



PROJECT REPORT

GROUP 1

Course of Spacecraft Attitude Dynamics

MSc in Space Engineering

Authors:

<i>Maccari Fabrizio</i>	10659231
<i>Menessini Matteo</i>	10689175
<i>Pecchioli Roberta</i>	10640484
<i>Pilo Elena</i>	10761604

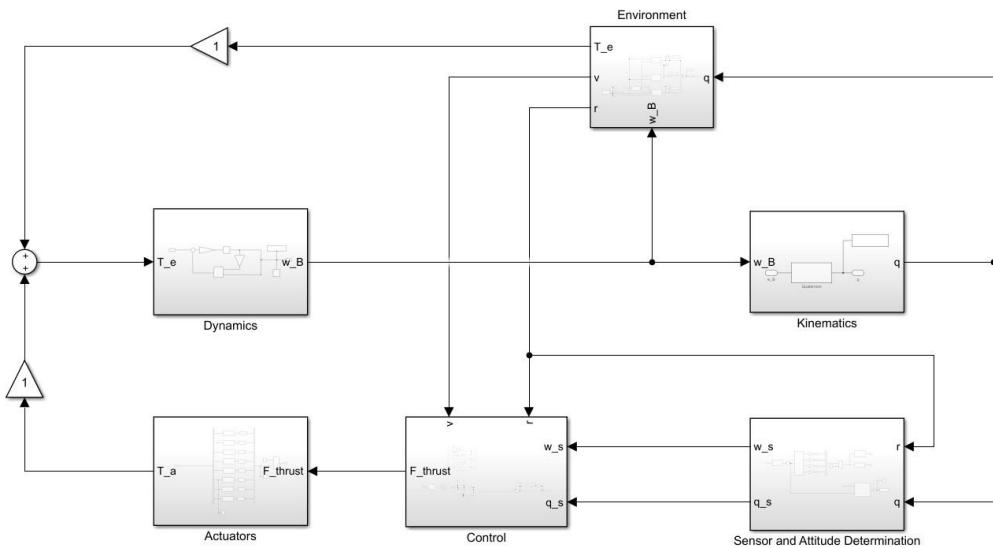
Index

1.	Introduction and Modelling	2
1.1	Block Diagram.....	2
1.2	Model description.....	2
1.3	Orbit and Pointing requirements.....	3
1.4	Satellite Description	4
1.5	Environment	5
2.	Attitude Determination	7
2.1	Sensor choice and Data.....	8
2.2	Measurement Simulation.....	8
2.3	Attitude Determination Algorithm	9
2.4	Attitude Determination Error.....	10
3.	Control	11
3.1	Actuators.....	11
3.2	Control Equation.....	12
3.3	Detumbling	12
3.4	Slew Manoeuvre	13
3.5	Pointing.....	14
	References.....	18

1. Introduction and Modelling

The aim of the following report is to present the results obtained from the simulation of an attitude dynamics control system, applied to the case of a minisat on a circular orbit around Earth. The implementation of a closed loop system allows the control of the attitude of the spacecraft in time, under the effect of environmental perturbations of different nature.

1.1 Block Diagram



1.2 Model description

The control system is composed by a series of blocks connected to model a closed loop, whose goal is to control the attitude in order to have it equal to the target one, with an acceptable value of uncertainties.

The block of Dynamics is describing an analytical model to get the evolution in time of the angular velocity of the spacecraft, obtained through the integration over time of the Euler Equations. The output of the dynamics is the angular velocity of the spacecraft. The reference frame used to describe the angular velocity is the body frame, with axes aligned with the principal inertia axes of the minisat.

The output of the Dynamic provides the input of the Kinematics block. This one is built in order to understand the attitude of the spacecraft with mathematical model. Quaternions are used as attitude parameters to describe the orientation of the spacecraft in space. This is still modelled working in body frame.

The evolution of angular velocities goes as input, together with the quaternions estimated by the Kinematics, into the block that models the environment. In the environment block the perturbations acting on the dynamic of the spacecraft are evaluated. This allows to determine by comparison which perturbation has important effects on the attitude and dynamic of the spacecraft. The main perturbations give as result the final torque that acts on the dynamic. This is the reason why the disturbing torques go as input to the integration of the Euler equations, together with the control torque. Moreover, in the Environment block, the propagation of the orbit of the satellite is performed.

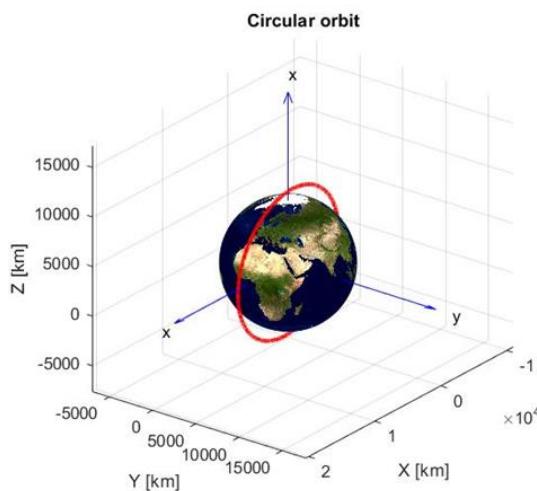
Merging of the Kinematics and the Dynamics provides the transfer function of the system, whose inputs are the torques and whose output is the orientation of the spacecraft, described in terms of quaternions.

To build a control system, the actual attitude of the satellite has to be known, starting from real measurements. A star sensor is used to get the real orientation of the spacecraft, starting from the direction of the stars in its field of view. These measurements are used to compute the attitude of the satellite. The difference between the measured attitude and the target one is used to determine the amount of torque needed. Both the measurement and attitude determination processes are modelled in the Sensor and Attitude Determination block.

The process of attitude determination is followed by the implementation of the control logic. For the first two phases of the mission, detumbling and slew manoeuvre, a proportional derivative control is exploited. For the last phase of target tracking, a Pulse Width Pulse Frequency Modulator is also implemented. The Control block gives as output the force needed to allow the spacecraft to reach the reference pointing.

The control torque needed, evaluated by the control, is provided by the actuators. In the model, actuators are represented by variable thrust jets. In the Actuators block, the way thrusters generate the torque needed is represented. The output of this block is the control torque to be added to the dynamics, together with the environmental torques.

1.3 Orbit and Pointing requirements



The minisat analysed is placed on a circular orbit with an altitude of 1200 km and a period of 6556 s. The orbit is nearly polar since the inclination is equal to 86.4° . The minisat is a telecommunication satellite meant to provide internet coverage on Earth. Therefore, Nadir pointing is needed.

The required pointing is reached once the x axis of the body frame is aligned with the opposite to the radial direction \hat{r} (line connecting the centre of Earth and the position of the

spacecraft). The y axis of the body frame needs to be aligned with the direction opposite to the velocity of the spacecraft. The z axis completes the righten side triad.

