Markov: Ejercicio de PageRank

```
N = matrix(RDF ,[[0, 0, 0, 0, 0.5 , 0.5, 0],[1/3, 0, 1/3, 0, 0, 1/3, 0],[0, 0, 0, 0.5, 0 , 0.5, 0],[0, 0, 0, 0, 0, 1, 0],[0.25, 0, 0,
0.25, 0, 0.25, 0.25],[0.5, 0.5, 0, 0, 0, 0, 0],[0, 0, 0, 0, 0 , 0, 0]])
```

```
N
[                0.0                0.0                0.0
0.0                0.5                0.5                0.0]
[0.333333333333333                0.0 0.333333333333333
0.0                0.0 0.333333333333333                0.0]
[                0.0                0.0                0.0
0.5                0.0                0.5                0.0]
[                0.0                0.0                0.0
0.0                0.0                1.0                0.0]
[               0.25                0.0                0.0
0.25                0.0               0.25               0.25]
[                0.5                0.5                0.0
0.0                0.0                0.0                0.0]
[                0.0                0.0                0.0
0.0                0.0                0.0                0.0]
```

```
Q = matrix(RDF ,[[0, 0, 0, 0, 0.5 , 0.5, 0],[1/3, 0, 1/3, 0, 0, 1/3, 0],[0, 0, 0, 0.5, 0 , 0.5, 0],[0, 0, 0, 0, 0, 1, 0],[0.25, 0, 0,
0.25, 0, 0.25, 0.25],[0.5, 0.5, 0, 0, 0, 0, 0],[1/7, 1/7, 1/7, 1/7, 1/7, 1/7, 1/7]])
```

```
Q
[                0.0                0.0                0.0
0.0                0.5                0.5                0.0]
[ 0.333333333333333                0.0  0.333333333333333
0.0                0.0  0.333333333333333                0.0]
[                0.0                0.0                0.0
0.5                0.0                0.5                0.0]
[                0.0                0.0                0.0
0.0                0.0                1.0                0.0]
[               0.25                0.0                0.0
0.25                0.0                0.25                0.25]
[                0.5                0.5                0.0
0.0                0.0                0.0                0.0]
[0.14285714285714285 0.14285714285714285 0.14285714285714285
0.14285714285714285 0.14285714285714285 0.14285714285714285
0.14285714285714285]
```

```
A = matrix(RDF ,[[1/7, 1/7, 1/7, 1/7, 1/7, 1/7, 1/7],[1/7, 1/7, 1/7, 1/7, 1/7, 1/7, 1/7],[1/7, 1/7, 1/7, 1/7, 1/7, 1/7, 1/7],[1/7, 1/7,
1/7, 1/7, 1/7, 1/7, 1/7],[1/7, 1/7, 1/7, 1/7, 1/7, 1/7, 1/7],[1/7, 1/7, 1/7, 1/7, 1/7, 1/7, 1/7],[1/7, 1/7, 1/7, 1/7, 1/7, 1/7, 1/7]])
A
```

[0.14285714285714285 0.14285714285714285 0.14285714285714285
0.14285714285714285 0.14285714285714285 0.14285714285714285
0.14285714285714285]
[0.14285714285714285 0.14285714285714285 0.14285714285714285
0.14285714285714285 0.14285714285714285 0.14285714285714285
0.14285714285714285]
[0.14285714285714285 0.14285714285714285 0.14285714285714285
0.14285714285714285 0.14285714285714285 0.14285714285714285
0.14285714285714285]
[0.14285714285714285 0.14285714285714285 0.14285714285714285
0.14285714285714285 0.14285714285714285 0.14285714285714285
0.14285714285714285]
[0.14285714285714285 0.14285714285714285 0.14285714285714285
0.14285714285714285 0.14285714285714285 0.14285714285714285
0.14285714285714285]
[0.14285714285714285 0.14285714285714285 0.14285714285714285
0.14285714285714285 0.14285714285714285 0.14285714285714285
0.14285714285714285]
[0.14285714285714285 0.14285714285714285 0.14285714285714285
0.14285714285714285 0.14285714285714285 0.14285714285714285
0.14285714285714285]

```
P=0.85*Q+0.15*A
P
```

[0.021428571428571425 0.021428571428571425 0.021428571428571425
0.021428571428571425   0.4464285714285714   0.4464285714285714
0.021428571428571425]
[0.30476190476190473 0.021428571428571425   0.30476190476190473
0.021428571428571425 0.021428571428571425   0.30476190476190473
0.021428571428571425]
[0.021428571428571425 0.021428571428571425 0.021428571428571425
0.4464285714285714 0.021428571428571425   0.4464285714285714
0.021428571428571425]
[0.021428571428571425 0.021428571428571425 0.021428571428571425
0.021428571428571425 0.021428571428571425   0.8714285714285714
0.021428571428571425]
[0.23392857142857143 0.021428571428571425 0.021428571428571425
0.23392857142857143 0.021428571428571425   0.23392857142857143
0.23392857142857143]
[0.4464285714285714   0.4464285714285714 0.021428571428571425

