
Evolutionary Game Theory

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Here is our super nice abstract with the solution which will bring peace to the world

Explanation of problem

why is it important to such things modelling of nature for forecasting, understanding, estimating get old paradigm of the behaviour in HD right - paradigm shift to the new paradigm that HD players could act in a way not known until now

super important for theoretical ecology both games 2 person-player games with two strategies both social dilemmas coop are exploited by defectors coop normally not persisting in PD, but in SD at moderate frequency

what is cooperation, defection in nature/ecology?

Looking at evolutionary game structures, the pay-off is crucial to determine each players fitness and the fitness of a whole group.

Spatial structure added to the game results in a population, where its individuals occupy patches on a spatial lattice (here two dimensional). Each tick (update) will be done by letting individuals play against their nearest neighbours. The resulting pay-offs will be used to decide upon the focal patch's future occupant. It could be an offspring of the last occupant resisting the invasion or from a neighbour spreading its strategy. The lattice is updated and the evolutionary process takes place with every update.

Prisoners Dilemma

Explanation of PD in general: The cooperators get exploited by defectors, subsequently defectors are naturally selected. The cost to the donor of fitness (pay-off?) is always higher than zero, but generally lower the benefit to the receiver of the pay-off ($b > c > 0$). The defectors pay-off is the highest pay-off b if the other player is cooperating. The lowest pay-off, namely only the cost, has then $(-c)$ the cooperator which is defected in the unilateral cooperation. Finally it is best to defect regardless of other players decision. Mutual defections result then in pay-off zero for both players, not reducing the fitness but also not increasing it (see 1). Here, the defector strategy is the ESS.

Table 1: Prisoner's Dilemma

	C	D
Payoff to C	$b - c$	$-c$
Payoff to D	b	0

Spatial PD: Spatial structure in Prisoners Dilemma is a potent promoter of cooperation. Cooperators stay on forming large compact clusters thus reducing the exploitation by defectors as one can see in figure 4. In this case the cooperators are not close to the threshold of extinction and survive in big clusters.

The big difference to non-spatial PD is, that cooperation can evolve as a stable proportion in the population, which would be not possible without the spatial component in the behaviour.

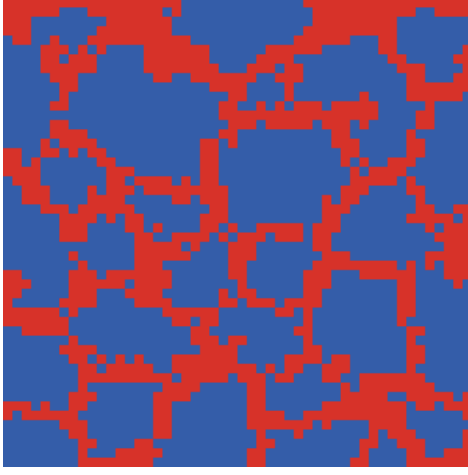


Figure 1: *Clustering of cooperators in spatial PD with $b=1$, $c=0.1$, $N=8$, blue=C, red=D*

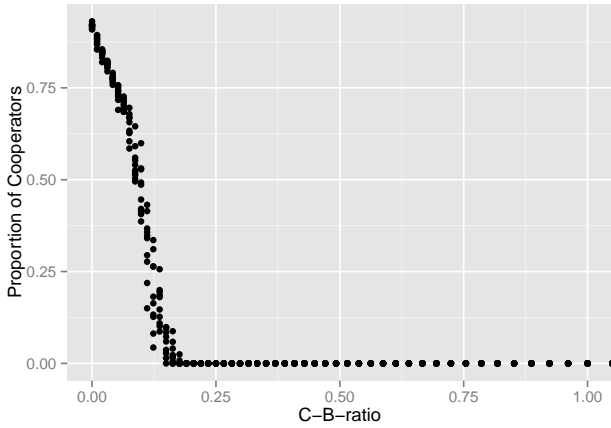


Figure 2: *Frequency of cooperators against c-b-ratio*

Snowdrift game

In general:

