PROJECT REPORT

GROUP 2112

GALILEO

(Group for the Assignment on Lunar and Interplanetary Launches chasing Efficient Orbits)

Course of Orbital Mechanics

MSc in Space Engineering

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1. Assignment 1

1.1 Mission Requirements

The objective is to perform an interplanetary transfer from Neptune to Venus exploiting a gravity assist by Earth. During Earth's fly-by, the minimum admissible altitude from the planet's surface is set to 400 km, to avoid meaningful energy losses due to drag. The transfer also must be performed starting no earlier than January 2026 and arriving before January 2066. The performance parameter is the required ΔV, which needs to be minimized within the constraints given. As this is a preliminary analysis, a patched conics method is employed and both escape and injection manoeuvres are not studied.

1.2 Preliminary Analysis

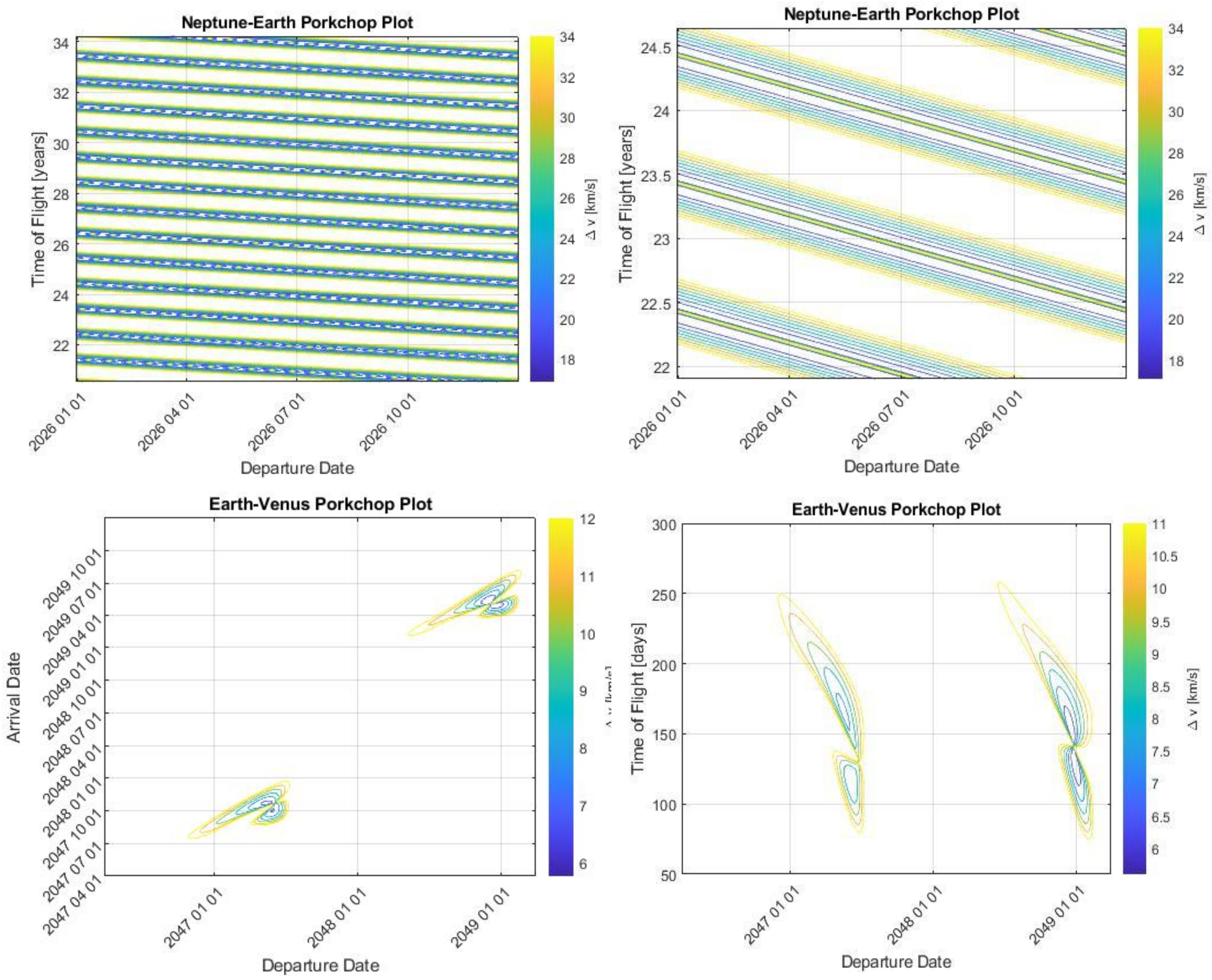
Under the assumption of circular and coplanar orbits and the planets being in the right positions at the right times, Hohmann transfers can be considered to perform preliminary analysis. Considering both a Hohmann transfer from Neptune to Venus, and a double Hohmann transfer (first leg from Neptune to Earth, second leg from Earth to Venus), the ΔV and time of flight required are reported in the table below.

Hohmann Transfer	Δv required [km/s]	Time of Flight [days]
Neptune-Venus	18.179	11'033
Neptune-Earth	15.707	11'182
Earth-Venus	5.210	146
Total (Neptune-Earth-Venus)	20.917	11'328

While not being applicable to a real mission, these values are used as a reference point for the analysis.
Next, the porkchop plots (presented at the end of the paragraph) of both legs of the transfer are computed for the entire time window. In particular, the minimum ΔV required for each leg can be identified within a certain range of times of flight, as reported in the table below. As this is only a first estimate, the data is rounded to two significant digits.

Lambert Arc	Minimum Δv [km/s]	Time of Flight [days]
Neptune-Earth	16	7'000 - 12'500
Earth-Venus	5	90 - 300
Total	21	7'090 – 12'800

The time window of the minima for the single transfers will not strictly coincide with the one of the most efficient overall transfer, but will be in the same range of times of flight. Also, since the gravity assist from Earth is exploited, a ΔV lower than 21 km/s is expected to be obtained, with the aim of bringing it lower than the 18 km/s required for the Hohmann transfer.



1.3 Strategy Choice

With the above considerations, the mesh to find the optimal transfer can be defined: to obtain a value close to the minimum for each Lambert leg, at least 7’000 days are required to go from Neptune to Earth and at least 90 days to go from Earth to Venus. This means that no less than 20 years are required to perform the mission to minimise the ΔV cost.

The departure window is thus spanning from 2026 to 2046. The flyby window is then selected as going from the earliest departure date plus 20 years (about 7’000 days) up to the last departure date plus 32 years (about 12’500 days). The arrival window goes from the earliest date of the fly-by to the latest one plus 1 year. The latest arrival dates are capped at year 2066.

The overall admissible time windows above are split into 4-year blocks: the departure window is split into 5 blocks, each of which must be considered with blocks of the fly-by window and arrival windows. For example, the 2034-2038 departure block has the 2054-2058, 2058-2062, 2062-2066 flyby window blocks. Each fly-by window block has a single arrival window block associated: in the example the 2054-2059, 2058-2063, 2062-2066 arrival window blocks respectively.