0.021428571428571425 0.021428571428571425 0.021428571428571425
0.021428571428571425]
[0.14285714285714285 0.14285714285714285 0.14285714285714285
0.14285714285714285 0.14285714285714285 0.14285714285714285
0.14285714285714285]

```
show("P=",P," Valores propios:", P.eigenvalues())
p=P.characteristic_polynomial(); show("p(x)=  ",p)
```

$$P=\begin{pmatrix} 0.021428571428571425 & 0.021428571428571425 & 0.021428571428571425 & 0.021428571428571425 & 0.4464285714285714 & 0.4464285714285714 & 0.021428571428571425 \\ 0.30476190476190473 & 0.021428571428571425 & 0.30476190476190473 & 0.021428571428571425 & 0.021428571428571425 & 0.30476190476190473 & 0.021428571428571425 \\ 0.021428571428571425 & 0.021428571428571425 & 0.021428571428571425 & 0.4464285714285714 & 0.021428571428571425 & 0.4464285714285714 & 0.021428571428571425 \\ 0.021428571428571425 & 0.021428571428571425 & 0.021428571428571425 & 0.021428571428571425 & 0.021428571428571425 & 0.8714285714285714 & 0.021428571428571425 \\ 0.23392857142857143 & 0.021428571428571425 & 0.021428571428571425 & 0.23392857142857143 & 0.021428571428571425 & 0.23392857142857143 & 0.23392857142857143 \\ 0.4464285714285714 & 0.4464285714285714 & 0.021428571428571425 & 0.021428571428571425 & 0.021428571428571425 & 0.021428571428571425 & 0.021428571428571425 \\ 0.14285714285714285 & 0.14285714285714285 & 0.14285714285714285 & 0.14285714285714285 & 0.14285714285714285 & 0.14285714285714285 & 0.14285714285714285 \end{pmatrix}$$

$\text{Valores propios: } [0.9999999999999996, -0$

$\mathbf{p(x)} = x^7 - 0.2714285714285714x^6 - 0.5264434523809524x^5 - 0.1345011160714286x^4 - 0.069180$

```
D = diagonal_matrix([P.eigenvalues()[0],
P.eigenvalues()[1],P.eigenvalues()[2],P.eigenvalues()[3],P.eigenvalues()[4],P.eigenvalues()[5],P.eigenvalues()[6]])
show("D=", D)
```

$$D=\begin{pmatrix} 0.9999999999999996 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & -0.07630724313020583 + 0.3867473130322981i & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & -0.07630724313020583 - 0.3867473130322981i & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.14764003194716535 + 0.13976875254877494i & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.14764003194716535 - 0.13976875254877494i \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \end{pmatrix}$$

```
show(P.eigenvectors_right())
```

$[(0.9999999999999996, [(0.37796447300922703, 0.3779644730092271, 0.3779644730092272, 0.3779644730092272, 0.3779644730092272, 0.3779644730092273, 0.3779644730092272)], 1), (-0.07630724313020583 + 0.3867473130322981i, [($

```
show(P.eigenmatrix_right()[0])
show(P.eigenmatrix_right()[1])
```

$$\begin{pmatrix}
0.9999999999999992 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & -0.5581138939084768 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & -0.31312311229687145 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.14764003194716557 + 0.13976875254877508i & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.14764003194716557 - 0.13976875254877508i \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -0.07630724313020547 + 0.3867473130322981 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0
\end{pmatrix}$$

