

# Exercise 4 - Game Theory

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We are given two properties:

- Monotonicity: "the more the better"  
let  $P' \subset P$  then  $P \succ P'$
- Continuity:  
let  $P \succ C \setminus P$  (which mean the whole cake without the "P" piece, for example because he likes the cherry a lot!) then there is a  $P' \subset P$  for which we have that  $P' \succ C \setminus P'$

We have to prove or disprove two statements.

- **Statement 1:** In any subgame perfect equilibrium player 2 is indifferent between the two pieces of cake offered by player 1 (which recall is the one cutting the cake).

**The statement is true! - prove by showing it could never be otherwise**

Step 1: Suppose that the statement is false, and hence that player 2 may not be indifferent between the two pieces. We can write this as  $P_1 \succ_2 P_2$ , signifying that player 2 prefers the first piece to the second. Given the preference, player 2 would want to pick  $P_1$ .

Step 2: Because of continuity we know that when a player has a preference for a given piece, in this case  $P_1$ , we can find a  $P'_1 \subset P_1$  s.t.  $P'_1 \succ C \setminus P'_1$ .

Step 3:  $P'_1 \subset P_1$  it is a "smaller piece" and hence player 1 would be better off to offer  $P'_1$  instead of  $P_1$ . This mean that whenever player 2 is not indifferent between the two pieces, under monotonicity and continuity, player 1 has a profitable deviation (offering the smaller piece).

Step 4: Because player 1 has a profitable deviation the cut of the cake when player 2 is not indifferent cant be a SBPNE. Eventually, we have shown that in any SBPNE player 2 is actually indifferent because otherwise there would be a profitable deviation.

- **Statement 2:** In any subgame perfect equilibrium, player 1 is indifferent between  $P_1$  and  $P_2$ .

**The statement is false! - disprove by finding an example for which is false**

Step 1: Suppose that a certain share of the cake is preferred to player 1 (for example he likes a certain decoration that is only on the bottom left of the cake). We write this as  $P'_1 \succ_1 C \setminus P'_1$ . Player 2 on the other hand only care about the size of the piece.

Step 2: Player 1 which is the "cutter" could then cut the cake into  $P'_1$  and  $C \setminus P'_1$ .

Step 3: Because of monotonicity player 2 would grab  $C \setminus P'_1$  and player 1 would get his preferred piece of cake  $P'_1$ . This is a SPNE but player 1 is not indifferent between the two pieces (he had a preference for the decoration part), hence the statement is false.