Departure Window	Fly-By Windows	Arrival Windows
2026-2030	2046-2050	2046-2051
	2050-2054	2050-2055
	2054-2058	2054-2059
	2058-2062	2058-2063
2030-2034	2050-2054	2050-2055
	2054-2058	2054-2059
	2058-2062	2058-2063
	2062-2066	2062-2066
2034-2038	2054-2058	2054-2059
	2058-2062	2058-2063
	2062-2066	2062-2066
2038-2042	2058-2062	2058-2063
	2062-2066	2062-2066
2042-2046	2062-2066	2062-2066

This leads to 14 different permutations to be considered. The grid for each permutation is selected to get at least one point per month.

Furthermore, to optimise computational efficiency, every solution requiring more than 12'500 or less than 7'000 days for the first leg and less than 90 days for the second transfer is automatically discarded. For every other transfer, the velocities required are computed and saved into a matrix.

For each of the 14 matrices, the minimum of the required velocities is evaluated, then rounded up and increased by 1 km/s. This is set as the 'threshold' velocity value for the entire matrix: all the velocities that are below this value in the matrix (the local minima) are saved in a vector, as are their corresponding dates of departure, fly-by and arrival. All the triplets of dates obtained are then used as initial guesses for the function *fmincon.m* in Matlab, where the constraint on the minimum altitude is imposed. As getting closer to the planet leads to a greater gain in heliocentric velocity, the minimisation function nearly always leads to a fly-by at the lowest altitude allowed: this constraint is set as 500 km in order to have some margin for the robustness of the solution (see paragraph 1.5).

The minimisation function often converges multiple times to the same transfer, indicating that the local constrained minima are being correctly identified. Finally, the set of dates with the lowest ΔV is selected as the best solution and checked for robustness.

Choosing a tighter mesh to evaluate the solution (two or more points per month) leads to the same results obtained with the above method, while being less computationally efficient.

1.4 Chosen Transfer

The solution has been chosen to be the mission with the lowest cost of ΔV to be provided by the spacecraft, with the constraint to robustly respect the limit on the minimum altitude of the fly-by, as further explained in paragraph 1.5. The transfer chosen has the following characteristics:

- Overall ΔV: $\Delta V_{tot} = 14.931\ km/s$
- Departure time: t_d = 2031/10/12 12:40:9
- Fly-by time: t_{fb} = 2061/5/22 16:10:43
- Arrival time: t_{arr} = 2065/3/10 14:28:24
- Overall mission duration: Δt_{tot} = 33 years 158.0752 days
- First heliocentric leg: $\Delta V_1 = 4.193\ km/s$ Δt_1 = 29 years 230.1462 days
- Fly-by: $\Delta V_{pfb} = 0.000\ km/s$ $\Delta t_{fb} =$ 28.01 h
  $\Delta V_{free} = 5.403\ km/s$

- Second Heliocentric leg: $\Delta V_2 = 10.738\ km/s$ Δt_2 = 3 years 292.9289 days

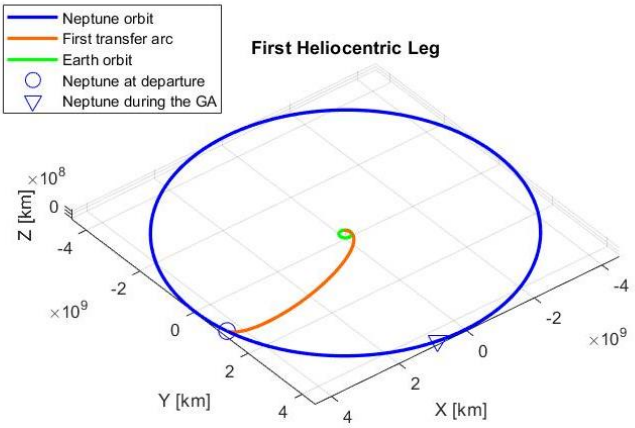
The overall ΔV is the sum of the 3 actual manoeuvres required to perform the transfer.

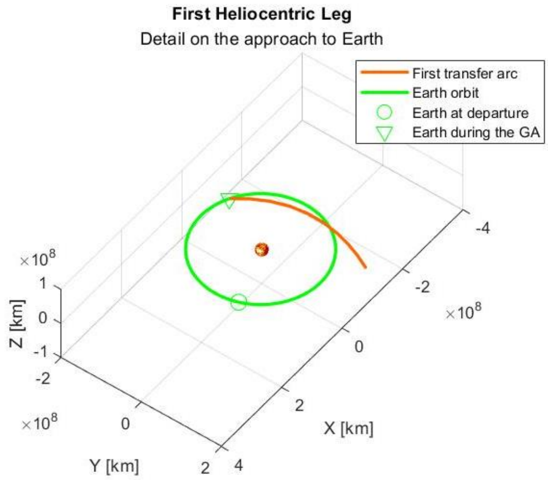
1st Heliocentric leg

The first manoeuvre of the mission (ΔV_1) is performed at departure, to leave Neptune and start the first Lambert arc. The transfer orbit from Neptune to Earth has the following parameters (defined with respect to the Heliocentric Ecliptic Inertial reference frame):

a (km)	e (km)	i (deg)	Ω (deg)	ω (deg)
2.2992e+9	0.9451	2.073	62.111	131.180

- True anomaly at departure on Neptune's orbit: 325.027°
- True anomaly at departure on the transfer orbit: 180.311°
- True anomaly of the fly-by on the first transfer orbit: 311.180°





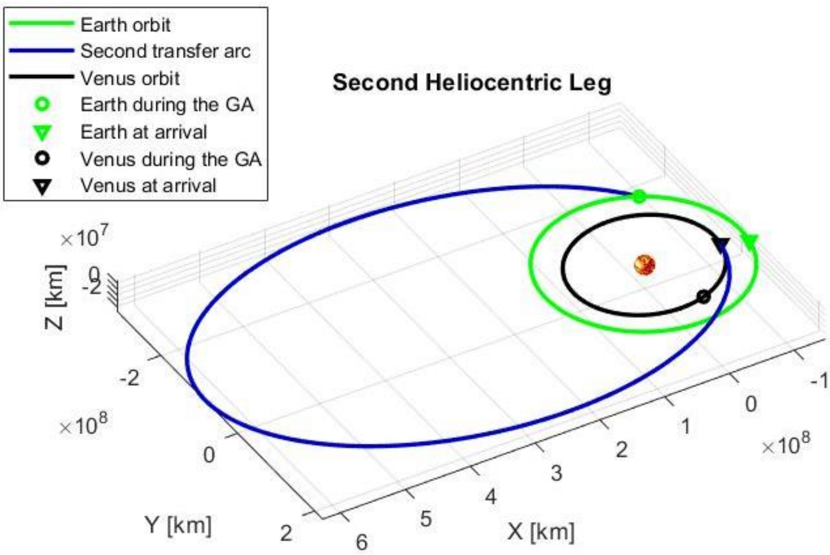
2^{nd} *Heliocentric leg*

The second heliocentric leg consists in an elliptic transfer orbit from Earth to Venus. It has the following parameters:

a (km)	e (km)	i (deg)	Ω (deg)	ω (deg)
3.7187e+8	0.7106	3.568	62.111	107.674

- True anomaly of the fly-by on the second transfer arc: 72.326°
- True anomaly at arrival on the transfer orbit: 359.560°
- True anomaly at arrival on Venus' orbit: 37.766°

The manoeuvre (ΔV_2) is performed at arrival, to match Venus' velocity, completing the mission.



