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EQUIVALENCE *VERSUS* INDIFFERENCE SCALES*

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Comparing income levels across families with different composition is a central issue for most welfare indicators and policies. Currently, the dominant method relies on income-based equivalence scales, which raise a host of theoretical, empirical and normative difficulties. However, a new approach, based on the notion of indifference scales, avoids most of these problems, while leading to empirically tractable estimation methods. I submit that time has come to abandon the traditional view in favour of the new concept.

According to the US Census Bureau, in 2014 a four-person family with two adults and two children was considered as poor if its annual cash income was below $24,000. If I take this level as a benchmark, how should the poverty level be defined for a couple without children, or for a single parent with one child, or for any other family composition?

Questions of this type are but an example of a paramount problem in applied welfare economics, namely the comparison of income levels across families with different composition. Poverty measurement is by no means the only issue for which such comparisons are needed. Consider, for instance, the construction of inequality indices. Several studies[1] argue that a significant fraction of the increase in household-level inequality (as measured by the Gini coefficient) over the recent decades comes from the rise of single-adult households. Implicitly, such a claim must rely on a comparison between per capita income for singles and couples; and its accuracy is likely to be quite sensitive to the particulars of the comparison. Replacing, in the global income distribution, a couple with $30,000 annual income with two divorced spouses with respective incomes, say, $18,000 and $12,000 will certainly have a mechanical effect on any inequality measure. But whether inequality, in an economic sense, has increased – and by how much – depends both on the magnitude of economies of scale generated by cohabitation and on the level of intra-household inequality in the initial couple. The change in inequality identified by standard measures must rely, at least implicitly, on strong assumptions regarding both aspects.

Of course, the policy implications of these questions are far from negligible. In most developed economies, some benefits are means-tested. Once the income threshold(s) for eligibility have been defined for some reference family, it has to be extended to all possible family sizes and compositions.[2] Again, whichever methodology is used for that

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[1] See for instance Burtless (1999).

[2] An alternative approach could be to define the relevant thresholds for each possible family composition independently (say, using a definition of 'basic needs'). Then one could, from a descriptive perspective, backwards induce the equivalence scales that are implicit in the definition of these thresholds. Such a 'revealed preferences' approach is sometimes called 'poverty relative equivalence scales'.

purpose, its justification, in terms of the very objectives pursued by the benefits under consideration, unavoidably relies on some implicit assumptions about the economic nature of cohabitation and the transfers it implies. Considerable amounts of money, with potentially significant impact on individual and collective welfare, are at stake – which makes the questions all the more important.

In practice, the methodologies used to answer these questions have been developed several decades ago, leading to various notions of equivalence scale, and have not been fundamentally altered since then. The actual outcomes may be quite surprising. Coming back to the US definition of poverty in 2014, the threshold equals \$12,316 for a childless single, \$15,853 for a childless couple, \$16,317 for a single with one child, \$24,000 for a couple with two children and \$24,091 for a single with three children. The scale, therefore, is highly non-linear. In the absence of children, a second adult increases the income threshold by less than 30%, suggesting important economies of scale. Still taking singlehood as reference, a child raises the threshold significantly more than a spouse (the difference exceeds \$400). Not so, however, for larger families. If the reference is a single parent with two children, the impact of an additional adult is basically the same as of an additional child; and the impact of a third child is almost nil (less than \$100). Obviously, some complex computations are involved in these numbers – but where do they come from?

Paradoxically, the intuitive notions these numbers try to capture are straightforward. That an adult's consumption should not have the same impact as a child's on the household budget is easy to see. More importantly, it seems intuitively clear that the 'cost of living' – whatever the precise definition of this notion – for a family should be less than the sum of these costs for the same number of single persons. Much consumption is (at least partly) shared within the household, generating economies of scale. I therefore expect that the cost of providing each individual of a group with some given level of well-being should be smaller when these individuals live together than when they are isolated. The problem is to translate these intuitions into economic methods that are both theoretically well defined and empirically implementable.

When considering the literature on inter-family comparisons of income, two striking facts emerge. One is their crucial policy relevance. They are needed for the implementation of basically all income-related policies; it is hard to think of any tax, benefit or other social programme that could consider income levels without adjusting them for family size. Even more important, perhaps, is the role they play in the understanding of economic and social inequalities, and generally in the micro-economic vision of most social structures – which lie at the core of the very definition of the type of policies we want to implement.

The second striking fact is how flawed their conceptual foundations are – particularly given their practical importance. A first claim of the present contribution is that the theoretical framework that underpins the current approaches is deeply unsatisfactory. It has been known for some time that the notion of equivalence scales requires strong and probably unrealistic assumptions, while leading to some deep identification problems. I argue that the situation is actually much worse. The dominant approach adopted by the equivalence scale literature relies on income effects; i.e., it postulates that the impact of family composition on welfare can be fully summarised by an income variation (so that families of all composition would have exactly the same demand, once income variations are compensated for). While this vision is by no means exclusive in the academic literature – alternative approaches, based on price variations, have been developed over the last decades – it constitutes the prominent reference for most policy-oriented applications. I argue in what follows that the income-based approach explicitly contradicts the very notion – i.e., the existence of economies of scale within families – it aims at capturing. Moreover, the conceptual foundations of equivalence scales of all types are extremely weak; in particular, they systematically rely on non-testable assumptions related to comparisons of utility levels across individuals. Last, but by no means least, these notions all but discard crucially important issues, such as intra-family allocation and inequality, which should instead play a central role in the design of relevant policies.

In a nutshell, what I have is a critically relevant question that is investigated using an inadequate theoretical framework, potentially leading to inappropriate policy conclusions. The picture, however, is not uniformly grim. Novel approaches have been developed recently, that avoid the flaws just mentioned. The second goal of this contribution is to summarise briefly these innovations, with a particular emphasis on the deep reconsideration of the basic concepts that they imply.

1. Equivalence Scales: The Current Situation

1.1. *Engel's Path-breaking Work*

The current approach to interfamily income comparisons, centred on the notion of equivalence scales, can be traced back to Engel (1895).[3] It is based on a simple but remarkably robust empirical observation: for any given family size, food expenditures tend to increase with income but less than proportionally – so that the share of income devoted to food decreases with income. The so-called Engel approach therefore proposes to consider two households with different compositions as 'equally well-off' if they devote the same share of their income to food expenditures. Interestingly, and despite the numerous problems raised by this approach, it remains by and large the dominant method for devising welfare-related policies and estimating their effects; for instance, it essentially describes how the US Census Bureau generates the numbers mentioned above.

