THE PARIKH PROPERTY

for Weighted Context-Free Grammars

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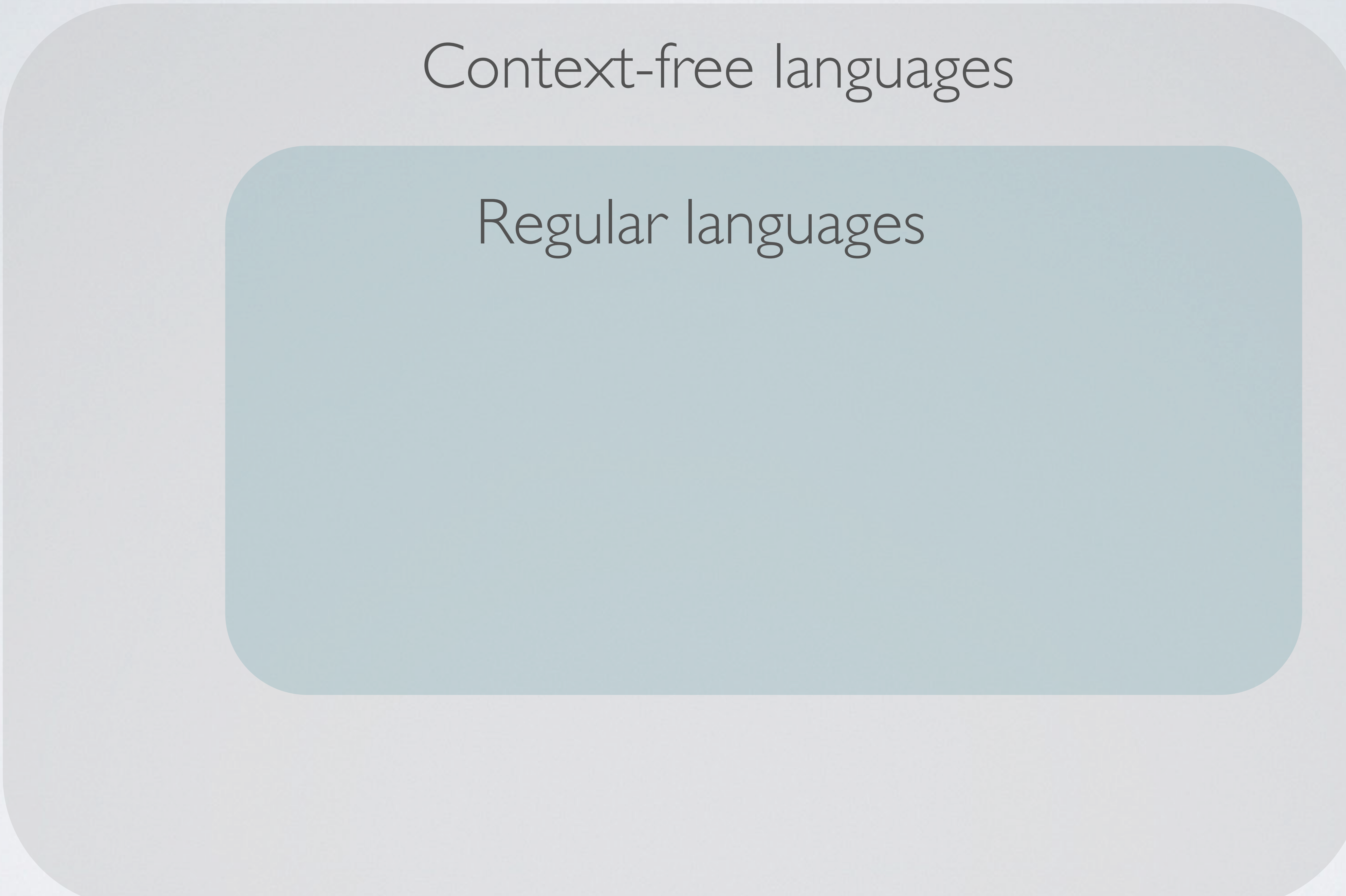
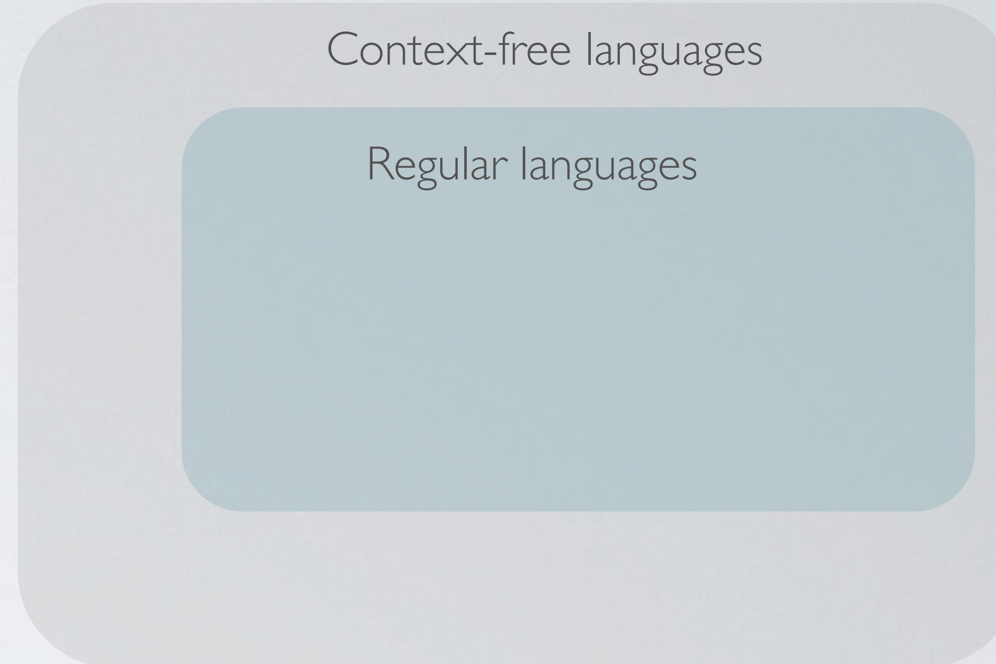
FSTTCS 2018

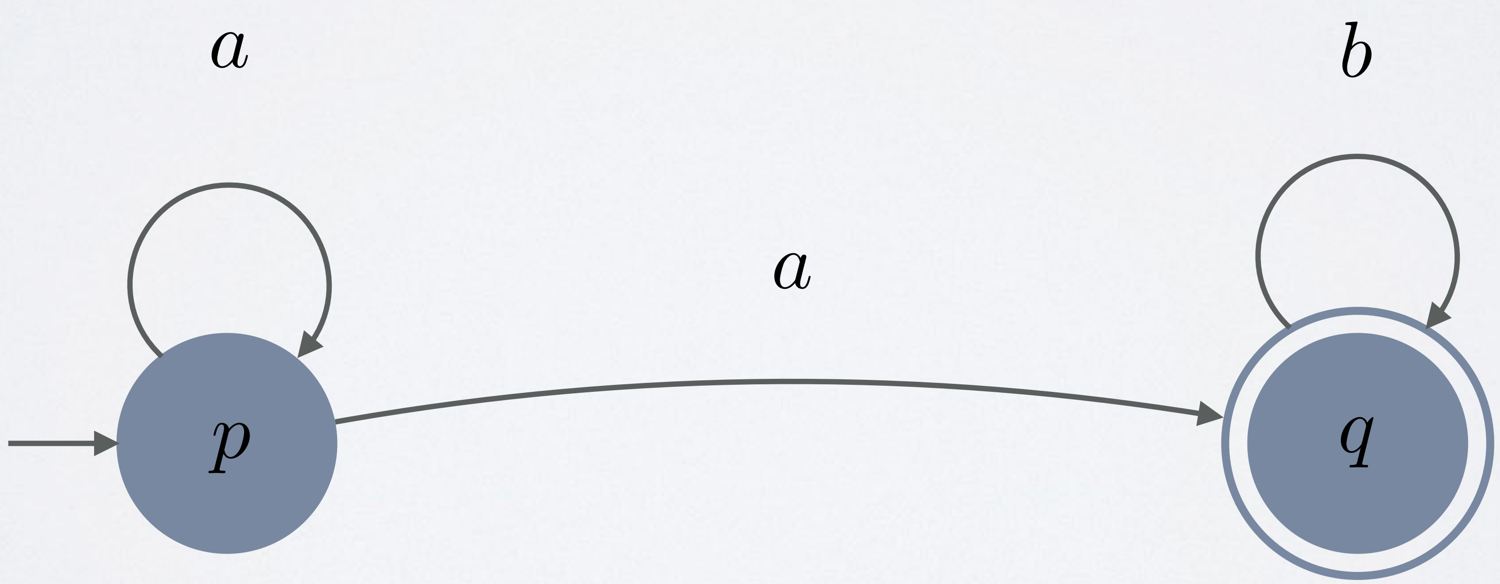

Context-free languages
Regular languages

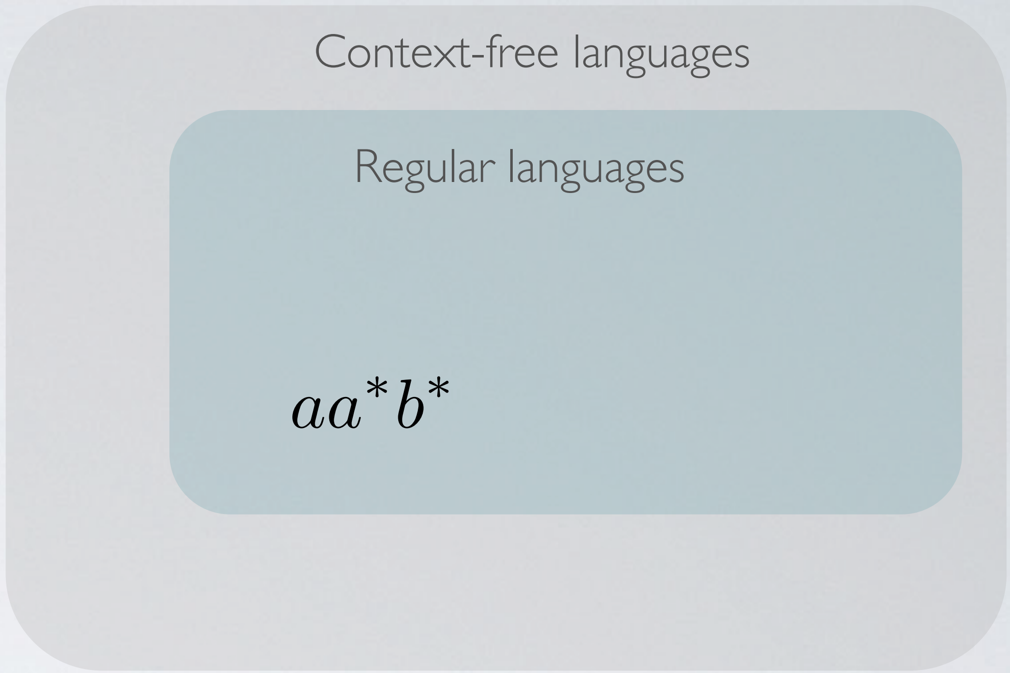
$$\{a^n b^m \mid n > 0, m \geq 0\}$$



$$\{a^n b^m \mid n > 0, m \geq 0\}$$

aa^*b^*




Context-free languages
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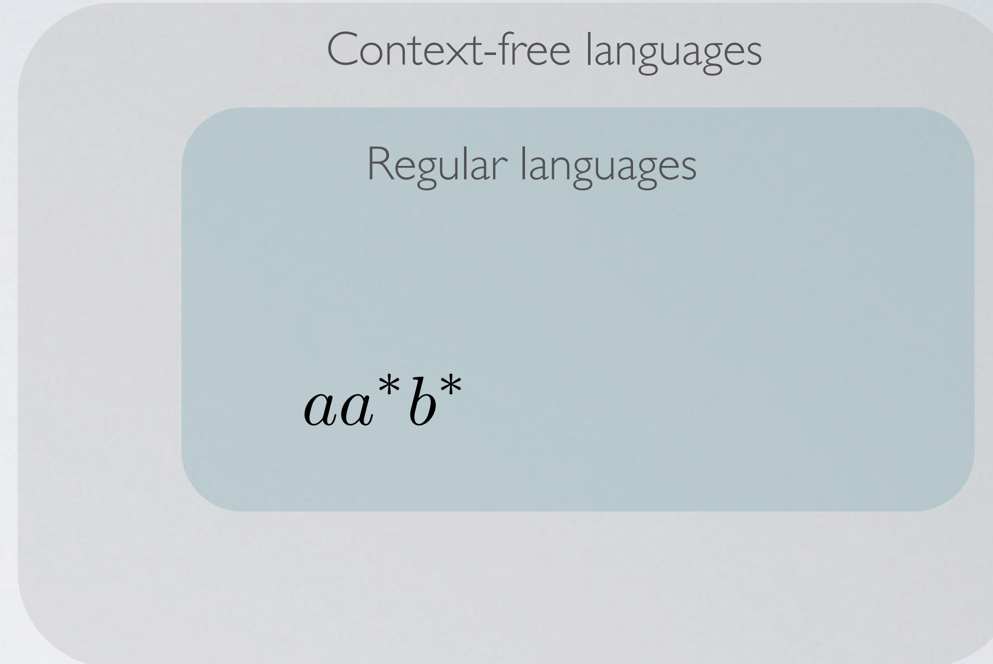
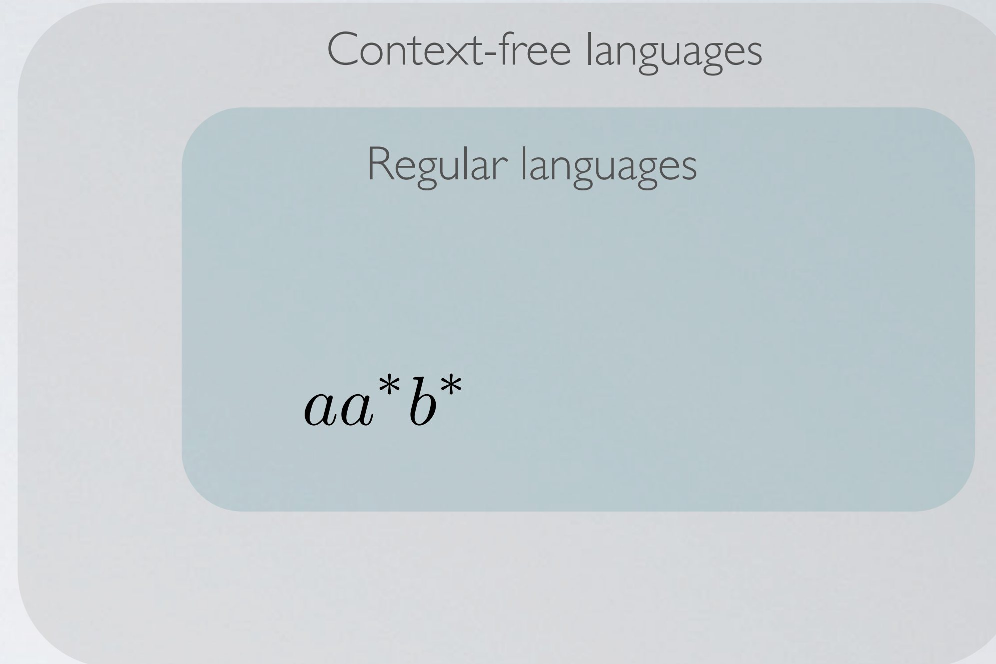
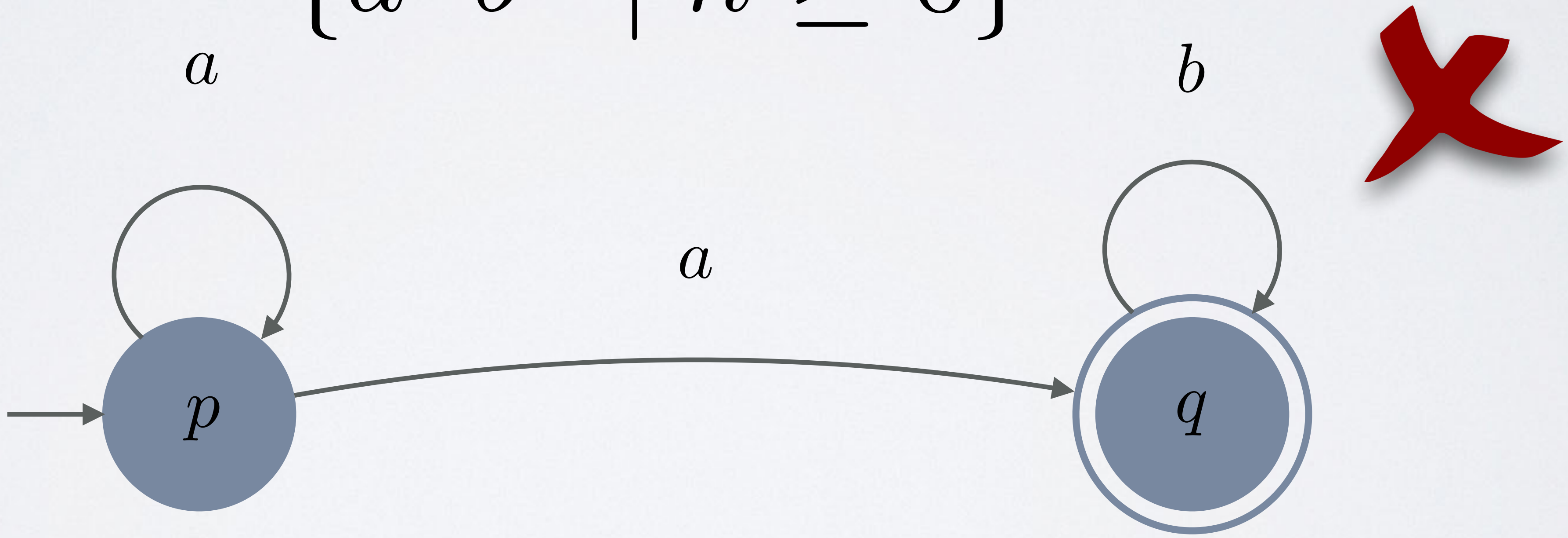
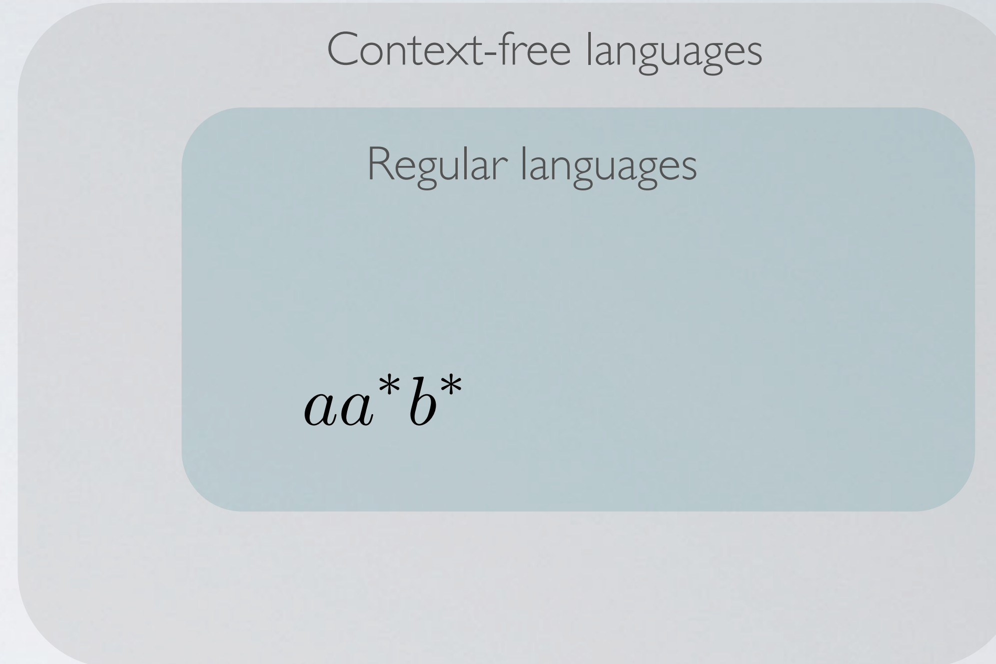


