
  
$$x \ge 0$$

starting at  $[0,0]^T$  after converting the problem to standard form. Use your judgement in reporting the behavior of the method

Firstly, we have to convert the problem into the standard form

The standard form for a linear program is

minimize 
$$c^t x$$
  
subject to  $Ax = b$   
 $x \ge 0$ 

If we convert the inequalities into equalities and we add slack variables, our problem becomes

minimize 
$$-5x_1 - x_2$$
  
subject to  $x_1 + x_2 + s_1 = 5$   
 $2x_1 + \frac{1}{2}x_2 + s_2 = 8$   
 $x, s \ge 0$ 

Let's define  $\hat{A},\,\hat{b},\,\hat{c},\hat{x}$  in order to define the problem in a standard form

$$\hat{A} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 2 & \frac{1}{2} & 0 & 1 \end{bmatrix}$$

$$\hat{b} = (5, 8)^{T}$$

$$\hat{c} = (-5, -1, 0, 0)^{T}$$

$$\hat{x} = (x_{1}, x_{2}, s_{1}, s_{2})^{T}$$

Let's define  $\hat{A}$ ,  $\hat{b}$ ,  $\hat{c}$ , as code in order to execute the simplex code and find the solution

# In [1]:

# Out[1]:

```
2×4 Matrix{Float64}:
1.0 1.0 1.0 0.0
2.0 0.5 0.0 1.0
```

Now, let's execute the simplex method code from class in order to get the solution

# In [2]:

```
1
 2
 3
   struct SimplexState
 4
        c::Vector
 5
        A::Matrix
        b::Vector
 6
 7
        bset::Vector{Int} # columns of the BFP
 8
   end
 9
10
   struct SimplexPoint
11
12
        x::Vector
        binds::Vector{Int}
13
14
        ninds::Vector{Int}
15
        lam::Vector # equality Lagrange mults
16
        sn::Vector # non-basis Lagrange mults
17
        B::Matrix
                    # the set of basic cols
                    # the set of non-basic cols
18
        N::Matrix
19
   end
20
   # These are constructors for
21
   function SimplexPoint(T::Type)
22
23
        return SimplexPoint(zeros(T,0),zeros(Int,0),zeros(Int,0),
24
            zeros(T,0), zeros(T,0), zeros(T,0), zeros(T,0))
25
   end
26
27
   function SimplexPoint(T::Type, B::Matrix, N::Matrix)
28
        return SimplexPoint(zeros(T,0),zeros(Int,0),zeros(Int,0),
29
            zeros(T,0), zeros(T,0), B, N)
30
   end
31
   function simplex_point(state::SimplexState)
32
33
        m,n = size(state.A)
34
        @assert length(state.bset) == m "need more indices to define a BFP"
        binds = state.bset # basic variable indices
35
        ninds = setdiff(1:size(A,2),binds) # non-basic
36
37
        B = state.A[:,binds]
38
        N = state.A[:,ninds]
39
        cb = state.c[binds]
        cn = state.c[ninds]
40
41
        c = state.c
42
43
        @show cn
44
45
        if rank(B) != m
46
            return (:Infeasible, SimplexPoint(eltype(c), B, N))
47
        end
48
49
        xb = B b
        x = zeros(eltype(xb),n)
50
51
        x[binds] = xb
52
        x[ninds] = zeros(eltype(xb),length(ninds))
53
54
        lam = B' \cb
55
        sn = cn - N'*lam
56
57
        @show sn
58
59
        if any(xb .< 0)</pre>
```

```
60
             return (:Infeasible, SimplexPoint(x, binds, ninds, lam, sn, B, N))
61
         else
             if all(sn \rightarrow 0)
62
                  return (:Solution, SimplexPoint(x, binds, ninds, lam, sn, B, N))
63
 64
                  return (:Feasible, SimplexPoint(x, binds, ninds, lam, sn, B, N))
65
             end
 66
         end
 67
 68
    end
 69
 70
71
     function simplex_step!(state::SimplexState)
72
         # get the current point from the new basis
73
         stat,p::SimplexPoint = simplex_point(state)
74
75
76
         if stat == :Solution
             return stat, p
 77
78
         elseif state == :Infeasible
79
             return :Breakdown, p
         else # we have a BFP
80
81
             #= This is the Simplex Step! =#
82
             # take the Dantzig index to add to basic
83
             qn = findmin(p.sn)[2]
84
85
             q = p.ninds[qn] # translate index
86
             # check that nothing went wrong
87
             @assert all(state.A[:,q] == p.N[:,qn])
88
89
             d = p.B \setminus state.A[:,q]
90
             #@show d
91
             # TODO, implement an anti-cycling method /
 92
             # check for stagnation and lack of progress
93
             # this checks for unbounded solutions
94
95
             if all(d .<= eps(eltype(d)))</pre>
96
                 return :Degenerate, p
97
             end
98
99
             # determine the index to remove
             xq = p.x[p.binds]./d
100
101
             @show xq
             ninds = d .< eps(eltype(xq))</pre>
102
103
             xq[d .< eps(eltype(xq))] .= Inf</pre>
104
             pb = findmin(xq)[2]
             pind = p.binds[pb] # translate index
105
106
             @show p.binds, pb, pind, state.bset, q
107
108
             # remove p and add q
109
110
             @assert state.bset[pb] == pind
111
112
             state.bset[pb] = q
113
114
             return stat, p
115
         end
116
    end
<del>017</del>[2]:
```