Several objections have nevertheless been formulated to the Engel method, leading over the years to both a generalisation of the approach and a more careful investigation of its theoretical foundations. Prais and Houthakker (1955) and Barten (1964) construct scales of the Engel type for a variety of goods, acknowledging the fact that, as a matter of principle, food may not be the only commodity on which the definition of equivalence scales could be based. An alternative method, initiated by Rothbarth (1943), concentrates on commodities that are only consumed by some members of the household (for instance, alcohol or tobacco are only consumed by adults). The idea, here, is that a couple with children would be deemed 'as well off' as a childless counterpart if they both devote the same amount to the purchase of such goods.

[3] See Lewbel (1997) and Lewbel and Pendakur (2008) for a detailed presentation.

1.2. *Equivalence Scales*: *Conceptual Issues*

On the theoretical front, several authors[4] have contributed to constructing a formally clean framework on which the notion of equivalence scale can be based. These contributions have been extremely useful, if only because they provided an explicit list of assumptions that any of the existing, equivalence scales approaches requires.

1.2.1. *Individual and family utility*

First, the concept of equivalence scale relies on the (admittedly vague) notion of 'family well-being'; remember, indeed, that Engel considered families with equal shares of food consumption as equally 'poor'.[5] The idea that a family can be represented by a single utility function – the so-called 'unitary' approach to family economics – does not necessarily contradict methodological individualism, which lies at the core of microeconomics; but individualism certainly requires that the 'utility of the family' be defined by reference to individual utilities. A first task of any serious theoretical foundation for equivalence scales is therefore to describe the relationship between individual utilities and family well-being explicitly.

In practice, this can be done in a number of ways. The simplest solution, adopted for instance by Blackorby and Donaldson (1993*a*, *b*), refers to Samuelson's welfare index; i.e. the household maximises some $W(u^1, \ldots, u^I)$ where u^i denotes the utility of member *i*. Note, however, that this formulation makes strong (if implicit) assumptions about the nature of the intra-family decision process. For instance, the respective 'weights' of each member (as summarised by the partial derivatives $\partial W/\partial u^i$) do not explicitly depend on prices, income, or any other aspect of the household's economic environment. Such an assumption is far from innocuous, particularly if one is interested in intra-household inequality. Moreover, identification of individual utilities from the household's aggregate behaviour is particularly problematic in that case. Lastly, this 'unitary' approach generates strong testable predictions (such as income pooling or Slutsky symmetry for household demand); these predictions have repeatedly been rejected by empirical analysis.[6]

1.2.2. *Identical preferences*

A second feature of the equivalence scales approach is the (mostly implicit) assumption that the preferences of a household are fully defined by its composition. The notion of 'composition' is admittedly flexible and may involve a series of detailed, observable characteristics (for instance, not only the number of children but also their

[4] See for instance Lewbel (1989), Blackorby and Donaldson (1993*a*, *b*), Donaldson and Pendakur (2004, 2006) and Pendakur (1999).

[5] Specifically, Engel states that, the poorer a family, the larger fraction of its income must be devoted to food expenditures, and that this pattern can be used for identification purposes:

> Man darf den auf dem Wege ächter Induction gefundenen Satz, dass, jeärmer eine Familie ist, einen desto grösseren Antheil von der Gesammtausgabe muss zur Beschaffung der Nahrung aufgewendet, ohne Gefahr einige Schritte weiter verfolgen und das erlangte Ergebniss dazu benutzen, die Grösse der Konsumtion in Sachsen zu bestimmen (Engel, 1895, Anlage II, pp. 28–9).

[6] See Browning *et al.* (2015) for a detailed presentation.

age and gender). Still, it must be the case that all families with identical composition, not only have the same ordinal preferences but also reach the same level of welfare when faced with the same consumption bundle. While assumptions of this flavour may be difficult to avoid whenever income comparisons are at stake (if only for empirical reason), it is important to note that, in the case of equivalence scales, identical preferences are part of the conceptual definition.

1.2.3. *Interpersonal comparison of welfare*

Third, the Engel approach fundamentally relies on welfare comparisons across individuals; as such, it requires a strong version of interpersonal comparison of utilities. This is needed at two distinct levels. First, the mere definition of a family utility, along the lines discussed above, typically involves such comparisons. This is particularly clear, for instance, for the maximin function:

$$W(u^1, \ldots, u^I) = \min(u^1, \ldots, u^I),$$

which explicitly assumes that one can compare utility levels across agents. Second, these indices must be comparable across families; that is, one must be able to decide whether two given households (with possibly different compositions) are 'as well off' – which, in practice, means equating utility levels across households. Not only is the comparability assumption debatable *per se*, but it generates well-known identification problems. I discuss this question below; for the time being, let me simply note that in their current form, the conceptual definitions of equivalence scales crucially depend on whether, and exactly how, family welfare depends on individual characteristics – something the sole observation of behaviour will never allow to assess.[7]

1.2.4. *The 'black box' fiction*

The previous criticisms are familiar and have been discussed extensively over the recent years. However, another problem may have been under-considered in these discussions. The various constructions supporting equivalence scales systematically adopt an axiomatic approach. As I show, they describe some abstract properties that household utilities must exhibit for the equivalence scale technology to be both theoretically justified and empirically implementable. In principle, there is general agreement on what the very notion of equivalence scale aims at capturing – namely the fact that economies of scale generate a discrepancy between the cost of living of a multi-person family and the sum of costs the same individuals would face, should they live independently from one another. Still, these aspects are never explicitly modelled – which would require a precise description of individual utilities and the household's consumption and decision processes. Instead, they are forced into the black box of axiomatic properties that the family's indirect utility is assumed to satisfy. This could be acceptable (although certainly not advisable) if the assumption made on the resulting household utility were consistent, or at least not incompatible, with the very notion – namely, economies of scale – they are supposed to capture. However, I argue below that the exact opposite is true, at least for income-related notions of equivalence scales

[7] This issue is related to the distinction that Pollak (1991) makes between situation comparison and welfare comparisons.

(which dominate the policy-oriented literature); and that a model of household behaviour that aims at capturing the presence of economies of scale must, except for highly specific preferences and consumption patterns, contradict the very properties needed for such equivalence scales to be usable.