$$\{a^n b^n \mid n \geq 0\}$$



$$\{a^n b^n \mid n \geq 0\}$$





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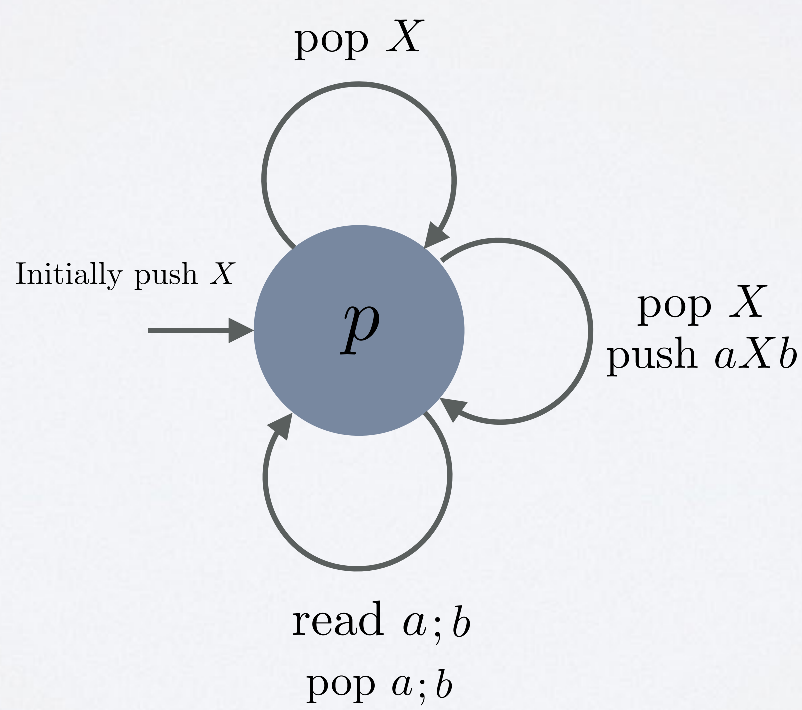

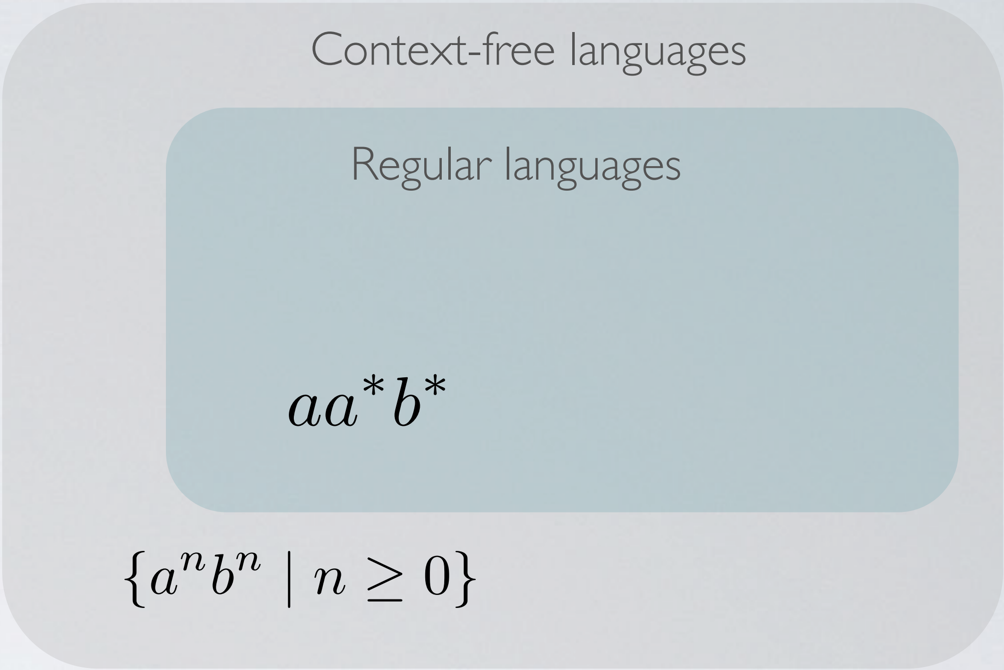
Context-free languages
Regular languages
aa^*b^*
$\{a^n b^n \mid n \geq 0\}$

$$\{a^n b^n \mid n \geq 0\} = \{\ \varepsilon,\ ab,\ aabb,\ aaabbb,\ \cdots\ \}$$

$$\{a^n b^n \mid n \geq 0\} = \{\ \varepsilon,\ ab,\ aabb,\ aaabbb,\ \cdots\ \}$$

$$\varepsilon \quad ab \quad abab \quad ababab \quad \cdots$$

$$\{a^n b^n \mid n \geq 0\} = \{\ \varepsilon,\ ab,\ aabb,\ aaabbb,\ \cdots\ \}$$

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$$\cdots$$

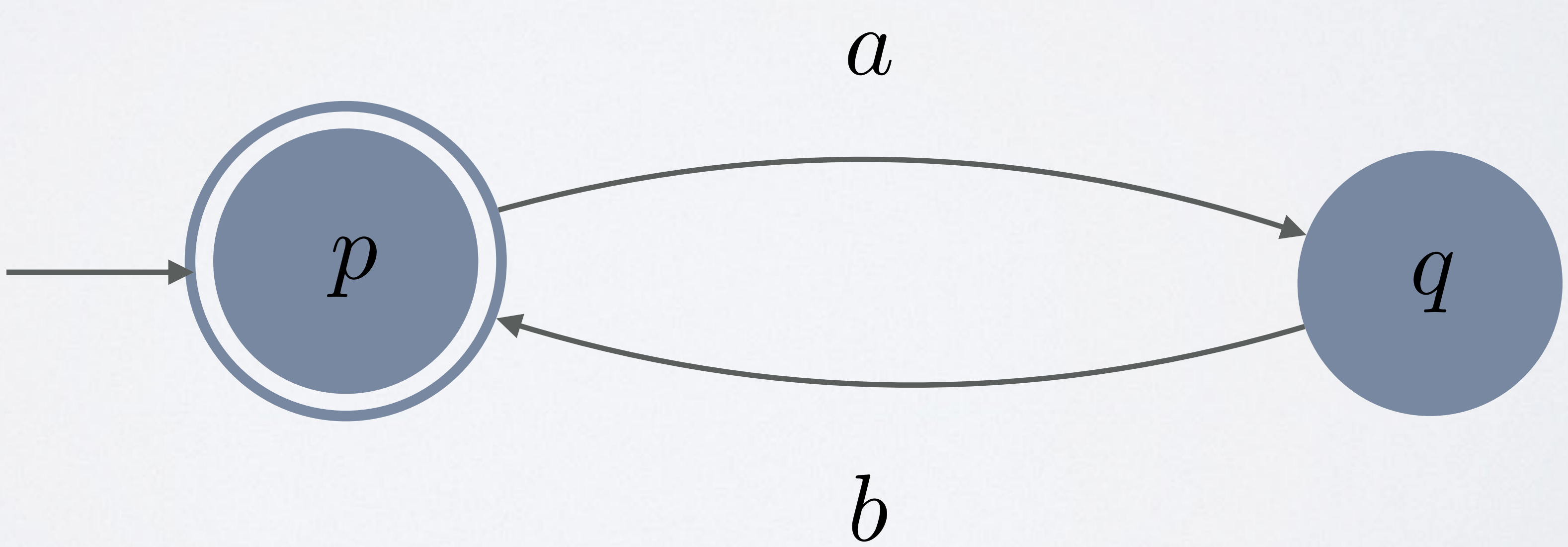
$$\{a^n b^n \mid n \geq 0\} = \{\ \varepsilon,\ ab,\ aabb,\ aaabbb,\ \cdots\ \}$$

$$\varepsilon \quad ab \quad abab \quad ababab \quad \cdots$$

$$\varepsilon \quad ba \quad abba \quad bbaaab \quad \cdots$$

$$\cdots$$

$$(ab)^*$$



PARIKH'S THEOREM

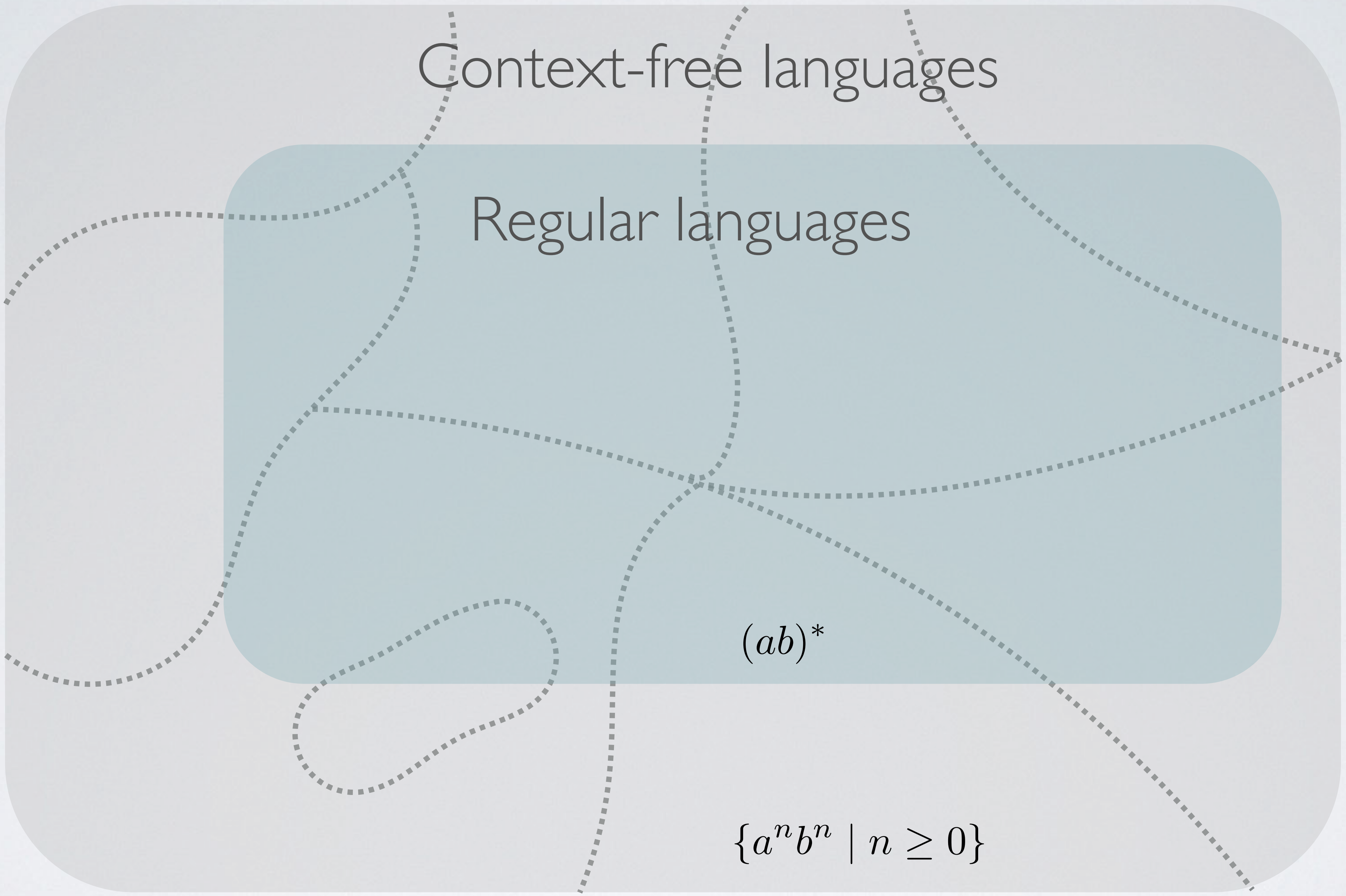
Parikh, 1966

Parikh's Theorem:

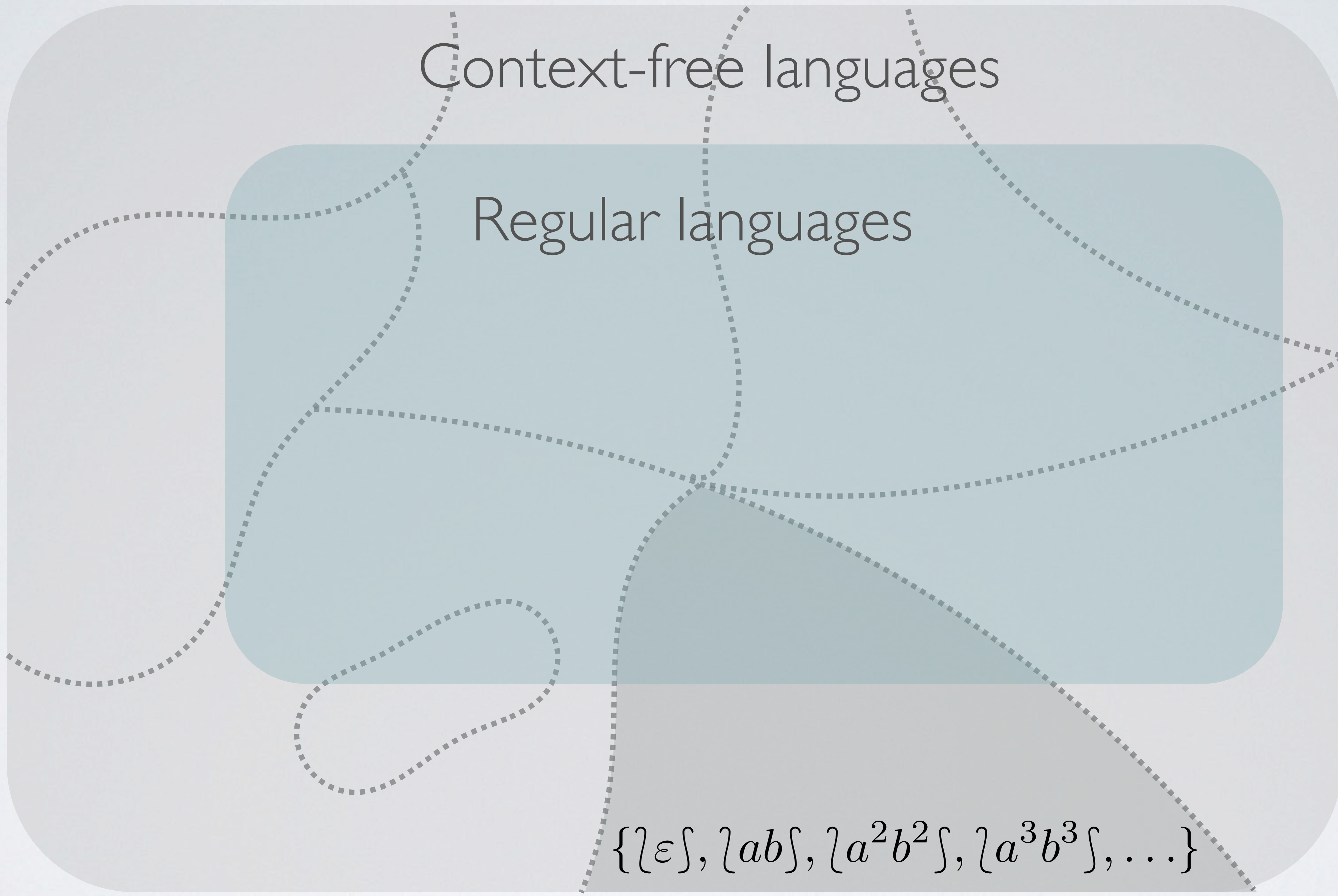
Every context-free language is equivalent to* a regular language.