Fly-by

In the chosen transfer, the two heliocentric legs are linked with what can be assumed as an unpowered fly-by of the Earth (the ΔV_{pfb} is in the order of the cm/s), that provides a free manoeuvre. The data before and after the fly-by are the following (vectors in the heliocentric inertial frame):

- Position vector: $\underline{R_{fb}} = [-7.084 \quad -13.385 \quad 0]^T * 10^7$ km

  $R_{fb} = 1.5144 * 10^8$
- Earth's heliocentric velocity: $\underline{V_E} = [25.847 \quad -14.054 \quad 0]^T$ km/s

  $V_E = 29.421$ km
- Heliocentric Velocity before the fly-by: $\underline{V_{s/c}^-} = [25.572 \quad -32.238 \quad -1.364]^T$ km/s

  $V_{s/c}^- = 41.170$ km/s
- Heliocentric Velocity after the fly-by: $\underline{V_{s/c}^+} = [20.291 \quad -31.305 \quad -2.031]^T$ km/s

  $V_{s/c}^+ = 37.3601$ km/s

Consequently, the fly-by hyperbola has the following characteristics:

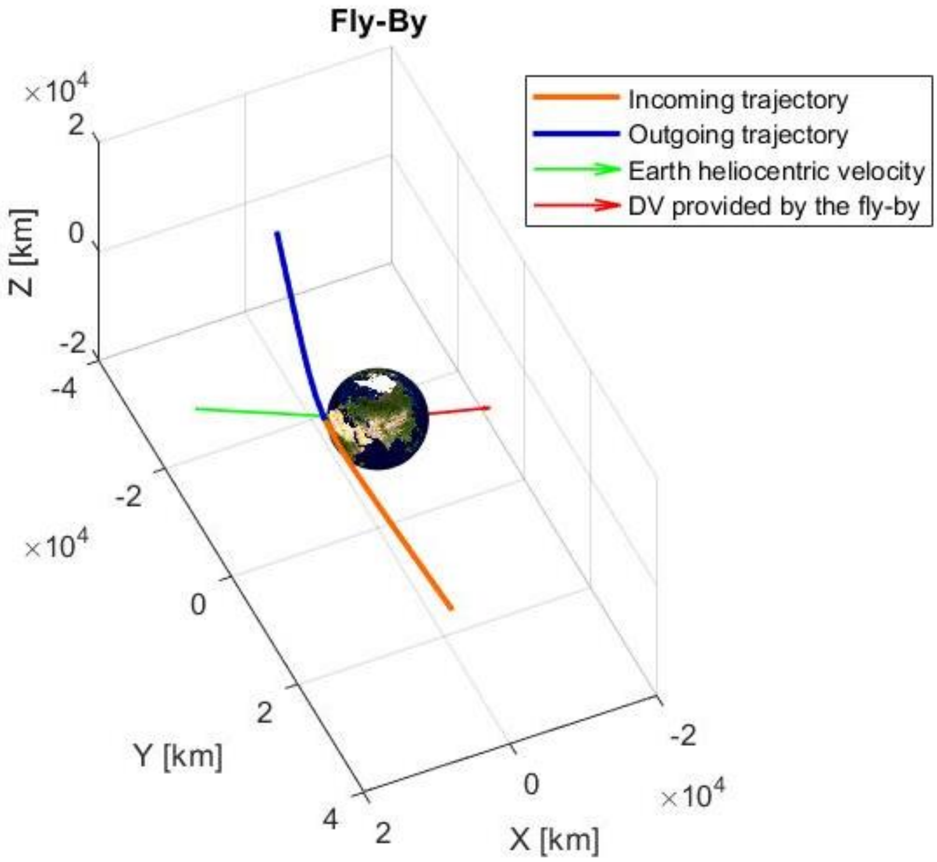
- Leading side fly-by
- Relative incoming velocity $\underline{v_\infty^-} = [-0.276 \quad -18.183 \quad -1.364]^T$ km/s

  $v_\infty = 18.236$ km/s
- Relative outgoing velocity $\underline{v_\infty^+} = [-5.556 \quad -17.250 \quad -2.031]^T$ km/s
- Altitude at the perigee $h_p = 521.227$ km
- Eccentricity $e = 6.7504$
- Turning angle $\delta = 17.040°$

The fly-by is unpowered, so the required ΔV at the pericentre is null. The free ΔV provided by the passage near Earth is: $\Delta \underline{V_{free}} = [-5.280 \quad 0.933 \quad -0.667]^T$ km/s

$$\Delta V_{free} = 5.403 \text{ km/s.}$$

The time required for the fly-by is measured as the interval of time between the 2 intersections of the geocentric trajectory with Earth's sphere of influence. The result is: $\Delta t_{fb} = 28.01$ h.



1.5 Robustness Check

The solutions computed are valid for specific time instants of departure, fly-by and arrival. To give a more valid solution, the transfer chosen with the optimization undergoes a further check. All the possible transfers inside small windows, built around the nominal time instants, can be computed. The goal is to see how the total cost in terms of ΔV and the minimum altitude of the fly-by vary in the transfers inside these time spans. A solution is defined robust if the minimum altitude of the fly-by is never below the set limit of 400 km, for all the possible transfers inside windows of 12h centred at each nominal time instant. Furthermore, the required ΔV and the fly-by altitude must be close to the nominal ones. This process gives more meaning to the result, proving the validity of a transfer strategy even if the manoeuvres are not performed at the exact computed instants. For the solution presented at paragraph 1.4, for the transfer windows $t_{dep} \pm 6\ h$, $t_{fb} \pm 6\ h$, $t_{arr} \pm 6\ h$:

- The lowest possible fly-by altitude is: $h_{low} = 447.210$ km
- The average fly-by altitude is: $h_{avg} = 521.190$ km
- The max possible cost in terms of ΔV is $\Delta V_{max} = 14.979$ km/s
- The average cost in terms of ΔV is $\Delta V_{avg} = 14.957$ km/s

Given that $h_{low} > 400\ km$ and that the nominal values of h_p and ΔV are $h_p = 521.227$ km and $\Delta V = 14.931$ km/s, the solution presented at 1.4 is considered robust.

1.6 Conclusions

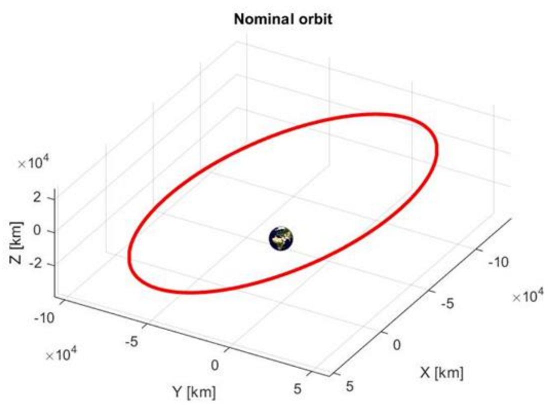
The solution has been chosen chasing the minimum possible ΔV. It features an unpowered fly-by, meaning that the gravity assist from Earth could be exploited more, but the overall transfer would end up being more expensive.

The most expensive manoeuvre of the mission is by far the rendezvous with Venus. Consequently, if the problem of the capture of the spacecraft by the planet is taken in account, it can be practical to also consider alternative heliocentric trajectories, with higher overall ΔV but with a less demanding rendezvous.