The pointing error is the parameter that measures the accuracy of the actual pointing of the spacecraft with respect to the target one. It is measured in terms of angular distance between the actual pointing of the spacecraft (x direction of the body frame) and the reference pointing. It is defined as:

$$PE = \cos^{-1} \left(\frac{-\hat{r} \cdot x}{\|\hat{r} \cdot x\|} \right) \quad (1.3.1)$$

The maximum admissible pointing error is set as equal to 1° .

When the spacecraft reaches the desired pointing, its angular velocity needs to be aligned with the z axis of the body frame and equal to $\omega_0 = [0 \ 0 \ n]^T \frac{rad}{s}$, where $n = \frac{2\pi}{T}$ and T is the period of the orbit.

1.4 Satellite Description

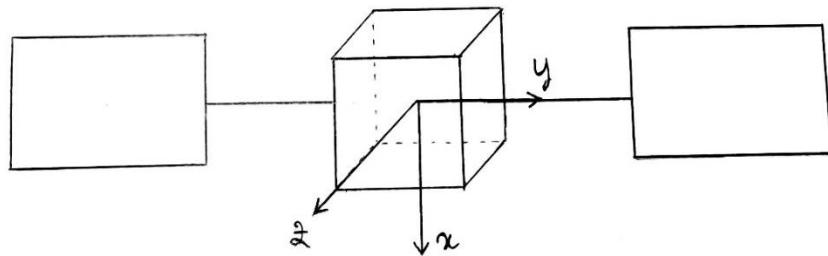
The satellite is a minisat of 150 kg, with two deployable solar panels. The main body is a cuboid of 1 m x 1 m x 1 m; the solar panels are 1.5 m x 1 m x 0.02 m, with their centre of mass at 2.5 m from the centre of mass of the main body. The weight of the solar panels is computed considering a density of $3.8 \frac{kg}{m^2}$.

The inertia matrix is then obtained for the stowed and the deployed configuration:

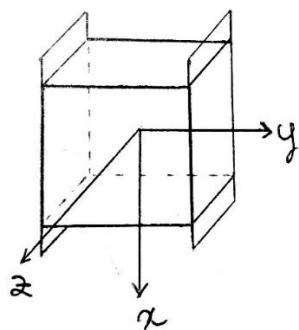
- Stowed:
$$J = \begin{bmatrix} 25.0395 & 0 & 0 \\ 0 & 25.0285 & 0 \\ 0 & 0 & 25.0554 \end{bmatrix} kg \cdot m^2$$

- Deployed:
$$J = \begin{bmatrix} 26.4488 & 0 & 0 \\ 0 & 25.0000 & 0 \\ 0 & 0 & 26.4678 \end{bmatrix} kg \cdot m^2$$

Deployed Configuration:



Stowed Configuration:



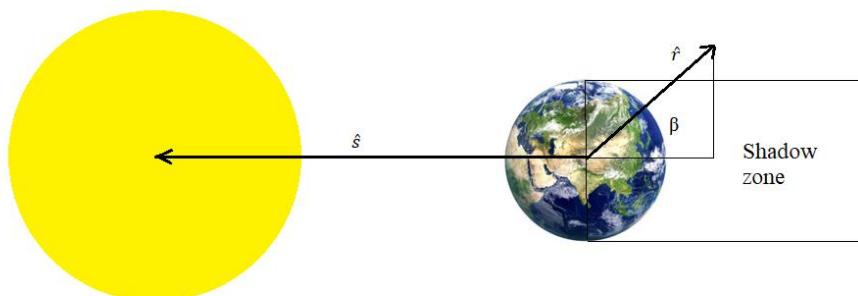
1.5 Environment

The main environmental disturbances to analyse are magnetic torque, gravity gradient, solar radiation pressure and drag. Nevertheless, being the orbit at an altitude of 1200 km, the perturbing effect related to drag is negligible.

For the magnetic disturbance, a simple dipole model is considered. In order to test the control in the worst-case scenario, the residual magnetic induction due to currents is set equal to

$$\mathbf{m} = [0.1 \ 0.1 \ 0.1]^T \text{ Am}^2$$

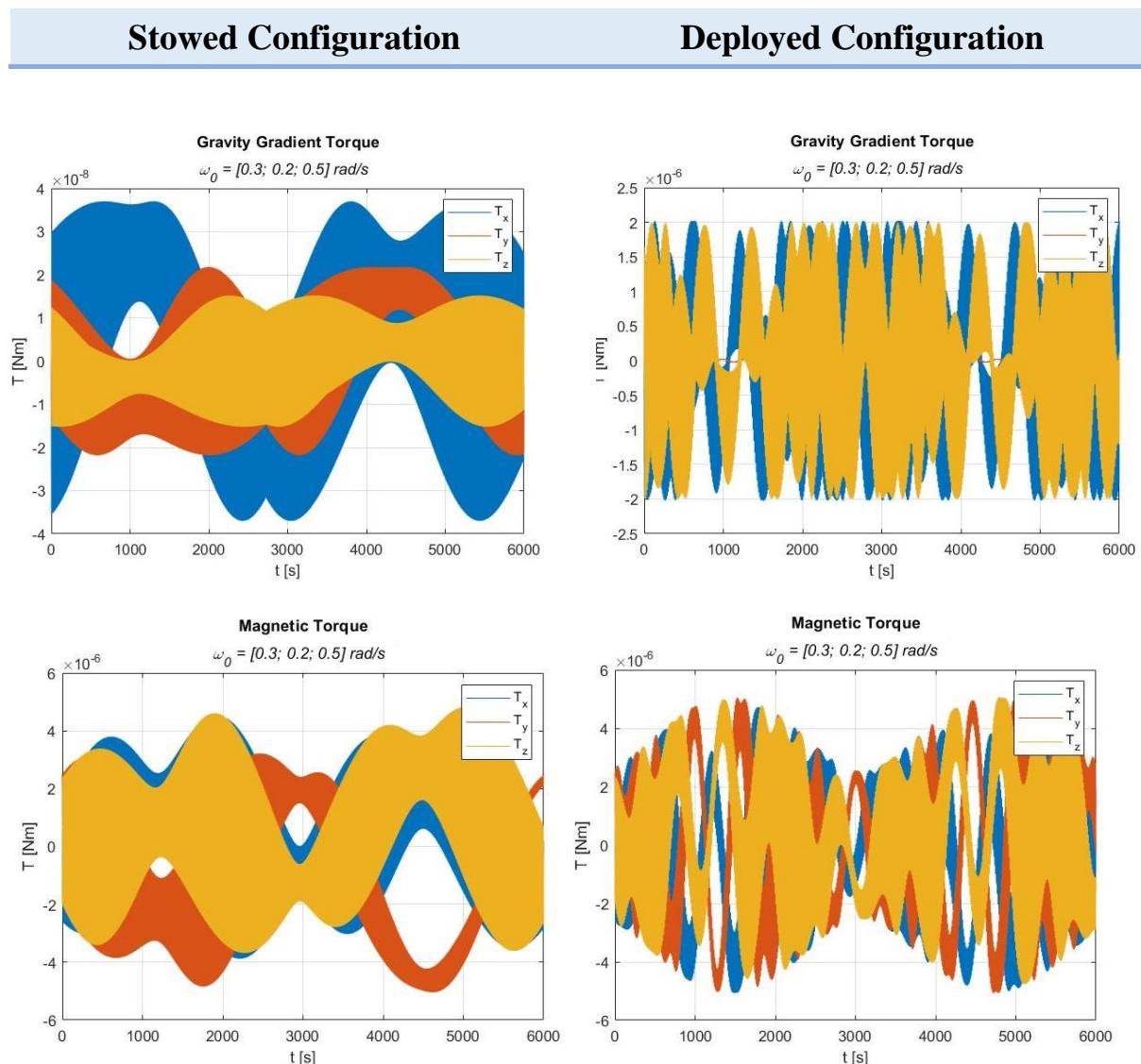
As far as it concerns the solar radiation pressure, a cylindrical model of the eclipse is also implemented:

