COM 3240

REINFORCEMENT LEARNING


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OPTIMISATION

GRADIENT DESCENT

Desirable: A system that performs a specific task

The system has parameters that we need to select appropriately (optimise) for that task

$$f(x_1, x_2, \ldots, x_n)$$

parameters

$$f$$

System's performance: maximise

System's error : minimise

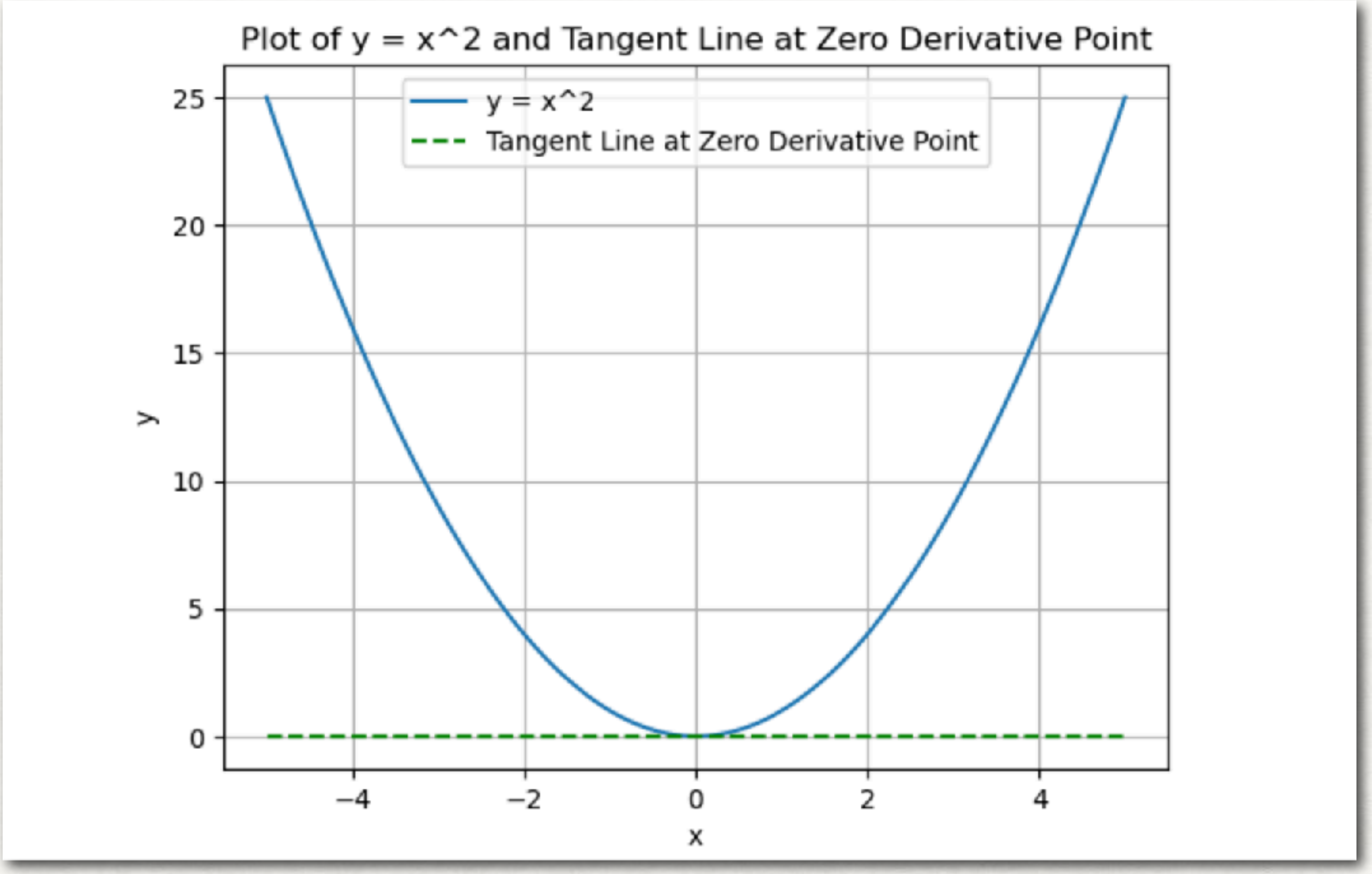