$*$: when we ignore the ordering of symbols in the words

PARIKH'S THEOREM



PARIKH'S THEOREM



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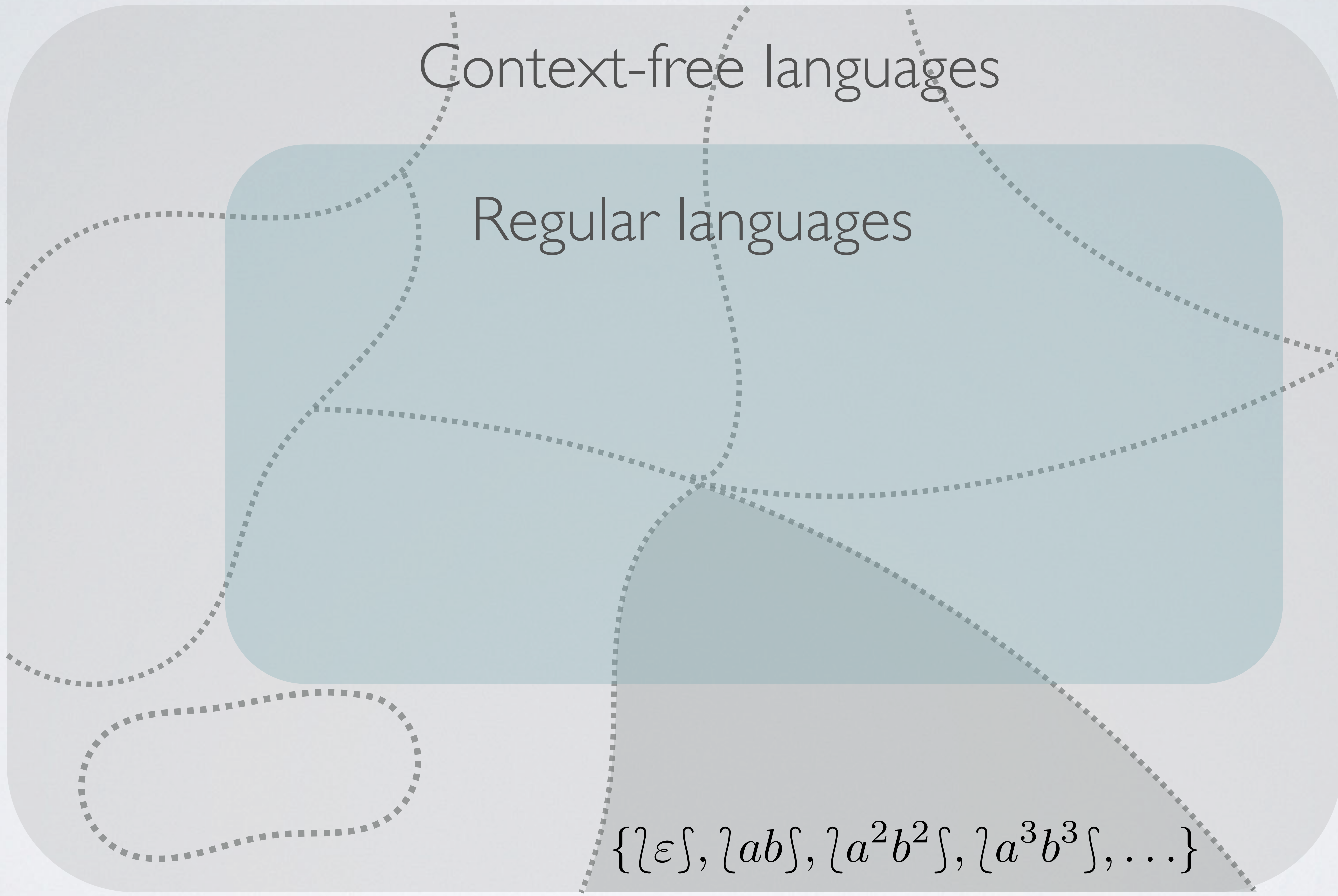
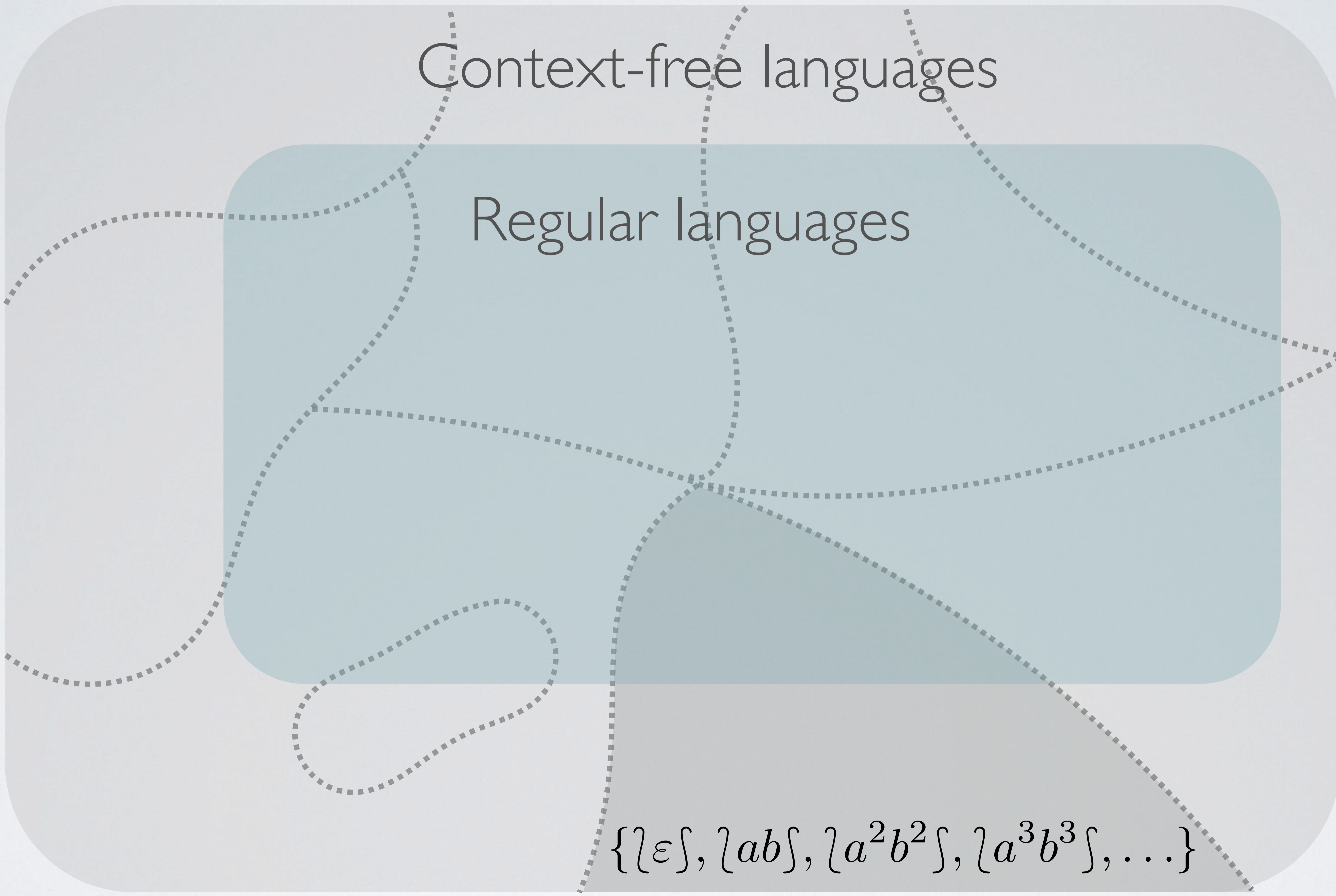
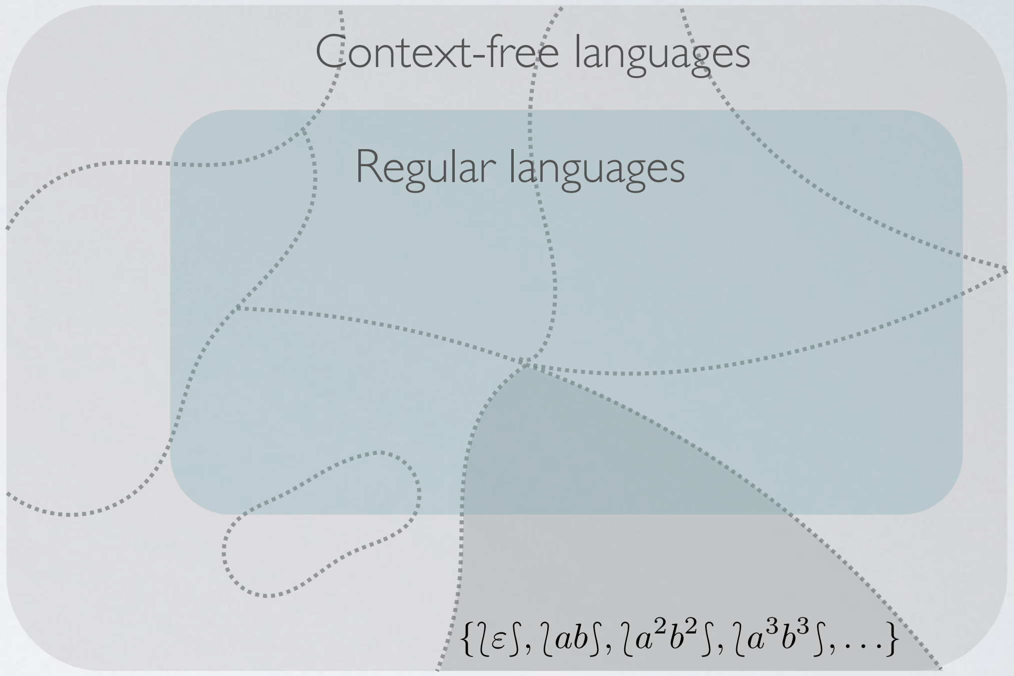


PARIKH'S THEOREM



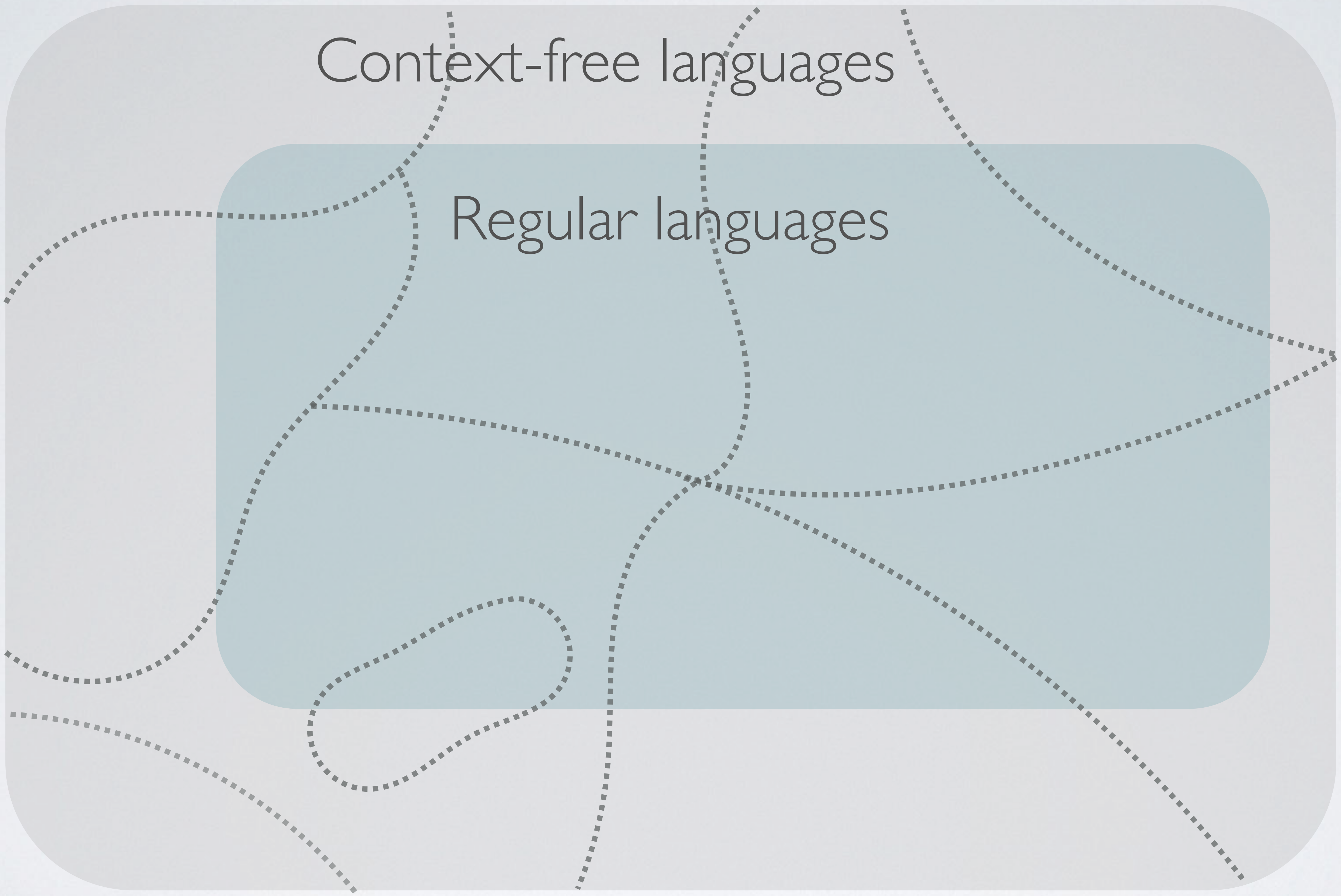
PARIKH'S THEOREM



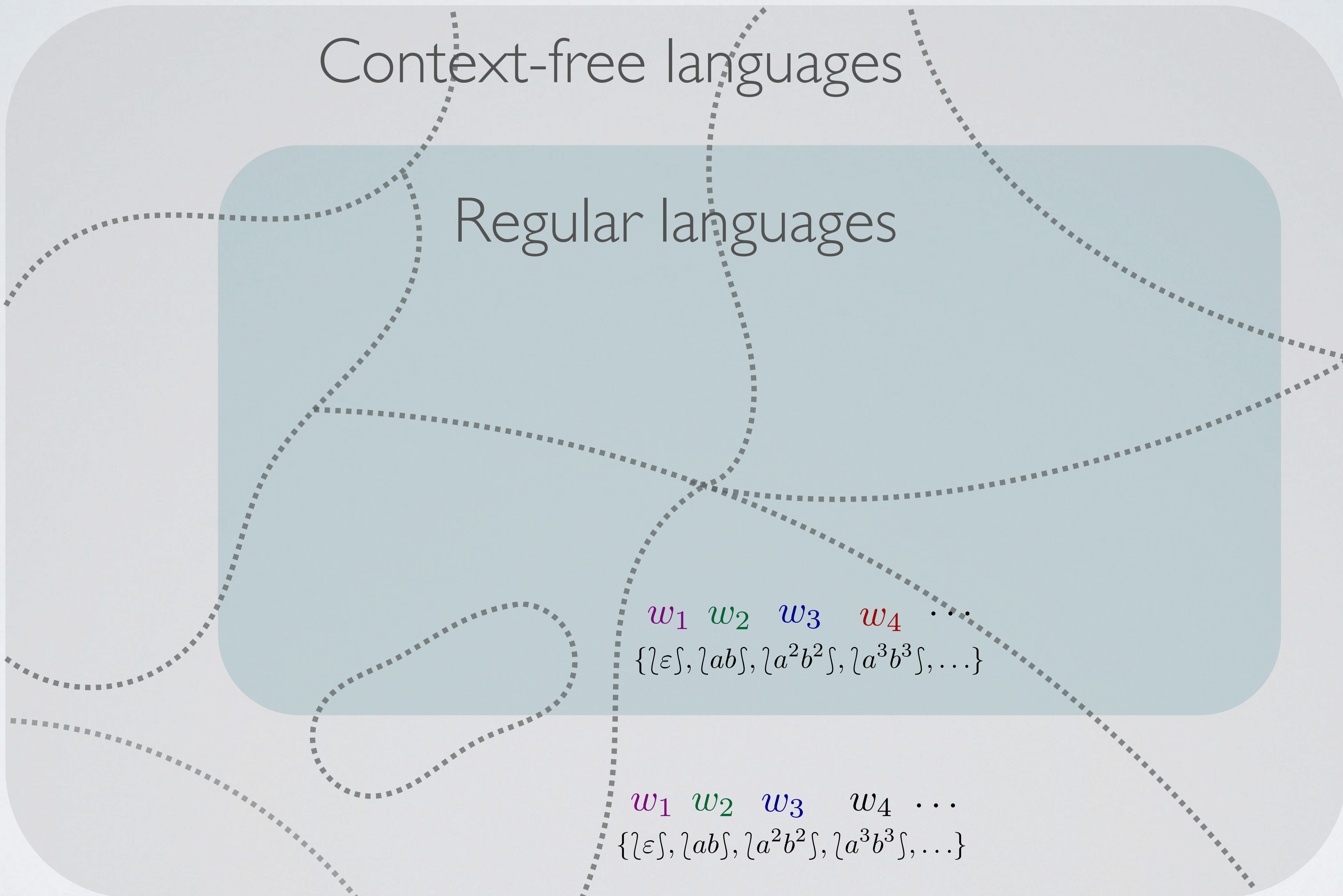
Parikh's Theorem:

For every context-free language, there exists a **regular** language defining the **"same set of bags"**.