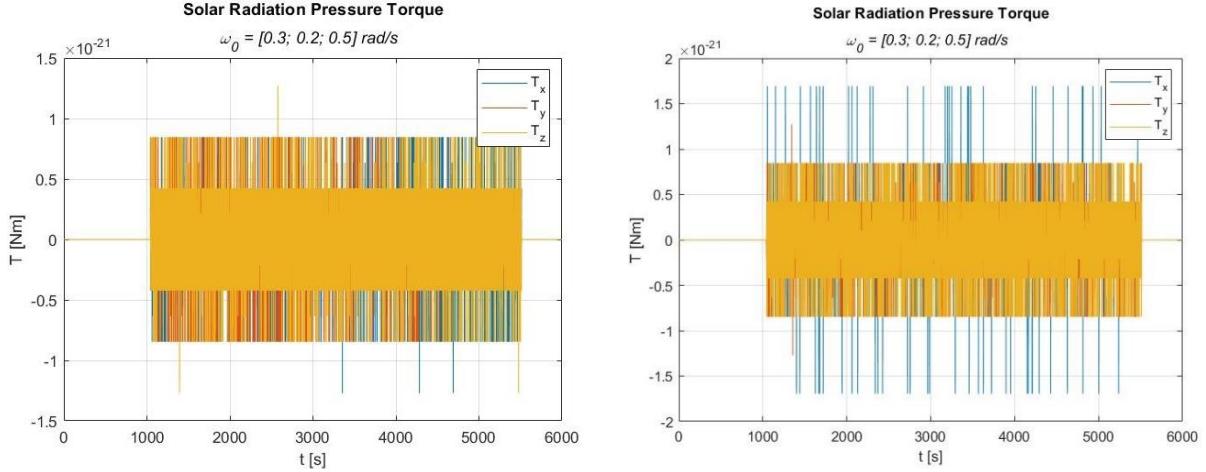


Given \hat{r} and \hat{s} , which are respectively the direction of the spacecraft and the Sun with respect to the Earth, it is possible to compute the angle β . Computing $r \cdot \sin \beta$ and comparing it with the Earth radius it's possible to state whether the satellite is in eclipse or not.

- If $r \cdot \sin \beta - R_E > 0$ the satellite is in sunlight
- If $r \cdot \sin \beta - R_E \leq 0$ the satellite is in eclipse

As a preliminary estimation of the order of magnitude of the three perturbations, the satellite attitude is evaluated as if no control acted on it, at an angular rate equal to the initial condition $\omega_0 = [0.3 \ 0.2 \ 0.5]^T \frac{\text{rad}}{\text{s}}$, in the stowed and deployed configurations.

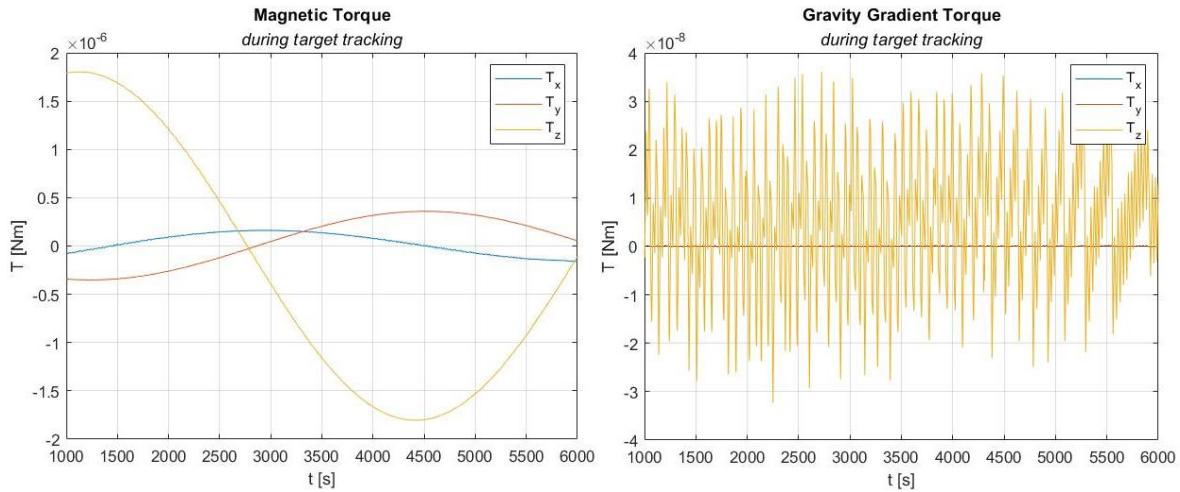




Being the order of magnitude of the SRP far lower than that of the other effects, its contribution to the disturbance torque is negligible. This is expected as the minisat is symmetric: the torques due to SRP tend to balance out.

An additional analysis is then performed with the control on, during the operational phase, i.e. the target tracking phase.

The nominal angular velocity will be $\omega_0 = [0 \ 0 \ n]^T \frac{rad}{s}$.



2. Attitude Determination

The attitude determination subsystem has the function to simulate the measurement and attitude computation processes done by the spacecraft's sensors and on-board computer. The outputs of the block are the attitude and angular velocity of the spacecraft, determined from the measurements coming from the sensor. All the subsystems modelling the behaviour of the on-board computer (Attitude Determination and Control) are built with discrete time blocks and signals, to correctly take into account the fact that the on-board computer works in discrete time, with a non-negligible step.

