# Multi-Agent Ergodic Exploration of Dynamic Environments

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*Abstract*— **The ability of multi-agent drone teams to intelligently explore dynamic environments is critical for applications including disaster response, environmental monitoring, and mapping. For these applications, information-based exploration methods are often preferred, meaning robot trajectories are planned according to how information is distributed across a space. Much work on informative exploration of dynamic settings focuses on how the distribution of information in a space changes according to a known model of an environmental process. However, an equally important problem is how a robot's sensing capabilities change according to a known process model. In this work, we consider the problem of multi-agent, informative path planning (IPP) for both dynamic information distributions and dynamic sensor models. We leverage ergodic trajectory optimization (ETO), which generates paths such that the amount of time an agent spends in an area is proportional to the expected information in that area. We focus specifically on the problem of multi-agent drone search of a wildfire, where the flow of smoke acts as an environmental process model. The environmental process model is used to calculate an expected information distribution (EID) to be used in the ETO algorithm. Our experiments show that our exploration method reduces uncertainty faster than both baseline search methods and naive ergodic search formulations that do not leverage the sensor process model to calculate expected information.**

## I. INTRODUCTION

The use of drones in informative exploration of extreme natural environments has become popular due to their relatively low cost and ability to reach areas too unsafe for humans [1]. Multi-drone systems are often preferred for the exploration of extreme natural environments for a few reasons: drones can be damaged or destroyed during the search process, and so using a multi-drone team provides critical redundancies; natural environments often have different types of information distributed on different physical scales, and so specialized, heterogeneous drone teams can be used to more effectively explore such environments; and the scale of extreme natural environments is frequently large, requiring many drones to effectively cover the exploration area. However, path planning for multi-drone systems in such environments is often complicated by the dynamic nature of the environment. Dynamic factors, like weather events or moving targets, can change both the composition of the exploration space as well as the capabilities of the multi-agent team. As such, dynamic environmental factors must be accounted for in the path planning algorithm in order for drones to successfully explore extreme environments. In this work, we focus on path planning with regard to two critical dynamic environmental factors: information distribution and sensor effectiveness.

In IPP problems, environmental changes can be encoded in environmental process models, which describe the evolution of a process in the exploration area over time [2]. Much prior work on IPP focuses on how information or obstacles in an exploration space will change according to an environmental process model [3]. However, given that robots are not isolated from the changes in the environment they are exploring, it is equally critical to consider how the effectiveness of a robot's sensor will change with some environmental process. Issues including decreased power availability, visibility occlusions, exposure to hazardous chemicals, or extreme temperatures can all dramatically reduce the performance of a robot's sensor [4].

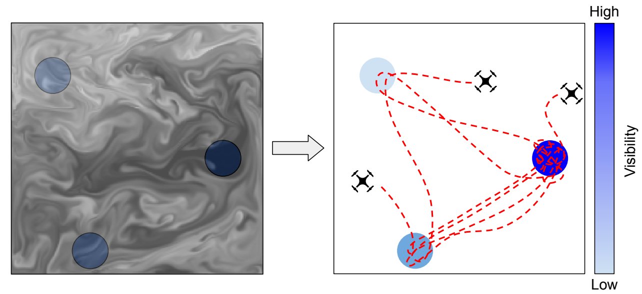


Fig. 1: **Multi-Agent Exploration with Dynamic Sensing.** Three targets have varying visibility according to the concentration of smoke in a search region. Given that less information can be captured when observing targets with low visibility, the multi-drone team prioritizes exploring targets with high visibility.

