Handout 2 **January 15th, 2020**

CSci 242. Algorithms and Data Structure **Dr. M. E. Kim**

Analysis of Running Time of Loops: the number of operations.

Case A:

Statement	Running Time (i.e. # of Operations)
1. for $i \leftarrow 0$ to n	$1 + (\sum_{i=0}^{n+1} 1) = (n+2)+1 = n+3$ -- initialization of i and comparison ($i < n+1$) until $i = (n+1)$
2. k ← k+1 ;	$\sum_{i=0}^{n} 2 = 2(n+1)$ – assignment & addition

Case B:

Statement	Running Time
1. for $i \leftarrow 1$ to n	$1 + (\sum_{i=1}^{n+1} 1) = n+2$ -- initialization of i and comparison ($i < n+1$) until $i = (n+1)$
2. k ← k+1 ;	$\sum_{i=1}^{n} 2 = 2n$ – assignment & addition

Case C:

Statement	Running Time
1. for $i \leftarrow 1$ to n	$1 + (\sum_{i=1}^{n+1} 1) = n+2$ -- initialization of i and comparison ($i < n+1$) until $i = (n+1)$
2. for $j \leftarrow 1$ to n	$\sum_{i=1}^{n}(1 + (\sum_{j=1}^{n+1} 1)) = \sum_{i=1}^{n}(n+2) = n(n+2)$
3. k ← k+1;	$\sum_{i=1}^{n}(\sum_{j=1}^{n} 2) = \sum_{i=1}^{n} 2n = 2n^2$

Case D:

Statement	Running Time
1. for $i \leftarrow 1$ to n	$1 + (\sum_{i=1}^{n+1} 1) = n+2$
2. for $j \leftarrow 1$ to i	$\sum_{i=1}^{n} (1 + (\sum_{j=1}^{i+1} 1)) = \sum_{i=1}^{n}(i+2) = n(n+5)/2$ - initialization of j and comparison ($j < i+1$) until $j = (i+1)$ within an outer for loop of i.
3. k ← k+1;	$\sum_{i=1}^{n} (\sum_{j=1}^{i} 2) = \sum_{i=1}^{n} 2i = n(n+1)$

Case E:

Statement	Running Time
1. for $i \leftarrow 1$ to s	$1 + (\sum_{i=1}^{s+1} 1) = s+2$
2. for $j \leftarrow 1$ to n	$\sum_{i=1}^{s} (1 + (\sum_{j=1}^{n+1} 1)) = \sum_{i=1}^{s}(n+2) = s(n+2)$
3. k ← k+1;	$\sum_{i=1}^{s} (\sum_{j=1}^{n} 2) = \sum_{i=1}^{s} 2n = 2sn$

Case F:

	Statement	Running Time	
	$i \leftarrow 0, k \leftarrow 0$		
1.	while $(i < n)$	$n+1$	-- comparison until $i=n$ from $i=0$..
2.	$k \leftarrow k + i;$	$\sum_{i=0}^{n-1} 2 = 2n$	-- assignment & addition.
3.	$i \leftarrow i + 1$	$\sum_{i=0}^{n-1} 2 = 2n$	-- assignment & addition.

Case G:

	Statement	Running Time	
	$j \leftarrow 0, k \leftarrow 0$		
1.	for $i \leftarrow 1$ to n	$1 + (\sum_{i=1}^{n+1} 1) = n + 2$-	- initialization of i and comparison $(i < n+1)$ until $i = (n+1)$
2.	while $(j \leq n)$	$\sum_{j=1}^{n} (n+2)$	- comparison until $j=n+1$ from $j=0$.
3.	$k \leftarrow k+1;$	$\sum_{i=1}^{n} \sum_{j=0}^{n} 2 = \sum_{i=1}^{n} 2(n+1) = 2n(n+1)$	
4.	$j \leftarrow j+1;$	$\sum_{i=1}^{n} \sum_{j=0}^{n} 2 = \sum_{i=1}^{n} 2(n+1) = 2n(n+1)$	

Case H:

	Statement	Running Time	
	$j \leftarrow 1$		
1.	while $(j \leq n)$	$n+1$	- comparison until j=n+1 from j=1
2.	for $i \leftarrow 1$ to j	$\sum_{j=1}^{n} (1 + \sum_{i=1}^{j+1} 1) = \sum_{j=1}^{n} (j+2) = n(n+5)/2$	
		- initialization of i and comparison $(i < j+1)$ until $i=(j+1)$ within an outer while loop of j.	
3.	$k \leftarrow k+1;$	$\sum_{j=1}^{n} (\sum_{i=1}^{j} 2) = \sum_{i=1}^{n} 2j = n(n+1)$	
4.	$j \leftarrow j+1$	$\sum_{j=1}^{n} (2) = 2n$	-- assignment & addition within while loop.

Case I:

	Statement	Running Time
1.	for $i \leftarrow 1$ to n	$1 + (\sum_{i=1}^{n+1} 1) = n+2$
2.	for $j \leftarrow i$ to n	$\sum_{i=1}^{n} ((1 + \sum_{j=i}^{n+1} 1)) = \sum_{i=1}^{n} (1 + (n + 1 - (i-1))) = \frac{n^2+5n}{2}$
3.	$k \leftarrow k+1;$	$\sum_{i=1}^{n} (\sum_{j=i}^{n} 2) = \sum_{i=1}^{n} 2(n - i + 1) = n(n+1)$