At the snowdrift game we have the difference to Prisoners Dilemma. The players can share the benefit and the cost, depending on their strategies. Another feature in this game is, that if one defects, the pay-off could be less than the sucker's pay-off of the unilateral cooperator. Still defecting if the other player cooperates is, according to the matrix, the best response choice. The other way round, if the opponent defects, it is still better to cooperate than to defect as well. This leads to the maintenance of cooperation to a certain extent in the population as an ESS. As one can see in table 2, the pay-off matrix is slightly different than the one of Prisoners Dilemma in table 1. If $2b > c > b > 0$, meaning that if costs are high, these pay-off structures change the game to a PD and affect the reverse pay-off structure. If $b > c > 0$, the best action depends

on co-players action resulting in a mixed strategy population, where rare strategies can invade, either defector or cooperator with an ESS at cooperator proportion is $1 - c/(2b - c)$. Here it gets important to investigate when the cooperation in the ESS is increased or decreased.

Table 2: *Snowdrift game*

	C	D
Payoff to C	$b - c/2$	$b - c$
Payoff to D	b	0

Spatial structure in snowdrift: In comparison to the spatial structure of the Prisoner's Dilemma's resulting cooperation lattice, the cooperators form only small filament-like clusters in the snowdrift game. The defectors have via the isolated cooperators structure the advantage to exploit them and break in those fragile clusters.

The individuals occupy patches on a regular lattice. The next update is done by populating this patch with the offspring. Which offspring depends on the game result of the present occupant and its nearest neighbour competing for the site. The competitive success depends therefore on the different pay-off that each player and possible parent gains from game interactions.

But after several updates spatial structure seems to fail to enhance cooperation in the snowdrift game. In moderate c-b-ratios, cooperation is actually proportionally lower than in non-spatial populations, what is a difference to the behaviour of players in spatial PD.

A small ratio, meaning high benefits with low costs can increase the proportion of cooperation compared to non-spatial populations. The frequency is higher than the $1 - r$ expected frequency in well-mixed populations. If the costs are increasing and the c-b-ratio is high, the defectors proportion increase, depending on lattice geometry. Cooperation is lost if sufficiently high ratio and disappears when the ratio is $1/N > 1 - r$, which you can see in the next chapter as different N (neighbourhood structures) are applied.

clusters small because of the different pay off matrix develop strategy contrary to the neighbour ones isolated coop: branchy structures possible but never large cluster defector: has advantage, can exploit coop, even more for high c-b-ratios = spatial structuring is not always advantageous for cooperation

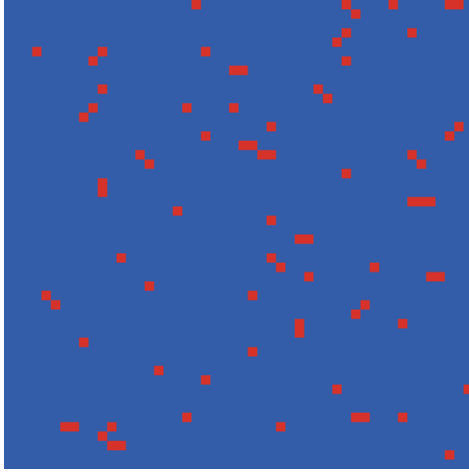


Figure 3: *Clustering*

task 1 the experiments

First we conducted two experiments to check the validity of our implementation. The first one runs 50 experiments with benefit 1 and cost 1.5. According to the paper of Hauert and Doebli (cite) this should result in a fast domination of defectors. The second experiment used benefit 1 and cost .75. This results in a stable distribution of defectors and cooperators. Both results could be achieved. It was important to check the plausibility of the experiment, especially to gain certainty that the equilibrium is stable and the variation will not change again. The verification, when to stop the calculations and repetitions was done via statistical testing (for detailed information see appendix). The second experiment can be visualized in further figures, with a stable average of 24,7% cooperators in the population as an ESS.