1.2.5. *Intra-family allocation*

A last concern raised by equivalence scales regards their normative implications. The issue, here, is the exclusive emphasis on family welfare. In the standard approach, while individual utilities may (or may not) be used for the definition of the household utility function, they are all but omitted for the relevant applications. Neglecting individual well-being is problematic from a double viewpoint. On the positive side, it may provide a biased view of reality. When used for measuring inequality, the equivalence scale approach leads to concentrating on inequality between households while ignoring inequality within households.[8] The resulting picture may be quite different from reality. Recently, Lise and Seitz (2011), using British data, have provided estimates of intra-household inequality and its evolution over several decades. Their conclusion is twofold. First, standard measures, by concentrating on the inter-household level, seriously underestimate the actual level of inequality among individuals. For instance, taking into account intra-household aspects increases the variance of log consumption by almost 50%. Second, and even more interestingly, the vision of the trends affecting inequality over time is totally reversed. The well-known surge in inequality over the last decades of the twentieth century only obtains at the inter-household level. Lise and Seitz (2011) show that it is compensated by a spectacular reduction of inequality within the household (reflecting women's increase in education, wages and participation in the labour market); as a result, they find that inequality between individuals did not change much (and if anything slightly declined) over the period. While the Lise and Seitz contribution relies on strong and sometimes debatable assumptions (as unavoidably would any attempt at measuring intra-household allocation), its main conclusion – namely that one should consider inequality between individuals, not between households, and that the two phenomena may evolve in quite different ways – is very hard to ignore.

These issues raise serious concerns regarding the validity of some apparently well-established facts. Take the 'demographic' causes of inequality and, in particular, the impact of the ever larger number of single-adult households. From an equivalence scale perspective, the comparison between a couple and two singles relies on the (implicit) assumption that resources are split equally within the couple. There is ample empirical evidence that such an assumption is grossly incorrect; consequently, the conclusions it generates should be considered with scepticism. Another example of a possible misleading policy conclusion by ignoring intra-household inequality is provided by Cherchye *et al.* (2015), who show that actual poverty levels, computed at the individual level, can substantially differ from the traditional poverty levels, as estimated by a household-based measure. For instance, poverty turns out to be more prevalent among women than among men for the particular sample.

[8] On this, see for instance Chiappori and Meghir (2015).

The normative implications of exclusively considering welfare at the family level are no less serious. In practice, using the equivalence scales approach amounts to assuming that the allocation of resources prevailing within households exactly coincides with what would be deemed optimal from whichever normative perspective one is willing to refer to. Again, ample empirical evidence contradicts such a statement (at least for most 'reasonable' normative criteria).

Recognising that the actual processes governing intra-household allocation may fail to be normatively optimal opens a new and crucially important set of issues that an equivalence scale approach imposes to disregard. Take the standard notion of compensating variation. For a reform that changes the economic environment (prices, incomes etc.) of an agent, the compensating variation is defined as the (positive or negative) amount the agent should receive to be exactly as well off, after the reform, as he was before it. When trying to extend this logic to multi-person families, one is faced with a thorny normative question: should we aim at preserving the well-being of all members, or simply at compensating the family 'on average'? Chiappori (2011) and Chiappori and Meghir (2015) introduce the notion of 'potentially compensating variation', defined as the amount that should be paid to allow the family, should it be inclined to do so, to compensate each of its members exactly. The problem, clearly, is that nothing guarantees the corresponding amount will be spent that way. On the contrary, it is quite likely, in many situations, that the hypothetical compensation paid to the 'family' would disproportionately benefit some members (who would then strictly gain from the reform) at the expenses of others (who would be net losers). However, any alternative (such as transferring enough for each member to be at least as well off as before, taking into account the allocation mechanism prevailing within the couple) raises other problems. Again, such issues are not particular to the equivalence scales approach; they mostly reflect the constraints stemming from the fact that, even when policies aim at targeting individual welfare, most of the tools (taxation, benefits etc.) they use only operate at the family level. Still, an immediate implication is that the process governing intra-household allocation of resources cannot possibly be disregarded in the design of such policies. If the 'compensation' for a given reform results in some members (e.g. adult males) gaining a large amount while others (say, women, children or both) being made miserable, the impact on individual poverty and inequality can hardly be ignored. The main weakness of equivalence scales, from that perspective, is that the mere definition of the concept explicitly rules out any analysis of this kind.

1.3. *Income-based Scales versus Price-based Scales*

As mentioned above, several notions of equivalence scales have been proposed. A first and basic distinction is between what can be called 'income-based scales' and 'price-based scales'.

Let me introduce some notations. First, accept for the moment the notion of household utility, and let $U(x, a)$ denote the household's utility function (as a function of the consumption vector x and a vector a describing the household's composition: number of adults, of children, ages etc.). Similarly, let $V(p, y, a)$ and $e(p, \bar{u}, a)$ respectively denote the corresponding household indirect utility and expenditure functions. That is, $V(p, y, a)$ is the maximum utility a household with composition a

can reach when endowed with an income y and faced with prices p; and $e(p, \bar{u}, a)$ is the income needed for a family with composition a and faced with prices p to reach some utility level $\bar{u}$. Formally:

$$V(p, y, a) = \max_x U(x, a) \text{ such that } p'x = y \text{ and}$$
$$e(p, \bar{u}, a) = \min_x p'x \text{ such that } U(x, a) = \bar{u}.$$

1.3.1. *Price-based scales: formal definition*

The notion of 'price equivalence scales' can be traced back to Barten (1964) and has been developed by Jorgenson and Slesnick (1987) and others. The core intuition is that the impact of cohabitation within a family is a form of price distortion: the prices individuals are facing within the family depend on family composition.

The basic notion, often referred to as 'Barten exactness', can be technically defined as follows. Take a particular family composition, say $\bar{a}$, as a reference; then there exists a diagonal matrix $B(a)$ such that the indirect utility of a household with composition $a \neq \bar{a}$ can be written as follows:

$$V(p, y, a) = V[B(a)p, y, \bar{a}].$$

Equivalently, the expenditures function is such that:

$$e(p, \bar{u}, a) = e[B(a)p, \bar{u}, \bar{a}].$$

A generalised version (generalized Gorman exactness) relaxes two restrictions. First, $B(a)$ is no longer assumed to be diagonal; second, family composition may have an additive income effect. Formally:

$$V(p, y, a) = V[B(a)p, y - p'A(a), \bar{a}],$$

for some vector $\boldsymbol{A}(a)$. Equivalently, the expenditures function is such that:

$$e(p, \bar{u}, a) = e[B(a)p, \bar{u}, \bar{a}] + p'A(a).$$

1.3.2. *Income-based scales: formal definition*

Alternatively, equivalence scales can be defined in terms of income effects. Then the notion of equivalence scale aims at answering the following question: taking the family composition $\bar{a}$ as reference, by how much should the income of a household with composition $a \neq \bar{a}$ be multiplied to allow that household to reach the same utility level $\bar{u}$ as the reference family? This formally translates into the following definition:[9]

$$D(p, \bar{u}, a) = \frac{e(p, \bar{u}, a)}{e(p, \bar{u}, \bar{a})}, \tag{ES}$$

[9] This formula corresponds to equivalence scale exactness. Other functional forms for household equivalence scales have been considered in the literature. For instance, Blackorby and Donaldson (1994) introduced absolute equivalence scale exactness (AESE); and both the generalised absolute equivalence scale exactness (AGESE) and the generalised equivalence scale exactness (GESE) of Donaldson and Pendakur (2006) generalises equivalence scale exactness. All these notions, however, exhibit the same feature – namely, the impact of a multi-person structure operates through an income effect. For the sake of simplicity, I only consider equivalence scale exactness in the following discussion (I am indebted to an anonymous referee for that clarification).

which can equivalently be expressed in terms of indirect utilities (with straightforward notations):

$$V(p, y, a) = V\left[p, \frac{y}{D(p, \bar{u}, a)}, \bar{a}\right], \tag{ES'}$$

where D is taken at the point $\bar{u} = V(p, y, a)$.