OPTIMISATION

ARTIFICIAL NEURAL NETWORKS


DALL·E

DERIVATIVES

OPTIMISATION - GRADIENT METHODS

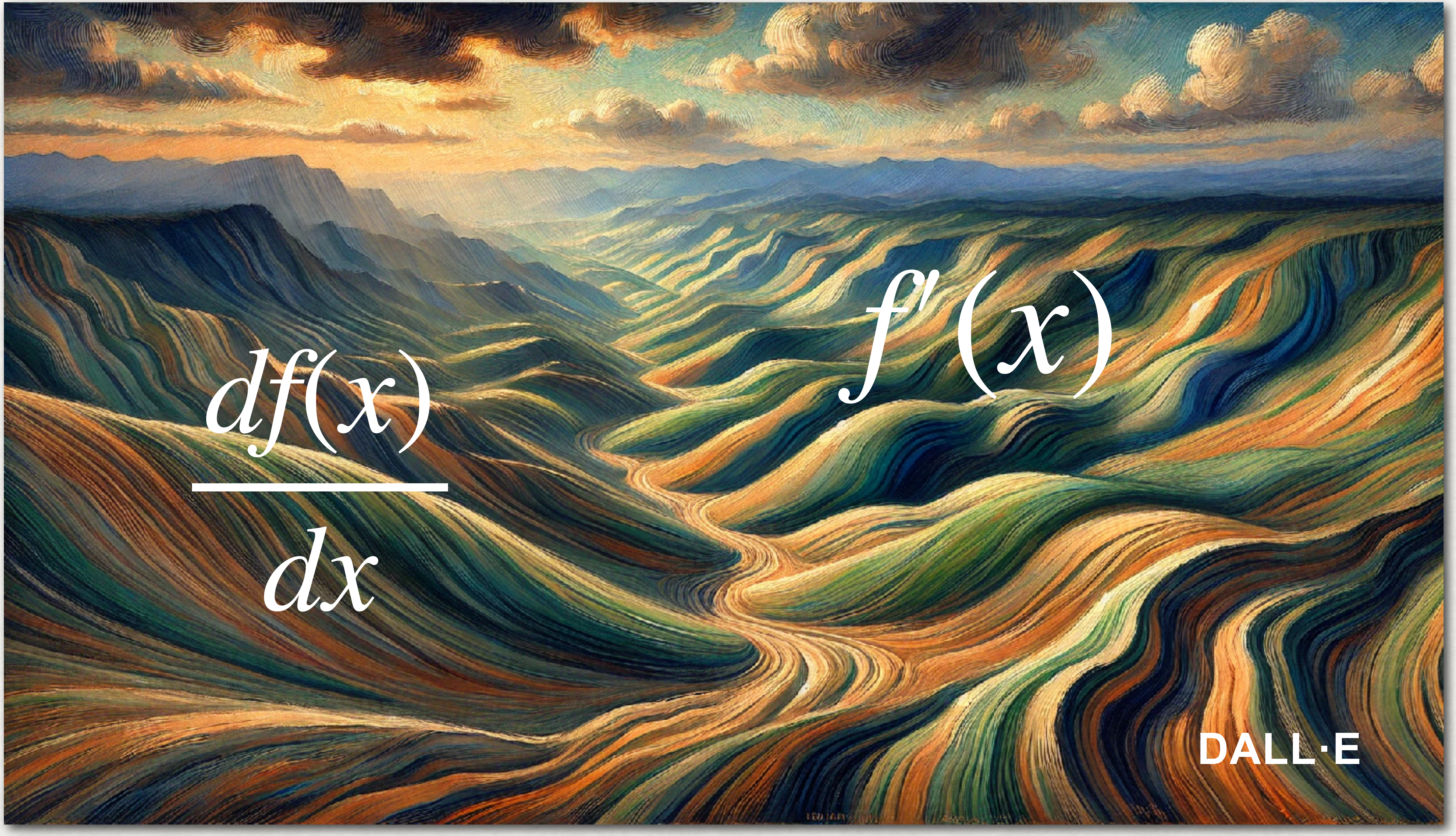


Slope

Error (Loss) function - Machine Learning

DERIVATIVES

OPTIMISATION - GRADIENT METHODS



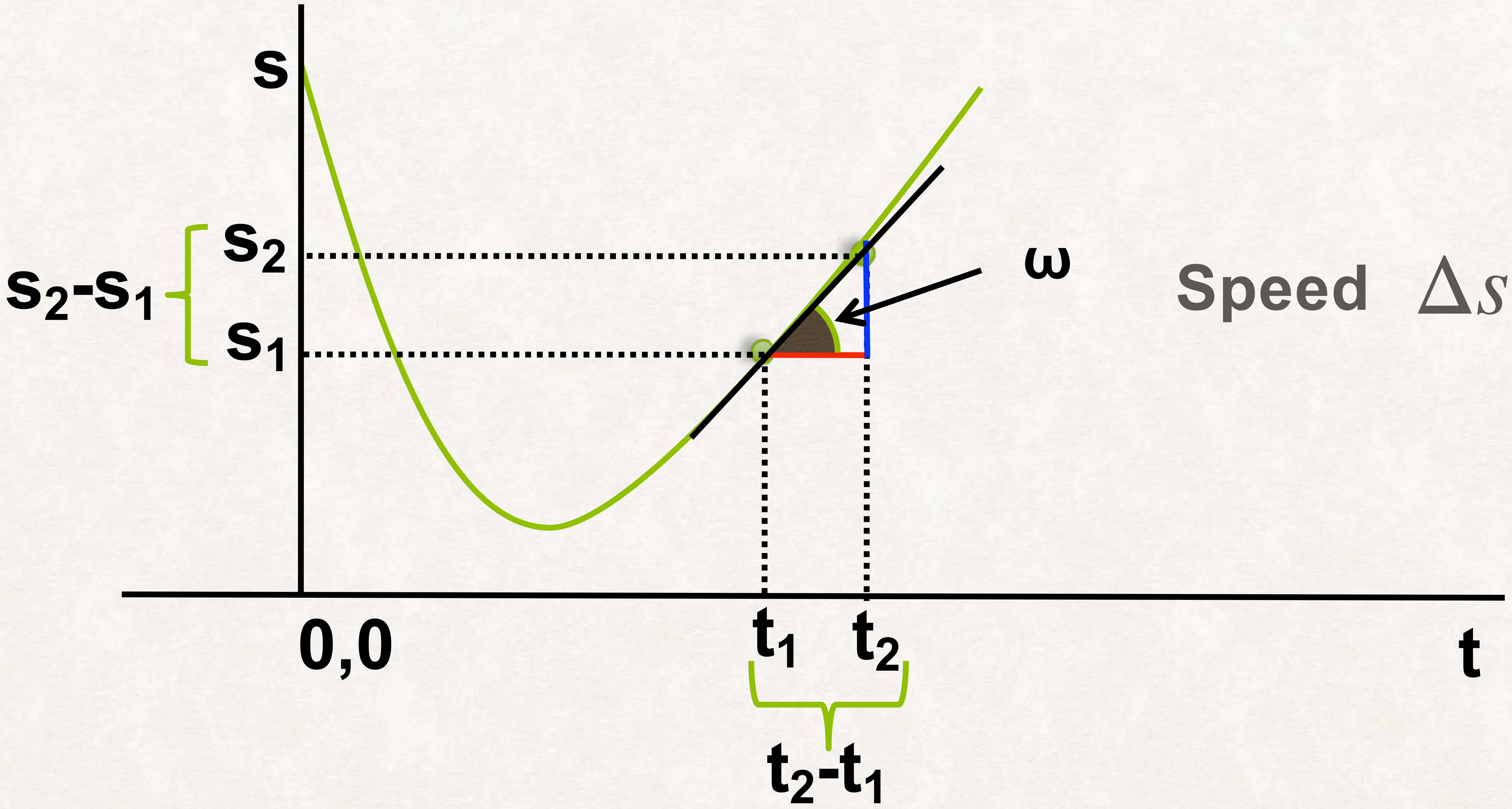
s_1 s_2

Speed $\Delta s/\Delta t$

t_1 t_2

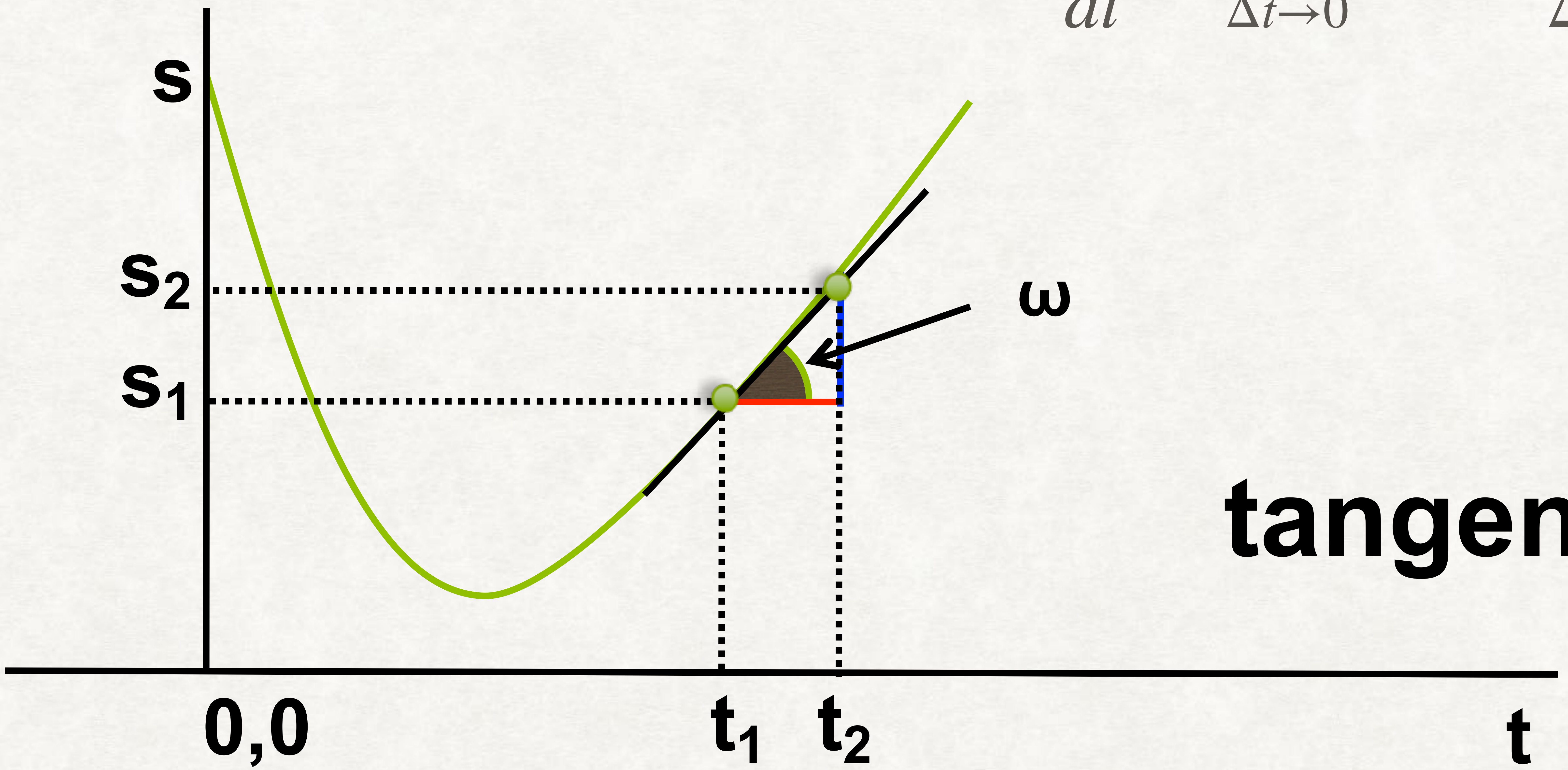
$$\Delta s = s_2 - s_1$$

$$\Delta t = t_2 - t_1$$



Speed $\Delta s/\Delta t$

$$\frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{s(t + \Delta t) - s(t)}{\Delta t}$$



tangent ω

DERIVATIVES

USE THE DEFINITION TO CALCULATE DERIVATIVES OF FUNCTIONS

More generally:

$$f'(x) = \frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

What is the effect of incrementing x by a small amount, Δx, on the value of f(x)?

$f(x) = 7$ $f(x) = 7x$ $f(x) = 7x^2$

DERIVATIVES IN PRACTICE

MEMORISE BASIC FUNCTIONS AND USE RULES

Power function: $f(x) = x^n$ $\quad f'(x) = nx^{n-1}$

Exponential function: $f(x) = e^x$ $\quad f'(x) = e^x$

Natural logarith: $f(x) = \ln(x)$ $\quad f'(x) = \dfrac{1}{x}$

DERIVATIVES - RULES

ADDITION

$$\frac{d}{dx}\left(f(x) + g(x)\right) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