simplex\_step! (generic function with 1 method)

## In [3]:

```
using Plots
 1
 2
 3
 4
 5
 6
7
   include("C:/Users/elepu/OneDrive/Escritorio/UNIVERSIDAD/3º DATA SCIENCE/2ºCUATRI/CS-
8
   PlotRegion.plotregion(A,b)
9
10
   # start off with the point (0,0)
11
   state = SimplexState(c,A,b,
12
       [3,4]
   @show state.bset
13
   status, p = simplex_step!(state)
15 | iter = 1
16
   while status != :Solution
       @show state.bset
17
       scatter!([p.x[1]],[p.x[2]],
18
            series_annotations=["$(iter)"],marker=(15,0.2,:orange),label="")
19
       status, p = simplex_step!(state)
20
21
       iter += 1
22 end
   scatter!([p.x[1]],[p.x[2]],
23
       series_annotations=["$(iter)"],marker=(15,0.2,:red),label="")
24
25
   @show p.x
   scatter!([p.x[1]],[p.x[2]],
26
       series_annotations=["$(iter)"],marker=(15,0.2,:red),label="")
27
```

```
state.bset = [3, 4]
cn = [-5, -1]
sn = [-5.0, -1.0]
xq = [5.0, 4.0]
(p.binds, pb, pind, state.bset, q) = ([3, 4], 2, 4, [3, 4], 1)
state.bset = [3, 1]
cn = [-1, 0]
sn = [0.25, 2.5]
p.x = [4.0, 0.0, 1.0, 0.0]
```

Out[3]:



After executing the simplex method starting at  $[0,0]^T$  with BFP columns 1 and 3, we can conclude that the algorithm moves to the right, to the optimum vertex, which is vertex 2.

### **PROBLEM 2:**

Using the codes from class (or your own implementations in another language) illustrate the behavior of the simplex method on the LP from problem 13.9 in Nocedal and Wright:

minimize 
$$-x_1 - 3x_2$$
  
subject to 
$$-2x_1 + x_2 \le 2$$
  
$$-x_1 + 2x_2 \le 7$$
  
$$x > 0$$

starting at  $[0,0]^T$  after converting the problem to standard form. Use your judgement in reporting the behavior of the method

Firstly, we have to convert the problem into the standard form.

If we convert the inequalities into equalities and we add slack variables, our problem becomes

minimize 
$$-x_1 - 3x_2$$
  
subject to  $-2x_1 + x_2 + s1 = 2$   
 $-x_1 + x_2 + s - 2 = 7$   
 $x, s \ge 0$ 

Let's define  $\hat{A},\,\hat{b},\,\hat{c},\hat{x}$  in order to define the problem in a standard form

$$\hat{A} = \begin{bmatrix} -2 & 1 & 1 \\ -1 & 1 & 0 \end{bmatrix}$$

$$\hat{b} = (2, 7)^{T}$$

$$\hat{c} = (-1, -3, 0, 0)^{T}$$

$$\hat{x} = (x_{1}, x_{2}, s_{1}, s_{2})^{T}$$

Let's define  $\hat{A}$ ,  $\hat{b}$ ,  $\hat{c}$  as code in order to execute the simplex code and find the solution