One can readily see how this definition captures, and actually generalises Engel's intuition. Assume for a moment that D does not depend on price p_j (where commodity j is, say, food). Then the Hicksian demand of household a for good j is:

$$h_j(p, \bar{u}, a) = \frac{\partial e(p, \bar{u}, a)}{\partial p_j} = D(p, \bar{u}, a)\frac{\partial e(p, \bar{u}, \bar{a})}{\partial p_j},$$

implying that the Hicksian budget shares of good j are:

$$\begin{aligned}\frac{p_j h_j(p, \bar{u}, a)}{e(p, \bar{u}, a)} &= \frac{p_j \partial e(p, \bar{u}, \bar{a})/\partial p_j}{e(p, \bar{u}, \bar{a})}\\ &= \frac{p_j h_j(p, \bar{u}, \bar{a})}{e(p, \bar{u}, \bar{a})}.\end{aligned}$$

In other words, the Hicksian budget share on food is the same for both households if and only if they are computed at the same utility level $\bar{u}$ – i.e., if the two households are equally well-off. However, (ES) is more general than the simple comparison of food expenditures, because:

(*i*) it considers all goods, not simply food; and
(*ii*) it allows for additional price effects (if the index D depends on p_j, the previous computations must include an additional term, corresponding to the derivative $\partial D/\partial p_j$).

1.3.3. *Common concerns*

The two approaches just described adopt different perspectives. One approach emphasises the impact of family composition on prices; *de facto*, the main consequence of being in a multi-person household is that individuals face a different price vector. The second approach, on the contrary, summarises the impact of family composition through an income effect: it postulates that a multi-person family behaves just as any single individual, only with a different income. In principle, the first approach is more general, because an income effect can always be interpreted in terms of price variations (this is a direct consequence of homogeneity of indirect utilities). The converse is obviously false; in fact, Lewbel (1991, Lemma 3) has shown that the Barten approach is in general incompatible, except for the special case of homothetic preferences, unless matrix B is proportional to the unitary matrix.

It is important to note, however, that the level $\bar{u}$ explicitly enters both definitions of equivalence scales. This fact validates the concerns described above about interpersonal (actually interhousehold) comparison of utilities. In the Barten approach, the crucial assumption is that the level of utility reached by the family of composition a is equal to the level that family $\bar{a}$ would reach, should it face the modified price vector. Note that this assumption is totally non-testable: observed behaviour may (or may not) confirm that ordinal preferences are the same but cannot say anything about utility levels.

In practice, most policy-oriented empirical applications (starting with the US Census Bureau's poverty definition (see for instance Jackson (1968)) rely on the second construct – which, in the remainder of the article, I refer to as 'Engel's equivalence scales'. The main motivation is tractability. Unlike price scales, income scales may (under a series of additional assumptions described below) be identified even without price variations, i.e. from the sole observation of Engel curves; and they do not require the estimation of a full-scale demand system. However, this apparent simplicity has a high price in terms of both normative implications and empirical identifiability, which I now discuss.

1.4. *Engel's Equivalence Scales*: *Empirical Issues*

1.4.1. *The basic problem*

That the mere definition of equivalence scales should lead to serious identification problems has been known for quite some time (see for instance Pollak and Wales, 1979; Deaton and Muellbauer, 1980 and Blackorby and Donaldson, 1991). A comprehensive analysis of these issues has been provided by Blundell and Lewbel (1991).

The basic intuition is easy to grasp from the previous definition. In principle, equivalence scales should be recoverable from data; in practice, they should therefore be identifiable from observed behaviour. That this is not the case is easy to see. Even with perfect data, household demand (or labour supply) allows at best to identify utility in an ordinal sense, i.e. up to an arbitrary increasing transform. However, definition (ES) requires a knowledge of the actual level of utility reached by the household.[10]

Formally, take any function $f(u,a)$ that is strictly increasing in its first argument and define a new expenditure function $\tilde{e}$ by:

$$\tilde{e}(p, \bar{u}, a) = e[p, f(\bar{u}, a), a]. \tag{1}$$

Clearly, e and $\tilde{e}$ are empirically equivalent – they generate exactly the same demand functions. In particular, no observed behaviour can possibly allow distinguishing between them. However, they correspond to totally different equivalent scales.

Elaborating on this fact, Blundell and Lewbel (1991) show that, given any demand system that is compatible with utility maximisation, for any choice of $(p, \bar{u})$ and any set of equivalence scales $D(p, \bar{u}, a)$ (defined for each possible value of the demographic vector a), there exists a well defined household utility that:

(*i*) generates the observed demand for all prices and incomes; and
(*ii*) generates equivalence scales which, at $(p, \bar{u})$, coincide with the arbitrarily chosen values.

[10] Note that even cardinal identification would not be sufficient, since (ES) is typically not invariant by an affine transform of utilities.

In other words, one can normalise equivalence scales at some given point to be any arbitrary function of family composition. For any particular normalisation, the changes in equivalence scales due to variations in prices can be identified. However, the value of equivalence scales remains arbitrary. Given that, for the measurement and policy purposes discussed above, one is primarily interested in values rather than changes, the unavoidable conclusion is that the previous construct simply fails to provide an empirically relevant conceptual tool.

Note, incidentally, that price-based equivalence scales suffer from a related weakness; namely, replacing e with $\tilde{e}$ (defined by (1)) has no impact on observed behaviour but will, in general, change the estimation of the price scale $B(p)$. The heart of the problem, here, is not the particular perspective adopted but the non-falsifiable assumption of equality of utility levels.

1.4.2. *Independence of base*

In the case of Engel's (income-based) equivalence scales, the standard solution to the identification problem just described has been introduced by Lewbel (1989), who coined the term 'Independence of base'; a similar notion, called 'equivalence-scale exactness', is due to Blackorby and Donaldson (1991, 1993*a*,*b*). The idea can be summarised as follows. Clearly, the problems just mentioned come in part from the fact that the scale D explicitly depends on the utility level u. What if that were not the case? In other words, is it possible that the scale D, defined as in (ES), does not depend on the utility level at which it is measured?