2. Assignment 2

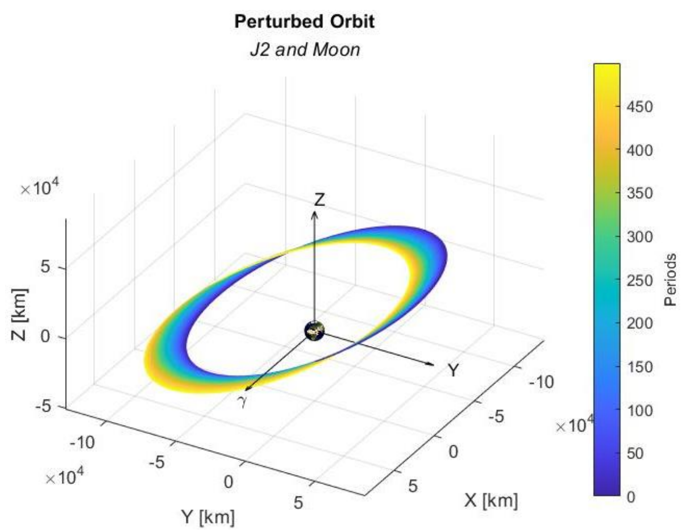
2.1 Mission Requirements and Data

The aim of the mission is to analyse the evolution in time of an orbit whose central planet is Earth. First, the propagation of the orbit is performed studying the unperturbed Two Body Problem. Then, the analysis is repeated including the modelling and the study of the effect of the main perturbations acting on the orbit. The success in the modelling of the perturbations is enforced making comparisons with a real case scenario.



The orbit analysed can be classified as HEO (Highly Elliptical Orbit) due to the high value of eccentricity e. The value of semi-major axis is significantly high. Different kind of perturbations acting on the orbit can be studied. Studying the Keplerian elements describing the nominal orbit, it can be stated which perturbations have relevant a effect on the orbit, to perform a valid preliminary analysis of the evolution of the perturbed orbit in time. In the specific case under analysis the most significant perturbations are due to Earth's oblateness and to the presence of the Moon as third body. The data of assignment 2 are shown in the following table:

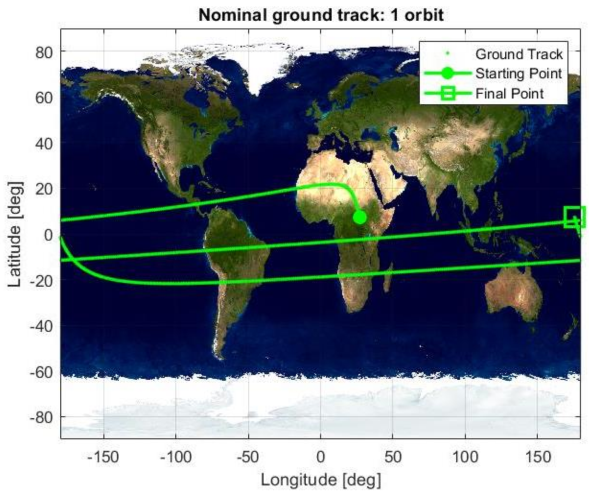
a [km]	e [km]	i [deg]	Ω [deg]	ω [deg]
98890	0.4846	21.776	10	20

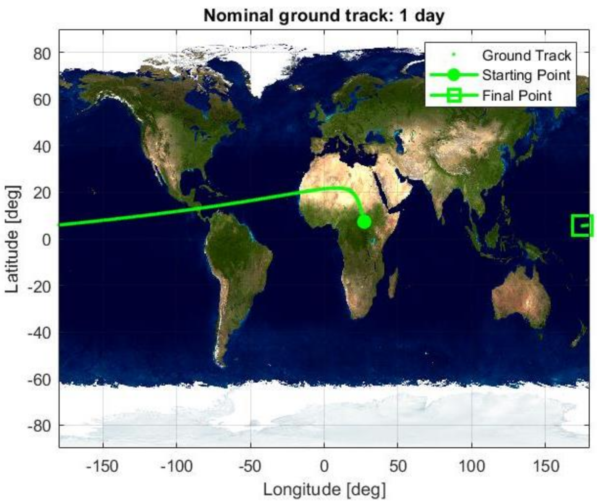


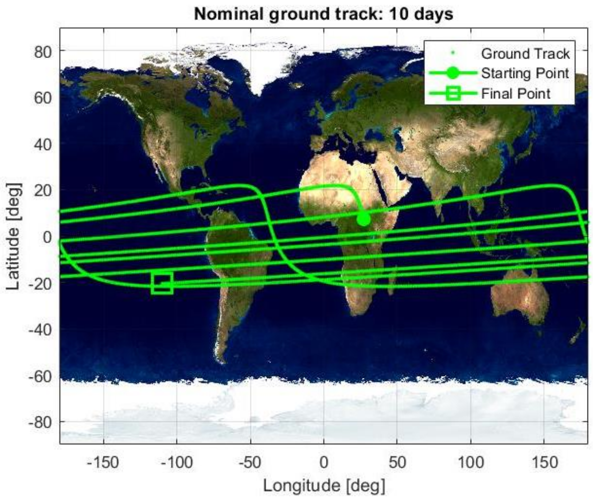
In the figure on the left the evolution of the orbit under the two considered perturbations is shown.

2.2 Ground Tracks

Ground Tracks – Unperturbed 2BP



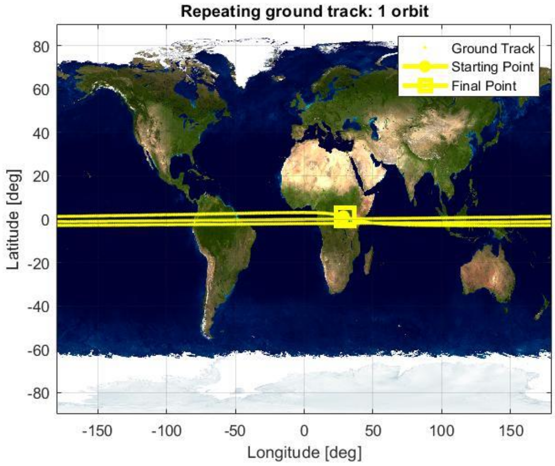


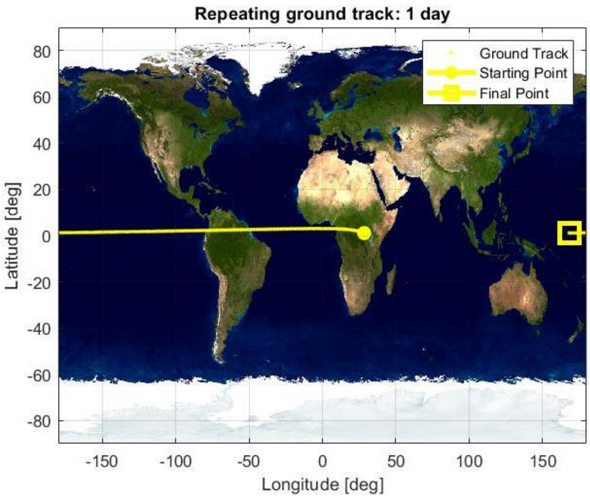


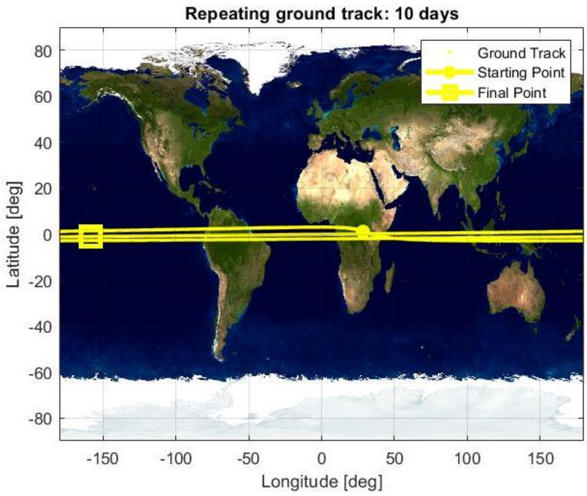
In the plots above the Ground Track of the orbit in the unperturbed Two Body Problem are shown, over time windows of 1 orbit, 1 day and 10 days. In the unperturbed case it can be noticed that the orbit remains constant, but the projection of it onto the Earth's surface drifts westward due to the fact that in the meantime the Earth is rotating around its own axis. This is the reason why the Ground Track is not repeating in time.

