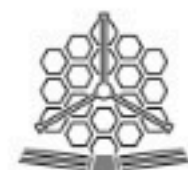




Task 1: The fast algorithm



$$\begin{pmatrix} x(\tau) \\ y(\tau) \end{pmatrix} = |b\rangle \quad \begin{pmatrix} x(\tau+t) \\ y(\tau+t) \end{pmatrix} = HHL \begin{pmatrix} x(\tau) \\ y(\tau) \end{pmatrix} = HHL |b\rangle$$

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} 0 & -\omega \\ \omega & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad \begin{pmatrix} 1 & \omega t \\ -\omega t & 1 \end{pmatrix} \begin{pmatrix} x(\tau+t) \\ y(\tau+t) \end{pmatrix} = \begin{pmatrix} x(\tau) \\ y(\tau) \end{pmatrix} \quad \sum \begin{pmatrix} 1 & \omega t \\ -\omega t & 1 \end{pmatrix} \begin{pmatrix} x(\tau+t) \\ y(\tau+t) \end{pmatrix} = \sum \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$$

Hermitian

$$\begin{pmatrix} 1 & \omega t \\ \omega t & -1 \end{pmatrix} \begin{pmatrix} x(\tau+t) \\ y(\tau+t) \end{pmatrix} = \begin{pmatrix} x(\tau) \\ -y(\tau) \end{pmatrix}$$

Eigenvalues

$$\lambda = \pm \sqrt{1 + \omega^2 t^2} = \pm v$$

Eigenvectors

$$|u\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{\frac{v+1}{2v}} \\ \pm \sqrt{\frac{v-1}{2v}} \end{pmatrix}$$

$$H = \begin{pmatrix} \sqrt{\frac{v+1}{2v}} & \sqrt{\frac{v-1}{2v}} \\ \sqrt{\frac{v-1}{2v}} & -\sqrt{\frac{v+1}{2v}} \end{pmatrix} \begin{pmatrix} v & 0 \\ 0 & -v \end{pmatrix} \begin{pmatrix} \sqrt{\frac{v+1}{2v}} & \sqrt{\frac{v-1}{2v}} \\ \sqrt{\frac{v-1}{2v}} & -\sqrt{\frac{v+1}{2v}} \end{pmatrix}$$

$$U^n = e^{iH\tau n} = \begin{pmatrix} \sqrt{\frac{v+1}{2v}} & \sqrt{\frac{v-1}{2v}} \\ \sqrt{\frac{v-1}{2v}} & -\sqrt{\frac{v+1}{2v}} \end{pmatrix} \begin{pmatrix} e^{iv\tau n} & 0 \\ 0 & e^{-iv\tau n} \end{pmatrix} \begin{pmatrix} \sqrt{\frac{v+1}{2v}} & \sqrt{\frac{v-1}{2v}} \\ \sqrt{\frac{v-1}{2v}} & -\sqrt{\frac{v+1}{2v}} \end{pmatrix} = \begin{pmatrix} \cos(v\tau n) + i\sin(v\tau n) & \frac{i\omega t}{v} \sin(v\tau n) \\ \frac{i\omega t}{v} \sin(v\tau n) & \cos(v\tau n) - i\sin(v\tau n) \end{pmatrix}$$



Task 1: The fast algorithm



Orbital period $T_{op} = 6 \text{ months} = 24 \text{ weeks}$

Rotational velocity $\omega = \frac{2\pi}{T_{op}} = \frac{\pi}{12}$ radian per week

Time step $t = 1 \text{ week}$

Angle step $\omega t = \frac{\pi}{12}$ radian

Eigenvalues $\lambda = \pm 1,0337..._{10} = \pm 1,00001001..._2$

Time evolution step $T = \pi$

QPE eigenvalue $\theta = \frac{\lambda T}{2\pi} = \pm 0, \boxed{100001} 001..._2$

6-qubit register $|l\rangle$ is required

QPE returns valid $|100001\rangle$ for positive θ

Otherwise QPE returns not valid $|011111\rangle$
corresponds to $1,000000_2 - 0,100001_2 = 0,011111_2$

Trick: $|\theta| > \frac{1}{2}$ since $T = \pi$ and $v > 1$ anyway.
That's why MSB of $|l\rangle$ ($l[0]$ in the circuit)
is a kind of flag ($|1\rangle$ for $\theta > 0$ and $|0\rangle$ for $\theta < 0$)
and it acts like an extra control qubits

The trick works if $|\theta|$ is strictly greater than $\frac{1}{2}$
and it is odd. So we need non-zero MSB and LSB.
It defines a length of $|l\rangle$. Time step 1 day
(angle step $\frac{\pi}{90}$ radians) requires 12 qubits in $|l\rangle$

