Handout 4: CSci 242. Algorithms and Data Structures **Spring, 2020**

Date: 1-22-2020 Instructor: Dr. M. E. Kim

Computation of Running Time: MaxSubSlow, MaxSubFast and MaxSubFastest

1. **MaxsubSlow:**

   (a)

   for $j \leftarrow 1$ **to** n **do**

       for $k \leftarrow j$ **to** n **do**

           $s \leftarrow 0$

           for $i \leftarrow j$ **to** k **do**

               $s \leftarrow s + A[i]$

           if $s > m$ **then**

               $m \leftarrow s$.

   (b) $\sum_{j=1}^{n}\sum_{k=j}^{n}\sum_{i=j}^{k} c$ where c is a positive constant

$$= \sum_{j=1}^{n}\sum_{k=j}^{n} c(k-j+1)$$

$$= \sum_{j=1}^{n}(c\sum_{k=j}^{n} k + c\sum_{k=j}^{n} (1-j))$$

where $c\sum_{k=j}^{n} k = c\sum_{k=1}^{n} k - c\sum_{k=1}^{j-1} k = c(\frac{n(n+1)}{2} - \frac{j(j-1)}{2}) = \frac{c}{2}(n^2+n-j^2+j)$,

and $c\sum_{k=j}^{n} (1-j) = c(1-j)\sum_{k=j}^{n} 1 = c(1-j)(n-j+1) = c(n+1-(n+2)j+j^2)$

$$= c\cdot\sum_{j=1}^{n} \frac{1}{2}j^2 - (n+\frac{3}{2})j + (\frac{1}{2}n^2+\frac{3}{2}n+1)$$

$$= c\cdot\sum_{j=1}^{n} \frac{1}{2}j^2 - c\cdot\sum_{j=1}^{n} (n+\frac{3}{2})j + c\cdot\sum_{j=1}^{n} \frac{1}{2}n^2+\frac{3}{2}n+1)$$

$$= c\cdot(\frac{1}{2}\cdot\frac{n(n+1)(2n+1)}{6}) - c\cdot((n+\frac{3}{2})\cdot\frac{n(n+1)}{2}) + c\cdot(\frac{1}{2}n^2+\frac{3}{2}n+1)\cdot n)$$

$$= c\cdot(\frac{1}{6}n^3+\frac{1}{2}n^2+\frac{1}{3}n)$$

$$= O(n^3).$$

2. **MaxsubFast:**

   (a)
   for $j \leftarrow 1$ **to** n **do**
   $\quad S_i \leftarrow S_{i-1} + A[i]$
   for $j \leftarrow 1$ **to** n **do**
   $\quad$ **for** $k \leftarrow j$ **to** n **do**
   $\quad\quad s \leftarrow S_k - S_{j-1}$
   $\quad\quad$ **if** $s > m$ **then**
   $\quad\quad\quad m \leftarrow s.$

   (b)
$$
\begin{aligned}
\sum_{j=1}^{n} c_1 + \sum_{j=1}^{n}\sum_{k=j}^{n} c_2 &= c_1 n + \sum_{j=1}^{n} c_2 \cdot (n-j+1) \\
&= c_1 n + \sum_{j=1}^{n} c_2 \cdot (n+1) - \sum_{j=1}^{n} c_2 \cdot j \\
&= c_1 n + c_2 \cdot (n+1)n - c_2 \cdot \tfrac{n(n+1)}{2} \\
&= c_1 n + \ c_2 \cdot \tfrac{n(n+1)}{2} \\
&= \tfrac{c_2}{2} n^2 + \ (c_1 + \tfrac{c_2}{2}) n \\
&= O(n^2).
\end{aligned}
$$

3. **MaxsubFastest:**

   (a)
   for $t \leftarrow 1$ **to** n **do**
   $\quad M_t \leftarrow \max\{0, M_{t-1} + A[t]\}$
   $m \leftarrow 0$
   for $t \leftarrow 1$ **to** n **do**
   $\quad m \leftarrow \max\{m, M_t\}$

   (b)
$$
\begin{aligned}
\sum_{t=1}^{n} c_1 + \sum_{t=1}^{n} c_2 &= (c_1 + c_2) \cdot \sum_{t=1}^{n} 1 \\
&= (c_1 + c_2) \cdot n \\
&= O(n)
\end{aligned}
$$