$$\begin{pmatrix}
0.3779644730092266 & 0.5248708240052737 & 0.6106823944521187 & -0.40023294601730247 + 0.10266435964507062i & -0.40023294601730247 - 0.10266435964507062i & 0.025468139840950456 + 0.042393535598 \\
0.3779644730092274 & 0.03380545585983735 & -0.618181132612792 & 0.3450842814367441 - 0.06359760423228189i & 0.3450842814367441 + 0.06359760423228189i & -0.017915206436619646 - 0.304718866045 \\
0.3779644730092271 & -0.1759162617514834 & 0.06449464096768785 & 0.573337436626127 & 0.573337436626127 & 0.627277635533 \\
0.3779644730092273 & 0.6532512280726186 & -0.06314833862727562 & 0.17664906660165494 + 0.14428458627115232i & 0.17664906660165494 - 0.14428458627115232i & 0.1362492403167753 + 0.55722453535 \\
0.3779644730092703 & -0.267028793650858 & -0.46555777220252603 & -0.19532089075152298 - 0.14022682211160617i & -0.19532089075152298 + 0.14022682211160617i & 0.2057241521357897 + 0.00197009006849 \\
0.3779644730092274 & -0.4356191165110211 & 0.030893936745950108 & -0.008606369300973441 + 0.06394939167733839i & -0.008606369300973441 - 0.06394939167733839i & -0.28265915654117896 + 0.0103441104798 \\
0.3779644730092764 & -0.06793908909205087 & 0.13810631563043868 & 0.12646193974117087 - 0.4974304195799171i & 0.12646193974117087 + 0.4974304195799171i & -0.024086728490666495 - 0.242754344571
\end{pmatrix}$$

```
q1=(P.eigenvectors_right()[0])[1][0];
q2=(P.eigenvectors_right()[1])[1][0];
q3=(P.eigenvectors_right()[2])[1][0];
q4=(P.eigenvectors_right()[3])[1][0];
q5=(P.eigenvectors_right()[4])[1][0];
q6=(P.eigenvectors_right()[5])[1][0];
q7=(P.eigenvectors_right()[6])[1][0];
show("q1=",q1," q2=",q2," q3=",q3," q4=",q4)
```

q1= $(0.377964473009922703, 0.3779644730092271, 0.3779644730092272, 0.3779644730092272, 0.3779644730092272, 0.3779644730092273, 0.3779644730092272)$ q2= $(0.02546813984094988 + 0.042393535598928805i, -0.01791520643661$

```
q1
```

$(0.37796447300922703, 0.3779644730092271, 0.3779644730092272,$
$0.3779644730092272, 0.3779644730092272, 0.3779644730092273,$
$0.3779644730092272)$

```
Q=column_matrix([q1,q2,q3,q4,q5,q6,q7]);
show("Q=",Q)
print norm(P-Q*D*Q.inverse())<=10^(-10)
```

$$Q=\begin{pmatrix}
0.37796447300922703 & 0.02546813984094988 + 0.042393535598928805i & 0.02546813984094988 - 0.042393535598928805i & 0.4002329460173027 - 0.10266435964507063i & 0.4002329460173027 + 0.1026643596450 \\
0.3779644730092271 & -0.017915206436619313 - 0.30471886604579157i & -0.017915206436619313 + 0.30471886604579157i & -0.3450842814367445 + 0.06359760423228192i & -0.3450842814367445 - 0.0635976042322 \\
0.3779644730092272 & 0.627277635533867 & 0.627277635533867 & -0.5733374366261274 & -0.57333743662 \\
0.3779644730092272 & 0.13624924031677563 + 0.5572245353500328i & 0.13624924031677563 - 0.5572245353500328i & -0.1766490666016553 - 0.1442845862711522i & -0.1766490666016553 + 0.14428458627 \\
0.3779644730092272 & 0.20572415213579 + 0.0019700900684927053i & 0.20572415213579 - 0.0019700900684927053i & 0.19532089075152242 + 0.1402268221116058i & 0.19532089075152242 - 0.14022682211 \\
0.3779644730092273 & -0.2826591565411786 + 0.01034411047981515i & -0.2826591565411786 - 0.01034411047981515i & 0.008606369300973235 - 0.063949391677733815i & 0.008606369300973235 + 0.06394939167 \\
0.3779644730092272 & -0.02408672849066648 - 0.24275434457122982i & -0.02408672849066648 + 0.24275434457122982i & -0.12646193974117104 + 0.4974304195799165i & -0.12646193974117104 - 0.49743041957
\end{pmatrix}$$

    True

```
mo=matrix(RDF,[1, 0, 0, 0, 0, 0, 0])
```

```
show(mo*Q*D^100*Q.inverse())
```

$\big(\ 0.22198000140591 8 - 2.5194642450745767 \times 10^{-17}i \quad 0.15271641500884678 - 3.67794187950405 \times 10^{-17}i \quad 0.07125255257058884 - 1.4937348970053177 \times 10^{-17}i \quad 0.08425917197181364 + 4.196248604 2513876 \times 10^{-17}i \quad 0.1$

```
z=
(0.22198000140591 8,0.15271641500884678,0.07125255257058884,0.08425917197181364,0.12232440224893072,0.29349061966454393,0.05397683712931 336)
```

```
z
```

    (0.221980001405918,
    0.1527164150088468,
    0.0712525525705888,
    0.0842591719718136,
    0.1223244022489307,
    0.2934906196645439,
    0.0539768371293134)

```
sum(z)
```