It can easily be seen that the answer is positive. Indeed, Lewbel and Blackorby and Donaldson show that D is independent of u if and only if the household's expenditure function is multiplicatively separable:

$$e(p, \bar{u}, a) = \phi(p, a) e(p, \bar{u}, \bar{a}), \tag{IB}$$

for some function ϕ. (IB) essentially implies that the coefficient ϕ by which family income should be inflated to keep utility constant when family composition is changed only depends on prices (and family composition), not on the level of utility itself. An equivalent formulation is in term of indirect utility; then it must be the case that:

$$V(p, y, a) = V\left(p, \frac{y}{\phi(p, a)}, \bar{a}\right).$$

Note that the sole independence of base assumption is not sufficient to solve the identification problem. After all, whether utilities can be compared across individuals remains an issue even when preferences exhibit the multiplicative separability just described. But the identifying assumption becomes simpler to express (if not necessarily easier to swallow); namely, it requires that two households with the same 'normalised' (or 'equivalent') income $y/\phi(p, a)$ have the same level of utility.

As I said before, the body of theory I have just summarised has an immense virtue: it clarifies the assumptions that are needed to implement the equivalence scales approach to income comparisons across families. In particular, the independence of base assumption has a clear implication – namely, that the impact of family composition on family demand goes through an income effect, in the sense that any family – say, a couple with two children – behaves like a single, only with a lower income. As soon as the income discrepancy is corrected – which is what the equivalence scale does – behavioural patterns become exactly identical.

Quite strikingly, nothing in the construct just described explicitly accounts for the existence of economies of scale – although these phenomena are clearly at the core of the economic mechanisms that the notion of equivalence scale aims at capturing. Actually, whether the (IB) assumption is even compatible with public goods or economies of scale is an open question. This is precisely the issue that I investigate in the next Section.

2. Economies of Scale and Engel's Equivalence Scales: Are They Compatible?

In this Section, I explicitly consider the main mechanism that explains why the cost of living of a family is less than the sum of costs of living of its members taken independently – namely, the presence of economies of scale. The central question I address is the following: to what extent are economies of scale compatible with the (IB) assumption – i.e., ultimately, with the income-based equivalence scale approach? The answer is quite clear-cut: they are incompatible in general. An explicit model of household behaviour that involves economies of scale cannot typically satisfy the (IB) assumption. Obviously, this conclusion has to be qualified, since it is always possible to construct knife-edge situations where incompatibility does not obtain; but such situations are not robust.

2.1. *Economies of Scale: Formal Representation*

Consider an I-person household in an n-commodities economy; I denote by x_j^i the consumption of commodity j by agent i, and by $x^i = (x_1^i, \ldots, x_N^i)$ the vector of i's private consumptions.[11]

Modelling economies of scale has become largely standard in the applied microliterature. The basic notion, which is directly related to Becker's (1965) notion of household production, is that of 'consumption technology', defined as the relationship between commodities consumed, x_j^i, and commodities purchased, z_j:

$$z_j = F_j\left(\sum_i x_j^i\right), \quad j = 1, \ldots, n. \tag{2}$$

Note that the notion of consumption technology is fully general and could be defined for singles as well. For instance, let good j be food, and suppose that if an individual buys a quantity of food z_j, then the total amount of food that the individual can actually consume (that is, get utility from) is $z_j - \delta_j$, where δ_j is waste in food preparation, spoilage etc. Then the individual's production function is:

$$z_j = x_j + \delta_j.$$

For practical purposes, however, it is convenient to normalise the production technologies so that it is the identity mapping for singles. In other words, in what

[11] The following presentation is directly borrowed from Browning *et al.* (2013).

follows x_j^i denotes the quantity of commodity j that i would purchase if single. The essential intuition, indeed, is that multi-person families have access to a more efficient technology – which is precisely the source of what we call 'economies of scale'. In particular, I will systematically adopt, as my reference household composition $\bar{a}$, a single person household. Also, note that this setting provides an explicit justification of Barten's price-based scales, as we now see.

In practice, the relationship is typically assumed to be affine (Gorman, 1976):

$$z = \Lambda \cdot \sum_i x^i - \Delta. \tag{3}$$

Moreover, empirical applications generally take Λ to be diagonal, as in Barten's initial model:[12]

$$z_j = \lambda_j \sum_i x_j^i - \delta_j. \tag{4}$$

For instance, coming back to our food preparation example, two individuals living apart would each waste an amount δ_j, so the total amount wasted would be $2\delta_j$. If, while living together, the total waste is only δ_j, then the economies of scale to food consumption from living together are a reduction in waste from $2\delta_j$ to δ_j, so the consumption technology function for food (under the normalisation mentioned above) takes the simple form $z_j = F_j(x) = \sum_i x_j^i - \delta_j$.

Economies of scale also arise directly from sharing. For example, let good j be automobile use, measured by distance travelled (or some consumed good that is proportional to distance, perhaps petrol). If x_j^a is the distance travelled by car by household member a, then the total distance the car travels is $z_j = \sum_a x_j^a/(1 + r)$, where r is the fraction of distance that the couple ride together. This yields a consumption technology function for automobile use of $z_j = F_j(x) = \sum_a x_j^a/(1 + r)$. This example is similar to the usual motivation for Barten (1964) scales but it is operationally more complicated, because Barten scales fail to distinguish the separate utility functions, and hence the separate consumptions, of the household members.

Following the now standard 'collective' approach to family behaviour,[13] I shall assume that the household makes Pareto-efficient decisions under the budget and production constraint. By the second welfare theorem, there exists a price vector π and a system of transfers ρ (usually called a 'sharing rule') such that household members each maximise their individual utility under an individual budget constraint of the form:

$$\sum_k \pi_k x_k^i = \rho^i,$$

where the ρ^is add up to household total income:

$$\sum_i \rho^i = y.$$

[12] The major restriction implied by this form is that it disregards the time spent on domestic production by family members. Few authors estimate models of domestic production with time spent on chores; a notable exception is Cherchye *et al.* (2012).

[13] See for instance Chiappori (1988, 1992) and Browning *et al.* (2015).

Note that, in general, the sharing rule $\rho = (\rho^1, \ldots, \rho^k)$ may depend on prices, total income and any other factor that influences the household decision process – such as the 'distribution factors' discussed below.