Repeating Ground Tracks – Unperturbed 2BP

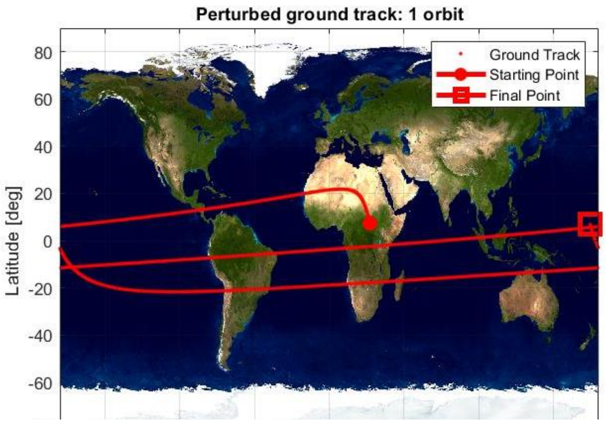
To achieve a repeating Ground Track with k=1 satellite revolutions and m=4 Earth rotations, the semi-major axis needs to be changed. The value of semi-major axis obtained is equal to 106250 km. The propagation of an orbit with such a semi-major axis leads to the repeating Ground Tracks shown in the following graphs, in the case of unperturbed two Body Problem.



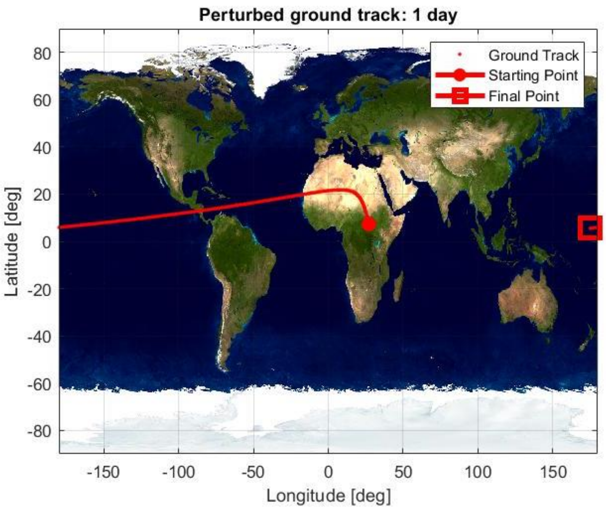


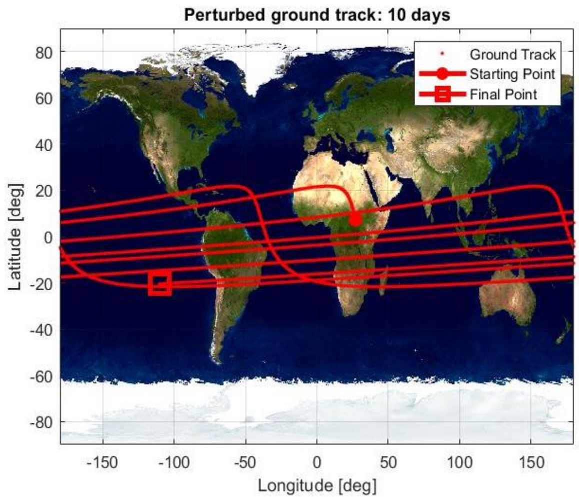


Ground Tracks – Perturbed 2BP



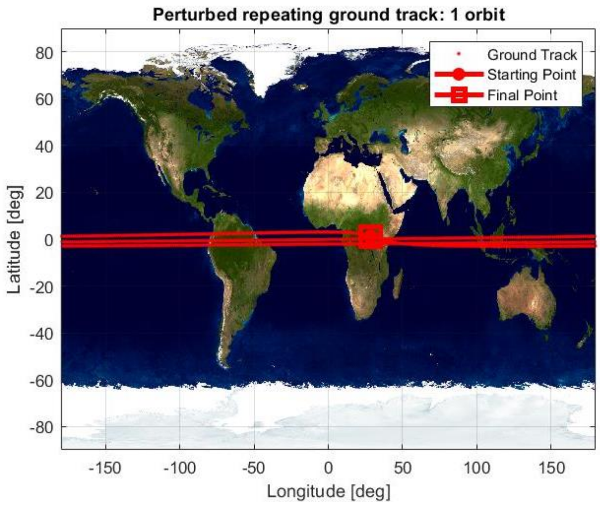
The introduction of the main perturbations acting on the orbit causes a change in the Ground Track with respect to the unperturbed case. In fact, since the nominal orbit is modified due to environmental disturbances, its track on Earth's surface is changed too. The addition of the total disturbing accelerations, due to the presence of the Moon and due to J2 perturbation, leads to the following plots.

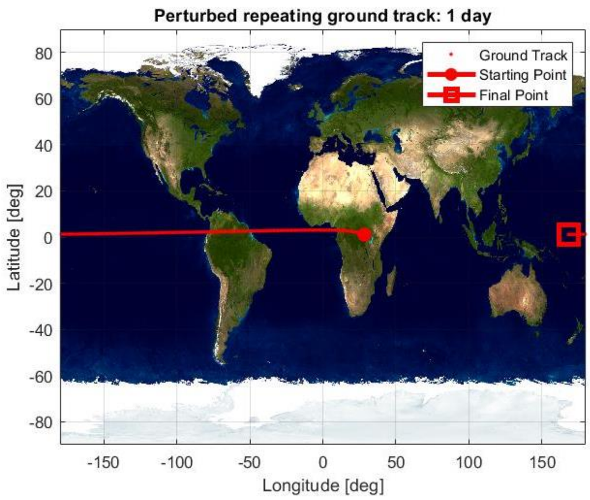


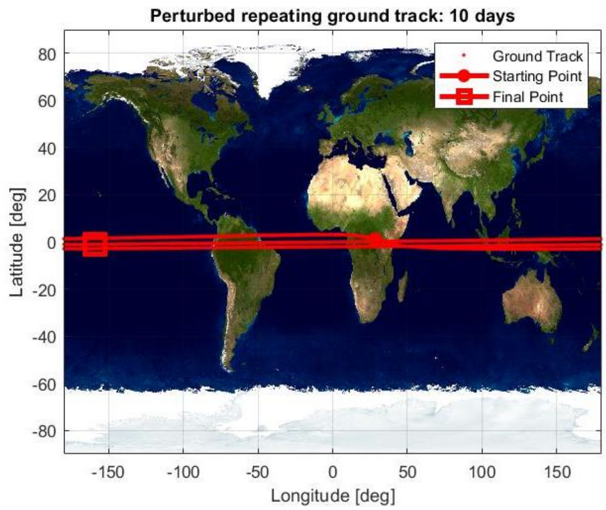


Ground Tracks Repeating Orbit – Perturbed 2BP

In the case of perturbed Two Body problem, the usage of the semi-major axis obtained to get a repeating Ground Track for the unperturbed case leads to the plot of a Ground Track which is not repeating anymore.