We consider the motivating problem of multi-drone exploration of a wildfire environment. The frequency and severity of wildfires has increased in recent years [5], and so the use of autonomous robots for wildfire monitoring or search and rescue is particularly critical [6]. For both of these problems, the aim of the multi-drone team is to gain confidence in the predicted state (position, velocity, etc) of a set of targets (humans, buildings, fire sources, etc). This is accomplished by taking sensor measurements that reduce uncertainty in the target belief state. However, the smoke emitted by the wildfire can occlude the drones' visual sensors, lessening their ability to capture new images and thus reduce uncertainty. In this work, we model the drones' changing sensor effectiveness by calculating expected visibility according to the density of smoke in an area. We then use multi-agent ergodic trajectory optimization (ETO) to plan paths for drones over a changing uncertainty map, leveraging the smoke model to calculate an expected information distribution (EID) used by the ETO algorithm

In ETO, trajectories are generated from an underlying information distribution so that the amount of time a robot spends in a region is proportional to the distribution of information in that region. As such, ETO balances exploitation of high-information regions with coverage of the exploration space. This is a valuable characteristic for wildfire exploration, as it both enables drones to take advantage of a-priori information for efficient information gain while still maintaining the ability to explore low-uncertainty areas which may contain unforeseen, valuable information.

The primary contributions of this paper are as follows:

1) The development of a multi-agent ETO algorithm that utilizes a dynamic uncertainty map and dynamic sensor measurements;
2) A general approach for calculating expected information from an environmental-process-driven sensor model; and
3) An illustration of our exploration method on simulated wildfire smoke.

## II. RELATED WORK

**Informative Path Planning:** Using information acquired in an environment to inform path planning is a widely investigated problem with applications to mapping, navigation, and localization [7]. In much prior work, the goal of IPP is to find and measure near the highest concentration of some spatial process [8] [9] [10]. This approach is useful for environmental process monitoring and feature detection. For the task of exploration, in which the robot system seeks to widely observe some spatially-distributed phenomenon, a more common goal of information-theoretic planning is to minimize entropy/uncertainty using sensor observations at optimally-selected locations [11], [12], [13], [14]. For the application of wildfire exploration, uncertainty reduction is a useful goal as the state of targets in the search environment is generally highly uncertain.

Some prior works that use IPP for uncertainty reduction leverage Bayesian Optimization (BO) [8] [15]. This approach iteratively samples from an objective to find a maximum without consideration of how an agent's movement constraints affect possible sampling locations. Many works use optimal rapidly-exploring random trees (RRT*) [16] for IPP problems [17] [18], which finds paths from start nodes to target nodes with regard to a control-effort cost function. More advanced than RRT* is Monte Carlo Tree Search (MCTS) [19] [20], which generates long-term measurement strategies by considering potential future measurements. Graph-based search methods, which discretize the exploration spaces into graphs and visit nodes within these graphs, have also widely been used for IPP problems [21] [22]. For the problem of wildfire exploration, we are interested in a planning method that does not solely prioritize information gain (like BO) or exploration space coverage (like graph-based methods), that does not neglect the movement constraints of a robot in planning sampling locations (like MCTS), and that does not require defined target node positions to plan paths (like RRT*) as the exploration space will continuously change.

For the sake of this work, we choose to use ergodic search. As previously described, in ergodic search, the time-averaged trajectories of a robotic system are planned to be proportional to an information distribution [23]. The benefits of ergodic search in comparison to other IPP methods are that it balances exploitation of high-entropy regions with coverage of low-entropy portions of the map, improving exploration outcomes; and it accounts for the robot's dynamic constraints, generating dynamically feasible trajectories. Additionally, ergodic search can be used to re-plan trajectories for an information map which is dynamically changing, which is critical for our work as the entropy distribution is continuously changing due to sensor measurement.

**Path Planning for Dynamic Maps:** Many prior works have focused on the problem of exploration of maps whose information or target distribution changes over time [10] [24] [25]. [2] presents a multi-agent exploration algorithm for predicting a dynamic environmental phenomenon modeled as a Dirichlet process mixture of multiple Gaussian processes. The authors aim to plan optimal sampling locations for prediction of a spatially-varying phenomenon. Meanwhile, we seek to leverage a known spatially-varying phenomenon to better plan a series of samples proportional to an uncertainty distribution.

Ergodic search has been extended to the problem of exploring dynamic information maps [26] [27] [28]. [29] uses multi-agent ergodic search to localize a single moving target by finding measurements that have a high probability of reducing uncertainty about target location. Our work also focuses on gaining knowledge of a target through measurements that reduce uncertainty in the state of the target, but we extend this approach to maps containing multiple static and moving targets. [30] proposes a Dynamic Multi-Objective Ergodic Search (D-MO-ES) algorithm for single agent exploration of dynamic information maps with multiple dynamic objectives. Our work extends exploration over dynamic uncertainty to multiple agents.