The crucial remark here is that the vector π of internal prices depends on the household's consumption technology; in that sense, it provides a theoretical justification of Barten's intuition. Moreover, one can explicitly describe the relationship between the coefficients of the production function, on the one hand, and Barten's matrix of price distortion on the other hand. Specifically, given the affine diagonal structure (4), I have that:

$$\pi_l = \frac{\lambda_l p_l}{1 + \frac{1}{y}\sum_j \delta_j p_j}. \tag{5}$$

For one thing, this result confirms Barten's main intuition: the impact of the more efficient technology operates through changes in prices. Moreover, the mechanisms generating these changes are fully explicit: those commodities for which economies of scales are larger (implying a smaller λ or a larger δ) become *de facto* cheaper.

2.2. *A Simple Counterexample*

I now proceed to show that, in general, the economies of scale just described are not compatible with the assumptions required by the income-based notion of equivalence scales – and specifically with the independence of base (IB) assumption.

That such a result can only hold 'in general' – that is, that one can construct specific examples in which economies of scale are compatible with (IB) – is easy to see. Assume, for instance, that the consumption technologies are linear (so that all the δ coefficients are equal to zero) and that, in addition, the proportionality factors λ are identical for all commodities (although they may differ by family composition). Then being part of a multi-person family essentially deflates all prices by the same factor – which, given the homogeneity of indirect utilities, is indeed equivalent to inflating income in the same proportion.

A slightly more sophisticated example is provided by Cobb–Douglas preferences. Assume the indirect utility of individual i is:

$$v^i(p, y) = \ln y - \sum_j \alpha_j^i \ln p_j,$$

generating a demand of the form:

$$x_j^i(p, y) = \frac{\alpha_j^i}{p_j} y.$$

Assume, moreover, that the coefficients α_j^i are identical across individuals:

$$\alpha_j^i = \alpha_j, \quad j = 1, \ldots, n, \; i = 1, \ldots, I,$$

and that the production function is linear in x:

$$\delta_1 = \ldots = \delta_n = 0,$$

implying that:

$$\pi_j = \lambda_j p_j, \quad j = 1, \ldots, n.$$

Consider now the same individuals within a multi-person family, facing prices π and a sharing rule $\rho(p, y)$; therefore individual demands for good j are:

$$x_j^i = \frac{\alpha_j}{\pi_j} \rho^i = \frac{\alpha_j}{\lambda_j p_j} \rho^i,$$

generating the aggregate demand:

$$X_j = \sum_i x_j^i = \frac{\alpha_j}{\lambda_j p_j} \left(\sum_i \rho^i \right) = \frac{\alpha_j}{\lambda_j p_j} y.$$

It follows that the household can be also represented by Cobb–Douglas preferences, with an indirect utility of the form:

$$V(p, y) = \ln y - \sum_j \alpha_j \ln \lambda_j - \sum_j \alpha_j \ln p_j.$$

Define $\phi(p)$ by:

$$\ln \phi(p) = \sum_j \alpha_j \ln \lambda_j,$$

then

$$\begin{aligned} V(p, y) &= \ln y - \ln \phi(p) - \sum_j \alpha_j \ln p_j \\ &= \ln \frac{y}{\phi(p)} - \sum_j \alpha_j \ln p_j, \end{aligned}$$

and (IB) is satisfied.

In summary, one can construct an example in which (IB) is compatible with economies of scale. But the example we have constructed requires very specific assumptions: preferences must be homothetic (and of course identical across agents), while the consumption technology must be purely linear.[14]

2.3. *General Result*

I then proceed to show that the previous example is much less peculiar than it seems. I start with a formal definition.

DEFINITION 1. *A set $(v^1, \ldots, v^I)$ of identical indirect utilities, together with a sharing rule $\rho = (\rho^1, \ldots, \rho^I)$ (satisfying $\sum_i \rho^i = y$) and an affine production function $z_j = \lambda_j \sum_i x_j^i - \delta_j$, $j = 1, \ldots, n$, satisfies the (IB) property if:*

(*i*) *the aggregate demand $X(\pi, y) = \sum_i x^i(\pi, \rho^i)$, where x^i is the Marshallian demand associated with v^i and the vector π is given by* (5), *is the Marshallian demand associated to some indirect utility $V(\pi, y)$*

[14] See for instance Lewbel (1991) and Pendakur (1994).

(*ii*) *there exists a function* ϕ *such that*:

$$V(\pi, y) = v^i\left[p, \frac{y}{\phi(p)}\right] \text{ for all } i. \tag{6}$$

The main result is the following:

THEOREM 1. *Assume that the (IB) property is satisfied for an open set of parameters* $(\lambda_1, \delta_1, \ldots, \lambda_k, \delta_k)$. *Then:*

(*i*) *the consumption technology must be linear*:

$$\delta_1 = \ldots = \delta_n = 0$$

(*ii*) *individual preferences must be homothetic*; *equivalently*, *individual demands must be linear in income.*

Proof. See Appendix.

Theorem 1 states that, except for cases similar to our example (i.e. involving linear technologies and homothetic preferences), while the (IB) property may be satisfied for particular values of the technology parameters, one cannot find an open set of parameters, however small, in which it is always satisfied. For instance, I have shown that the (IB) property obtains when all δs are zero and all λs are identical; but, even if I maintain the assumption that the δs are all nil, this property crucially depends on the knife-edge equality between all λs – unless preferences are homothetic, in which case it holds in general.

The general message of this result is straightforward. On the one hand, (IB) requires that the impact of family composition goes through a pure income effect: families essentially behave as singles, only with a different income. On the other hand, any realistic representation of the phenomenon at stake generates an intuitively natural result, namely that economies of scale distort the price vector; *de facto*, larger economies of scale for some commodities translate, for these commodities, into cheaper shadow prices within the household. In general, therefore, economies of scale are not equivalent to a pure income effect, as postulated by the (IB) assumption. The only exception occurs for homothetic preferences, because they imply strong links between price and income effects. Even then, however, the consumption technology must have a very specific form; for instance, it must be linear instead of affine.

Lastly, it should be noted that the previous argument assumes that all commodities are privately consumed; in that sense, the argument is in line with Browning *et al.* (2013). Alternatively, one could consider the presence of commodities that are publicly consumed within the household. While the precise structure of the model would be quite different, the main intuition remains valid. Indeed, a standard result in public economics states that in the presence of public consumption, any Pareto efficient allocation can be decentralised using individual (Lindahl) prices for the public goods. Again, an implication is that the household consumption technology changes the price vector faced by individuals within the household. Public goods become cheaper, since each individual internally faces a personal price that is smaller than the market price (efficiency actually requires that individual prices add up to the market price, the standard Bowen–Lindahl–Samuelson condition); the main difference with domestic production is that, in addition, personal prices are now endogenous and depend on market prices, income, individual preferences and (in general) intra-household allocation. Again, one cannot expect the resulting impact to be equivalent to a pure income effect, except for highly specific preferences.