WEIGHTED CASE



WEIGHTED CASE



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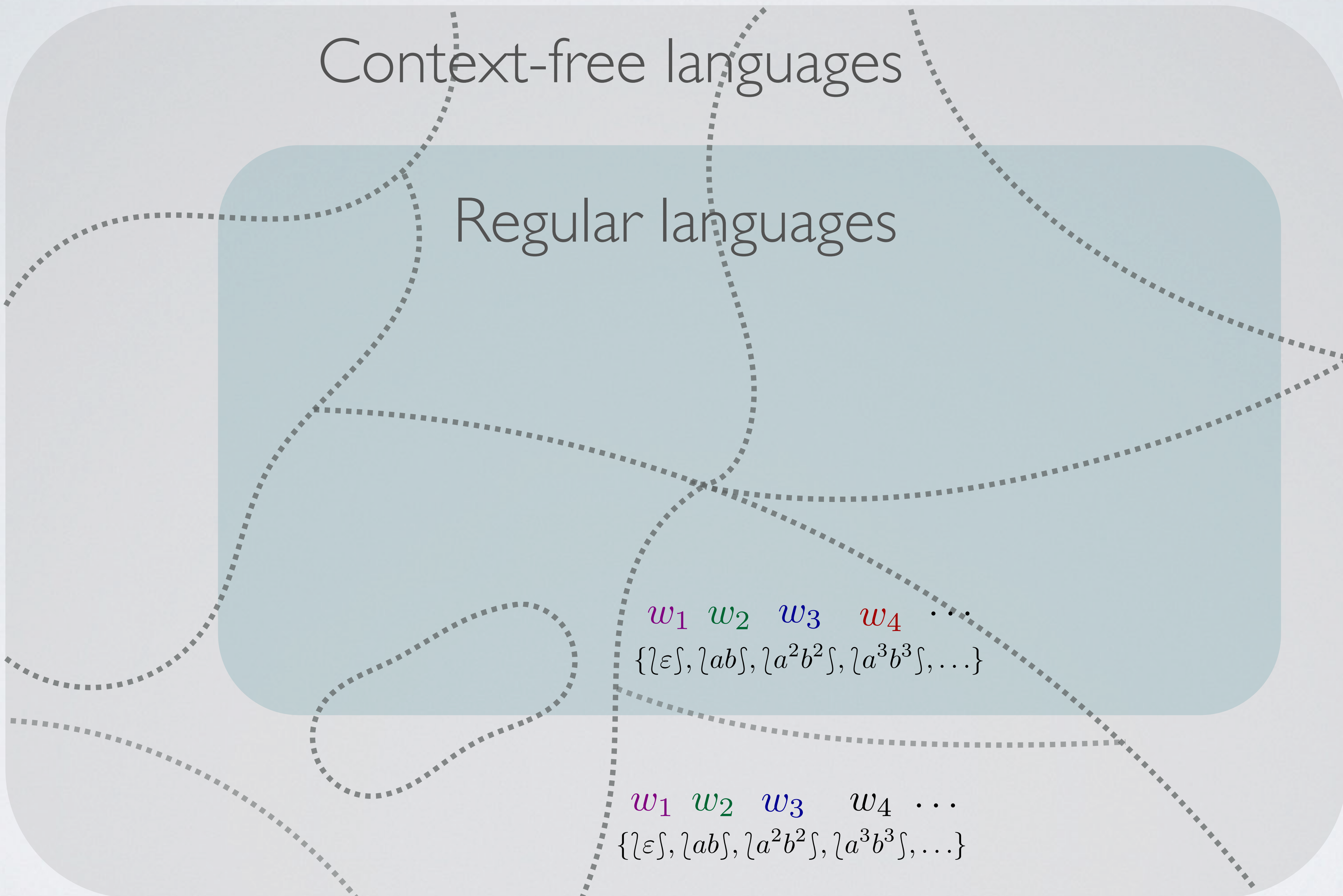

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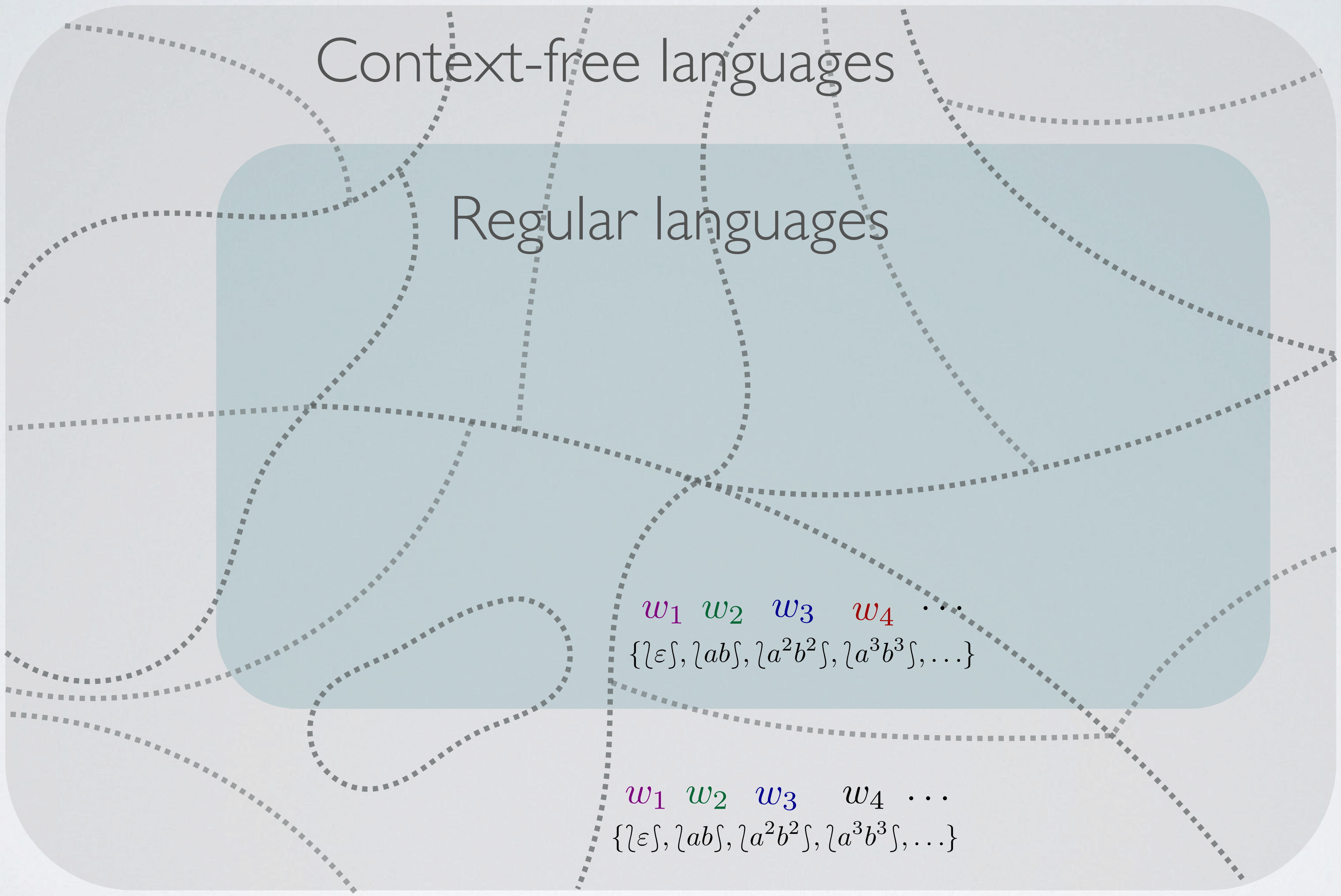

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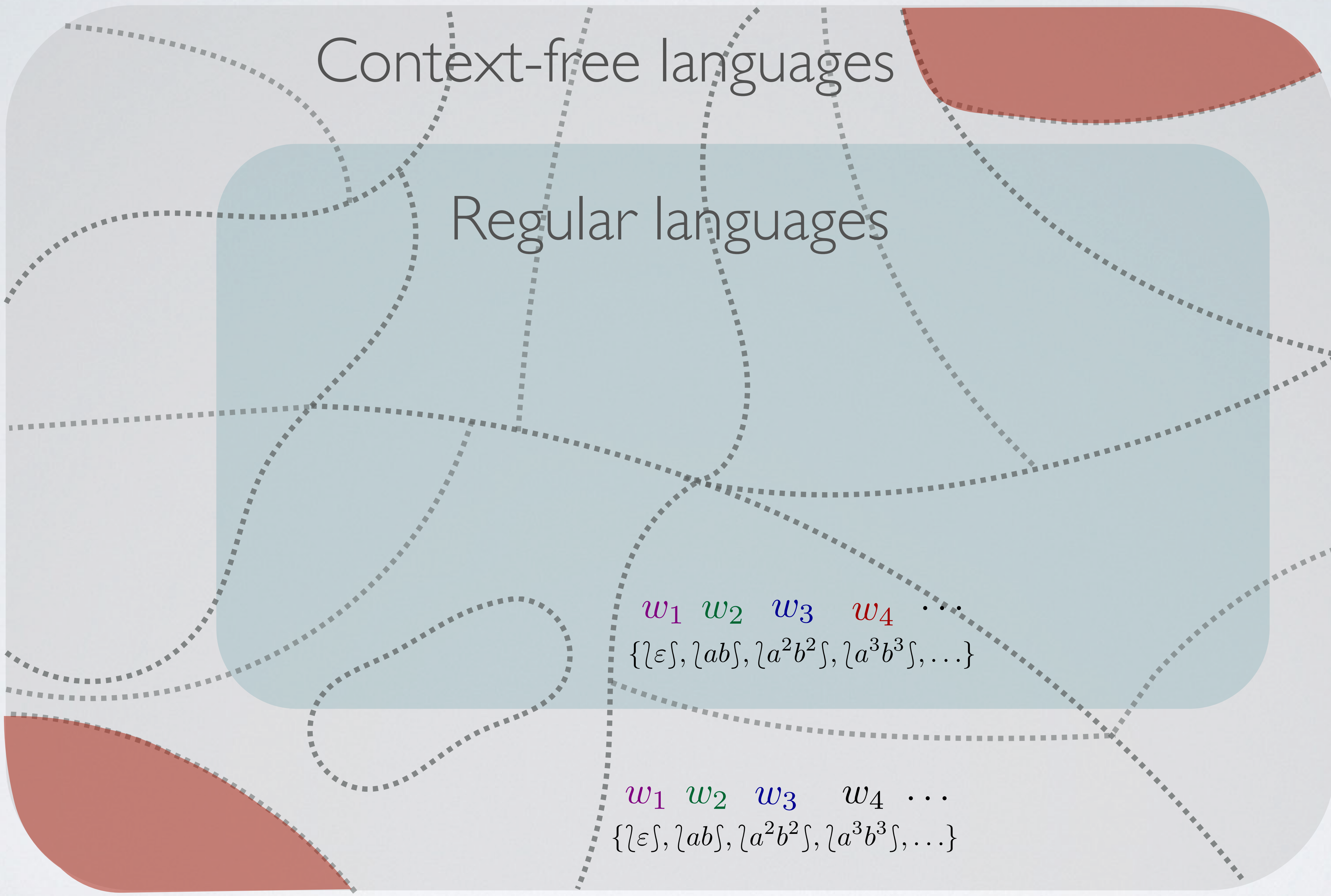
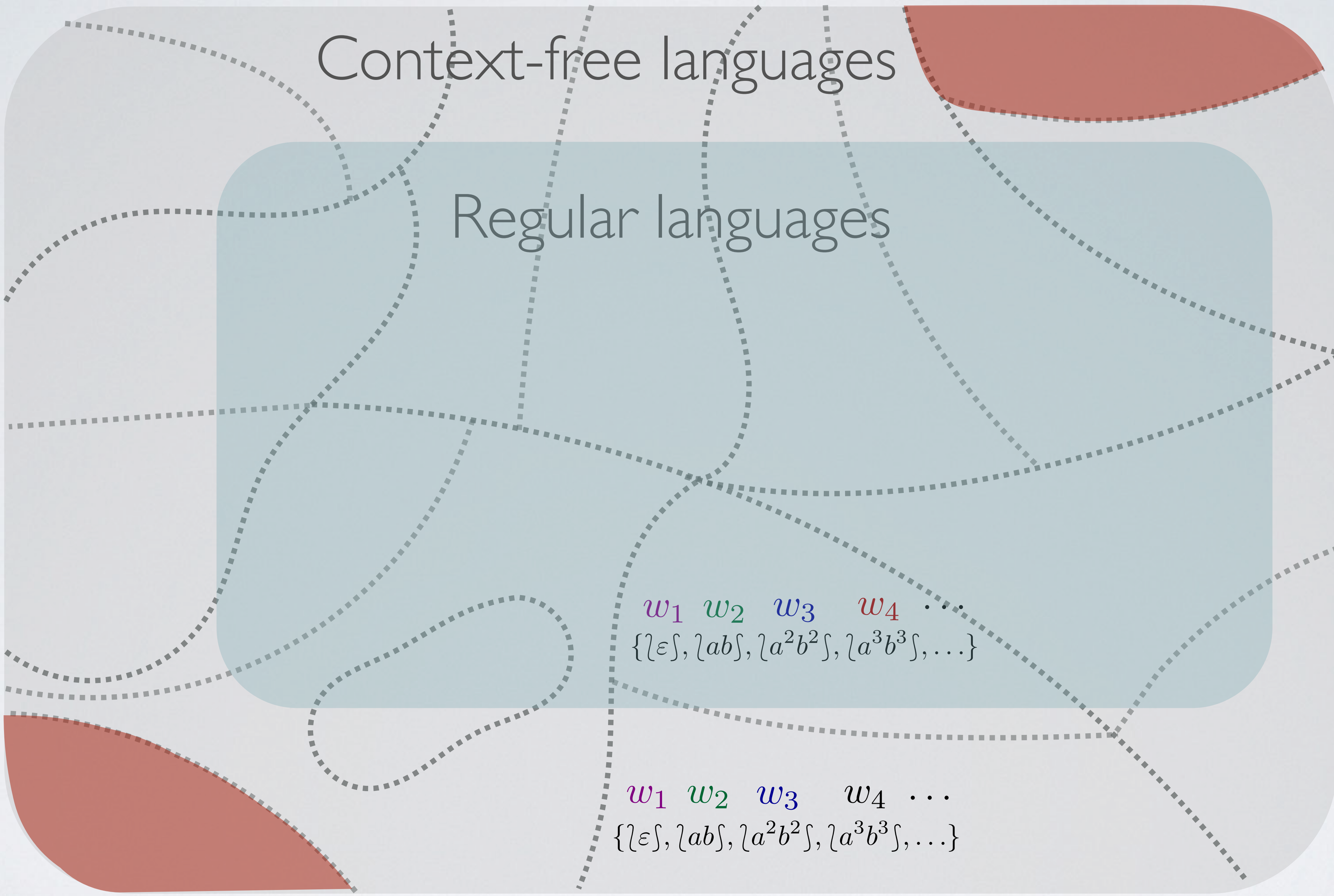


WEIGHTED CASE



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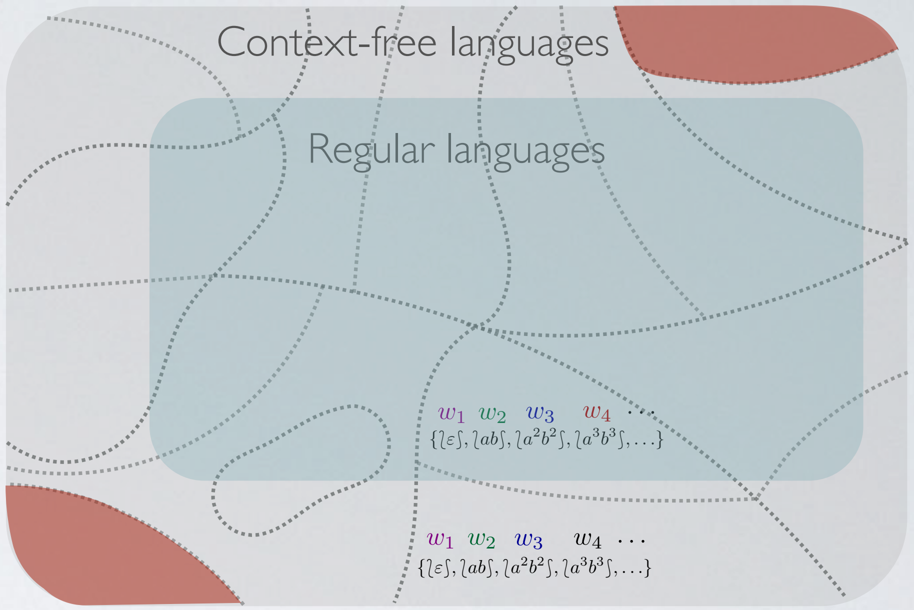
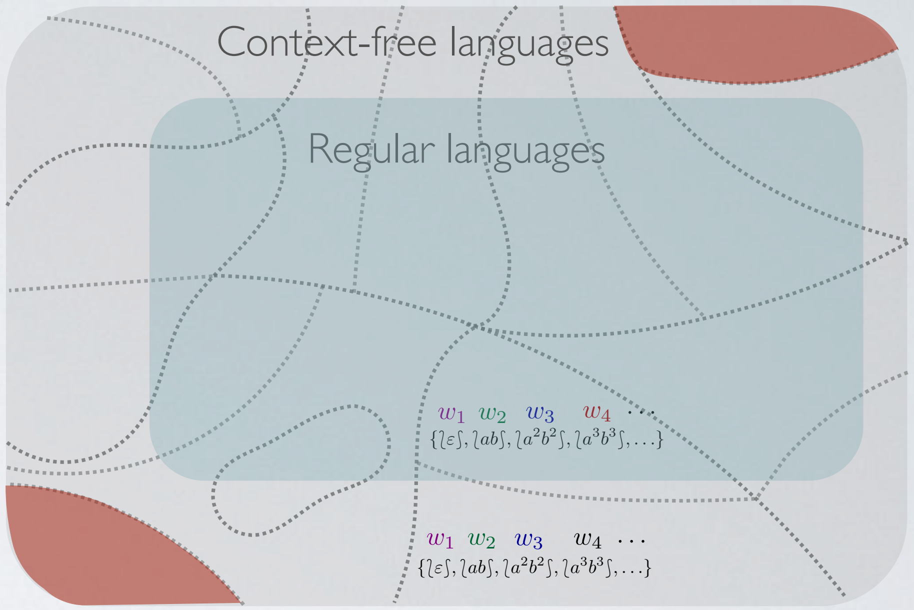