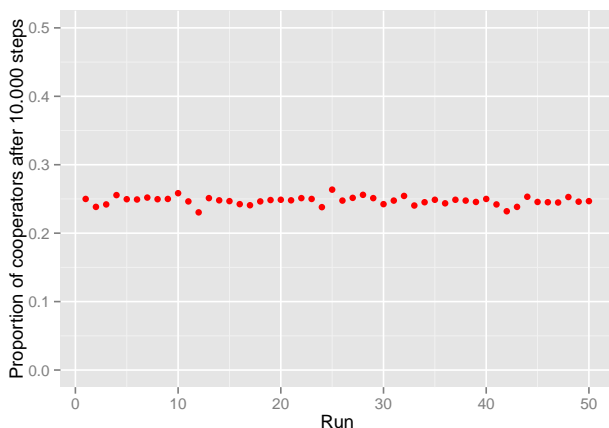


Figure 4: *Proportion of cooperators in the population after 10000 steps*

task 2 the neighbourhood problem

We choose 3 different types of neighbourhoods to be experimented on. The first one is the Moore neighbourhood with the default value of $N=8$ neighbours. The neighbours surround the focal patch in 8 directions. The second neighbourhood is the Von Neumann neighbourhood with the default value of $N=4$ neighbours which are orthogonally surrounding the focal patch on a two dimensional square lattice. The last type of neighbourhood introduces a parameter instead of a defined neighbourhood, having the attribute of a radius equal or less than 3 patches.

To increase the certainty of our results, the behaviour space model is used to compare different cooperation frequencies against the ratio of cost and benefit within different neighbourhood features and between the snowdrift game and the prisoners dilemma game. The mutual cooperation ratio in the snowdrift game is calculated as $r = c/(2b - c)$, cost to benefit ratio. The mutual cooperation ratio in the Prisoner's Dilemma is calculated as $r = c/(b - c)$. If one applies the Hamilton's rule to spatial PD with differing neighbours, the general assumption of $r = c/(b - c)$ must be improved with the rule that cooperators can invade if $b - n*c > 0$ and if $r = 1/N$. In the experiment of the neighbourhood structure of V.Neumann with 4 neighbours, it would mean, that the ratio needs to be higher than 0.2 ($b > 4c$, $r = c/(b - c) < 0.2$) what you can see in the plots of figure ??.

With the different neighbourhood interactions in this experiment, the cooperator proportion is always rather small. The probability that the neighbours are related to the invading cooperator is small, the neighbours tend to be defectors.

The different neighbourhood experiments verify the Hamilton's rule, as the threshold of an ESS favouring cooperators corresponds roughly to the calculations given above. It is thus possible to change the defecting ESS by adding a spatial structure to the PD.

HD: for moderate c-b-ratios: spatial structure results in less cooperators than in well-mixed non spatial SD – those results remain stable at any neighbourhood experiments very contrary effect on cooperation

task 3 the mixed strategy

The Algebra of the Nash Equilibrium

The simulation for the third task ended in a Nash Equilibrium for the snow drift game. In order to

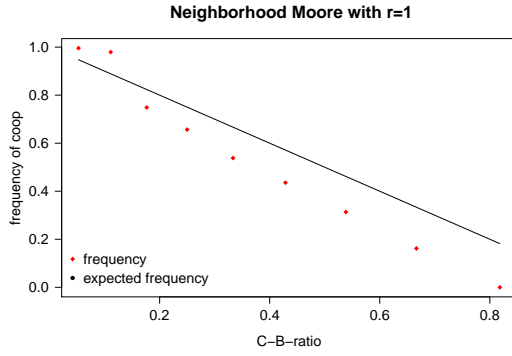


Figure 5: $N=8$

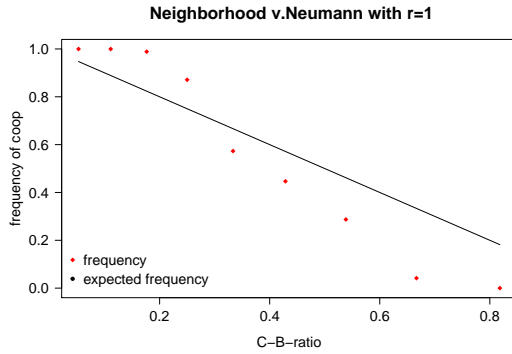


Figure 6: $N=4$

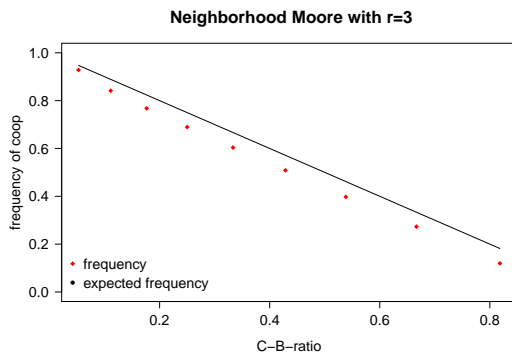


Figure 7: $N=3$

verify the result of our simulation we will derive the probability for the mixed strategy for the “defector player”.