3. Indifference Scales

In this Section, I describe a new concept, indifference scales, recently introduced by Browning *et al.* (2013) and developed by Dunbar *et al.* (2013, 2014). I shall argue that this notion provides a credible alternative to equivalence scales that avoids most of the weaknesses of the standard approach while remaining empirically implementable. Indifference scales do not rely on household utilities, only on individual preferences; they require neither identical preferences nor interpersonal comparisons of utilities; and they emphasise issues related to intra-household allocation and inequality.

3.1. *Indifference Scales*: *Definition*

3.1.1. *The basic question*

The notion of indifference scales relies on a simple intuition, namely that any conceptual approach based on interpersonal comparisons of welfare is unavoidably flawed, not only because this notion is theoretically disputable but, more importantly, because it raises insurmountable empirical challenges, reflecting the fact that only ordinal representations of preferences can be empirically recovered. It is therefore natural to submit that the question that lies at the core of the equivalence scale approach:

> How much income would an individual living alone need to attain the same utility *level* as a family of given composition?

should be replaced with the following:

> How much income would an individual living alone need to attain the same *indifference curve* over goods that this individual attains as a member of a family of given composition?

A few remarks are in order. First, the new phrasing does not require any notion of family utility; the basic operational concept is that of individual well-being. Second, it does not require utility comparison across individuals – it only compares welfare of the same individual in different situations. Third, it is a purely ordinal concept: what is required is that the individual reaches the same indifference curve, whatever cardinal utility is attached to it. In particular, it can in principle be identified from observed behaviour. Fourth, I do not assume identical preferences across individuals; in principle, the approach is compatible with any type of heterogeneity between agents. Lastly, the notion is fully compatible with the existence of intra-family inequality, since it does not assume that the allocation of resources within the household is optimal in any normative sense. In a nutshell, while specific assumptions are still needed (e.g. that an individual's indifference curves are not altered by family composition) and while some normative problems remain unsolved (for instance, how should altruism be introduced? how should we take into account the direct utility generated by cohabitation, in addition to any consumption?), the new notion avoids most of the weaknesses of more traditional approaches.

3.1.2. *Formal definition*

A formal translation is the following. Consider an *I*-person household consuming n private goods; let u^i and x^i_j respectively denote i's utility and consumption of commodity j, while y is the household's total income. The consumption technology is as described above; in particular, and for expositional sake, I assume it takes the affine form (4), although this assumption could readily be relaxed.

Maintaining the collective approach, assume that the household's decision process, whatever its exact nature, always generates Pareto-efficient outcomes. As before, the second welfare theorem implies that any efficient allocation $x = (x^1_1, \ldots, x^I_n)$ can be decentralised as an equilibrium. That is, there exists a price vector π and a sharing rule $\rho = (\rho^1, \ldots, \rho^I)$, with $\sum_i \rho^I = y$, such that x^i solves:

$$\max u^i(x^i) \text{ s.t. } \sum_j \pi_j x^i_j = \rho^i.$$

In practice, the ith component ρ^i of the sharing rule denotes the total value of i's consumption bundle, evaluated at the internal prices π. Note that the sharing rule fully summarises intra-household inequality. Also, the sharing rule typically depends on prices, income and all factors that may influence the household's decision process ('distribution factors', see Browning *et al.*, 2015). And, as before, the affine nature of the consumption technology implies that internal prices are independent of preferences; indeed, we have that:

$$\pi_j = \frac{\lambda_j p_j}{1 + \frac{1}{y}\sum_j \delta_j p_j}, j = 1, \ldots, n.$$

As discussed above, internal prices are typically not colinear to market prices: economies of scale distort relative prices. A comparison of an individual's welfare within and outside the household must account for the change in prices induced by the superior consumption technology available within the household (which is precisely the source of economies of scale). In particular, if an individual reaches the same indifference curve in both situations, it will typically be for two different consumption bundles.

I can now define individual i's 'indifference scale':

$$S^i = \frac{e^i(p, v^i(\pi, \rho^i))}{y},$$

where v^i is i's indirect utility. In words, $v^i(\pi, \rho^i)$ is the utility reached by i within the household, where he faces prices π and receives a monetary endowment ρ^i. Then $y^i = e^i[p, v^i(\pi, \rho^i)]$ is the income i would need if single (i.e., facing prices p) to achieve the same level of utility; and S^i is the ratio of y^i to the income of the family. Note that, as required, the price variation generated by the family's consumption technology plays a central role in this definition; actually, the mechanism through which economies of scale affect the indifference scales is precisely the change in internal prices they generate. Also, i's indifference scale depends on i's preferences but also on the consumption technology and on the intra-household allocation process, as summarised by the sharing rule – in sharp contrast with standard, price-based equivalence scales.

3.2. *Identification*

While the theoretical soundness of the notion of indifference scale is clear, its empirical relevance seems less obvious. The main difficulty concerns identification: to what extent is it possible to recover indifference scales from the sole observation of household behaviour?

A first answer was given by Browning *et al.* (2013, from now on BCL). The main identifying assumption is that individual preferences are the same whether single or married; thus, regarding individuals' indifference curves over goods (and therefore the resulting consumption patterns), the only impact of living in a family is the implied access to a more efficient consumption technology and a specific allocation process.[15] Under this assumption, BCL show that, in general, the sole observation of singles' and families' (aggregate) consumption patterns is sufficient to identify, for each possible family composition, both the consumption technology and the sharing rule.

In practice, however, the BCL approach is quite demanding in terms of data. First, it requires price variations; indeed, the technology is (in part) identified by analysing the differential impact of identical price variations on the consumption patterns of families with different compositions. Second, and more damaging, BCL requires observing similar individuals both as single and as part of a multi-person family. This requirement may be problematic for adults (in particular in case of selection into marriage), but particularly for children, since one cannot observe consumption patterns of single children.

An important progress is due to Dunbar *et al.* (2013) from now on DLP), who consider a different set of identifying assumptions. First, the restrictions they impose on preferences are milder; technically, it must be the case that the budget semi-elasticities of shadow budget shares do not depend on the number of children (although they may be affected by the presence of children). Second, they assume scale independence for the sharing rule; that is, the fraction of expenditures going to each member is independent of income, at least for low income levels. And third, there must an assignable good observed for each person in the household (that is, for each individual, the econometrician must be able to observe the person's consumption of at least one good). Under these assumptions, DLP show identification of economies of scale and the sharing rule. A particularly remarkable aspect is that this result does not

[15] This does not preclude utility from being directly affected by family composition; but such an impact is assumed to change utility levels, not the shape of indifference curves. Technically, i's utility within a family of composition a can be any function $F(u^i, a)$ where u^i is i's utility when single and F is strictly increasing in its first argument.

require price variations; identification obtains from the sole comparison of Engel curves for families of different compositions. Finally, a recent contribution by Dunbar *et al.* (2014) further extends the result by showing that if a distribution factor is observed, then one can relax the restriction on preferences.