2.1 Sensor choice and Data

The mandatory sensor assigned is a star sensor, the only type of attitude sensor that is sufficient to provide the attitude by itself. Consequently, in this simplified model, the only attitude determination sensor on board is a star mapper. Looking at the characteristics of the satellite, the Extended NST [1] sensor has been chosen, with the following characteristics:

Uncertainty	6 arcsec (cross boresight), 40 arcsec (about boresight)
Mass	0.9 kg
Volume	25x10x10 cm
Peak Power	< 1.5 W
Field of view	10x12 deg

The sensor is mounted on the satellite in a way that in nominal pointing conditions is always pointing at the deep space, to be able to recognise the stars in its field of view. Consequently, the orientation matrix of the sensor's reference system, with respect to the body frame is:

$$A_{S2B} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

The sensor's reference system has the z axis along the boresight while x and y define a plane perpendicular to it. The columns of the matrix define the orientation of the sensor's axis in the body frame. The mounting of the sensor has been chosen to always point in the same direction in space during the target tracking phase.

2.2 Measurement Simulation

To simulate the measurements, 4 stars directions have been defined, belonging to the field of view of the sensor in the nominal nadir pointing conditions. To simulate the uncertainty in the measurements each star direction output of the sensor (b_{m_i}) is computed multiplying the real one (b_{r_i}) for an error matrix defined as:

$$A_\varepsilon = \begin{bmatrix} 1 & \alpha_z & -\alpha_y \\ -\alpha_z & 1 & -\alpha_x \\ \alpha_y & \alpha_x & 1 \end{bmatrix}$$

So:

$$\underline{v}_{m_i} = A_\varepsilon * v_{r_i} \quad (2.2.1)$$

The elements of the matrix identify the error measurement error along each axis of the sensor frame. To realistically simulate this error values a random white noise is not sufficient: the uncertainty comes from the slow drifting of the stars between the pixels in the field of view of the sensor. Consequently, the values have been modelled with a random white noise signal filtered with a low pass filter, making the time variations of the error values smoother, closer

to the real case. The maximum values of the error are set as the attitude knowledge errors from the datasheet of the sensor:

- $\alpha_x = 6 \text{ sec}$
- $\alpha_y = 6 \text{ sec}$
- $\alpha_z = 40 \text{ sec}$

The directions v_{m_i} and \underline{v}_{r_i} are expressed in the inertial frame.

2.3 Attitude Determination Algorithm

The algorithm chosen to determine the attitude is the algebraic method:

Defining the matrices containing all the measured star directions in the inertial (V) and body (S) reference frames as:

$$V = [\underline{v}_{m_1} | \underline{v}_{m_2} | \underline{v}_{m_3} | \underline{v}_{m_4}] \quad S = [s_1 | s_2 | \underline{s}_3 | \underline{s}_4]$$

The following relation is valid

$$S = A_{I2B}^S * V \quad (2.3.1)$$

with A_{I2B}^S that is the DCM matrix describing the orientation of the body frame with respect to the inertial frame, hence defining the attitude of the satellite. A_{I2B}^S can be determined inverting eq. (2.3.1):

$$A_{I2B}^S = S * V^* \quad (2.3.2)$$

With V^* that is the pseudo inverse of V . This matrix is directly computed in Matlab, using the function *pinv.m* since the stars considered to determine the attitude are always the same (and in the real case they are always in view during the target tracking phase). This assumption allows to reduce the computational cost of the steps if the Simulink simulation.

The computed DCM matrix defines the determined attitude of the satellite. It is then converted into the correspondent quaternion q_s choosing the best possible mapping to minimize the numerical error (therefore choosing the mapping associated to the maximum quaternion component).

The angular velocity is estimated too, from the computed DCM matrix, inverting the relation of the kinematics. In particular:

$$\dot{A}_{I2B}^S \cdot {A_{I2B}^S}^T = -[\omega^\wedge] \quad (2.3.3)$$

Where:

$$[-\omega^\wedge] = \begin{bmatrix} 0 & -\omega_w & \omega_v \\ \omega_w & 0 & -\omega_u \\ -\omega_v & \omega_u & 0 \end{bmatrix}$$

The estimated angular velocity vector in the body frame is assembled extracting the components from the matrix:

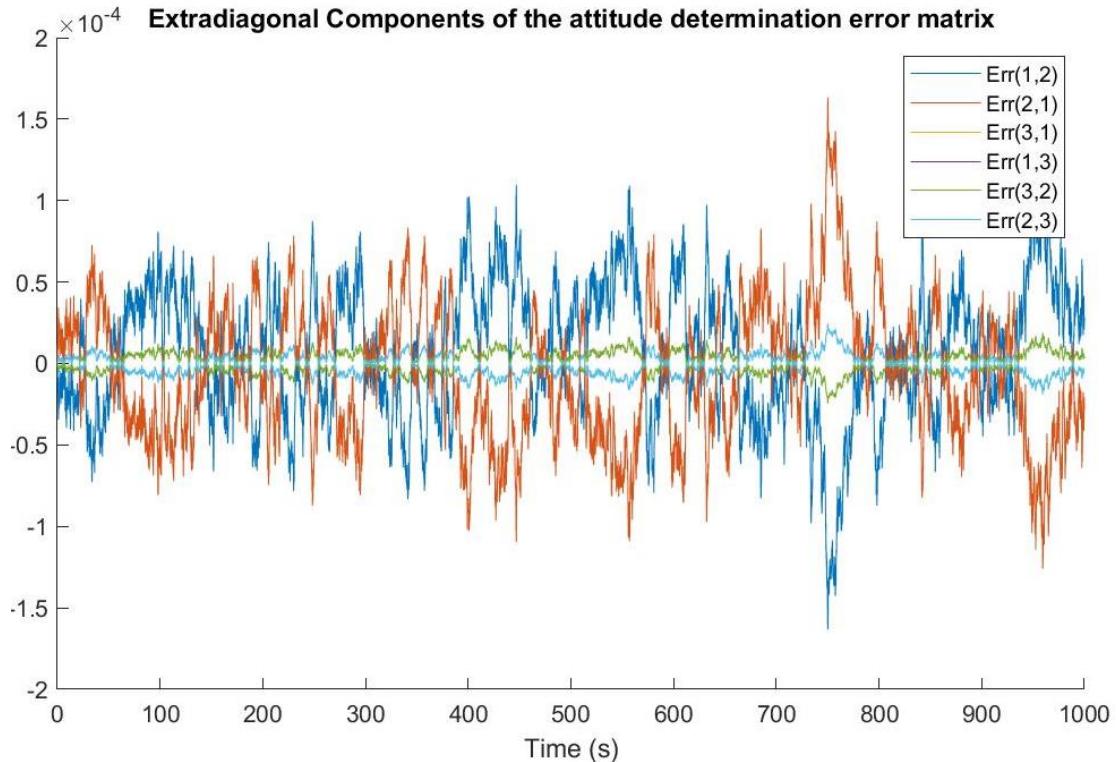
$$\omega_B^S = [\omega_u \quad \omega_v \quad \omega_w]^T.$$