# In [4]:

# Out[4]:

```
2×4 Matrix{Float64}:
-2.0 1.0 1.0 0.0
-1.0 1.0 0.0 1.0
```

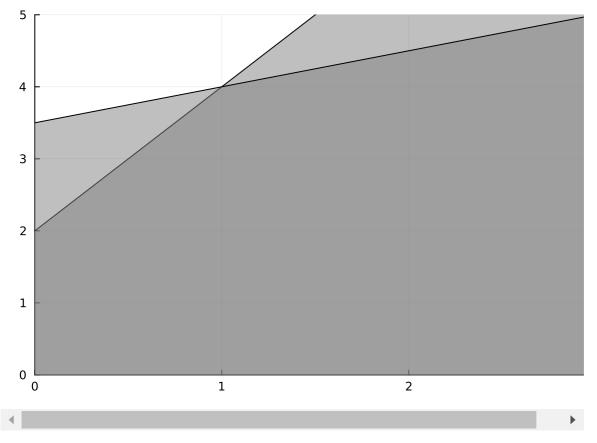
Now, let's execute the simplex method code from class in order to get the solution

## In [5]:

```
using Plots
    include("C:/Users/elepu/OneDrive/Escritorio/UNIVERSIDAD/3º DATA SCIENCE/2ºCUATRI/CS-
    PlotRegion.plotregion(A,b)
 5
    # start off with the point (0,0)
    state = SimplexState(c,A,b,
 7
        [3,4])
    @show state.bset
 9
    status, p = simplex_step!(state)
    iter = 1
    while status != :Solution
11
12
        @show state.bset
13
        scatter!([p.x[1]],[p.x[2]],
14
            series_annotations=["$(iter)"],marker=(15,0.2,:orange),label="")
15
        status, p = simplex_step!(state)
16
        iter += 1
17
    end
    scatter!([p.x[1]],[p.x[2]],
18
        series_annotations=["$(iter)"],marker=(15,0.2,:red),label="")
19
20
    @show p.x
    scatter!([p.x[1]],[p.x[2]],
        series_annotations=["$(iter)"],marker=(15,0.2,:red),label="")
22
state.bset = [3, 4]
cn = [-1, -3]
sn = [-1.0, -3.0]
xq = [2.0, 7.0]
(p.binds, pb, pind, state.bset, q) = ([3, 4], 1, 3, [3, 4], 2)
state.bset = [2, 4]
cn = [-1, 0]
sn = [-7.0, 3.0]
xq = [-1.0, 5.0]
(p.binds, pb, pind, state.bset, q) = ([2, 4], 2, 4, [2, 4], 1)
state.bset = [2, 1]
cn = [0, 0]
sn = [-4.0, 7.0]
state.bset = [2, 1]
cn = [0, 0]
sn = [-4.0, 7.0]
state.bset = [2, 1]
cn = [0, 0]
sn = [-4.0, 7.0]
```

## In [6]:

```
using Plots
 1
 2
 3
   fs1(x1) = 2*x1 +2
 4
   fs2(x1) = 0.5*x1 +3.5
 5
 6
   x1 = 0:0.01:3
 7
   p = plot(x1, [fs1.(x1), fs2.(x1)], fill=(0, 0.5, :grey),
8
             fillalpha=0.5,
9
             xlim=(0, 3),
             ylim=(0, 5),
10
             linecolor=:black,
11
             legend=false)
12
13
   display(p)
14
15
```



The region plot is above, the region is the one in dark grey. After executing the code, we notice that starting at  $[0,0]^T$ , the algorithm moves to the vertex (0,2) and then to (1,4). But the problem is unbounded. For this reason, the objective function decrease in the unbounded direction, so the solution is noton a vertex. And The algorithm cycles and stays at vertex (1,4) indefinitely.

### **PROBLEM 3**

Using the codes from class (or your own implementations in another language) illustrate the behavior of the simplex method on the LP.

minimize 
$$-3/4x_1 + 150x_2 - 1/50x_3 + 6x_4$$
subject to 
$$1/4x_1 - 60x_2 - 1/25x_3 + 9x_4 \le 0$$

$$1/2x_1 - 90x_2 + 1/50x_3 + 3x_4 \le 0$$

$$x_3 \le 1$$

$$x \ge 0$$

# starting at $[0,0,0,0]^T$ after converting the problem to standard form. Use your judgement in reporting the behavior of the method.