Parikh's Theorem does not hold in the **weighted case** Petre, 1998

WEIGHTED CASE



Parikh property

~~Parikh's Theorem~~ does not hold in the **weighted case**

Petre, 1998

QUESTIONS

1. When does the Parikh property hold?

2. Is the Parikh property decidable?

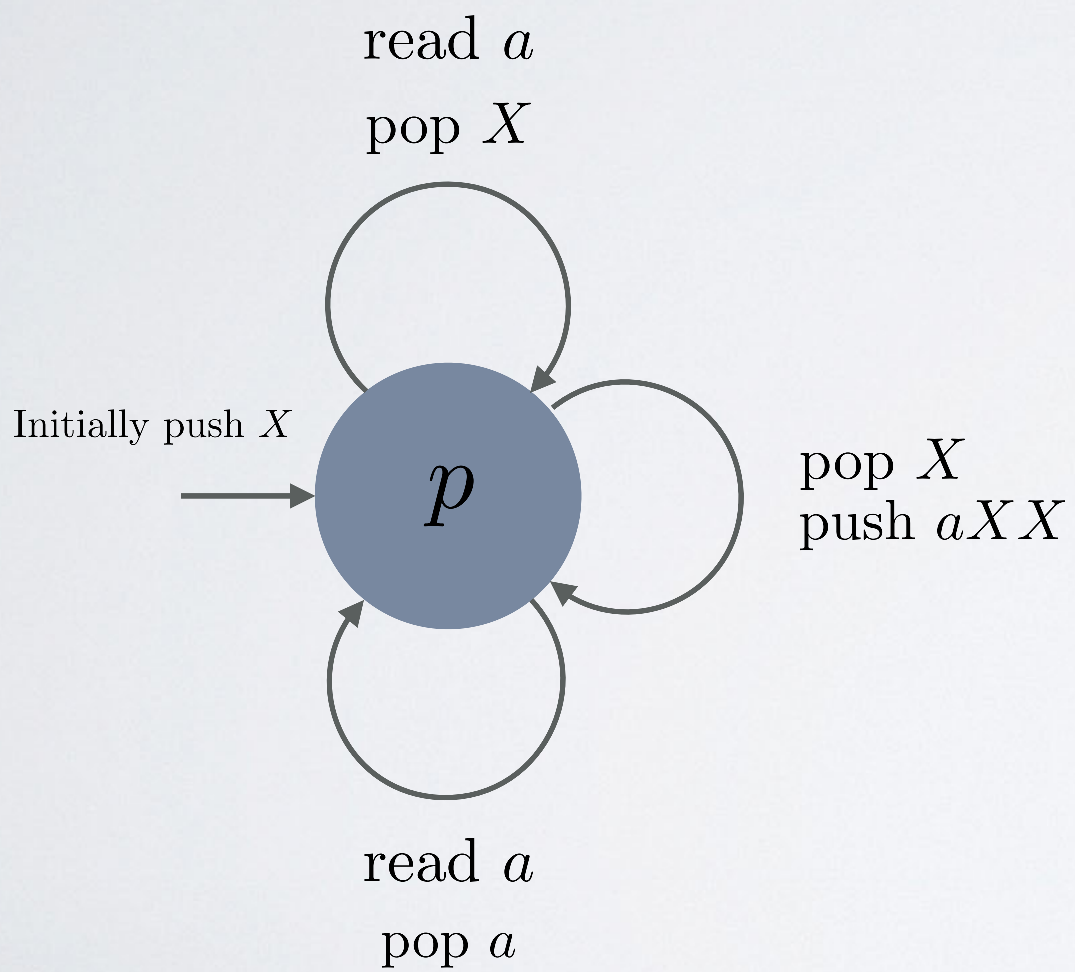
COUNTEREXAMPLE[1]

1	1	2	5	14	42	132	429	
a	a^3	a^5	a^7	a^9	a^{11}	a^{13}	a^{15}	$\dots$

1: Parikh's Theorem does not hold in the weighted case

COUNTEREXAMPLE[1]

1	1	2	5	14	42	132	429	
a	a^3	a^5	a^7	a^9	a^{11}	a^{13}	a^{15}	$\dots$



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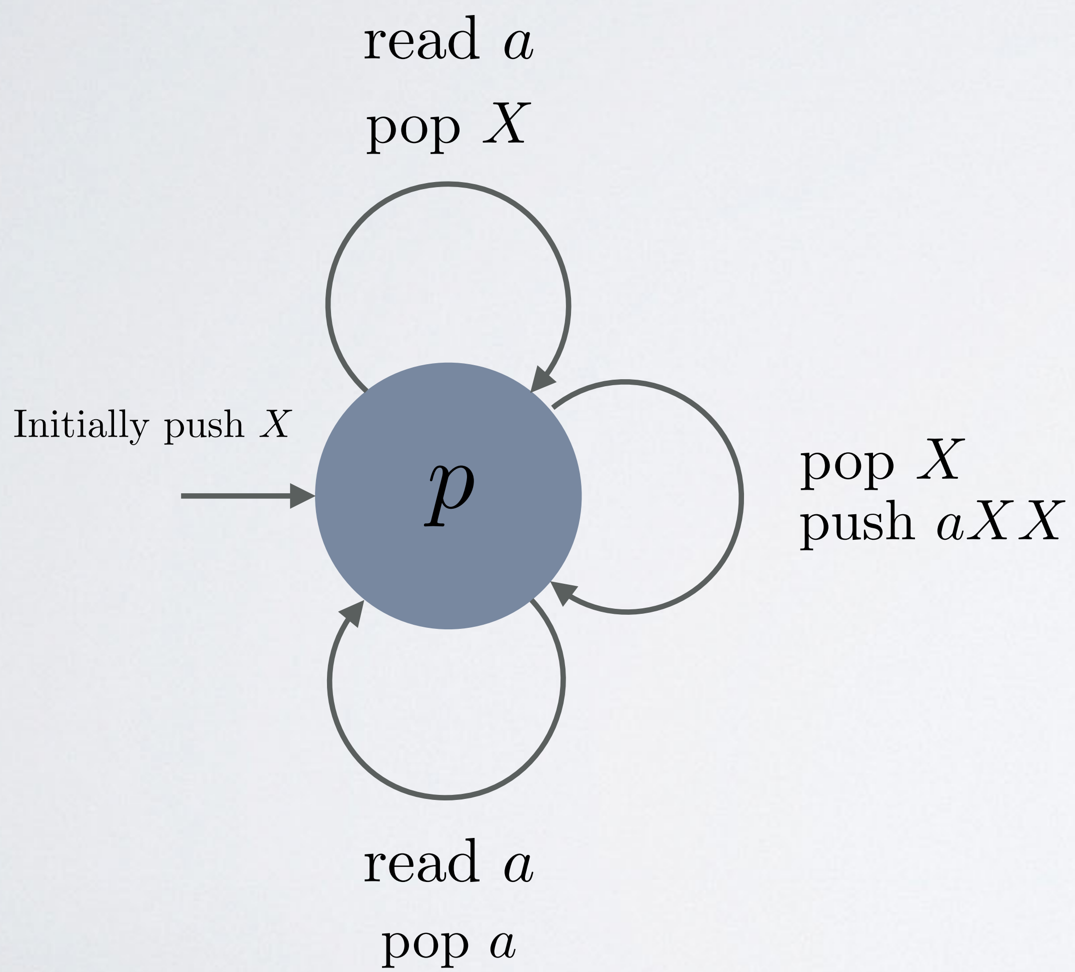

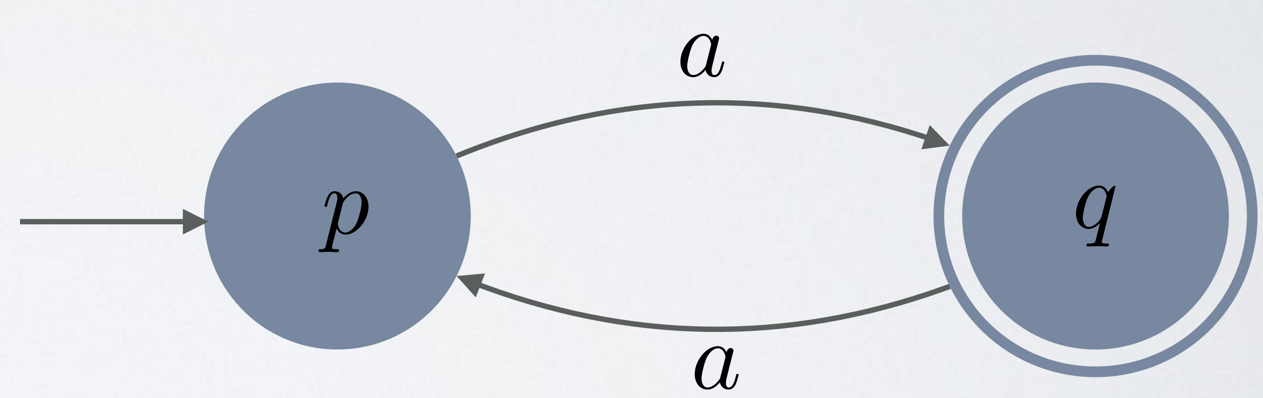

1: Parikh's Theorem does not hold in the weighted case

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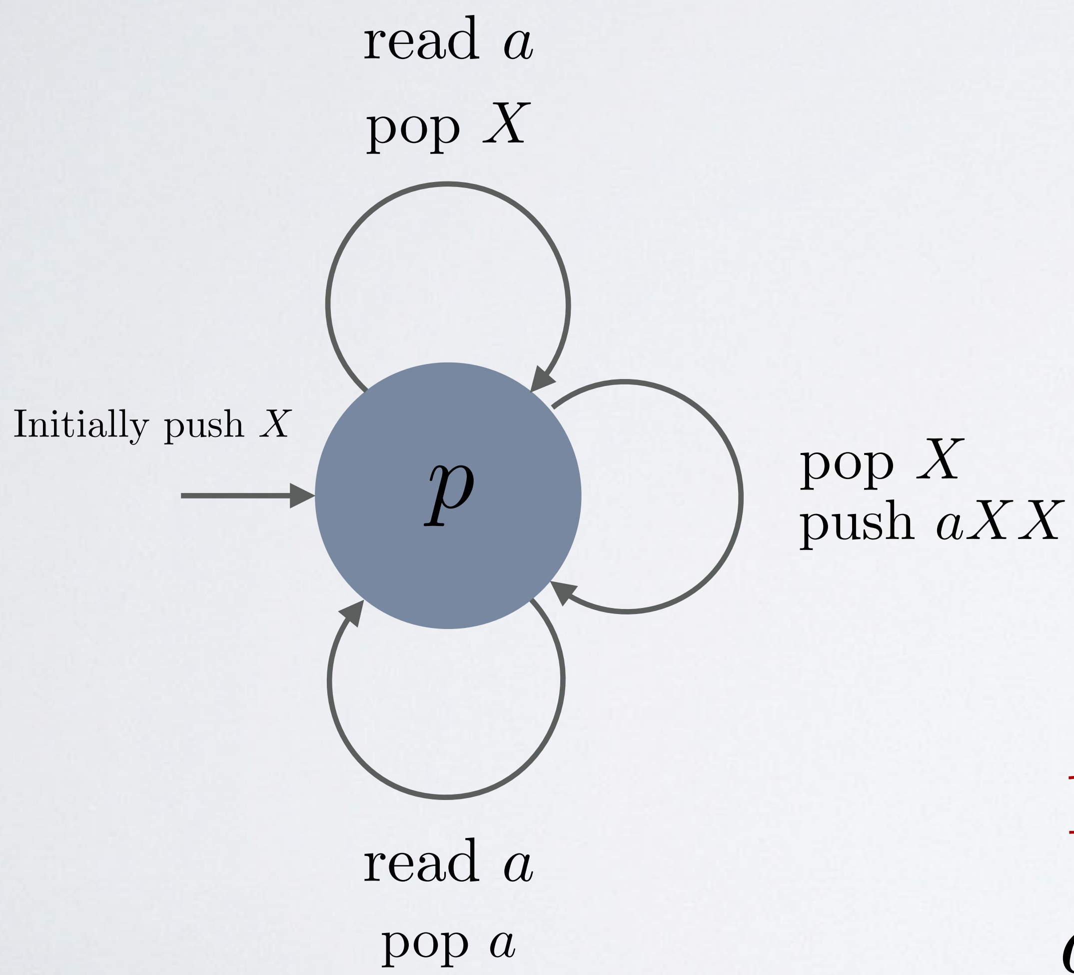
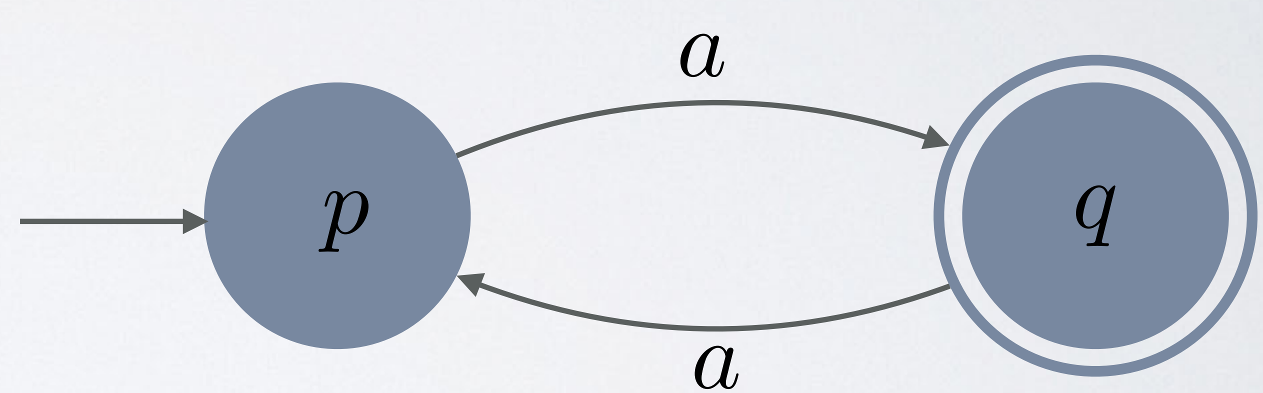




1	1	1	1	1	1	1	1	
a	a^3	a^5	a^7	a^9	a^{11}	a^{13}	a^{15}	$\ldots$

1: Parikh's Theorem does not hold in the weighted case

THE MODEL

Grammar Model

Regular Grammar

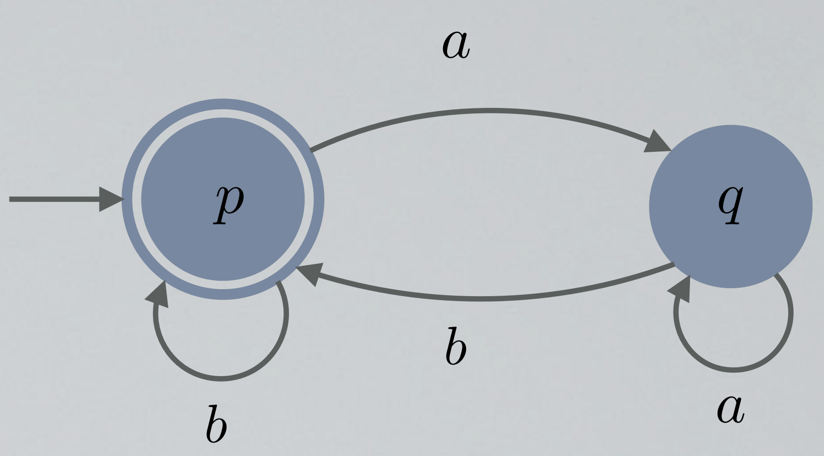
$$S \rightarrow aS$$
$$S \rightarrow bS$$
$$S \rightarrow b$$
$$S \rightarrow \varepsilon$$

Context-Free Grammar

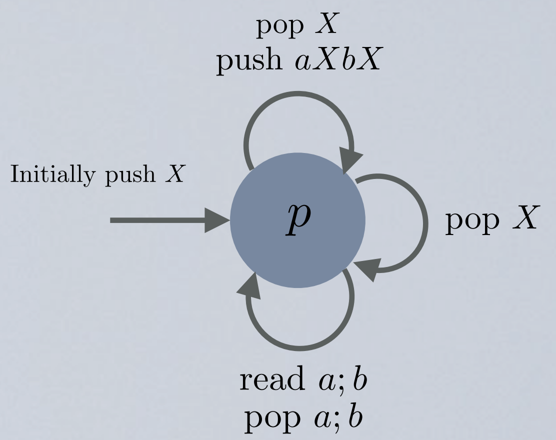
$$S \rightarrow aSbS$$
$$S \rightarrow \varepsilon$$

Automata Model

Finite-State Automata



Pushdown Automata



THE MODEL

Grammar Model

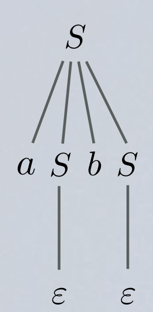
Regular Grammar

$$S \rightarrow aS$$
$$S \rightarrow bS$$
$$S \rightarrow b$$
$$S \rightarrow \varepsilon$$

$$S \underset{lm}{\Rightarrow} aS \underset{lm}{\Rightarrow} abS \underset{lm}{\Rightarrow} ab$$

Context-Free Grammar

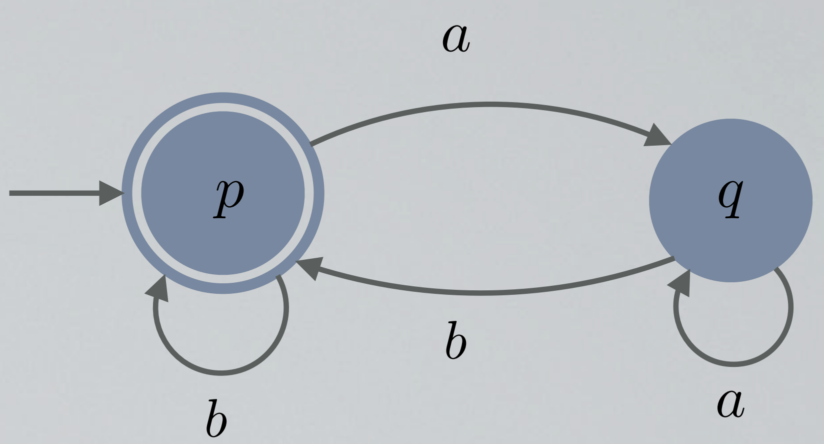
$$S \rightarrow aSbS$$
$$S \rightarrow \varepsilon$$



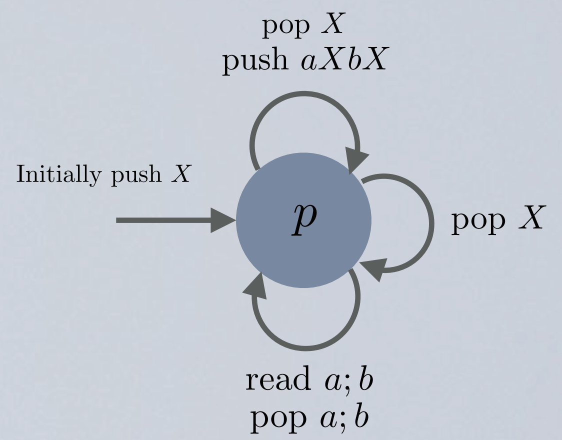
$$S \underset{lm}{\Rightarrow} aSbS \underset{lm}{\Rightarrow} abS \underset{lm}{\Rightarrow} ab$$