$$h(x) = x + x^2$$

DERIVATIVES - RULES

MULTIPLICATION

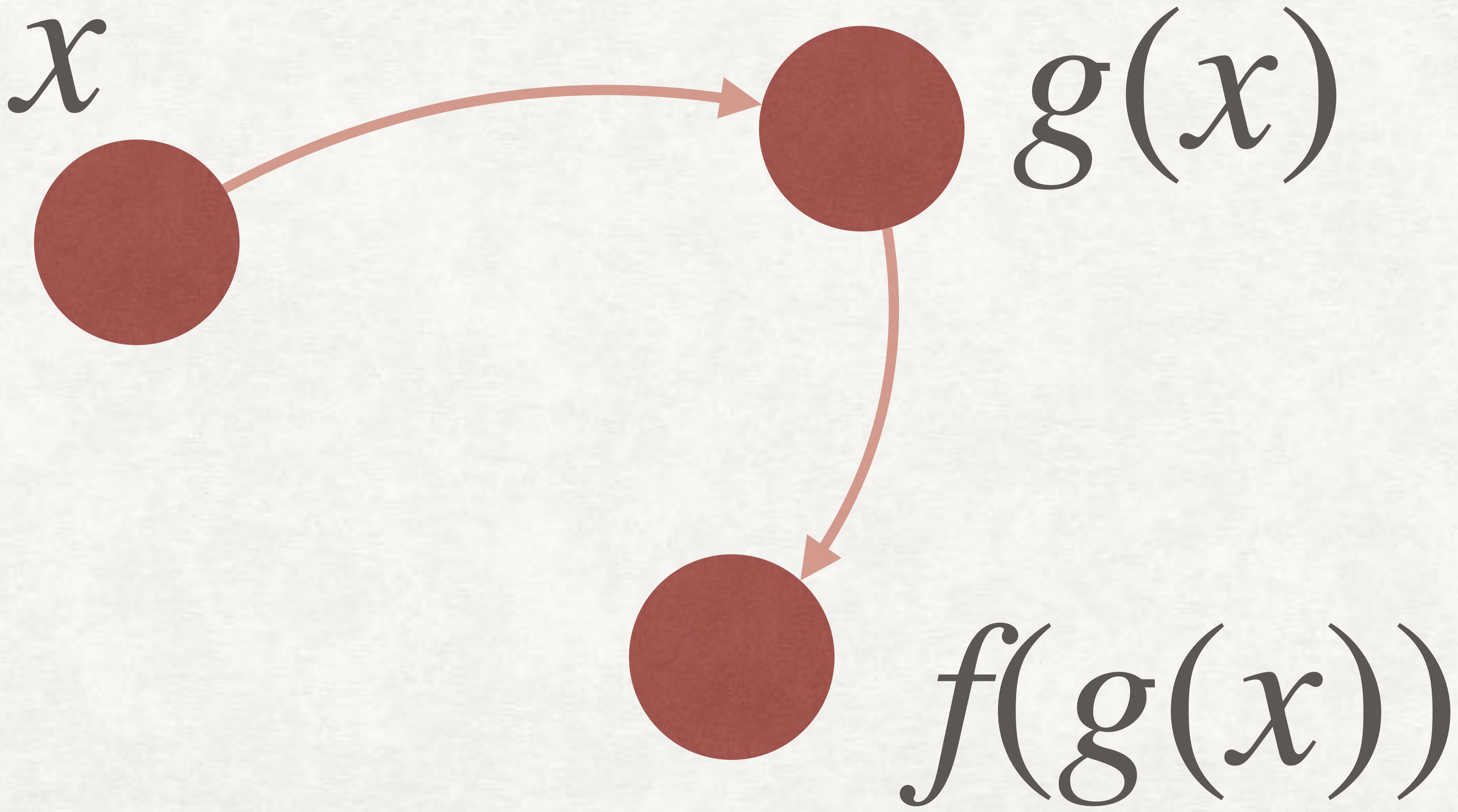
$$\frac{d}{dx}\left(f(x) \cdot g(x)\right) = \frac{df(x)}{dx} \cdot g(x) + f(x) \cdot \frac{dg(x)}{dx}$$

$$h(x) = x \cdot ln(x)$$

DERIVATIVES - RULES

THE CHAIN RULE

$$\frac{df\left(g(x)\right)}{dx} = \frac{df\left(g(x)\right)}{dg(x)} \cdot \frac{dg(x)}{dx}$$



$$h(x) = (x - c)^2$$

MOTIVATION FOR PARTIAL DERIVATIVES

LIGHTS AND BUTTONS PUZZLE

$$f(x, y) = x^2 y + y^2$$

A more general way to write the same:

$$f(x_1, x_2) = x_1^2 x_2 + x_2^2$$

How do we estimate the gradient?