**Dynamic Sensor Modeling:** [31] plans optimal waypoints for measurement of a spatial phenomenon in a time-varying field, in which the utility of a measurement captured at a location is time-dependent. [32] proposes a Gaussian Processes occupancy mapping method that uses mutual information calculated from future map predictions based on a 2D range-finder sensor model. [33] introduces a terrain monitoring approach with an altitude-dependent sensor model in which uncertainty increases with camera distance to a target. Each of these works uses a variation of greedy search, attempting to find the next-best point for information gain. For our application, imperfect knowledge of the state of targets or the environmental process model can make it so that the EID does not always reflect the true distribution of information in the environment. Any pockets of information not encoded in the EID would be entirely missed by greedy search, which targets expected information peaks. Our method overcomes this issue by encouraging coverage alongside exploitation, increasing the odds that unexpected regions of useful information will be discovered.

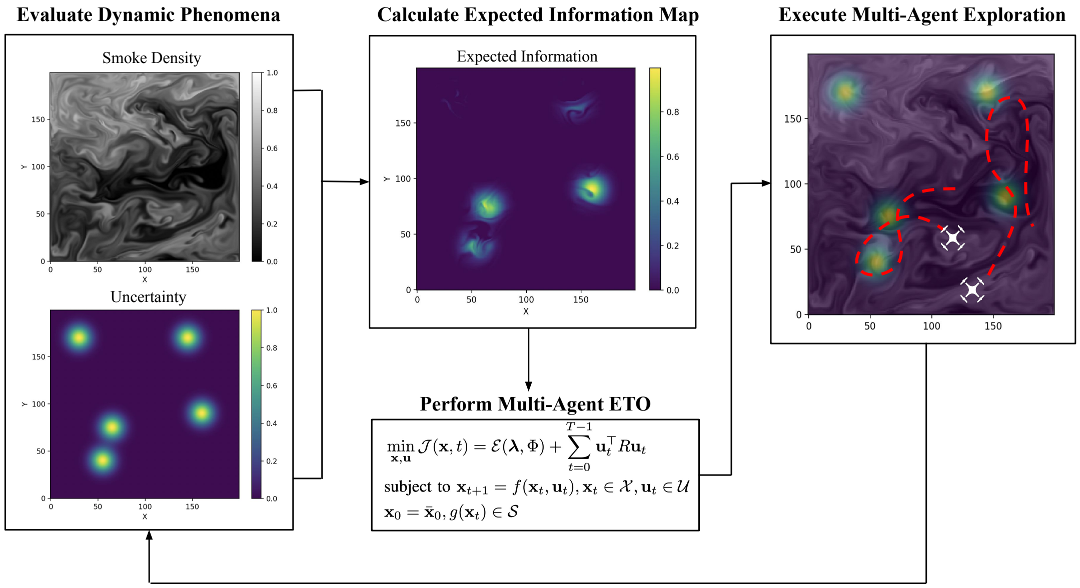


Fig. 2: **Dynamic Multi-Agent Ergodic Exploration.** Shown is a flow chart of the dynamic ergodic exploration process. To begin, the underlying uncertainty distribution and smoke model are used to calculate the EID. Using this distribution, multi-agent ETO is performed. The agents follow these generated trajectories, and the information map is updated as agents measure new information.

In [26], Miller and Murphey propose an ergodic exploration of distributed information (EEDI) algorithm that plans trajectories for robots with nonlinear sensor dynamics. The EEDI algorithm is tested on a fish-inspired robot which uses electrolocation as a sensing technique. The authors use Fisher information to calculate predicted measurement utility from sensor efficacy at a point and re-plan trajectories according to the evolving expected information distribution. The EEDI algorithm is tested on a single robot for the task of localization of stationary targets. Our work extends the problem of ergodic exploration with dynamic sensor modeling to the task of multi-agent exploration of moving targets.