2.4 Attitude Determination Error

The attitude determination error of the algorithm can be estimated in the model, comparing the estimated DCM matrix (A_{I2B}^S) with the real one (A_{I2B}) computed from the quaternion vector coming from the kinematics. In particular, in absence of error, the product of the two should be:

$$A_{I2B}^S * A_{I2B}^T = I.$$

Consequently, the extra diagonal terms of this product can be seen as a measure of how far the estimated attitude is from the real one. For example, with the simulation of the uncontrolled motion of the satellite over 1000 s, the extra diagonal terms have the following behaviour:



The plot shows peaks in the order of 10^{-4} , but in general the errors are even lower: the term with the highest errors is $Err_{1,2}$, which has as mean and standard deviation:

Component	Mean	Standard Deviation
$Err_{1,2}$	6.602e-06	4.111e-05

The low order of magnitude of the errors shows the accuracy of the attitude determination control system, that is enough to guarantee the required pointing performance. Thanks to the high accuracy of the star tracker, the choice of the algebraic method is justified, more sophisticated attitude determination algorithms are not necessary in this case.

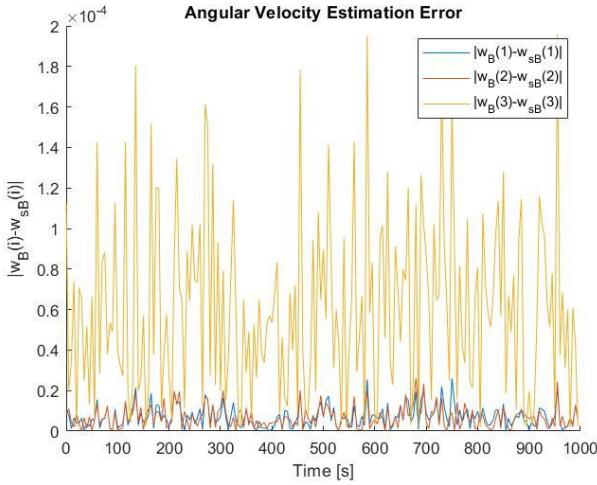
A similar procedure can be applied to validate the estimation of the angular velocity: in this case, the error parameter to be studied is the difference between the estimated (ω_B^S) and real (ω_B) angular velocity vectors:

$$\omega_{Err} = \omega_B - \omega_B^S$$

As an example, the evolution in time of the components of ω_{err} can be studied in the case of an uncontrolled rotation of the satellite around its major inertia axis, with two small perturbations in angular velocity around the other two. In this way the accuracy of the estimation can be studied for both relatively high and low rotational velocities. Therefore, setting as an example:

$$\omega_B = [1e - 4 \quad 1e - 4 \quad 0.3]^T$$

The error has the following behaviour:



As it can be seen from the graph, the error for each component is always at least one order of magnitude lower than the real value. The algorithm of estimation is therefore considered to be reliable.

3. Control

3.1 Actuators

The actuators chosen for the satellite are variable force thrusters with the following characteristics (plausible for the actuators of a minisat):

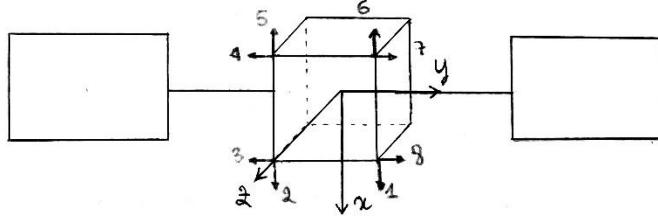
Maximum Thrust: F_{\max} [N]	Minimum Thrust: F_{\min} [N]	Minimum Impulse Bit [s]
0.65	0.16	0.2

The minimum impulse bit is modelled by forcing the thrusters to fire for a minimum of two-time steps (that is, keeping them on even if the control requires to turn them off after 0.1 seconds).

The configuration used is a typical 8-thruster configuration, with two thrusters positioned at each corner on one face of the main body of the S/C, resulting in the following configuration matrix:

$$R = \frac{L}{2} \cdot \begin{bmatrix} 0 & 0 & 1 & 1 & 0 & 0 & -1 & -1 \\ 1 & 1 & 0 & 0 & -1 & -1 & 0 & 0 \\ -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \end{bmatrix}$$

Where L is the length of the side of the (cubic) main body.



3.2 Control Equation

The goal of the control is to drive the quaternion and angular velocity given by the sensor to a commanded quaternion and angular velocity imposed by the mission requirements. This implies taking the quaternion error to the identity quaternion [0;0;0;1] and reducing to zero the angular velocity error (evaluated as the difference between the sensed and the commanded velocities).

Choosing a simple linear model, the control equation employed is:

$$M = -k_p \text{sign}(q_{4e}) q_e - k_d \omega_e \quad (3.2.1)$$

where q_e and q_{4e} are the vector and the scalar component of the quaternion error, respectively. The use of sign function introduces a positive feedback term whenever q_{4e} is negative, allowing faster convergence to the equilibrium point. The candidate Lyapunov function is taken as:

$$V = \frac{1}{2} \omega_e I \omega_e + \frac{1}{2} k_p (1 - q_{4e}^2) \quad (3.2.2)$$

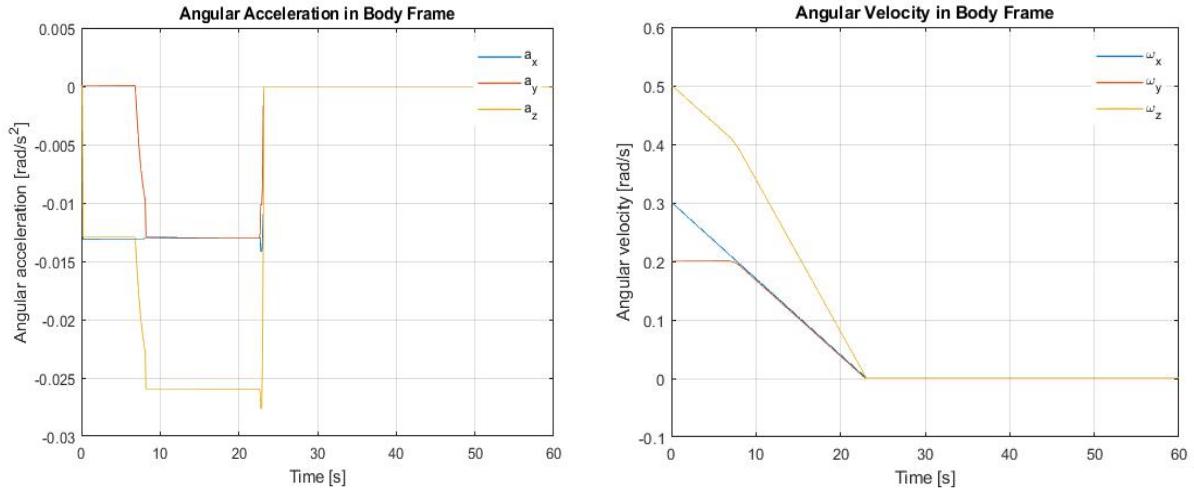
It can be easily shown that $V = 0$ for the equilibrium point (null angular velocity difference and identity quaternion) and that $\dot{V} < 0$, meaning that the system is stable.

