Handout 5 **January 22nd , 2020**

CSci 242. Algorithms and Data Structure **Dr. M. E. Kim**

Running Time of the *Recursive* algorithm: Recurrence

Algorithm recursiveMax(A, n):
  Input: An array A storing $n \geq 1$ integers.
  Output: The maximum element in A.
 if $n = 1$ **then**
  return $A[0]$
 return $\max\{$recursiveMax$(A, n-1),\ A[n-1]\}$

The number of primitive operations:

Comparison	: 1	: n=1
Indexing	: 2	: A[0], A[n-1]
Recursive call	: 1	: recursiveMax(A, n-1)
Max calculation:	1	: max
Return	: 2	: return, return

Running time of Recursive algorithm in Recurrence Equation: recursiveMax.

$$T(n) = \begin{cases} 3 & \text{if } n = 1 \\ T(n-1) + 7 \text{ (or } 5\text{?)} & \text{otherwise} \end{cases} \qquad \text{(eq. 1)}$$

$T(n) = T(n\text{-}1) + 7$

$= \{\, T(n\text{-}2) + 7 \,\} + 7 \quad = T(n\text{-}2) + 2 \cdot 7$

$= \{\, T(n\text{-}3) + 7 \,\} + 2 \cdot 7 = T(n\text{-}3) + 3 \cdot 7$

$= \{\, T(n\text{-}4) + 7 \,\} + 3 \cdot 7 = T(n\text{-}4) + 4 \cdot 7$

$= \{\, T(n\text{-}5) + 7 \,\} + 4 \cdot 7 = T(n\text{-}5) + 5 \cdot 7$

$= \ \ldots$

$= \{\, T(n - (n\text{-}1)) + 7\} + (n\text{-}2) \cdot 7 \ = T(1) + (n\text{-}1) \cdot 7$

$= 3 + 7\,(n\text{-}1)$

$= 7n - 4$

Therefore, the ***solution*** of the recurrence equation in (eq. 1) is: $T(n) = 7n - 4.$

Proof of the Solution by Mathematical Induction:

For the recurrence equation $T(n) = \begin{cases} \mathbf{3} & \text{if } n = 1 \\ \boldsymbol{T(n-1) + 7} \text{ (or } \mathbf{5}?\text{)} & \text{otherwise} \end{cases}$

the solution is $\mathbf{T(n) = 7n - 4}$

Base Case: $n = 1$

$T(n) = 7n - 4 = 7 \cdot 1 - 4 = 3$

which is equal to $T(n) = 3$ if $n=1$ in the given recurrence equation.

Thus, $T(n) = 7n - 4$ is true for $n=1$

Inductive Hypothesis:

Assume that $T(n) = 7n - 4$ is true for any $k \le n$ in the given recurrence equation.

i.e. $T(k) = 7k - 4$ is true for any $k \le n$.

Inductive Step:

Let's prove that $T(n) = 7n - 4$ is true for $k+1$ where $k \le n$.

$T(k+1) = T(k) + 7$ by means of the definition of given recurrence equation

$= (7k - 4) + 7$ by Inductive Hypothesis

$= 7k + 7 - 4$

$= 7(k+1) - 4$.

Thus, $T(n) = 7n - 4$ hold for $n = k+1$.

Therefore, $T(n) = 7n - 4$ holds for any $n \ge 1$, which is a solution of the given recurrence equation.