Automata Model

Finite-State Automata



Pushdown Automata



THE MODEL

Grammar Model

Regular Grammar

$S \to aS$

$S \to bS$

$S \to b$

$S \to \varepsilon$

Context-Free Grammar

$S \to aSbS$

$S \to \varepsilon$

THE MODEL

Grammar Model

Weighted Regular Grammars

$S \rightarrow aS$	1
$S \rightarrow bS$	1
$S \rightarrow b$	1
$S \rightarrow \varepsilon$	1

Weighted Context-Free Grammar

$S \rightarrow aSbS$	1
$S \rightarrow \varepsilon$	1

THE MODEL

Grammar Model

Weighted Regular Grammars

$S \rightarrow aS \quad 1$

$S \rightarrow bS \quad 1$

$S \rightarrow b \quad 1$

$S \rightarrow \varepsilon \quad 1$

$$S \underset{lm}{\overset{1}{\Rightarrow}} aS \underset{lm}{\overset{1}{\Rightarrow}} abS \underset{lm}{\overset{1}{\Rightarrow}} ab \qquad 1$$

Weighted Context-Free Grammar

$S \rightarrow aSbS \quad 1$

$S \rightarrow \varepsilon \quad 1$

$$S \underset{lm}{\overset{1}{\Rightarrow}} aSbS \underset{lm}{\overset{1}{\Rightarrow}} aaSbSbS \underset{lm}{\overset{1}{\Rightarrow}} aabSbS \underset{lm}{\overset{1}{\Rightarrow}} aabbS \underset{lm}{\overset{1}{\Rightarrow}} ababb \qquad 1$$

THE MODEL

Grammar Model

Weighted Regular Context-Free Grammar

$$S \to aS \quad 1$$
$$S \to bS \quad 1$$
$$S \to b \quad 1$$
$$S \to \varepsilon \quad 1$$

$$S \overset{1}{\underset{lm}{\Rightarrow}} aS \overset{1}{\underset{lm}{\Rightarrow}} abS \overset{1}{\underset{lm}{\Rightarrow}} ab \quad 1$$

Weighted Context-Free Grammar

$$S \to aSbS \quad 1$$
$$S \to \varepsilon \quad 1$$

$$S \overset{1}{\underset{lm}{\Rightarrow}} aSbS \overset{1}{\underset{lm}{\Rightarrow}} aaSbSbS \overset{1}{\underset{lm}{\Rightarrow}} aabSbS \overset{1}{\underset{lm}{\Rightarrow}} aabbS \overset{1}{\underset{lm}{\Rightarrow}} aabb \quad 1$$

THE MODEL

Grammar Model

Weighted Regular Context-Free Grammar

$S \rightarrow aS$ 1

$S \rightarrow bS$ 1

$S \rightarrow b$ 1

$S \rightarrow \varepsilon$ 1

ab

$S \underset{lm}{\overset{1}{\Rightarrow}} aS \underset{lm}{\overset{1}{\Rightarrow}} abS \underset{lm}{\overset{1}{\Rightarrow}} ab$ 1

$S \underset{lm}{\overset{1}{\Rightarrow}} aS \underset{lm}{\overset{1}{\Rightarrow}} ab$ 1

2

Weighted Context-Free Grammar

$S \rightarrow aSbS$ 1

$S \rightarrow \varepsilon$ 1

$aabb$

$S \underset{lm}{\overset{1}{\Rightarrow}} aSbS \underset{lm}{\overset{1}{\Rightarrow}} aaSbSbS \underset{lm}{\overset{1}{\Rightarrow}} aabSbS \underset{lm}{\overset{1}{\Rightarrow}} aabbS \underset{lm}{\overset{1}{\Rightarrow}} aabb$ 1

1

THE MODEL

Grammar Model

Weighted Regular Context-Free Grammar

$S \rightarrow aS$	1
$S \rightarrow bS$	1
$S \rightarrow b$	1
$S \rightarrow \varepsilon$	1

$\wr ab \int$

ab

$S \xRightarrow[lm]{1} aS \xRightarrow[lm]{1} abS \xRightarrow[lm]{1} ab$ 1

$S \xRightarrow[lm]{1} aS \xRightarrow[lm]{1} ab$ 1

2

ba

$S \xRightarrow[lm]{1} bS \xRightarrow[lm]{1} baS \xRightarrow[lm]{1} ba$ 1

1

3

Weighted Context-Free Grammar

$S \rightarrow aSbS$	1
$S \rightarrow \varepsilon$	1

$\wr a^2b^2 \int$

$aabb$

$S \xRightarrow[lm]{1} aSbS \xRightarrow[lm]{1} aaSbSbS \xRightarrow[lm]{1} aabSbS \xRightarrow[lm]{1} aabbS \xRightarrow[lm]{1} aabb$ 1

1

$abab$

$S \xRightarrow[lm]{1} aSbS \xRightarrow[lm]{1} abS \xRightarrow[lm]{1} abaSbS \xRightarrow[lm]{1} ababS \xRightarrow[lm]{1} abab$ 1

1

2

THE MODEL

Grammar Model

Weighted Regular Context-Free Grammar $(\mathbb{N}, +, \times)$

$S \to aS$ 1

$S \to bS$ 1

$S \to b$ 1

$S \to \varepsilon$ 1

$\{\!\!\{ ab \}\!\!\}$

ab

$S \overset{1}{\underset{lm}{\Rightarrow}} aS \overset{1}{\underset{lm}{\Rightarrow}} abS \overset{1}{\underset{lm}{\Rightarrow}} ab$ 1

$S \overset{1}{\underset{lm}{\Rightarrow}} aS \overset{1}{\underset{lm}{\Rightarrow}} ab$ 1

2

ba

$S \overset{1}{\underset{lm}{\Rightarrow}} bS \overset{1}{\underset{lm}{\Rightarrow}} baS \overset{1}{\underset{lm}{\Rightarrow}} ba$ 1

1

3

Weighted Context-Free Grammar $(\mathbb{N}, +, \times)$

$S \to aSbS$ 1

$S \to \varepsilon$ 1

$\{\!\!\{ a^2 b^2 \}\!\!\}$

$aabb$

$S \overset{1}{\underset{lm}{\Rightarrow}} aSbS \overset{1}{\underset{lm}{\Rightarrow}} aaSbSbS \overset{1}{\underset{lm}{\Rightarrow}} aabSbS \overset{1}{\underset{lm}{\Rightarrow}} aabbS \overset{1}{\underset{lm}{\Rightarrow}} aabb$ 1

1

$abab$

$S \overset{1}{\underset{lm}{\Rightarrow}} aSbS \overset{1}{\underset{lm}{\Rightarrow}} abS \overset{1}{\underset{lm}{\Rightarrow}} abaSbS \overset{1}{\underset{lm}{\Rightarrow}} ababS \overset{1}{\underset{lm}{\Rightarrow}} abab$ 1

1

2

THE MODEL

Grammar Model

Weighted Regular Context-Free Grammar

$(\mathbb{S}, \oplus, \otimes)$

$S \rightarrow aS$	1
$S \rightarrow bS$	1
$S \rightarrow b$	1
$S \rightarrow \varepsilon$	1

$\wr ab \int$

ab

$S \underset{lm}{\overset{1}{\Rightarrow}} aS \underset{lm}{\overset{1}{\Rightarrow}} abS \underset{lm}{\overset{1}{\Rightarrow}} ab$ 1

$S \underset{lm}{\overset{1}{\Rightarrow}} aS \underset{lm}{\overset{1}{\Rightarrow}} ab$ 1

2

ba

$S \underset{lm}{\overset{1}{\Rightarrow}} bS \underset{lm}{\overset{1}{\Rightarrow}} baS \underset{lm}{\overset{1}{\Rightarrow}} ba$ 1

1

3

Weighted Context-Free Grammar

$S \rightarrow aSbS$	1
$S \rightarrow \varepsilon$	1

$\wr a^2b^2 \int$

$aabb$

$S \underset{lm}{\overset{1}{\Rightarrow}} aSbS \underset{lm}{\overset{1}{\Rightarrow}} aaSbSbS \underset{lm}{\overset{1}{\Rightarrow}} aabSbS \underset{lm}{\overset{1}{\Rightarrow}} aabbS \underset{lm}{\overset{1}{\Rightarrow}} aabb$ 1

1

$abab$

$S \underset{lm}{\overset{1}{\Rightarrow}} aSbS \underset{lm}{\overset{1}{\Rightarrow}} abS \underset{lm}{\overset{1}{\Rightarrow}} abaSbS \underset{lm}{\overset{1}{\Rightarrow}} ababS \underset{lm}{\overset{1}{\Rightarrow}} abab$ 1

1

2

THE MODEL

Grammar Model

Weighted Regular Context-Free Grammar $(\mathbb{Q}, +, \times)(\mathbb{N}, \min, +)$

$S \to aS$ 1
$S \to bS$ 1
$S \to b$ 1
$S \to \varepsilon$ 1

$\wr ab \int$

ab

$S \overset{1}{\underset{lm}{\Rightarrow}} aS \overset{1}{\underset{lm}{\Rightarrow}} abS \overset{1}{\underset{lm}{\Rightarrow}} ab$ 1

$S \overset{1}{\underset{lm}{\Rightarrow}} aS \overset{1}{\underset{lm}{\Rightarrow}} ab$ 1

2

ba

$S \overset{1}{\underset{lm}{\Rightarrow}} bS \overset{1}{\underset{lm}{\Rightarrow}} baS \overset{1}{\underset{lm}{\Rightarrow}} ba$ 1

1

3

Weighted Context-Free Grammar

$S \to aSbS$ 1
$S \to \varepsilon$ 1

$\wr a^2b^2 \int$

$aabb$

$S \overset{1}{\underset{lm}{\Rightarrow}} aSbS \overset{1}{\underset{lm}{\Rightarrow}} aaSbSbS \overset{1}{\underset{lm}{\Rightarrow}} aabSbS \overset{1}{\underset{lm}{\Rightarrow}} aabbS \overset{1}{\underset{lm}{\Rightarrow}} aabb$ 1

1

$abab$

$S \overset{1}{\underset{lm}{\Rightarrow}} aSbS \overset{1}{\underset{lm}{\Rightarrow}} abS \overset{1}{\underset{lm}{\Rightarrow}} abaSbS \overset{1}{\underset{lm}{\Rightarrow}} ababS \overset{1}{\underset{lm}{\Rightarrow}} abab$ 1

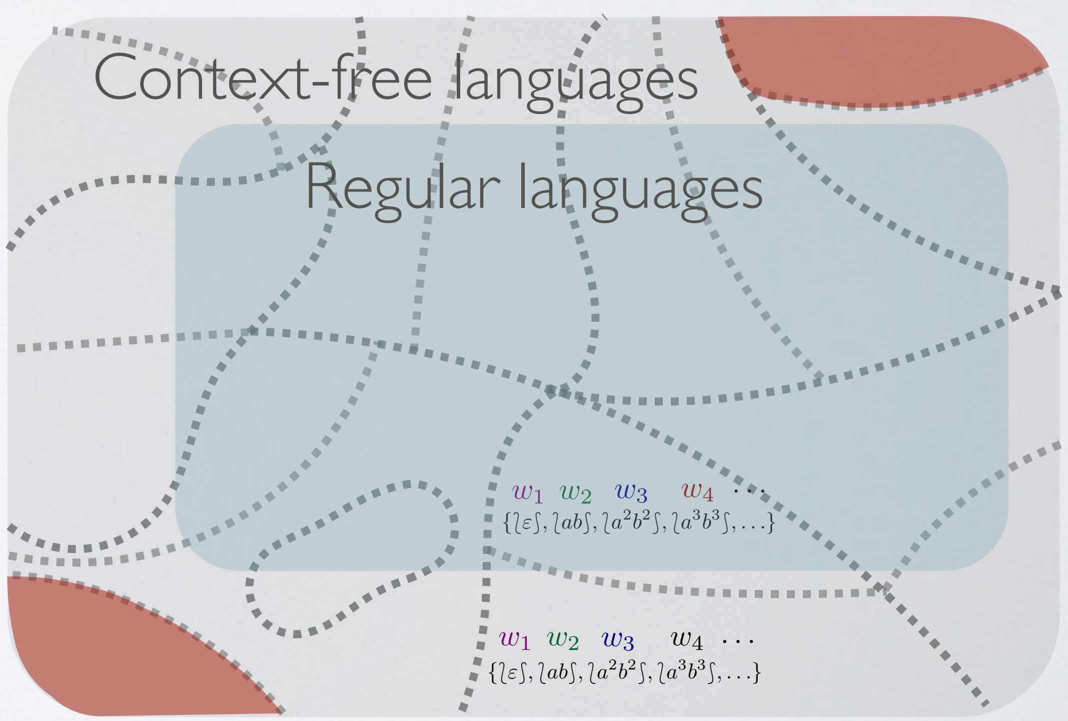
1

2

THE PARIKH PROPERTY

<u>Definition (the Parikh property)</u>

A **weighted** context-free grammar (WCFG) **satisfies the Parikh property if there exists a regular WCFG** that defines the "**same set of weighted bags**".



PREVIOUS WORK

1. When does the Parikh property hold?

Kuich, 1986

Petre, 1999

Luttenberger et al., 2016

Bhattiprolu et al., 2017

Sufficient condition on the **grammar**

Sufficient condition on the **weight domain**

2. Is the Parikh property decidable?

?

CONTRIBUTION

1. When does the Parikh property hold?

2. Is the Parikh property decidable?

CONTRIBUTION

1. When does the Parikh property hold?

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Nonexpansive grammars over **any** weight domain

Construction of an **equivalent regular** WCFG

CONTRIBUTION

1. When does the Parikh property hold?

Nonexpansive grammars over **any** weight domain

Construction of an **equivalent regular** WCFG

$$S \Rightarrow aX \Rightarrow abSS \Rightarrow \dots$$

CONTRIBUTION

1. When does the Parikh property hold?

Nonexpansive grammars over **any** weight domain

Construction of an **equivalent regular** WCFG

The condition is not necessary (**counterexample**)

COUNTEREXAMPLE [1]

Dyck language: $\varepsilon \quad () \quad ()() \quad (()) \quad ()()() \quad ()(()) \quad \ldots$

1 : Nonexpansiveness is not necessary for the Parikh property

COUNTEREXAMPLE [1]

Dyck language: $\varepsilon \quad a\overline{a} \quad a\overline{a}a\overline{a} \quad aa\overline{aa} \quad a\overline{a}a\overline{a}a\overline{a} \quad a\overline{a}aa\overline{aa} \quad \ldots$

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G_{Dyck} :

$$D \rightarrow aD\overline{a}D \mid \varepsilon$$

$$\begin{array}{ccccc} 1 & 1 & 1 & 1 & \\ \varepsilon & a\overline{a} & a\overline{a}a\overline{a} & aa\overline{a}\overline{a} & \cdots \end{array}$$

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G_{Dyck} :

$$D \to aD\overline{a}D \mid \varepsilon$$

$G_{\overline{Dyck}}$:

$$\overline{D} \to D\,\overline{a}\,Y \mid D\,a\,Z$$

$$Y \to aY \mid \overline{a}\,Y \mid \varepsilon$$

$$Z \to D\,a\,Z \mid D$$

1	1	1	1	
ε	$a\overline{a}$	$a\overline{a}a\overline{a}$	$aa\overline{a}\overline{a}$	$\cdots$

1	1	1	1	1	1	
a	$\overline{a}$	aa	$\overline{a}a$	$\overline{a}\overline{a}$	aaa	$\cdots$

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Dyck language: $\mathcal{E} \quad a\overline{a} \quad a\overline{a}a\overline{a} \quad aa\overline{aa} \quad a\overline{a}a\overline{a}a\overline{a} \quad a\overline{a}aa\overline{aa} \quad \ldots$

$G_{Union}:$

$$S \to D \mid \overline{D}$$

$$D \to aD\overline{a}D \mid \varepsilon$$

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$$\begin{array}{cccccccccccc} 1 & 1 & 1 & 1 & & 1 & 1 & 1 & 1 & 1 & 1 & \\ \mathcal{E} & a\overline{a} & a\overline{a}a\overline{a} & aa\overline{aa} & \ldots & a & \overline{a} & aa & \overline{a}a & \overline{aa} & aaa & \cdots \end{array}$$

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$G_{Regular}$:

$$X \to aX \mid \overline{a}X \mid \varepsilon$$

$$\begin{array}{cccccccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & & \\ \varepsilon & a\overline{a} & a\overline{a}a\overline{a} & aa\overline{a}\overline{a} & \ldots a & \overline{a} & aa & \overline{a}a & \overline{a}\overline{a} & aaa & \ldots \end{array}$$

1 : Nonexpansiveness is not necessary for the Parikh property

CONTRIBUTION

1. When does the Parikh property hold?

Nonexpansive grammars over **any** weight domain

Construction of an **equivalent** WCFG

The condition is not necessary (**counterexample**)

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2. Is the Parikh property decidable?