The importance of these findings is easy to illustrate. Table 1 comes from DLP; it describes the outcome of an empirical analysis of indifference scales in Malawi. For families of different compositions (here, different number of children), the first four columns describe the sharing rule, while the last two columns compute implied poverty rates. Specifically, the last column gives poverty rates as computed using a standard, equivalence scales approach (which implicitly assumes equal allocation of resources within the household); this should be contrasted with the previous column, in which poverty rates are computed at the individual level, using estimated sharing rules. What comes out quite clearly from these results is that intra-family inequality is a major phenomenon and benefits mostly men, while women and especially children are net losers. In particular, the standard approach considerably underestimates poverty for children, while overestimating poverty for adults (particularly men).

Table 1
Estimated Resource Shares and Poverty Rates

	Mean	SD	Min	Max	Pov. rate *Unequal*	Pov. rate *Equal*
One child						0.850
Man	0.463	0.087	0.245	0.762	0.686	
Woman	0.402	0.071	0.168	0.587	0.766	
Children	0.135	0.047	0.008	0.260	0.954	
Each child	0.135	0.047	0.008	0.260		
Two children						0.916
Man	0.516	0.078	0.282	0.786	0.547	
Woman	0.273	0.063	0.075	0.475	0.885	
Children	0.211	0.044	0.059	0.326	0.970	
Each child	0.105	0.022	0.029	0.163		
Three children						0.948
Man	0.521	0.081	0.219	0.795	0.522	
Woman	0.244	0.065	0.002	0.512	0.889	
Children	0.236	0.042	0.112	0.374	0.996	
Each child	0.079	0.014	0.037	0.125		
Four children						0.972
Man	0.441	0.080	0.170	0.701	0.538	
Woman	0.267	0.066	0.043	0.532	0.838	
Children	0.293	0.037	0.178	0.402	0.989	
Each child	0.073	0.009	0.044	0.101		
All households						0.913
Man	0.489	0.088	0.170	0.795	0.582	
Woman	0.304	0.093	0.002	0.587	0.842	
Children	0.207	0.070	0.008	0.402	0.974	
Each child	0.103	0.038	0.008	0.260		
All persons						0.924
All	0.235	0.177	0.008	0.795	0.855	

Source. Dunbar *et al.* (2013).

4. Conclusion

The conclusion of my analysis can easily be summarised. Equivalence scales are at best severely biased, at worst totally misleading. Approaches based on income effects – which constitute in practice the main reference of policy-related empirical works – are structurally incompatible with the very notions (economies of scale) they try to capture. Moreover, the empirical estimation of both price and income-based equivalence scales is largely arbitrary, not only because of the counterfactual assumptions on which they rely but also and more deeply due to their reliance on interpersonal comparison of utility levels. Lastly, their normative implications are deceptive, if not plain nefarious, in particular, because they totally disregard issues linked with intra-family allocation and inequality.

Given this pessimistic diagnosis, it may seem awkward that equivalence scales keep being widely used. Until recently, the main justification for this persistence was the absence of any credible alternative. As I argued in the Introduction, comparing incomes across households with different composition is an indispensable prerequisite for the design of many policies; an inadequate tool may still be better than no tool at all. However, this situation is currently being reversed. The concept of indifference scales avoids practically all the weaknesses of the standard approach, while allowing the introduction of intra-family inequality concerns into the analysis. Moreover, it can be taken to data in a relatively simple way; and the identifying assumptions it requires, while certainly strong, are much milder than what equivalence scales need for a much inferior outcome. Given these asymmetries, elementary cost–benefit analysis strongly suggests that the time has come for a change in paradigm.

Appendix A

Proof of Theorem 2. Start from condition (6) and fix p. The following conditions should be satisfied for all y and all (λ, δ) in some open set:

$$v\left(\frac{y}{\phi(\lambda_1,\ldots,\lambda_n,\delta_1,\ldots,\delta_n)}\right) = V\left(\frac{y\lambda_1 p_1}{y+\sum_j \delta_j p_j},\ldots,\frac{y\lambda_n p_n}{y+\sum_j \delta_j p_j}, y\right). \quad \text{(A1)}$$

Note first that the right-hand side depends on the δs only through the sum $\delta = \sum_j \delta_j p_j$. This should also be the case for the left hand side, hence for ϕ:

$$\phi(\lambda_1,\ldots,\lambda_n,\delta_1,\ldots,\delta_n) = \bar{\phi}(\lambda_1,\ldots,\lambda_n,\delta).$$

Next, note that since V is zero-homogeneous, (A1) is equivalent to:

$$v\left(\frac{y}{\bar{\phi}(\lambda_1,\ldots,\lambda_n,\delta)}\right) = V(\lambda_1 p_1,\ldots,\lambda_n p_n, y+\delta). \quad \text{(A2)}$$

For $y = 0$, this gives that:

$$v(0) = V(\lambda_1 p_1,\ldots,\lambda_n p_n,\delta), \quad \text{(A3)}$$

which is impossible unless $\delta = 0$, since otherwise it would imply that the indirect utility is constant for an open set of prices and income (just vary the λs and δs while maintaining p constant). Since the conclusion $\delta = 0$ does not depend on the particular p initially chosen, it must be the case that $\sum_j \delta_j p_j = 0$ for all p, implying that $\delta_1 = \ldots = \delta_n = 0$.

Finally, I am left with:

$$v\left(\frac{y}{\bar{\phi}(\lambda_1, \ldots, \lambda_n)}\right) = V(\lambda_1 p_1, \ldots, \lambda_n p_n, y), \tag{A4}$$

or equivalently:

$$v\left(\frac{y}{\bar{\phi}(q_1/p_1, \ldots, q_n/p_n)}\right) = V(q_1, \ldots, q_n, y), \tag{A5}$$

where $q_i = \lambda_i p_i$, $i = 1, \ldots, n$.

Still keeping p fixed, change the λs; then this changes the vector $\boldsymbol{q} = (q_1, \ldots, q_n)$. Considering $\boldsymbol{q}$ as a price vector, (A5) implies that V is an increasing transform v of the ratio $y/\bar{\phi}$, where $\bar{\phi}$ is a function of prices q only. It follows that V is, up to an increasing transform, linear in y for all prices, hence that the corresponding direct utility is homothetic.

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