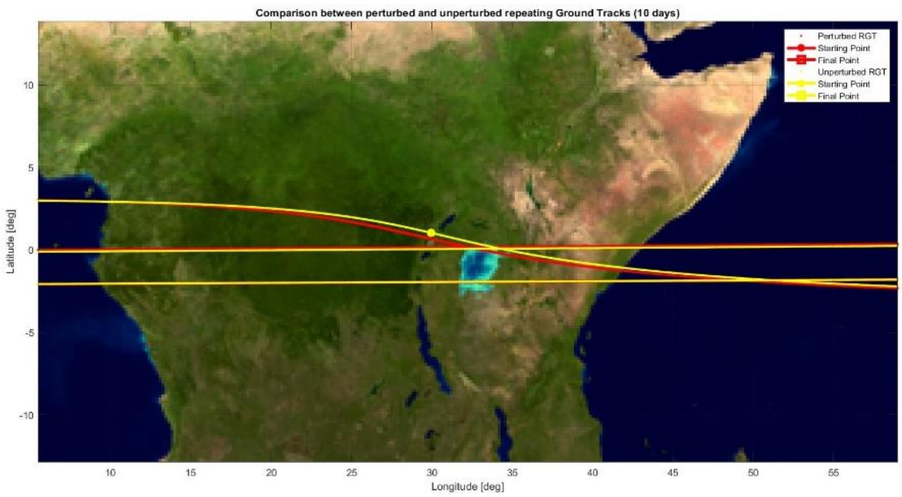






In order to get a repeating perturbed Ground Track after k revolutions of the satellite and m revolutions of Earth, it would be necessary to evaluate again the appropriate semi-major axis, including also the effect of both the perturbations.

In the graph below the difference between the perturbed and unperturbed repeating Ground Tracks is evident.



2.3 Perturbations Study

In the assigned orbital region, the main perturbing effects come from J2 and the Moon's gravity.
The J2 perturbation takes into account the non-spherical geometry of the Earth. Indeed, due to the oblateness of the Earth, the gravitational field depends, as a first approximation, on the latitude. The resultant gravity force on a satellite will not be directed towards the centre of the Earth because of an additional force exerted by the equatorial bulges.
The overall effect is a secular variation of the node, which will advance or regress depending on the inclination, and on the apsidal motion. Being the orbit prograde, in this case the ascending node is expected to regress.
The perturbation caused by the Moon, tends to make the orbit regress about an axis normal to the Moon's orbital plane. On the short term this affects all the orbital elements, while on the long term it causes a variation mainly on the eccentricity, inclination, right ascension of ascending node and argument of perigee.
The perturbing acceleration caused by the moon in the geocentric equatorial reference frame, can be expressed as:

$$\boldsymbol{a_m} = \mu_m \left(\frac{\boldsymbol{r_{m/s}}}{r_{m/s}{}^3} - \frac{\boldsymbol{r_m}}{r_m{}^3} \right) \tag{2.3.1}$$

Where:

- $\boldsymbol{r_{m/s}}$ relative position of the Moon with respect to the satellite [km]
- $\boldsymbol{r_m}$ relative position of the Moon with respect to the Earth [km]
- μ_m Moon's planetary constant [km^3/s^2]

Therefore, the overall acceleration acting on the satellite is:

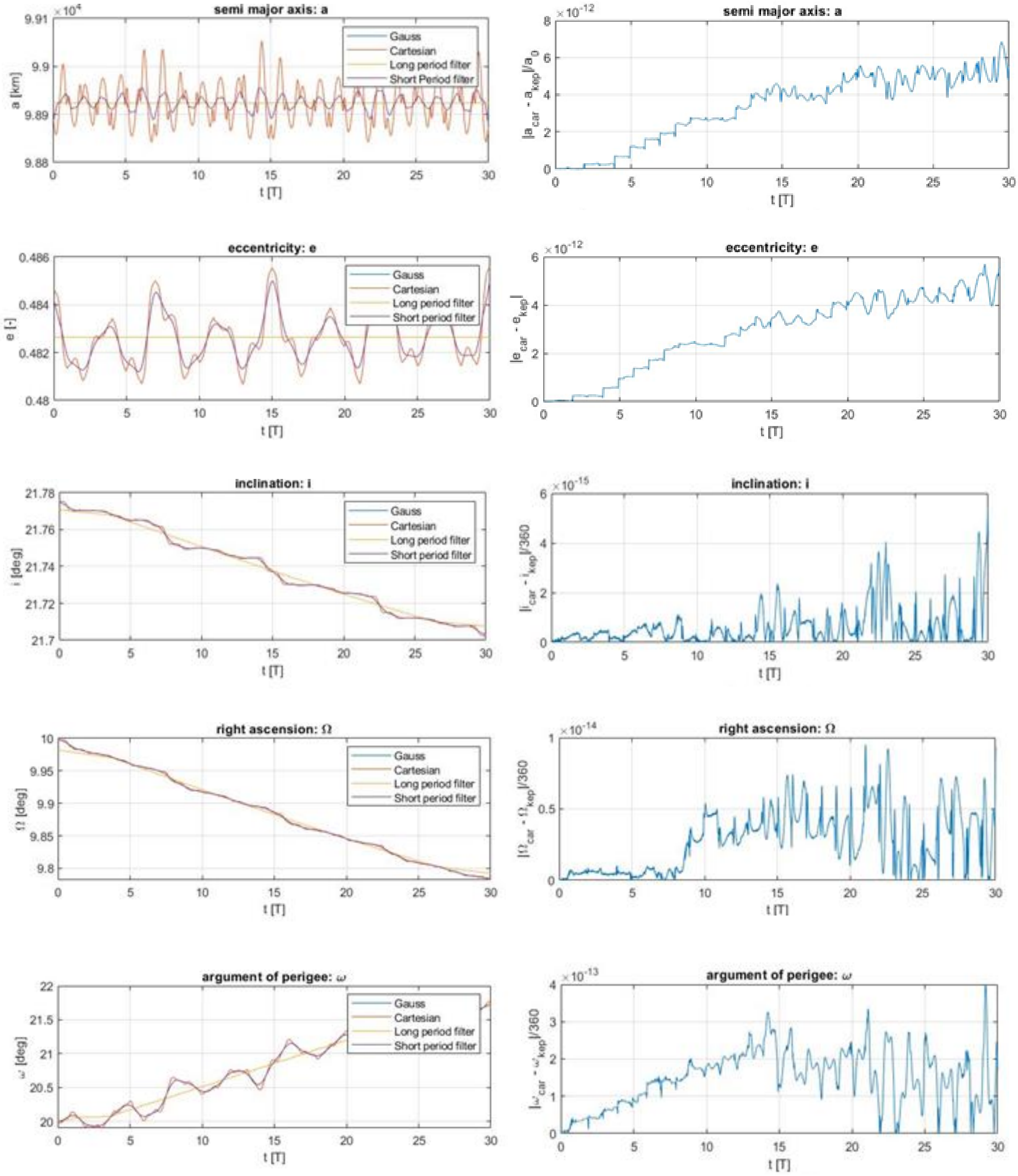
$$\ddot{\boldsymbol{r}} = -\frac{\mu_E}{r^3}\boldsymbol{r} + \boldsymbol{a_{J2}} + \boldsymbol{a_m} \tag{2.3.2}$$

In the analysis this equation is integrated using the ode133 which is suitable if a stringent error tolerance is required.
The other method implemented regards the use of Gauss' planetary equations. This method requires a further step to transfer the computed acceleration from the inertial to the radial-transversal-out of plane reference frame *(r, s, w)*.
From the use of the two different methods to propagate the orbit it turned out that Gauss' method is more computationally efficient.
The results of the integration are showed in figures of paragraph 2.4.