## III. BACKGROUND

### A. Multi-Agent Ergodic Trajectory Optimization

To begin, for a system of $N$ robots, we define the robots' states at time $t$ as $\mathbf{x}_t = \{x_1, x_2, ..., x_N\}_t = [\boldsymbol{\lambda}, \boldsymbol{\xi}]_t^T \in \mathcal{X} \subset \mathbb{R}^n$, where $\boldsymbol{\lambda}$ are trajectories and $\boldsymbol{\xi}$ are states that are not in the exploration domain (e.g., velocities). We define control inputs to the robots as $\mathbf{u_t} = \{u_1, u_2, ..., u_N\}_t \in \mathcal{U} \subset \mathbb{R}^m$. The set of states of the robots over a time horizon $T$ are given as

$$\mathbf{x}_{t+1} = f(\mathbf{x}_t, \mathbf{u}_t) \tag{1}$$

where $f : \mathcal{X} \times \mathcal{U} \rightarrow \mathcal{X}$ are the dynamics of the robots. Additionally, we define the exploration space of the robots as $\mathcal{S} \in [0, L_0] \times \ldots [0, L_{v-1}]$ where the space is bounded by $L_i$ and has a dimension of $v \leq n$. Associated with this exploration space is an information density $\Phi(\mathcal{S})$; methods for calculating information density are described later in this section. Formally, the trajectories $\boldsymbol{\lambda}$ are ergodic when the following equation holds:

$$\lim_{T\to\infty} \frac{1}{T} \sum_{i=1}^{N} \sum_{t=0}^{T-1} h(\lambda_t) = \sum_{i=1}^{N} \int_{\mathcal{S}} \Phi(\lambda) h(\lambda) d\lambda \tag{2}$$

for all Lebesque integrable functions $h \in \mathcal{L}^1$ [34].

To quantify the ergodicity of a set of trajectories $\boldsymbol{\lambda}$, we consider the time-averaged statistics $C_k$ of $\boldsymbol{\lambda}$:

$$c_k(t) = \frac{1}{T} \sum_{i=1}^{N} \int_0^T F_k(\lambda_i(t)) dt \tag{3}$$

where $F_k(\lambda) = \prod_{i=0}^{v-1} \cos(\lambda_i k_i \pi / L_i)/h_k$ is the cosine Fourier transform for the $k^{\text{th}}$ mode and $h_k$ is a normalization factor [35].

To drive trajectories to ergodicity, we optimize over $\boldsymbol{\lambda}_{0:T-1}$ and control inputs $\mathbf{u}_{0:T-1}$ to minimize an error metric called the ergodic metric, defined as

$$\mathcal{E}(\boldsymbol{\lambda}, \Phi) = \sum_{k \in \mathbb{N}^v} \Lambda_k \left( c_k(\boldsymbol{\lambda}_{0:T-1}) - \int_{\mathcal{S}} \Phi(\boldsymbol{\lambda}) F_k(\boldsymbol{\lambda}) d\boldsymbol{\lambda} \right)^2 \tag{4}$$

where $\Lambda_k = \frac{1}{1+\|k\|^{\frac{v+1}{2}}}$ are weights assigned to the Fourier coefficients $F_k$. Finally, to find the set of states and controls which both minimize the ergodic metric and consider the system's control limitations, we minimize over the augmented cost function $\mathcal{J}$:

$$\begin{gathered}\min_{\mathbf{x},\mathbf{u}} \mathcal{J}(\mathbf{x},t) = \mathcal{E}(\boldsymbol{\lambda},\Phi) + \sum_{t=0}^{T-1} \mathbf{u}_t^\top R \mathbf{u}_t \\ \text{subject to } \mathbf{x}_{t+1} = f(\mathbf{x}_t,\mathbf{u}_t), \mathbf{x}_t \in \mathcal{X}, \mathbf{u}_t \in \mathcal{U} \\ \mathbf{x}_0 = \bar{\mathbf{x}}_0, g(\mathbf{x}_t) \in \mathcal{S}\end{gathered} \tag{5}$$

where $\bar{\mathbf{x}}_0$ is the initial set of conditions of the robots and $R$ is a weight controlling the relative priority of minimizing the control input to the system. Eq. (5) is solved using an interior point constraint solver [36], [37].