The payoff matrix for the snowdrift game is calculated as follows:

	Left	Right
Up	$b - (c/2)$	$b - c$
Down	b	0

We simulated the game for benefit $b = 1$ and cost $c = 0.75$. So the actual payoff matrix is:

	Left	Right
Up	0.625	0.25
Down	1	0

Let σ be the probability for the mixed strategy

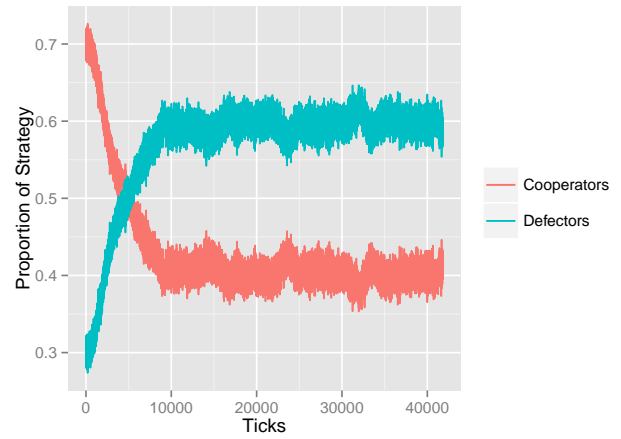


Figure 8: Resulting evolution in a mixed strategy snow-drift game

to play C and let U_x be the utility function for the player when playing strategy “Left” (L) or “Right” (R) respectively.

We get the equation system:

$$U_L = U_R$$

$$U_L = f(\sigma)$$

$$U_R = f(\sigma)$$

We can further define (this is just the probability of the strategy multiplied with the payoff, for both strategies):

$$U_L = 0.625\sigma + 1 \cdot (1 - \sigma)$$

$$U_R = 0.25\sigma + 0 \cdot (1 - \sigma)$$

Set the two equations for $U_L = U_R$, we get:

$$0.625\sigma + (1 - \sigma) = 0.25\sigma$$

The resulting σ is 0.60.

discussion

where can we give in more time, where are options to prolong this topic? relation to nature, where is the importance here ?

Discussion: Paper: <http://iopscience.iop.org/0253-6102/57/4/04> Not included the noise, like in this paper - could it make even more realistic but heavier to implement Our study more straightforward to understand the snowdrift game and the players evolution better Afterwards you could add those noise to our model to make it more exact

”Some analytical results have been obtained using geometrical arguments about cluster formation

(Nowak & May 1992; Killingback et al. 1999; Hauert 2001), and Schweitzer et al. (2002) recently gave a classification of the dynamic regimes in the spatial PD.”

<http://www.math.pitt.edu/~bard/classes/mth3380/spatialgame.pdf>

continuous spatial SD - change of function of cost and benefit (investment) leading somehow to the tragedy of commune as contrast to tragedy of commons

tragedy of commons in PD: cooperation will be advantageous only for others, elimination of altruism
tragedy of commune in SD: coop system, all individuals contribute to group benefit, but option to asymmetric stable state where some invest a lot and others less or nothing: stichwort fairness, diversification

PD and SD both social dilemma PD spatial assortment crucial for coop, where defectors dominant SD spatial assortment irrelevant, coop can invade if rare

all experiments are very theoretically and neglecting a lot of ecological dynamics and parameters which are important for evolution in reality missing input of empirical studies, raw data

Appendix - the plausibility verification

We were determining whether 500 steps are sufficient for reaching a rather stable state with the two-sided t-test and the F-test to compare two variances of the different final proportions of cooperators (the final proportion and the proportion of cooperators after 500 updates).

	p-value	NULL hyp
F-Test	> 0.05	<i>accepted</i>
t-test	> 0.05	<i>accepted</i>

Also we tested whether the initial proportion of defectors has some impact on the final cooperation proportion with a starting value of 0.4 and 0.6. The tests confirmed the NULL hypotheses that variances are the same and there is no difference in means between the two possible initial defector proportions. So we can assume that if cost and benefit structures are given, the stable distribution of the final cooperation proportion is evolving.

References