The proportional and derivative gains are real, positive numbers to be computed for each phase of the mission.

3.3 Detumbling

In detumbling, the objective is to zero out the angular velocity of the spacecraft, with no requirements on the final attitude for pointing. The proportional gain can hence be set to zero, while the derivative term has to have an adequate value, determined in this case by trial and error. Therefore:

$$\begin{aligned} K_p &= 0 \\ K_d &= 100 \end{aligned}$$



This choice of parameters allows the spacecraft to detumble in less than 25 seconds, with contained angular velocities and tolerable angular accelerations.

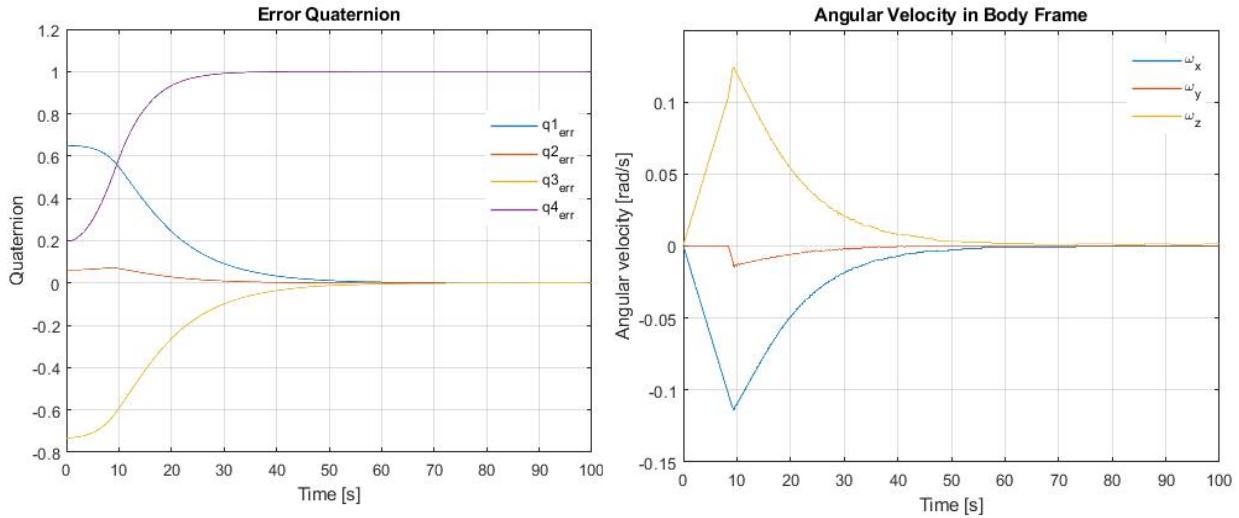
3.4 Slew Manoeuvre

The slew manoeuvre changes the pointing of the satellite, taking it from one pointing condition to the nominal one (or vice versa): both proportional and derivative terms need to be taken into account. A compromise is then to be found between angular acceleration and time required to perform the slew.

By trial and error, this is obtained by setting:

$$K_p = 50$$

$$K_d = 250$$



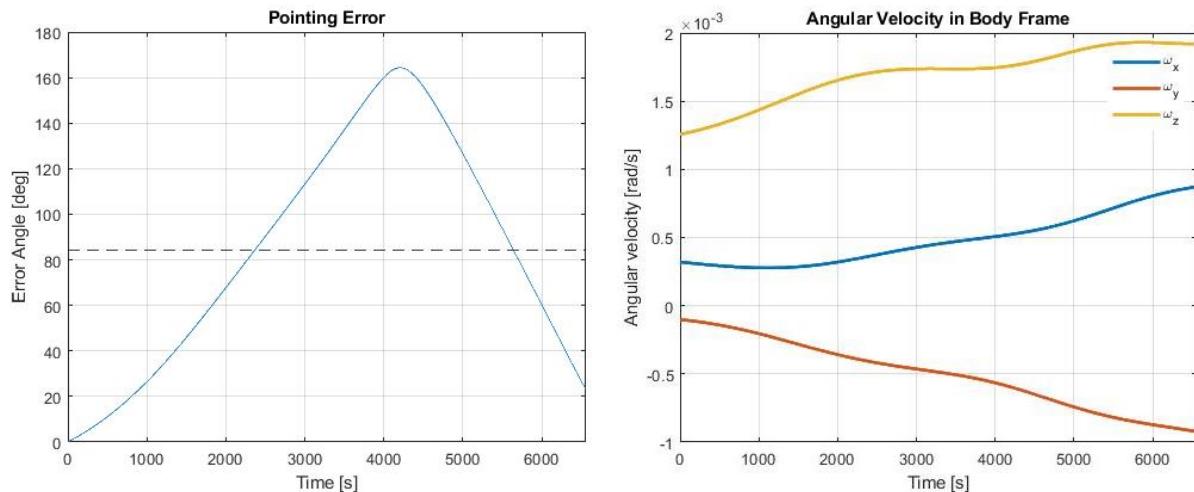
The slew manoeuvre performs the rotation in less than 60 seconds quite smoothly.

3.5 Pointing

The pointing phase of the mission requires the spacecraft to perform Earth-pointing, with a constant angular velocity and a given accuracy.

Starting by simulating the open-loop behaviour of the S/C for one orbital period:

	Mean [deg]	Variance [deg^2]
Pointing Error	84.11	2501



Since the minisat is not controlled, Earth pointing is not maintained as the S/C keeps about the same inertial pointing as the error increases and decreases during one orbit. The angular velocity starts diverging from the nominal values in slower way with respect to the pointing error.