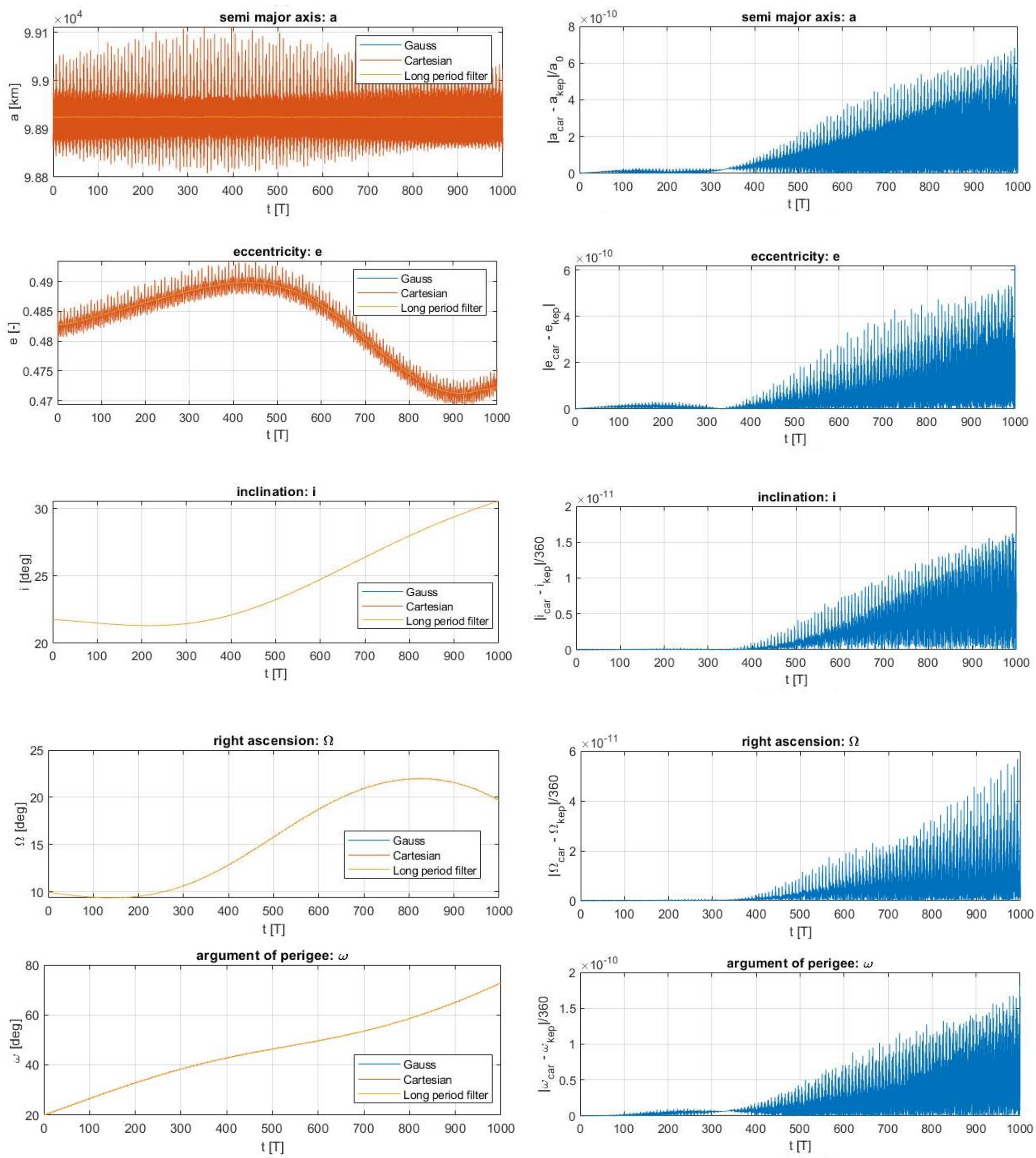
2.4 Perturbation Effect Filtering

The perturbing effects can be filtered with different frequencies, in order to show secular and long period variations of orbital elements.
The time window associated to the maximum period of revolution of the satellite around Earth can be used to filter high frequency oscillations, highlighting the short period variations. The

filter used is in fact a low-pass filter with a cut-off frequency associated to the maximum period. All the frequencies higher than the set one are filtered. The plots show the results of filtering out high frequency variations along 30 periods of all the orbital elements.
Moreover, to show the trend of long period variation, it is needed to filter out short period oscillations. This effect is obtained using as reference the sidereal period of the Moon, equal to 27.322 days. This works in the case of inclination, right ascension of the ascending node and pericentre anomaly, but the time window is not enough to filter in the case of eccentricity and semi-major axis. In the last two cases a multiple of the sidereal period of the Moon is used.



With the aim of showing how orbital elements vary on the secular term, the propagation of orbital elements in time is plotted over a high number of periods equal to 1000, avoiding to perform short period filtering.



In all the plots the relative error between the orbital elements obtained from the propagation in cartesian elements and with Gauss' planetary equations is shown in the right column.

2.5 Comparison with Real data

The model is validated against the actual ephemerides of an artificial celestial body. This is chosen to be in the same orbital region as the previous prescribed satellite, to have similar characteristic perturbations.