### B. Expected Information

To begin, we define the uncertainty distribution across the exploration space $S$ to be $\mathcal{V}$. Given an environmental process model $E(s)$ for $s \in S$, we can determine a sensor performance model $M : E \to [0, 1]$. This sensor performance model evaluates $E$ at a given point of the exploration space and a discrete time, then returns a coefficient describing the effectiveness of the sensor in capturing information at that location/moment. We can construct a preliminary EID, $\Phi_1$, by weighting $\mathcal{V}$ by the sensor effectiveness coefficients $M$:

$$\Phi_1 = \mathcal{V} \odot M \tag{6}$$

A more advanced EID, $\Phi_2$, can be determined through the use of Shannon entropy. We define a probability density function

$$P(d_s, m_s) = \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-(\frac{d_s}{d_{s,avg}} - m_s)^2}{2}\right) \tag{7}$$

where $d_s$ is an observation at a point $s$ and $m_s = M(s)$. The observation $d_s$ arises from applying a Gaussian noise model to the value of the uncertainty map $\mathcal{V}$ at the point $s$ being measured:

$$d_s \sim \mathcal{N}(\mathcal{V}(s), \sigma^2) \tag{8}$$

where $\mathcal{N}(\mathcal{V}(s), \sigma^2)$ is a normal distribution centered at the uncertainty value $\mathcal{V}(s)$. We define $D_s$ to be the range of potential observations at $s$. To reduce the effect on sensor noise before calculating Shannon entropy, we apply a Gaussian filter to $D_s$. Then, we can calculate the Shannon entropy at $s$ to be

$$H(D) = -\sum_{d \in D} P(d, m_s) \log_2 P(d, m_s) \tag{9}$$

Finally, we can calculate the second EID $\Phi_2$ to be

$$\Phi_2 = H(D_S) \tag{10}$$

## IV. ERGODIC EXPLORATION OF A WILDFIRE

### A. Smoke Modeling

Smoke simulations were obtained using the Jet framework from [38]. The motion of any constant-density fluid is described by the incompressible Navier-Stokes equation:

$$\begin{aligned} &\rho \frac{D\mathbf{V}}{D\mathbf{t}} = \rho \mathbf{g} - \boldsymbol{\nabla} p + \mu \boldsymbol{\nabla}^2 \mathbf{V} \\ &\nabla \cdot \mathbf{V} = 0 \end{aligned} \tag{11}$$

where $\mathbf{V}$ is velocity, $\rho$ is density, $p$ is pressure, and $\mu$ is viscosity. As described in [38], fluid is modeled as a multidimensional grid, the cells of which store information about the fluid's physical properties. To simulate the motion of a fluid, the gravity, pressure, and viscosity forces of Eq. (11) are calculated and used to update the fluid's velocity field $\mathbf{u}$. To transfer data along grid points as the simulation evolves (a process called advection), grid points are traced backwards by linearly interpolating on nearby grid values, yielding

$$f(\mathbf{s})^{n+1} - \tilde{f}(\mathbf{s} - \Delta t \mathbf{u})^n \tag{12}$$

for a quantity $f$ at time step $n$ and position $\mathbf{s}$.

To simulate smoke, one additional force must be considered. Due to differences in the temperature of the smoke and the surrounding air, smoke undergoes a buoyancy force, which is approximated as

$$\mathbf{f}_b = -\alpha \rho \mathbf{y} + \beta (T - T_{amb}) \mathbf{y} \tag{13}$$

where $\alpha, \beta$ are scaling factors, $T$ is temperature, and $\mathbf{y}$ is vertical position. Adding this buoyant force into Eq. (11), the velocity field $\mathbf{u}$ can be solved. Then, the density field $\boldsymbol{\rho}$ can be determined by applying Eq. (12) with $f(\mathbf{s}) = \rho(\mathbf{s})$.