Yes, when the weights are over $\mathbb{Q}$

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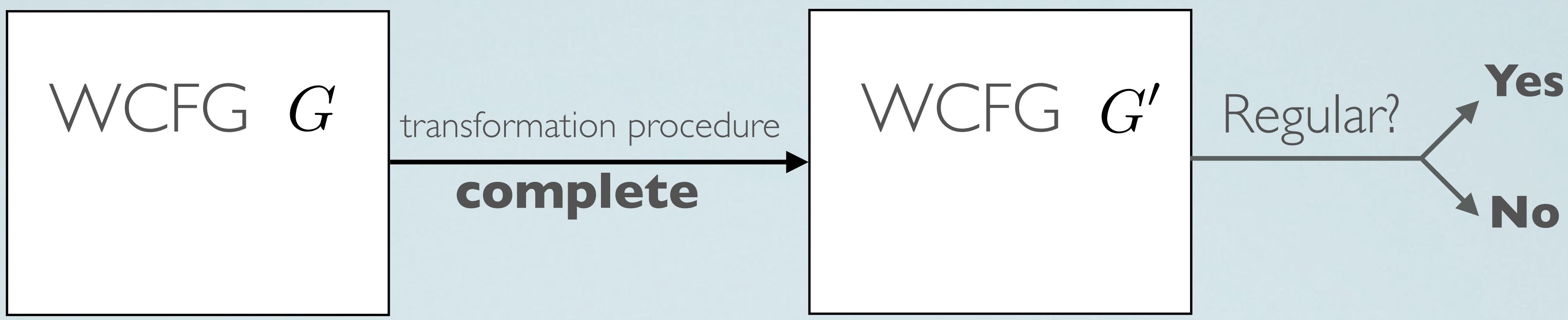

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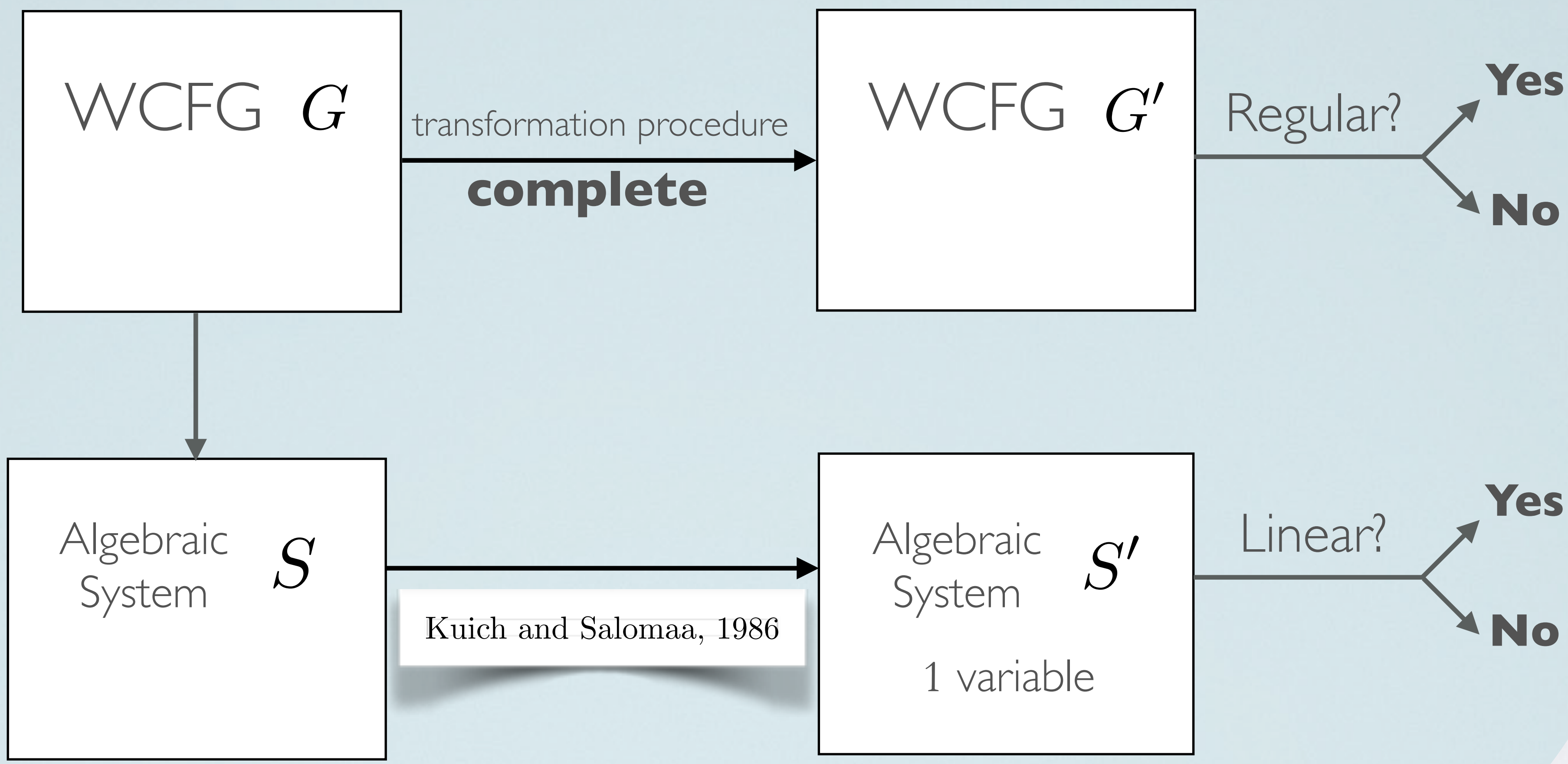

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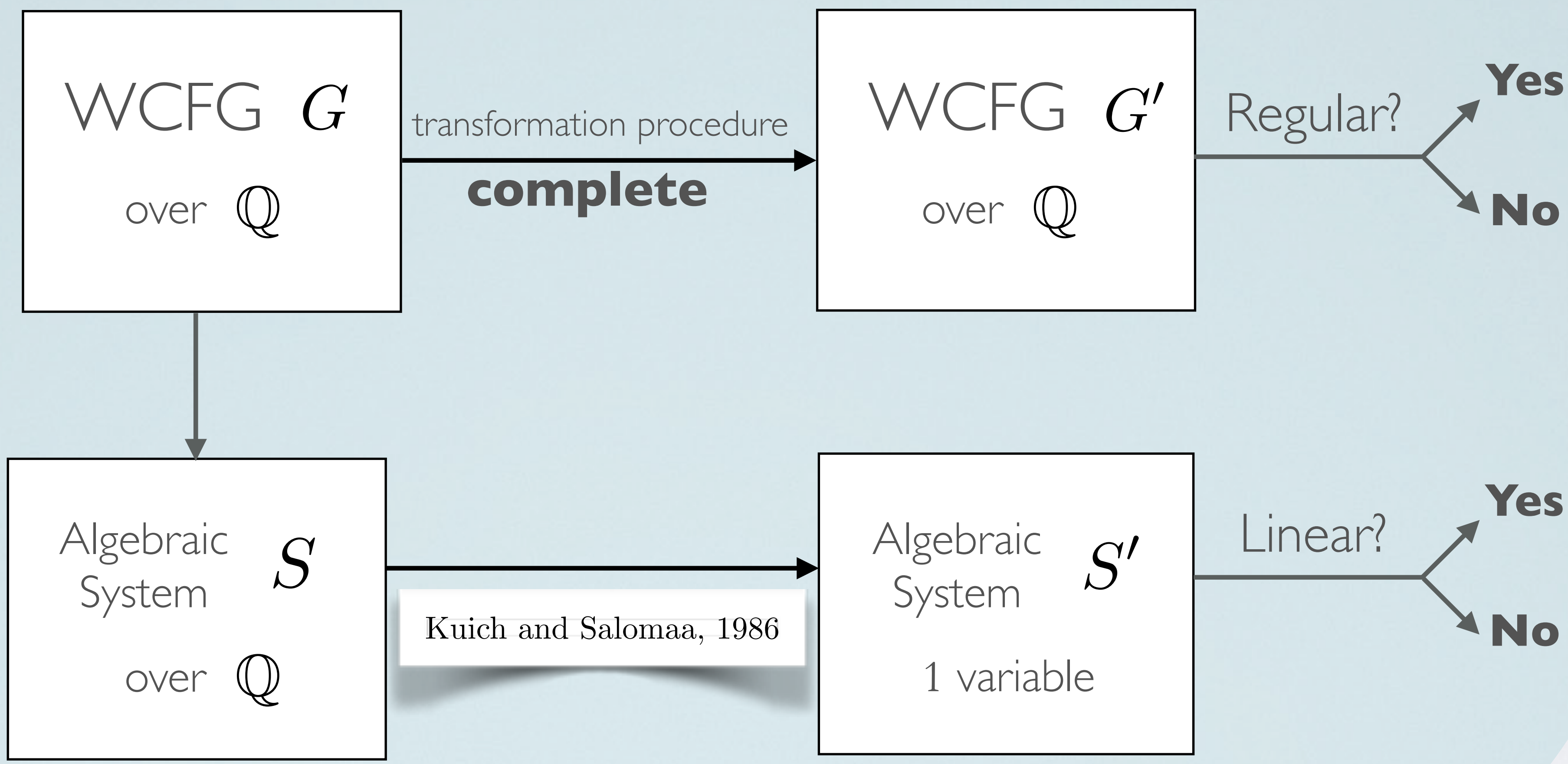

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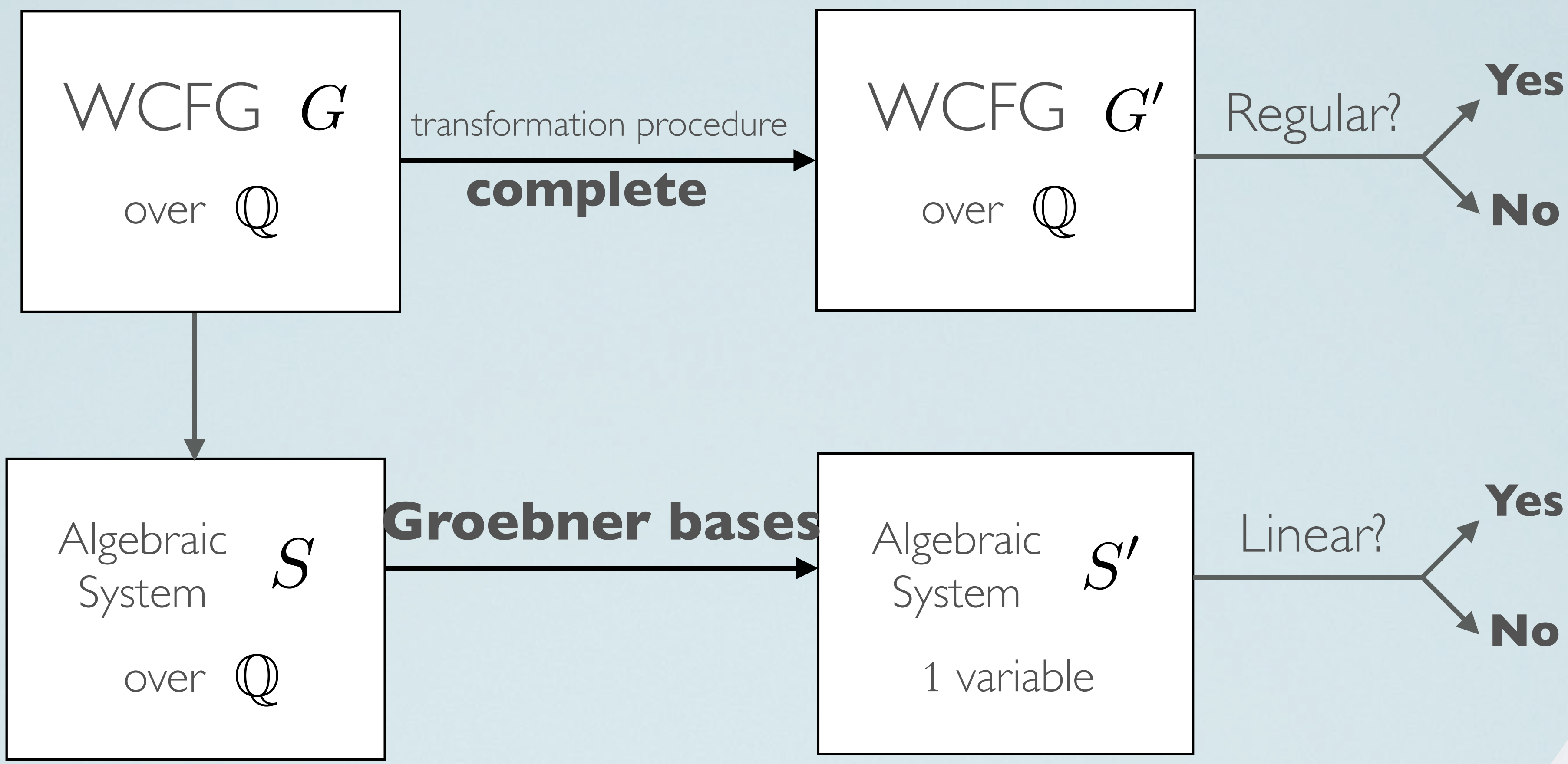

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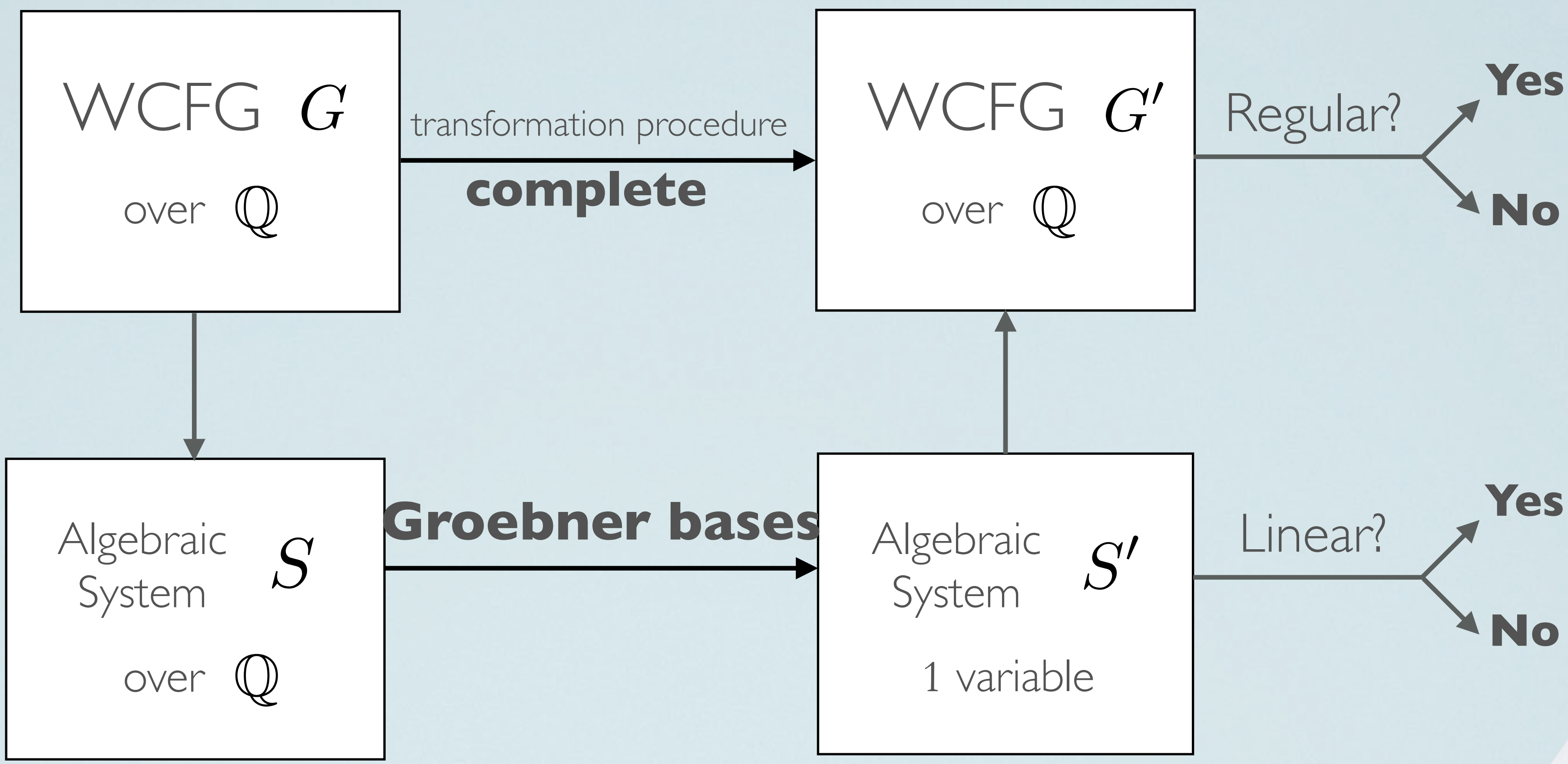