Firstly, we have to convert the problem into the standard form

If we convert the inequalities into equalities and we add slack variables, our problem becomes

minimize 
$$-3/4x_1 + 150x_2 - 1/50x_3 + 6x_4$$
 subject to 
$$1/4x_1 - 60x_2 - 1/25x_3 + 9x_4 + s_1 = 0$$
$$1/2x_1 - 90x_2 + 1/50x_3 + 3x_4 + s_2 = 0$$
$$x_3 + s_3 = 1$$
$$x, s \ge 0$$

Let's define  $\hat{A}$ ,  $\hat{b}$ ,  $\hat{c}$ ,  $\hat{x}$  in order to define the problem in a standard form

$$\hat{A} = \begin{bmatrix} \frac{1}{4} & -60 & -\frac{1}{25} & 9 & 1 & 0 & 0 \\ \frac{1}{2} & -90 & \frac{1}{50} & 3 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\hat{b} = (0, 0, 1)^{T}$$

$$\hat{c} = (-\frac{3}{4}, 150, -\frac{1}{50}, 6, 0, 0, 0)^{T}$$

$$\hat{x} = (x_{1}, x_{2}, x_{3}, x_{4}, s_{1}, s_{2}, s_{3}, s_{4})^{T}$$

Let's define  $\hat{A}, \hat{b}, \hat{c}$ , as code in order to execute the simplex code and find the solution

## In [7]:

```
using LinearAlgebra
A1 = [0.25 -60.0 -(1.0/25.0) 9.0;
0.50 -90.0 (1.0/50.0) 3.0;
4 0.0 0.0 1.0 0.0];

b = [0.0; 0.0; 1.0];
c=[-(3.0/4.0); 150.0; -(1.0/50.0); 6.0;0.0;0.0;0.0]
A = [A1 Matrix{Float64}(I,3,3)] # form the problem with slacks.
```

## Out[7]:

```
3×7 Matrix{Float64}:
    0.25    -60.0    -0.04    9.0    1.0    0.0    0.0
    0.5    -90.0    0.02    3.0    0.0    1.0    0.0
    0.0    0.0    1.0    0.0    0.0    1.0
```

Now, let's execute the simplex method code from class in order to get the solution

## In [8]:

```
2
    # start off with the point (0,0)
 3
    state = SimplexState(c,A,b,
 4
        [5,6,7]
 5
    @show state.bset
    status, p = simplex_step!(state)
 7
    iter = 1
 8
    while status != :Solution
 9
        @show state.bset
10
        scatter!([p.x[1]],[p.x[2]],
11
            series_annotations=["$(iter)"],marker=(15,0.2,:orange),label="")
12
        status, p = simplex_step!(state)
13
        iter += 1
14
    end
15
    scatter!([p.x[1]],[p.x[2]],
        series_annotations=["$(iter)"],marker=(15,0.2,:red),label="")
16
17
    @show p.x
18
state.bset = [5, 6, 7]
```

```
cn = [-0.75, 150.0, -0.02, 6.0]
sn = [-0.75, 150.0, -0.02, 6.0]
xq = [0.0, 0.0, Inf]
(p.binds, pb, pind, state.bset, q) = ([5, 6, 7], 1, 5, [5, 6, 7], 1)
state.bset = [1, 6, 7]
cn = [150.0, -0.02, 6.0, 0.0]
xq = [-0.0, 0.0, Inf]
(p.binds, pb, pind, state.bset, q) = ([1, 6, 7], 2, 6, [1, 6, 7], 2)
state.bset = [1, 2, 7]
cn = [-0.02, 6.0, 0.0, 0.0]
sn = [-0.04, 18.0, 1.0, 1.0]
xq = [0.0, -0.0, 1.0]
(p.binds, pb, pind, state.bset, q) = ([1, 2, 7], 2, 2, [1, 2, 7], 3)
state.bset = [1, 3, 7]
cn = [150.0, 6.0, 0.0, 0.0]
sn = [12.0, 11.9999999999999, 0.19999999999973, 1.400000000000000]
p.x = [0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 1.0]
```

## Out[8]:

1.0

```
7-element Vector{Float64}:
0.0
0.0
0.0
0.0
0.0
0.0
0.0
```

The transverse vertices are  $state.\ bset = [5, 6, 7],\ state.\ bset = [1, 6, 7],\ state.\ bset = [1, 2, 7],\ state.\ bset = [1, 3, 7]$  in all of these  $p.\ x = [0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 1.0]$ 

After executing the code, we know that starting at  $[0,0,0,0]^T$ , the value of x at the optimum is  $[0.0,0.0,0.0,0.0,0.0,0.0,1.0]^T$ . The first BFP is optimal, but the algorithms move to other BFP, until it finds a BFP that satisfies the condition.