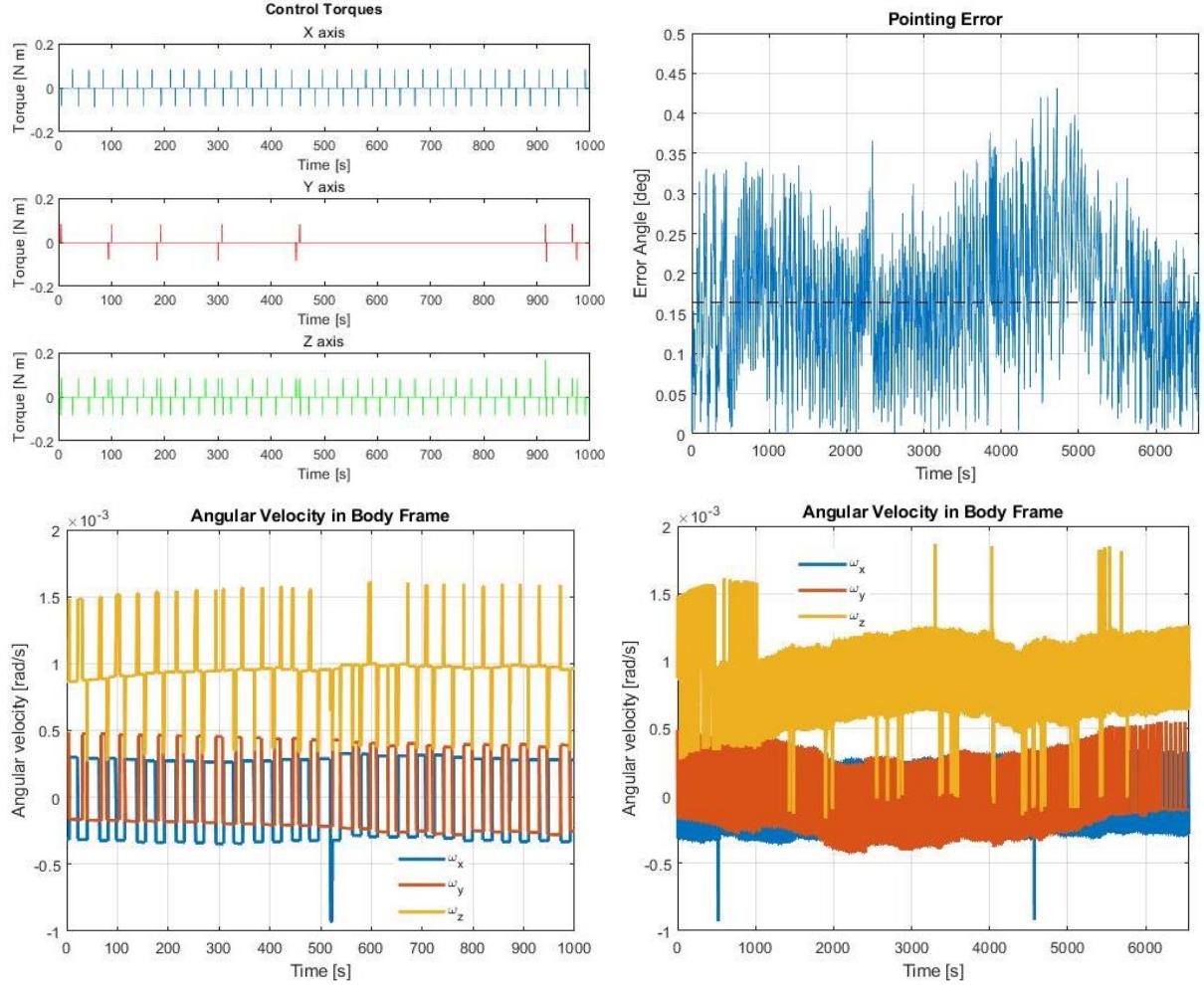
It is observed, however, that choosing a zero derivative gain would lead to rapid oscillations, as the control tries to achieve precise pointing while the angular velocity increases without control as the thrusters keep bringing energy into the system. Hence, by choosing as gains:

$$K_p = 40$$

$$K_d = 200$$

and simulating for one orbit (the angular velocity is represented both for one period and 1000 seconds to clearly show its behaviour):

	Mean [deg]	Variance [deg^2]
Pointing Error	0.1673	2.872e-3



The pointing obtained with these conditions is quite accurate, however the number of firings is rather high. Furthermore, as the thrusters are powerful enough, the torques generated are mostly due to the thrusters firing at minimum output F_{min} . Thusly, a Pulse Width Pulse Frequency modulator is implemented, using the actuators at the minimum thrust as non-variable thrust jets. This way, the number of firings can be contained, while keeping the error under the admissible value for pointing (set at 1° , in this case). The PWPF modulator consists of a negative feedback loop with a discrete-time filter and a Schmitt trigger (characterised by hysteresis and a dead band).

The PWPF is implemented on the three components of the commanded torque, with slightly different parameters on each axis for better flexibility.

The constant output command of the trigger is computed as:

$$u_{max} = F_{min} \cdot R \cdot vec(i)$$

where $vec(i)$ is an 8-element vector with 1s where the thruster acts with a positive torque around the considered axis and 0s everywhere else. Due to the configuration matrix chosen, this leads to a value on the z axis twice that of the x and y axes.

As the dead band of the Schmitt trigger is selected, two problems may incur:

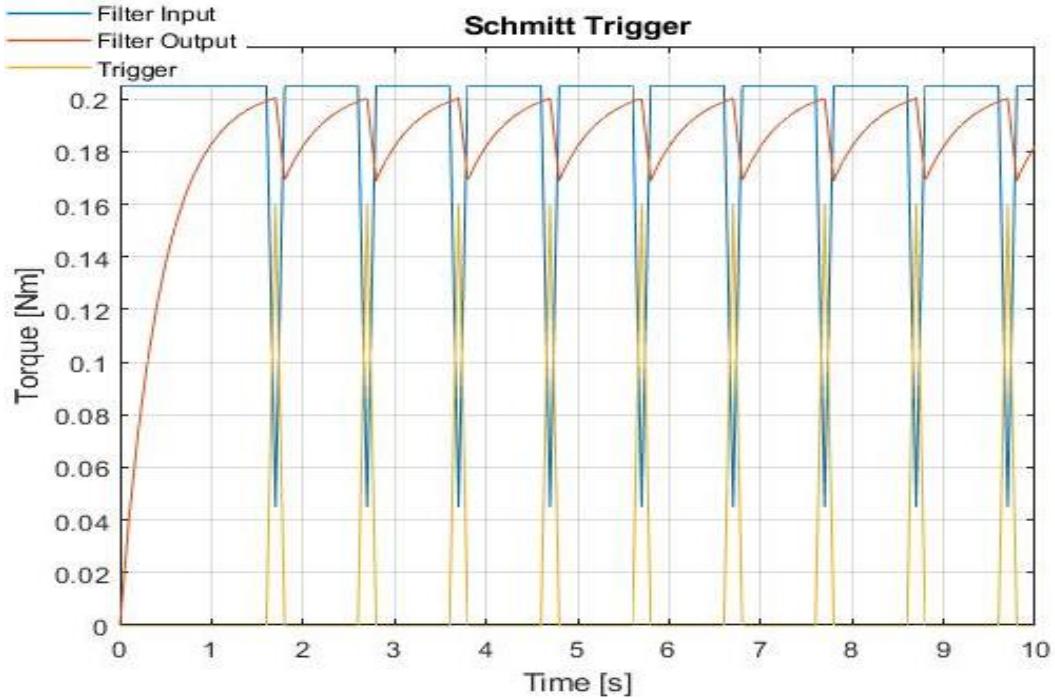
- i. u_{max} is too low and unable to keep the input contained in the dead band, resulting in an error outside the given margins
- ii. u_{max} is too high and inverts the sign of the input with a single pulse, resulting in a rapid succession of pulses with alternating sign

The values for u_{c_on} and u_{c_off} are carefully adjusted to avoid these possible issues, from the starting value of 0.25, associated to an error of 1° with the proportional and derivative terms of choice.