From $\boldsymbol{\rho}$, we can construct a simple model for the drones' sensor visibility. For a sensor observing the point $s$ at time $t$, the visibility coefficient of the sensor is

$$m(s,t) = \begin{cases} 1 - \boldsymbol{\rho}(s,t) & \boldsymbol{\rho}(s,t) \le \mathrm{c} \\ 0 & \boldsymbol{\rho}(s,t) > \mathrm{c} \end{cases} \tag{14}$$

where $\mathrm{c}$ is the cutoff density. Eq. (14) encodes a sensor performance model where visibility decreases linearly with smoke density until a cutoff density is reached; at that cutoff, the smoke is considered too dense for any information to be obtained, and so the visibility coefficient becomes zero.

### B. Dynamic Ergodic Exploration

To test our ergodic exploration method, we performed simulations of drone exploration of a wildfire on two different types of uncertainty maps. The first set of maps encoded uncertainty in the position of static targets, and the second set of maps encoded uncertainty in the position of moving targets. The static-target uncertainty maps were composed of randomly-placed Gaussian peaks, which varied in both size and number of peaks. The moving-target uncertainty maps were also initialized as randomly-placed Gaussian peaks of varying size and quantity. When observations were taken near the position of a target, the uncertainty peak associated with that target was reduced proportional to the visibility coefficients from Eq. (14). When a target moved, the uncertainty peak associated with that target was reset to its initial value as movement corresponds with a loss in confidence about the position of that target.

For the environmental process model, smoke density data was generated offline for a point source fire at random initial positions. The number of drones ranged from 2 to 10, with larger drone teams used on larger information maps. In the simulation, measurements were collected by the drones at each step along their trajectories, and the uncertainty map was updated according to the visibility coefficient at that step. The trajectories were then re-planned according to the current value of the EID after a specified amount of time, $t_u$, until the program reached its final time, $t_f$. The full process of our ergodic exploration method is described in Algorithm 1.

**Algorithm 1** Multi-Agent Dynamic Ergodic Exploration

**init:** $\mathcal{V}, E, N, \mathbf{x}, t_f, t_u$
$\Phi \leftarrow$ expectedInformation($\mathcal{V}, E$)
$\mathbf{x}, \mathbf{u}$ = ergodicPlanner($\mathbf{x}$, $\Phi$)
**while** $t < t_f$ **do**
  **for** $\mathbf{x}_i, \mathbf{u}_i \in \mathbf{x}, \mathbf{u}$ **do**
    step trajectory
    **for** $s \in \lambda_i$ **do**
      $m_s \leftarrow$ getVisibilityCoeffs()
      $\mathcal{V}(s) \leftarrow (1 - m_s)\mathcal{V}(s)$
    **end for**
    **if** $t \mod t_u$ **then**
      $E \leftarrow$ getSmokeDen($t$)
      $\Phi \leftarrow$ expectedInformation($\mathcal{V}, E$)
      $\mathbf{x}, \mathbf{u}$ = ergodicPlanner($\mathbf{x}$, $\Phi$)
      break
    **end if**
    $t \leftarrow t + 1$
  **end for**
**end while**

## V. RESULTS

### A. *Stationary Targets*

To begin, we tested the performance of the multi-agent ETO algorithm on a series of randomly-generated uncertainty maps for static targets. We also compared the ETO algorithm's performance between the two EIDs, $\Phi_1$ and $\Phi_2$, described in Section IV. These two approaches to calculating expected information are referred to as the 'smoke mask approach' and the 'Shannon entropy approach', respectively. Additionally, we evaluated the ETO algorithm on a baseline information measure, for which the uncertainty map $\mathcal{V}$ is used directly as the EID $\Phi$. In other words, the baseline approach does not consider the smoke model in planning trajectories, only the uncertainty distribution. The relative performance of these three methods is shown in Fig. 3. The trajectories generated by the baseline and Shannon entropy approaches are compared in Fig. 4.

We find that, on average, the Shannon entropy approach minimizes the ergodic metric in 11.7% and 5.42% fewer iterations than the baseline and smoke mask approaches, respectively. We also find that, on average, the Shannon entropy approach reduces uncertainty by 93.8% and 38.1% more than the baseline and smoke mask approaches, respectively. We note that the smoke mask and Shannon entropy approaches were also compared for a simulation in which the sensor measurements were not subject to Gaussian noise. In this ideal sensor scenario, the iterations to minimize the ergodic metric and uncertainty reduction for the two methods both differed by less than 5%. However, in real-world applications, there will always be some amount of noise in the values measured by a drone's sensor, and so we conclude that the Shannon entropy approach the best means of calculating expected information for our wildfire exploration problem.