### **PROBLEM 4**

Show that if we have:

minimize 
$$c^T x$$
  
subject to  $Ax \le b$   
 $x \ge 0$ 

and  $b \ge 0$ , then x = 0 is always a vertex after converting to standard form.

Firstly, we have to convert the problem into the standard form

If we convert the inequalities into equalities and we add slack variables, our problem becomes

minimize 
$$c^T x$$
  
subject to  $Ax + s = b$   
 $x, s \ge 0$ 

Let's define  $\hat{A}$ ,  $\hat{b}$ ,  $\hat{c}$ ,  $\hat{x}$  in order to define the problem in a standard form

$$\hat{A} = \begin{bmatrix} A & I \end{bmatrix}$$

$$\hat{b} = b$$

$$\hat{c} = (c, 0)^{T}$$

$$\hat{x} = (x, s)^{T}$$

So, our problem is

minimize 
$$\hat{c}^T \hat{x}$$
  
subject to  $\hat{A}\hat{x} = b$   
 $\hat{x} > 0$ 

Firstly, let's show that, the point with x = 0 is a **BFP**.

A BASIC FEASIBLE POINT (BFP) is a feasible point  $(Ax = b, x \ge 0)$  where  $AP = [B \ N]$ , B is non-

singular and 
$$\hat{p}^T = \begin{bmatrix} x_B \\ 0 \end{bmatrix} = \begin{bmatrix} \hat{B}^{-1}b \\ 0 \end{bmatrix}$$

It is feasible because Ax = b and  $x \ge 0$ . The columns of B we choose for the BFP are the last column of A, so that  $AP = [I \ A]$ . Thus, B = I, so B is non-singular.

As explained above,  $x_B = B^{-1}b$ . As B = I,  $x_B = I^{-1}b = b$ , As  $b \ge 0$ ,  $x_B \ge 0$ ,  $x_B$  is positive. We proved that the point with x = 0 is a BFP.

As, it is A BFP, it is a vertex of the polytope. So, we have shown that the point with x=0 is always a vertex after converting to standard form.

#### **PROBLEM 5**

The goal here is to develop an LP-based solver for a 1-norm regression problem. Consider the problem:

$$\underset{x}{\text{minimize}} \qquad ||Ax - b||_1 = \sum_i |a_i^T x - b_i|$$

where  $a_i^T$  is the ith row of A. This can be converted into a linear program and solved via the simplex method. Compute the solution for the sports ranking problem

Firstly, we have to convert the problem into the LP standard form in order to solve it later via the simplex method

Let's introduce an auxiliary veriable  $t_i = a_i x - b_i$ . So now, we have:

minimize 
$$\sum_{i} t_{i}$$
 subject to 
$$t_{i} \geq a_{i}^{T} x - b_{i} \leq t_{i}$$

If we separate the inequalities, we have:

minimize 
$$\sum_{i} t_{i}$$
subject to 
$$a_{i}^{T} x - b_{i} \leq t_{i}$$

$$a_{i}^{T} x - b_{i} \geq -t_{i}$$

This can be rewritten as

minimize 
$$\sum_{i} t_{i}$$
subject to 
$$t_{i} - a_{i}^{T} x + b_{i} \ge 0$$

$$t_{i} + a_{i}^{T} x - b_{i} \ge 0$$

Let's introduce slack variables in order to convert the inequalities into equalities

minimize 
$$\sum_{i} t_{i}$$
subject to 
$$t_{i} - a_{i}^{T} x + b_{i} + s_{1} = 0$$

$$t_{i} + a_{i}^{T} x - b_{i} + s_{2} = 0$$

$$s_{1}, s_{2}, t \ge 0$$

As x is unconstrained in sign, we convert it into a pair of positive variables.

$$x = x^+ - x^-$$

Now, our problem becomes

minimize 
$$\sum_{i} t_{i}$$
subject to 
$$t_{i} - a_{i}^{T} x^{+} + a_{i}^{T} x^{-} + b_{i} + s_{1} = 0$$

$$t_{i} + a_{i}^{T} x^{+} - a_{i}^{T} x^{-} - b_{i} + s_{2} = 0$$

$$s_{1}, s_{2}, x^{+}, x^{-}, t \ge 0$$

This problem can be rewrittwn as follows:

which as follows:  
minimize 
$$te$$
  
subject to  $-Ax^+ + Ax^- + te + s_1 = -b$   
 $Ax^+ - Ax^- + te + s_2 = b$   
 $s_1, s_2, x^+, x^-, t \ge 0$ 