The parameters for each axis are set as:

	X axis	Y axis	Z axis
u_{c_on} [Nm]	0.20	0.24	0.36
u_{c_off} [Nm]	0.18	0.216	0.324
u_{max} [Nm]	0.16	0.16	0.32

The correct behaviour of the PWPF can be easily verified by looking at its output to a constant input just above u_{c_on} :



The input to the filter (in blue) is periodically reduced by the output (in yellow) of the trigger due to the negative feedback. The frequency of the pulses is dictated by the low-pass filter (here in red).

As the control torque is still powerful enough to invert the input in a few time steps, u_{c_off} is set to the high value of 90% of u_{c_on} , to minimise the length of the thruster's pulse.

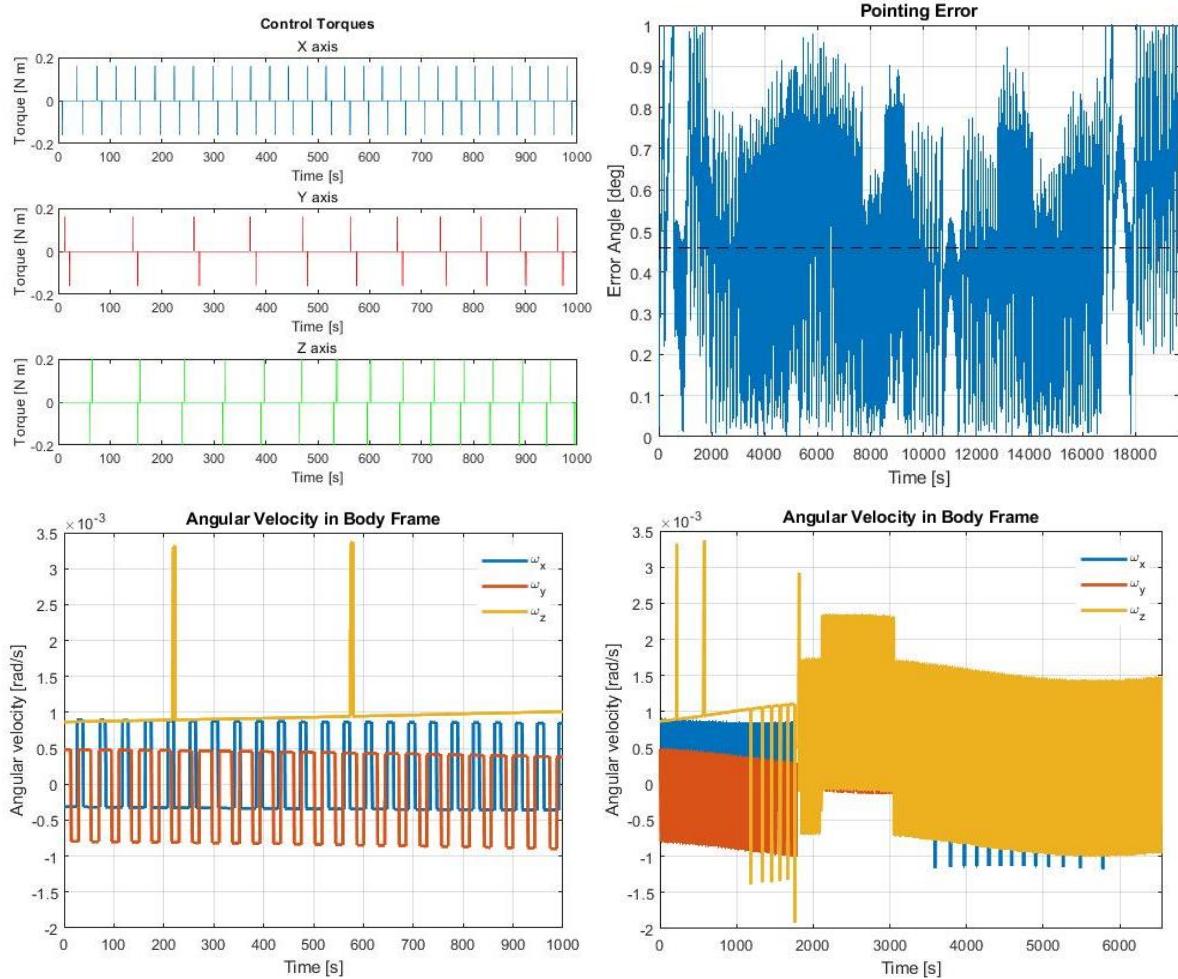
The parameters of the filter are set as follows, to obtain a filter that damps out high frequencies while being responsive enough to control the system:

$$\tau = 0.5 \text{ s}$$

$$K = 1$$

Simulating the model, the following results are obtained (the angular velocity is represented both for one period and 1000 seconds to clearly show its behaviour):

	Mean [deg]	Variance [deg^2]
Pointing Error	0.4871	4.749e-2



The introduction of the PWPF keeps the error under its maximum allowed value while reducing the number of firings required. However, the number of thruster firings is still high, especially when considering the x axis, but this is excepted as precise pointing is hardly ever achieved with thrusters alone in real-life missions.

References

- [1] Blue canyon Technologies, “NST Datasheet,” [Online]. Available: https://satsearch.s3.eu-central-1.amazonaws.com/datasheets/satsearch_datasheet_vifdac_bluecanyontech_extended_nst_star_tracker.pdf?X-Amz-Algorithm=AWS4-HMAC-SHA256&X-Amz-Credential=AKIAJLB7IRZ54RAMS36Q%2F20220105%2Feu-central-1%2Fs3%2Faws4_request&X-Amz-D.
- [2] Markley L. F., Crassidis L. J., *Fundamentals of Spacecraft Attitude Determination and Control*, Springer, 2019
- [3] Matlab & Simulink, Mathworks
- [4] *Lecture Slides of Spacecraft Attitude Dynamics Course*, Prof. Bernelli, Politecnico di Milano, 2021.