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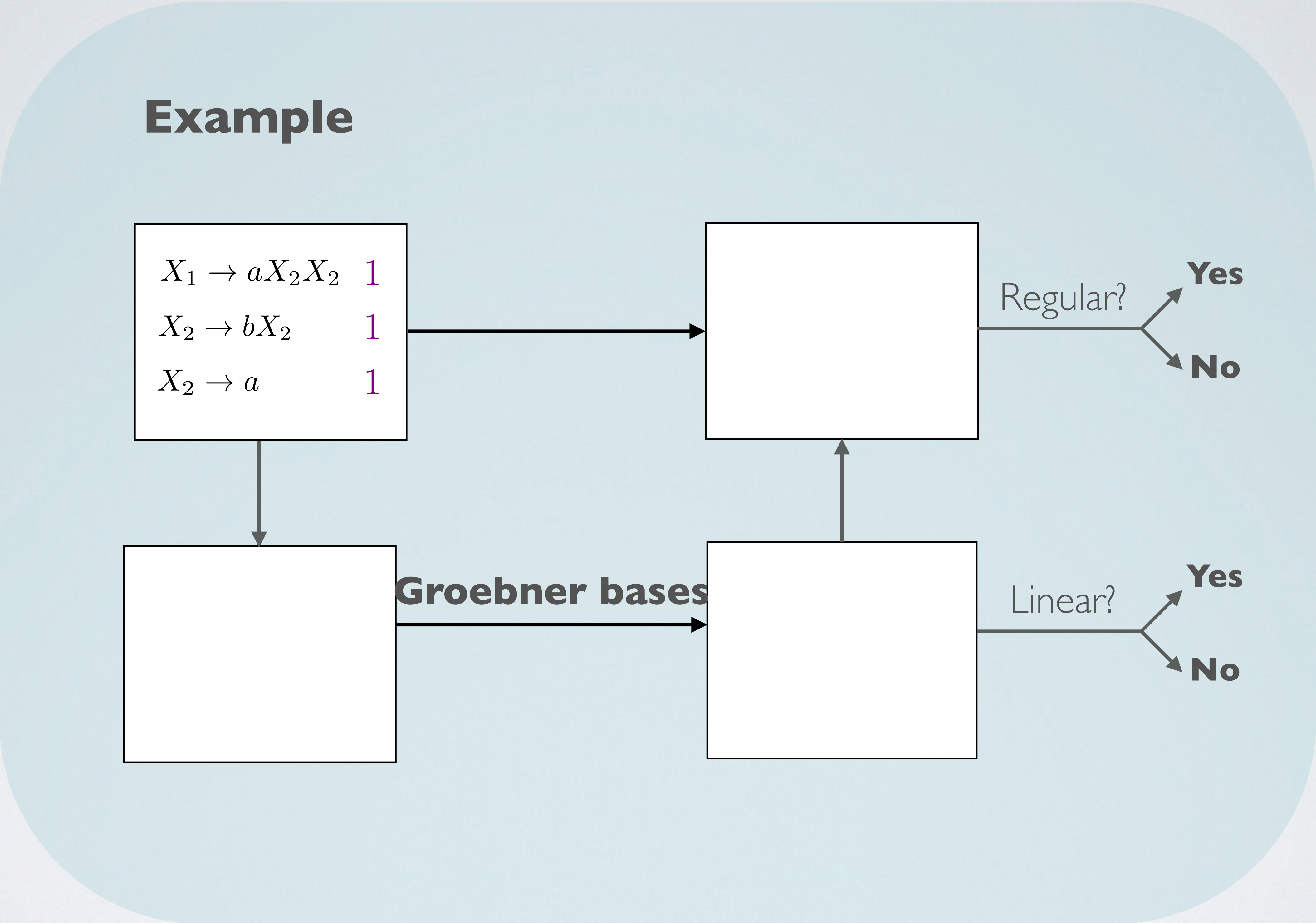
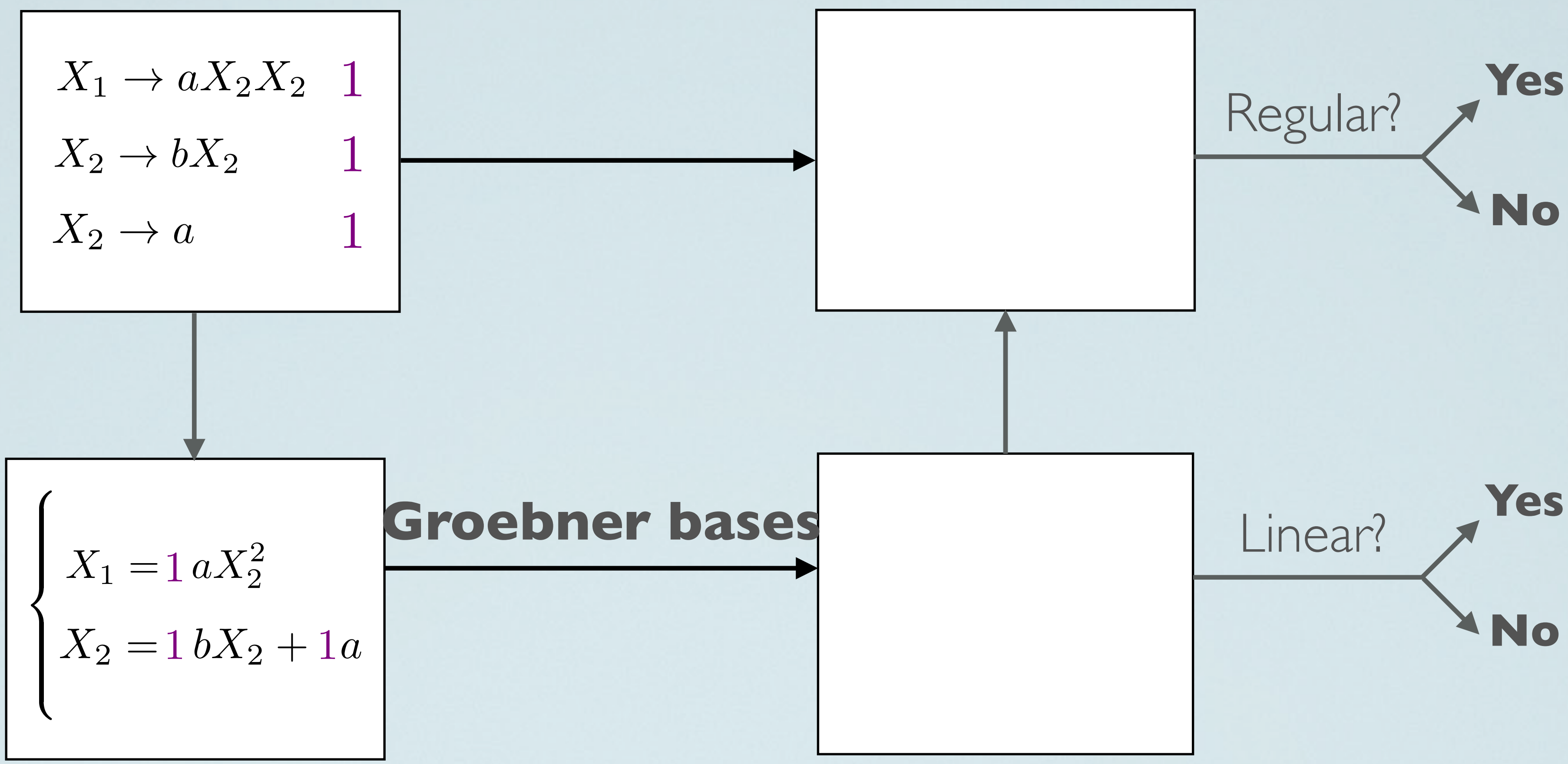


CONTRIBUTION

Example



CONTRIBUTION

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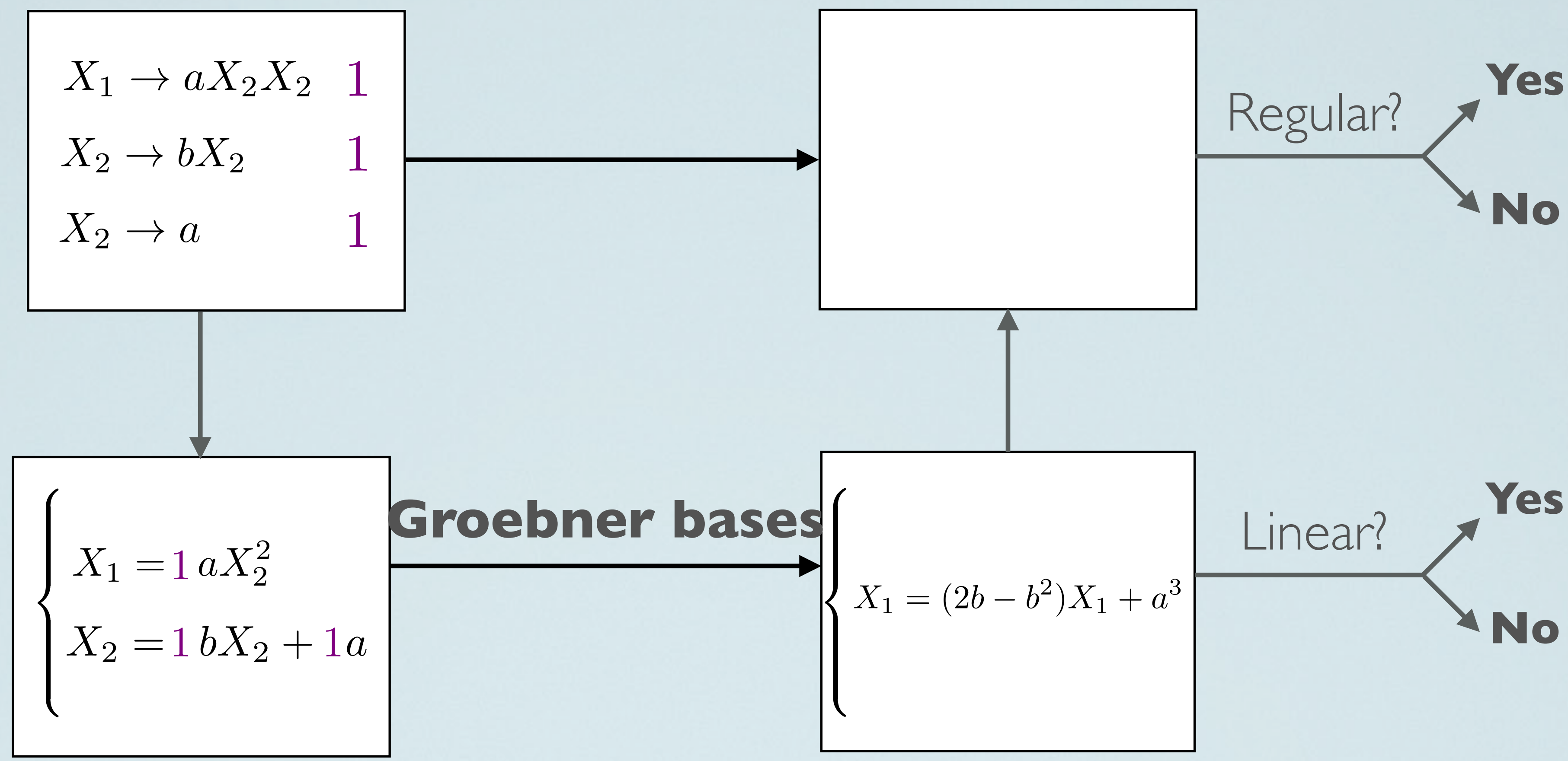

CONTRIBUTION

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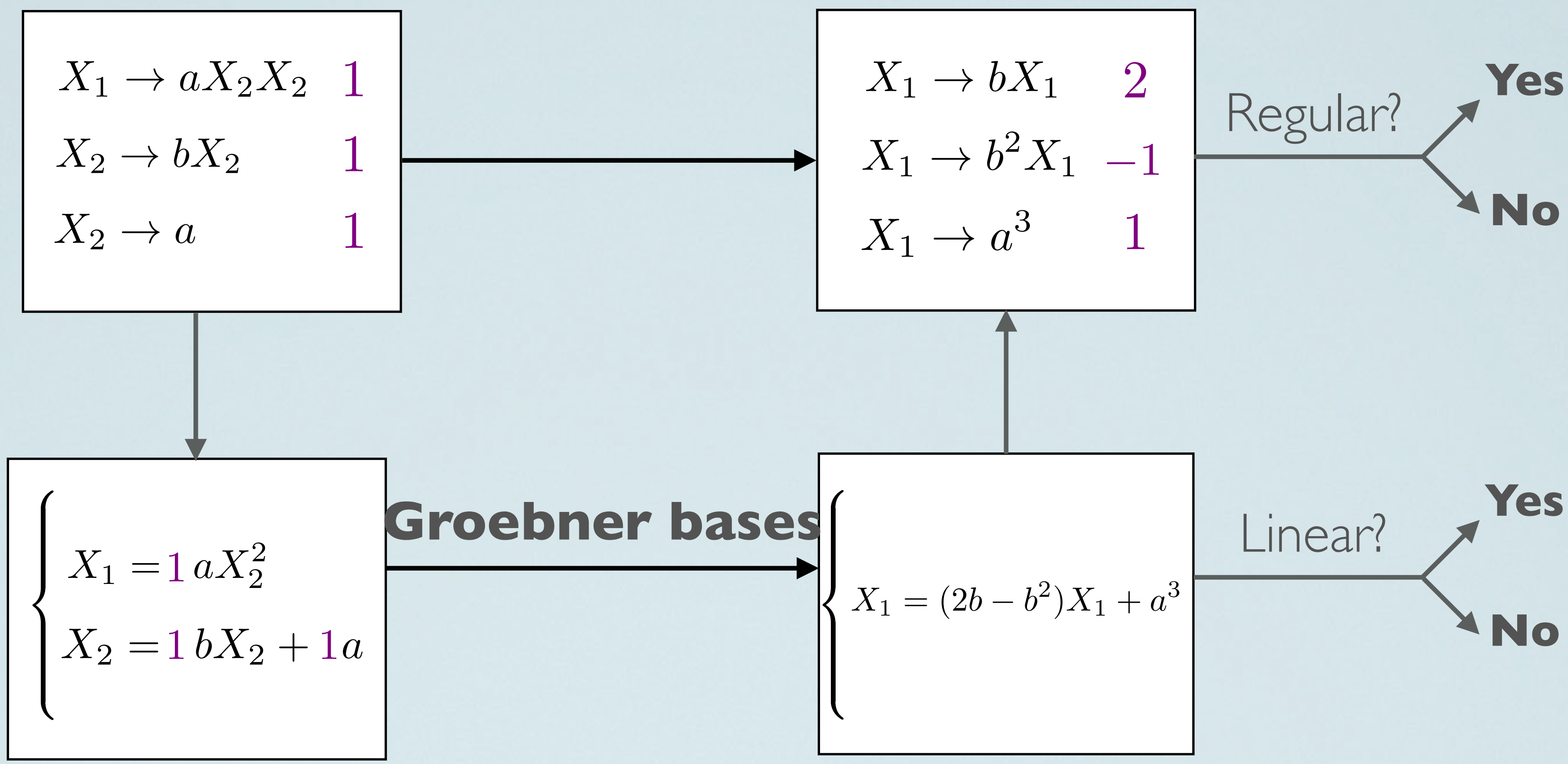

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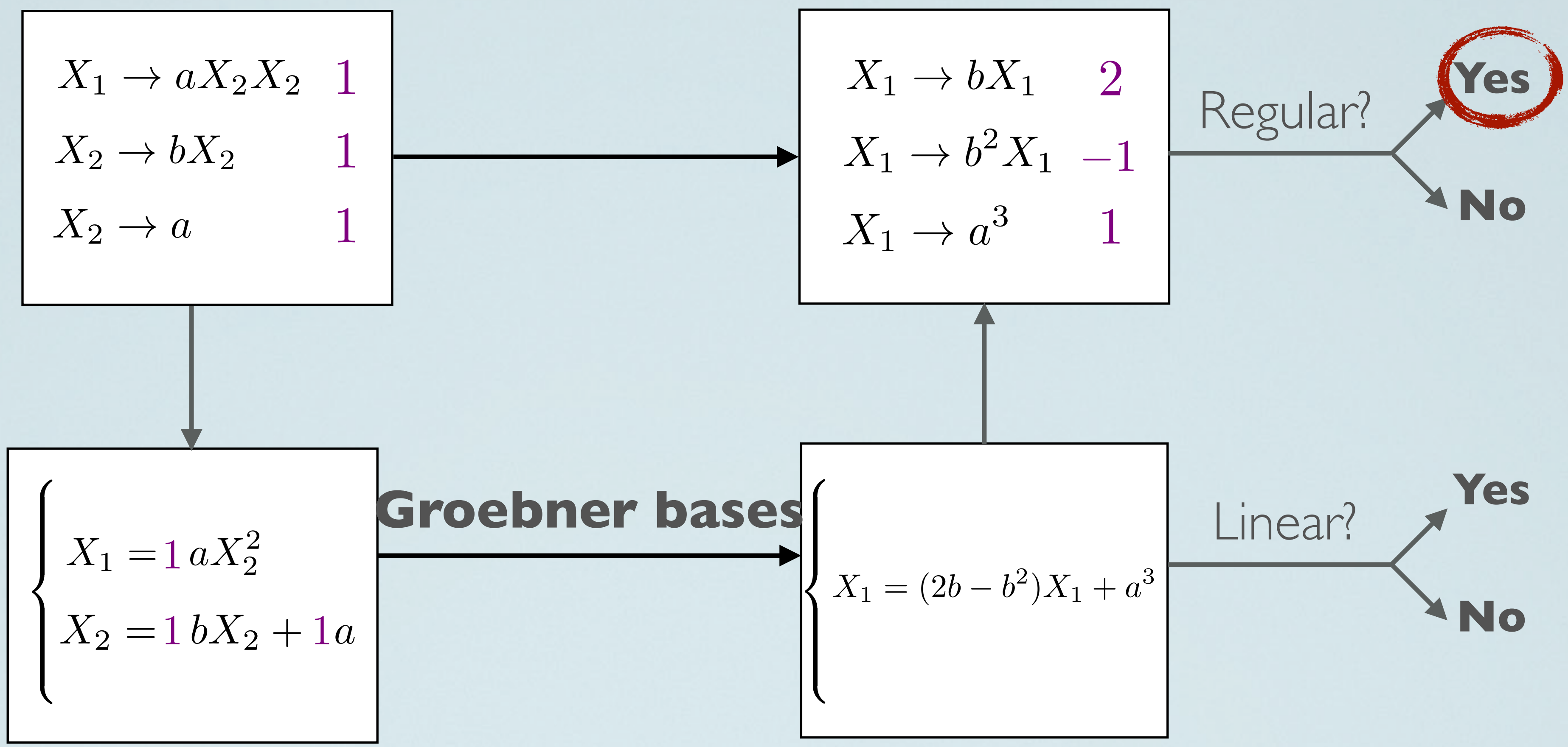
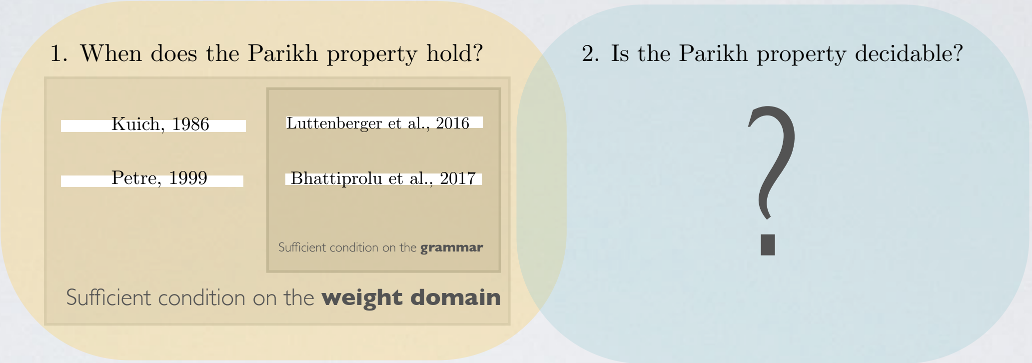
Example



CONCLUSIONS

Previous Work



1. When does the Parikh property hold?

Extend previous sufficient condition
+
construction

Open question:

Sufficient and necessary condition?

2. Is the Parikh property decidable?

Yes, when the weights are over $\mathbb{Q}$

Open question:

Arbitrary weight domains?

CONCLUSIONS

Previous Work

1. When does the Parikh property hold?

Kuich, 1986

Petre, 1999

Luttenberger et al., 2016

Bhattiprolu et al., 2017

Sufficient condition on the **grammar**

Sufficient condition on the **weight domain**

2. Is the Parikh property decidable?

?

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Thank you