Let's define  $\hat{A}$ ,  $\hat{b}$ ,  $\hat{c}$ ,  $\hat{x}$  in order to define the problem in a standard form

$$\hat{A} = \begin{bmatrix} -A & A & e & I & 0 \\ A & -A & e & 0 & I \end{bmatrix}$$

$$\hat{b} = (-b, b)^{T}$$

$$\hat{c} = (0, 0, e, 0, 0)^{T}$$

$$\hat{x} = (x^{+}, x^{-}, t, s_{1}, s_{2})^{T}$$

We have to use the sport data that is the following one:

## In [9]:

```
using LinearAlgebra, Plots, Random, SparseArrays
   teams = ["duke","miami","unc","uva","vt"]
   data = [ # team 1 team 2, team 1 pts, team 2 pts
       1 2 7 52 # duke played Miami and lost 7 to 52
       1 3 21 24 # duke played unc and lost 21 to 24
 5
 6
       1 4 7 38
 7
       1 5 0 45
 8
       2 3 34 16
9
       2 4 25 17
10
       2 5 27 7
11
       3 5 3 30
12
13
       4 5 14 52]
14 ngames = size(data,1)
   nteams = length(teams)
15
16
   G = zeros(ngames, nteams)
17
18 p = zeros(ngames, 1)
19 for g=1:ngames
20
       i = data[g,1]
      j = data[g,2]
21
22
      Pi = data[g,3]
23
      Pj = data[g, 4]
24
       G[g,i] = 1
25
26
       G[g,j] = -1
       p[g] = Pi - Pj
27
28
   end
```

Let's define  $\hat{A}$ ,  $\hat{b}$ ,  $\hat{c}$ ,  $\hat{x}$  as code in order to compute the simplex method. IN the sport data G=A and p=b

Let's define  $\hat{A}, \, \hat{b}, \, \hat{c}, \hat{x}$  in order to define the problem in a standard form

$$\hat{A} = \begin{bmatrix} -A & A & e & I & 0 \\ A & -A & e & 0 & I \end{bmatrix}$$

$$\hat{b} = (-b, b)^{T}$$

$$\hat{c} = (0, 0, e, 0, 0)^{T}$$

$$\hat{x} = (x^{+}, x^{-}, t, s_{1}, s_{2})^{T}$$

### In [10]:

```
using LinearAlgebra
e=[1,1,1,1,1,1,1,1,1]
A1 = [-G G e;
G -G e;];

b = [-p;p];
c=[0.0,0.0,e,0.0,0.0]
A = [A1 Matrix{Float64}(I,20,20)] # form the problem with slacks.
```

## Out[10]:

```
20×31 Matrix{Float64}:
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```

Once that everything is defined, let's compute the Simplex method

### In [11]:

```
# start off with the point (0,0)
    state = SimplexState(c,A,b,
        [1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20])
 4
    @show state.bset
 5
    status, p = simplex_step!(state)
    iter = 1
    while status != :Solution
 7
 8
        @show state.bset
 9
        scatter!([p.x[1]],[p.x[2]],
            series_annotations=["$(iter)"],marker=(15,0.2,:orange),label="")
10
11
        status, p = simplex_step!(state)
12
        iter += 1
13
    end
    scatter!([p.x[1]],[p.x[2]],
14
        series_annotations=["$(iter)"],marker=(15,0.2,:red),label="")
15
16
    @show p.x
17
MethodError: no method matching (Vector)(::Matrix{Float64})
Closest candidates are:
```

```
MethodError: no method matching (Vector)(::Matrix{Float64})
Closest candidates are:
   (Array{T, N} where T)(::AbstractArray{S, N}) where {S, N} at boot.jl:481
   (Vector)() at baseext.jl:38
   (Vector)(::AbstractSparseVector{Tv}) where Tv at C:\Users\elepu\AppData
\Local\Programs\Julia-1.8.5\share\julia\stdlib\v1.8\SparseArrays\src\spars
evector.jl:950
   ...
Stacktrace:
   [1] convert(#unused#::Type{Vector}, a::Matrix{Float64})
    @ Base .\array.jl:617
   [2] SimplexState(c::Vector{Any}, A::Matrix{Float64}, b::Matrix{Float64},
bset::Vector{Int64})
   @ Main .\In[2]:4
   [3] top-level scope
   @ In[11]:2
```

I can't get a numeric solution because I don't know how I have to define the indices. I think that my error is in the vector e, I have been a long time working on this exercise, but I am not ablo to fin my error and solve it