    0.99999999999955

OTRO MÉTODO

```
G=P.transpose()
```

```
show("G=",G," Valores propios:", G.eigenvalues())
p=G.characteristic_polynomial(); show("p(x)=  ",p)
```

$$G=\begin{pmatrix} 0.021428571428571425 & 0.30476190476190473 & 0.021428571428571425 & 0.021428571428571425 & 0.23392857142857143 & 0.4464285714285714 & 0.14285714285714285 \\ 0.021428571428571425 & 0.021428571428571425 & 0.021428571428571425 & 0.021428571428571425 & 0.021428571428571425 & 0.4464285714285714 & 0.14285714285714285 \\ 0.021428571428571425 & 0.30476190476190473 & 0.021428571428571425 & 0.021428571428571425 & 0.021428571428571425 & 0.021428571428571425 & 0.14285714285714285 \\ 0.021428571428571425 & 0.021428571428571425 & 0.4464285714285714 & 0.021428571428571425 & 0.23392857142857143 & 0.021428571428571425 & 0.14285714285714285 \\ 0.4464285714285714 & 0.021428571428571425 & 0.021428571428571425 & 0.021428571428571425 & 0.021428571428571425 & 0.021428571428571425 & 0.14285714285714285 \\ 0.4464285714285714 & 0.30476190476190473 & 0.4464285714285714 & 0.8714285714285714 & 0.23392857142857143 & 0.021428571428571425 & 0.14285714285714285 \\ 0.021428571428571425 & 0.021428571428571425 & 0.021428571428571425 & 0.021428571428571425 & 0.23392857142857143 & 0.021428571428571425 & 0.14285714285714285 \end{pmatrix}$$ Valores propios: $[0.9999999999999992, -0.5$

$p(x){=}x^7 - 0.2714285714285714x^6 - 0.5264434523809524x^5 - 0.13450111607142856x^4 - 0.0691$

```
show(G.eigenmatrix_right()[0])
show(G.eigenmatrix_right()[1])
```

$$\left(\begin{array}{ccccc}
0.9999999999999996 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & -0.07630724313020583 + 0.3867473130322981i & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & -0.07630724313020583 - 0.3867473130322981i & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.14764003194716535 + 0.13976875254877494i & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.14764003194716535 - 0.13976875254877494i \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0
\end{array}\right.$$

$$\left(\begin{array}{ccccc}
0.510889853011873 & 0.29815234795008116 - 0.19623610088094673i & 0.29815234795008116 + 0.19623610088094673i & -0.30072315688536944 + 0.1762104527211478i & -0.30072315688536944 - 0.176210452721147 \\
0.351478810352939392 & 0.481737695976532 & 0.481737695976532 & -0.2541976388501714 - 0.16712176886840552i & -0.2541976388501714 + 0.167121768868405 \\
0.163988674109918182 & -0.01648160138551999 - 0.32894400823653536i & -0.01648160138551999 + 0.32894400823653536i & -0.13331627845223273 - 0.19451172919423057i & -0.13331627845223273 + 0.194511729194230 \\
0.193923573984166 & -0.41051293054690313 + 0.24030142791665232i & -0.41051293054690313 - 0.24030142791665232i & 0.17299482023396617 - 0.10369017491117423i & 0.17299482023396617 + 0.103690174911174 \\
0.281531198706429 & -0.21924550903864273 - 0.263656019766981i & -0.21924550903864273 + 0.263656019766981i & 0.08078361906087259 + 0.4307667500826174i & 0.08078361906087259 - 0.43076675008261 \\
0.675472468650242434 & -0.06763201220275353 + 0.39431568033619746i & -0.06763201220275353 - 0.39431568033619746i & -0.22046553609164982 - 0.1416535298299549i & -0.22046553609164982 + 0.141653529829954 \\
0.124228390901701033 & -0.06601799075279421 + 0.15421902063161347i & -0.06601799075279421 - 0.15421902063161347i & 0.6549241709845856 & 0.65492417098458
\end{array}\right.$$

```
e1=(G.eigenvectors_right()[0])[1][0];
```

```
e1
```

(0.510889853011387, 0.35147881035293954, 0.16398867410991863, 0.19392357739841667, 0.28153119870642523, 0.6754724686502435, 0.12422839090170118)

```
sum(e1)
```

2.301512973131032

```
SOL=1/(sum(e1))*e1
```

```
SOL
```

(0.22198000140592758, 0.15271641500885375, 0.07125255257059213, 0.08425917197181754, 0.1223244022489361, 0.293490619664557, 0.05397683712931584)