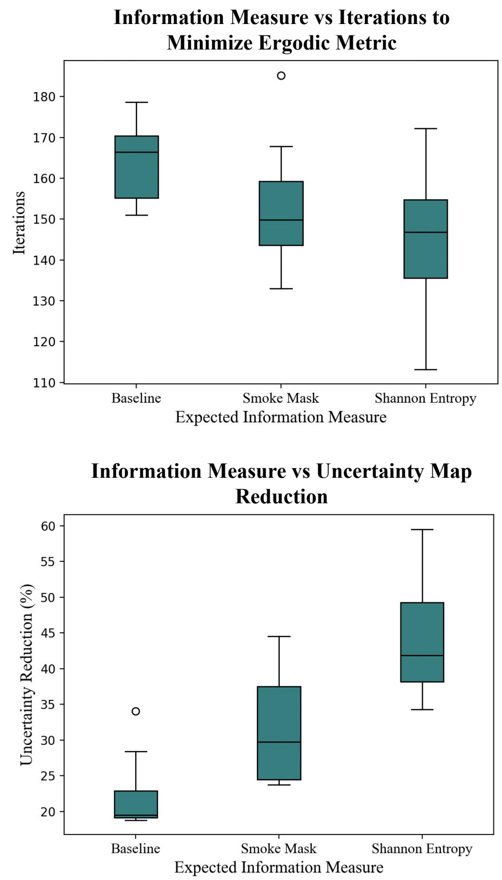


Fig. 3: **Stationary Targets: Expected Information Measure Performance Comparison.** The average iterations required to minimize the ergodic metric (top) and the percent reduction in uncertainty (bottom) for multi-agent ETO are compared across three different approaches for calculating expected information.

Next, we compare the performance of our ergodic exploration method to two baseline search methods. The first is 'lawnmower search', which prioritizes coverage of the exploration map. The second is 'greedy search', which prioritizes exploitation of areas of high uncertainty. For fair comparison, the three methods are limited to the same time horizon, trajectory re-planning frequency, and maximum trajectory step size. The performance of these three methods is shown in Fig. 4.

We find that our ergodic exploration method, on average, reduces 443% more uncertainty than the lawnmower method and 40.7% more uncertainty than the greedy method. Given that our ergodic exploration approach substantially increases the amount of uncertainty that can be reduced over the course of exploration while maintaining a compute time low enough for online, real-time search, our approach is clearly advantageous for the problem of wildfire exploration.

### B. *Moving Targets*

Next, we tested the performance of the multi-agent ETO algorithm on random uncertainty maps for moving targets.