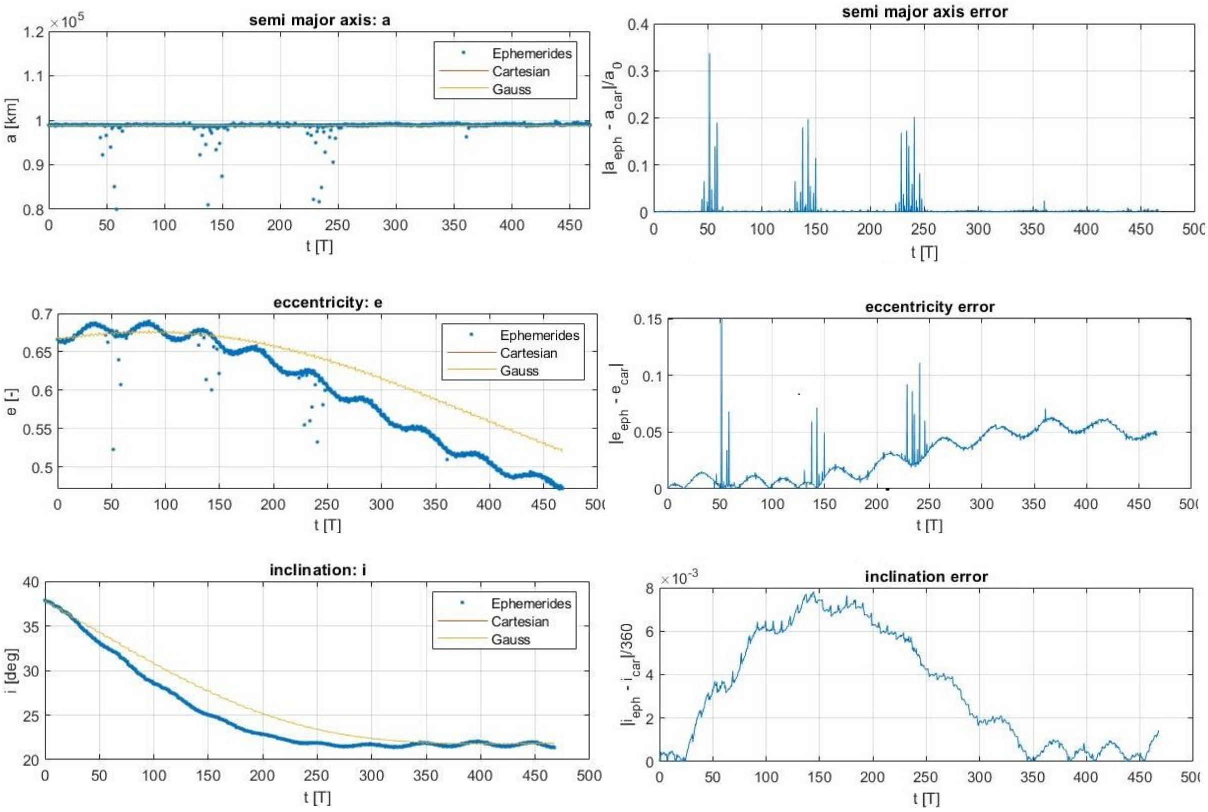
The satellite selected for the comparison is the rocket body *YZ-1 R/B (SAT ID: 14929)*, launched in 2015 by PCR. The TLEs are collected for four years, from January 1, 2017, to January 1, 2022, using Space-Track software [2]. These are then propagated with NASA-JPL's Horizon [3] to compute highly accurate ephemerides.

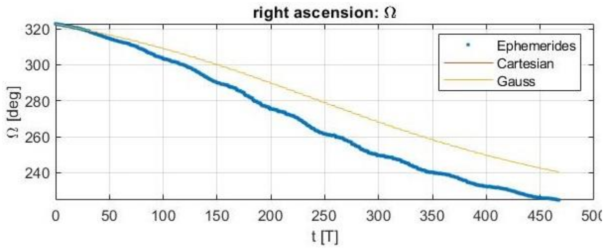
The orbit is propagated for the same time span, using as initial condition the real orbital elements at the initial time.

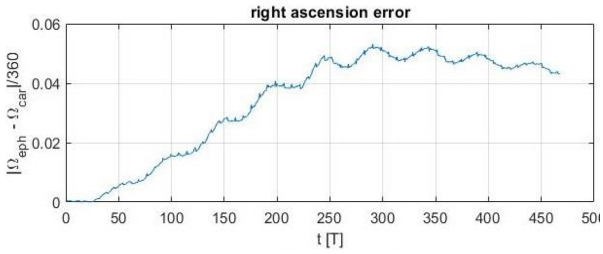
The following orbital elements for the *YZ-1 R/B* are referred to 2021.

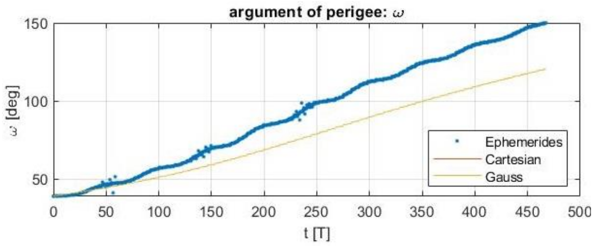
	Assigned	YZ-1 R/B
a [km]	98890	98897.5
e	0.4846	0.4780
i [deg]	21.7756	21.6206
T [days]	3.58	3.58

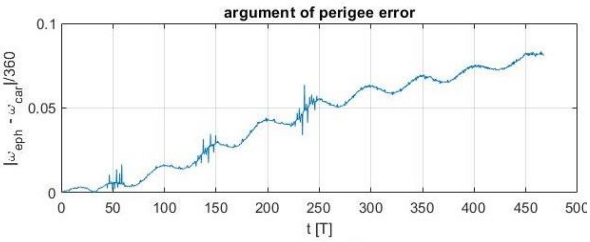
The resultant evolution of the Keplerian elements is showed in the following graphs:



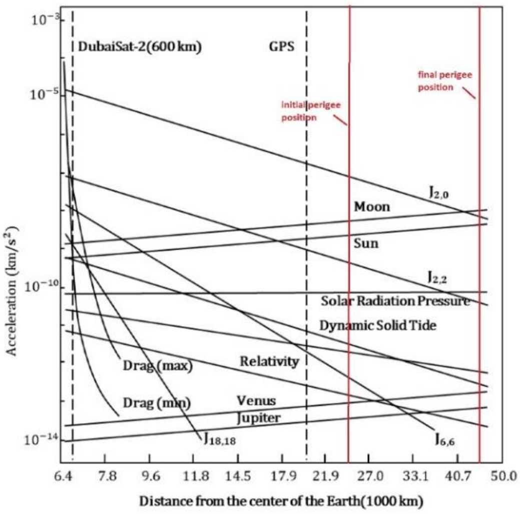






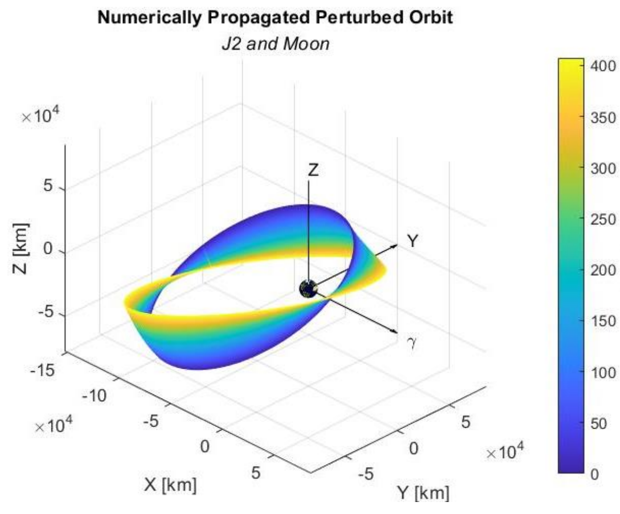


As expected, there are nor secular neither long-period variations in the semi major axis under the effect of the Moon and J2 perturbations. The secular variations on the right ascension Ω and the argument of perigee ω are caused by both effects, while the Moon is responsible for the secular variation on the eccentricity e and inclination i.



The divergence between the predicted variation and the real ephemerides is due to the fact that the model doesn't take into account all possible perturbing effects.

At the altitudes in which the satellite operates, the effect of the solar gravity has the same order of magnitude as that of the Moon. The Luni-Solar gravity becomes even more important as the satellite passes through the apogee, which has an altitude around 140000-160000 km, while the effect of J2 decreases. In this region the effect of J2 decreases significantly with respect to that of the third body, reaching an order of magnitude lower than SRP effect.



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