PARTIAL DERIVATIVES

AND THE NABLA NOTATION

$$f(x_1, x_2, \ldots)$$

$$\frac{\partial f}{\partial x_1} = \lim_{\Delta x_1 \to 0} \frac{f(x_1 + \Delta x_1, x_2, \ldots) - f(x_1, x_2, \ldots)}{\Delta x_1}$$

$$\nabla f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2} \right)$$

$$\frac{\partial f}{\partial x_2} = \lim_{\Delta x_1 \to 0} \frac{f(x_1, x_2 + \Delta x_2, \ldots) - f(x_1, x_2, \ldots)}{\Delta x_2}$$

PARTIAL DERIVATIVES

AND THE NABLA NOTATION

One variable at the time, treat others as constants!

PARTIAL DERIVATIVES

AND THE NABLA NOTATION

$$f(x_1, x_2) = x_1^2 x_2 + x_2^2$$

$$\frac{\partial f}{\partial x_1} = \frac{\partial}{\partial x_1}(x_1^2 x_2 + x_2^2) = 2x_1 y_2$$

$$\frac{\partial f}{\partial x_2} = \frac{\partial}{\partial x_2}(x_1^2 x_2 + x_2^2) = x_1^2 + 2x_2$$

INTRODUCE (PARTIAL) DIFFERENTIAL EQUATIONS

EVERYTHING CHANGES

$$f\left(x, \frac{d}{dx}x, \frac{d^2}{dx^2}x, \ldots \right) = 0$$

x: **variable**

$$f\left(x_1, x_2, \ldots, x_n, \frac{\partial u}{\partial x_1}, \frac{\partial u}{\partial x_2}, \ldots, \frac{\partial^m u}{\partial x_m} \right) = 0$$

DIFFERENTIAL EQUATIONS

RELAXATION

$$\tau \frac{dx(t)}{dt} = -x(t)$$

$x(t)$: **here it is a function of time $f(t) = x(t)$; not a variable!**

τ : **time constant, if a large value you relax slow (you have a long memory:)**

DIFFERENTIAL EQUATIONS

LEAKY INTEGRATOR

$$\tau \frac{dx(t)}{dt} = -x(t) + s(t)$$

$x(t)$: **a function**

$s(t)$: **sample, input**

t : **variable**

DEFINITION OF DERIVATIVES

AND ITS APPLICATION TO DIFFERENTIAL EQUATIONS

$$\frac{dx(t)}{dt} = \lim_{\Delta t \to 0} \frac{x(t + \Delta t) - x(t)}{\Delta t}$$

Approximate the derivative as difference.

$$\tau \frac{dx(t)}{dt} = -x(t) + s(t)$$

You need one initial point and a time step.

$e.g., x(0), \Delta t = 0.1$ **(seconds)**

Its discrete form is known as "exponential moving average"

DEFINITION OF DERIVATIVES

AND ITS APPLICATION TO DIFFERENTIAL EQUATIONS

$$\frac{dx(t)}{dt} \approx \frac{x(t+\Delta t) - x(t)}{\Delta t}$$

Approximate the derivative as difference.

You need one initial point and a time step.

$$\tau \frac{x(t+\Delta t) - x(t)}{\Delta t} = -x(t) + s(t)$$

e.g. $x(t=0), \Delta t = 0.1$ **(seconds)**

Its discrete form is known as "exponential moving average"

DEFINITION OF DERIVATIVES

AND ITS APPLICATION TO DIFFERENTIAL EQUATIONS

$$x(t+\Delta t) - x(t) = \frac{\Delta t}{\tau}\Big(-x(t) + s(t)\Big)$$

Only unknown!

e.g. $x(t=0), \Delta t = 0.1$ **(seconds)**

Euler's method: I can start from, e.g. time 0 (sec) and iteratively calculate the value of x at any time

DEFINITION OF DERIVATIVES

AND ITS APPLICATION TO DIFFERENTIAL EQUATIONS

$$x(t + \Delta t) = x(t) + \frac{\Delta t}{\tau}\Big(-x(t) + s(t)\Big)$$

e.g. $x(t = 0), \Delta t = 0.1$ **(seconds)**

Useful equation for “forgetful” averaging, filtering high frequencies (noise)

SUMMARY

PREREQUISITES

- Concept of Optimisation, Gradient Learning
- Derivatives & partial derivatives
- Differential equations, Euler’s method & leaky integrators

WARNING!

GENERATIVE AI AND THEORY

Unless specialised in maths and reasoning, answers to questions regarding theory may be wrong!

Safe to use in programming because it is possible to test the answer.

THANK YOU!