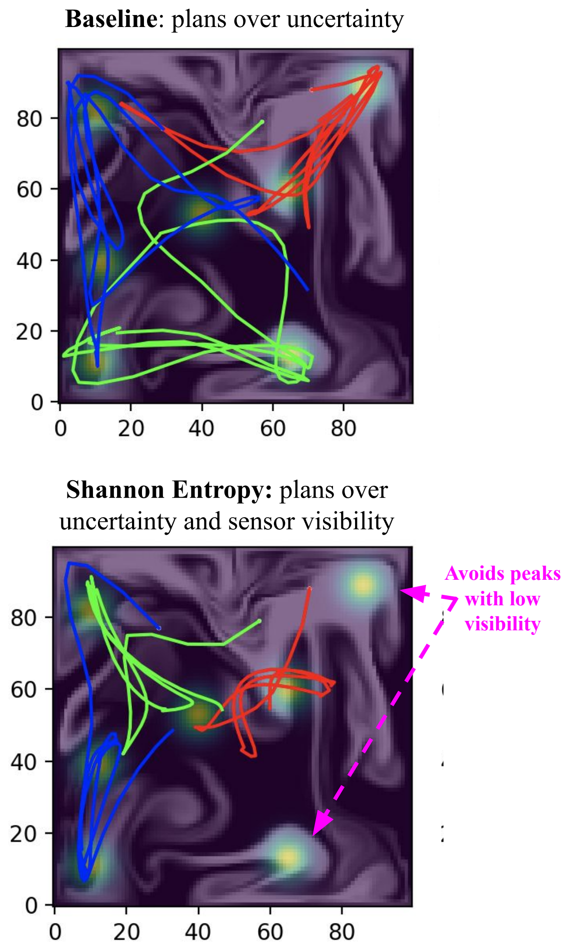


Fig. 4: **ETO Trajectory Comparison: with and without Sensor Modeling** For a sample uncertainty map, ergodic trajectories are generated over a baseline EID, which encodes only uncertainty (top) and a EID generated from Shannon entropy, which encodes uncertainty and the sensor model (bottom). The baseline approach visits all peaks regardless of visibility, while the Shannon entropy approach only visits peaks which are visible, improving information gain efficiency.

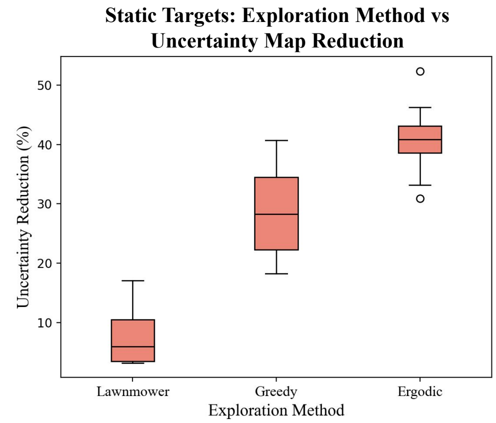


Fig. 5: **Stationary Targets: Exploration Method Performance Comparison.** The percent reduction in uncertainty of our ergodic search method is compared to that of lawnmower and greedy search methods over randomly-generated uncertainty maps for stationary targets.

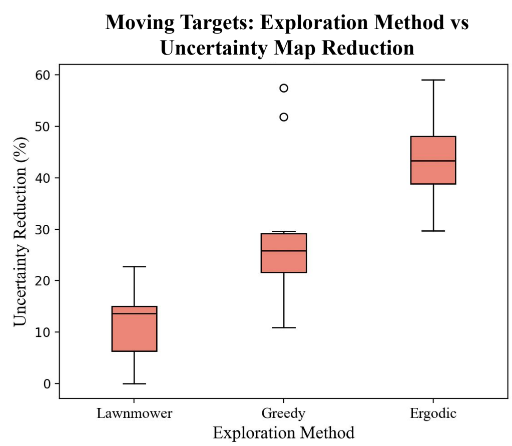


Fig. 6: **Moving Targets: Exploration Method Performance Comparison.** The percent reduction in uncertainty of our ergodic search method is compared to that of lawnmower and greedy search methods over randomly-generated uncertainty maps for moving targets.

Further comparison of the ETO algorithm on different information measures showed that, on average, the Shannon entropy approach minimized the ergodic metric in 21.7% and 16.9% fewer iterations and reduced 235% and 97.1% more uncertainty than the baseline and smoke mask approaches, respectively. So, we continue to use the Shannon entropy approach to calculate the EID for our method. The performance of our ergodic exploration method is then compared to lawnmower and greedy exploration methods in Fig. 6.

We find that our ergodic exploration method, on average, reduces 275% more uncertainty than the lawnmower method and 51.9% more uncertainty than the greedy method. Once again, we find that our ergodic search approach significantly outperforms the two baseline methods in reducing uncertainty, indicating that our exploration method is advantageous wildfire exploration for both stationary and moving targets.

## VI. CONCLUSIONS

In this paper, we introduced an algorithm for multi-agent ETO that accounts for information maps that change dynamically, both due to interactions with robots and to changes in target state. We also demonstrated that ETO outcomes can be substantially improved by accounting for the limitations of a sensor while calculating expected information. The main algorithm of this work could be applied to multi-agent exploration of any extreme environment which is dynamic in information distribution and/or sensor capabilities. In future work, we plan to test this algorithm on entropy maps derived from real-world wildfire datasets. Additionally, we are interested in extending this exploration method to heterogeneous multi-agents teams.

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