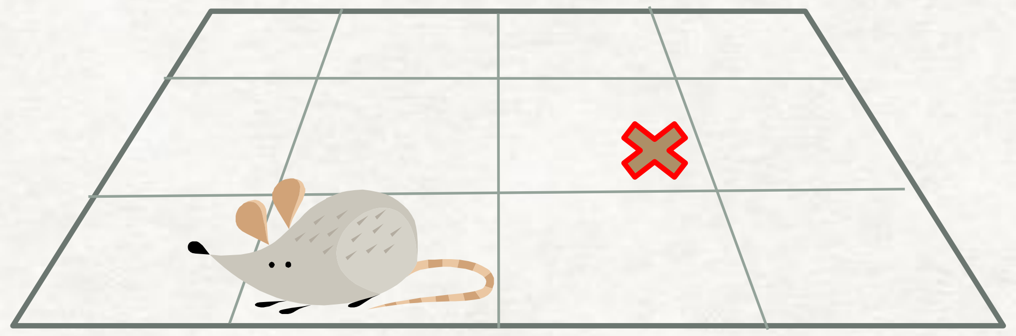
COM 3240

REINFORCEMENT LEARNING

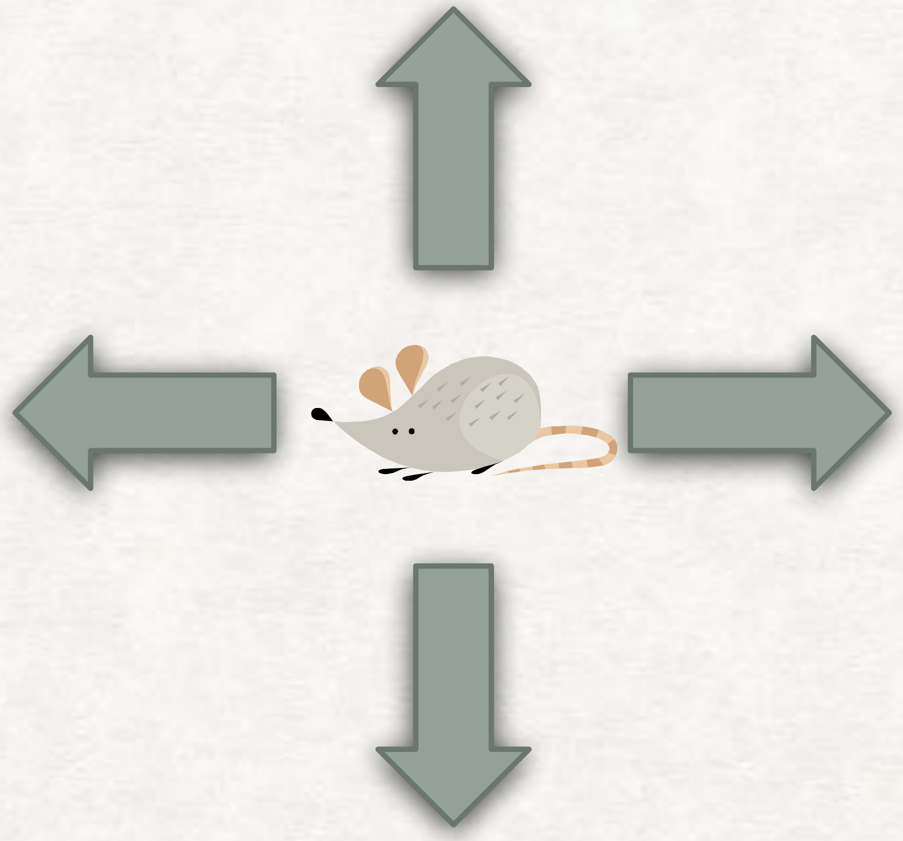
Create me a painting in the style of Dali with the title "waiting for the discounted expected returns"

FUTURE REWARDS

MARKOVIAN PROPERTY



Squares = State (in this case)



Actions

The future state depends ONLY on the current state and not on the sequence of events that preceded it.

FUTURE REWARDS

TOTAL RETURN

$$G_t = R_{t+1} + R_{t+2} + R_{t+3} + \ldots + R_T$$

$$G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \ldots$$

FUTURE REWARDS

TOTAL RETURN

$$0 \leq \gamma < 1$$

$$G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \ldots$$

$$G_t = R_{t+1} + \gamma G_{t+1}$$

FUTURE REWARDS

BELLMAN "EXPECTATION" EQUATION FOR STATE-ACTION-VALUES

$$q^{\pi}(s, a) = \mathbb{E}_{\pi}[R_{t+1} + \gamma\, q^{\pi}(s', a') \,|\, S_t = s, A_t = a]$$

Simplify notation: $q^{\pi}(s, a) = \mathbb{E}_{\pi}[r + \gamma\, q^{\pi}(s', a') \,|\, s, a]$

Current state-action

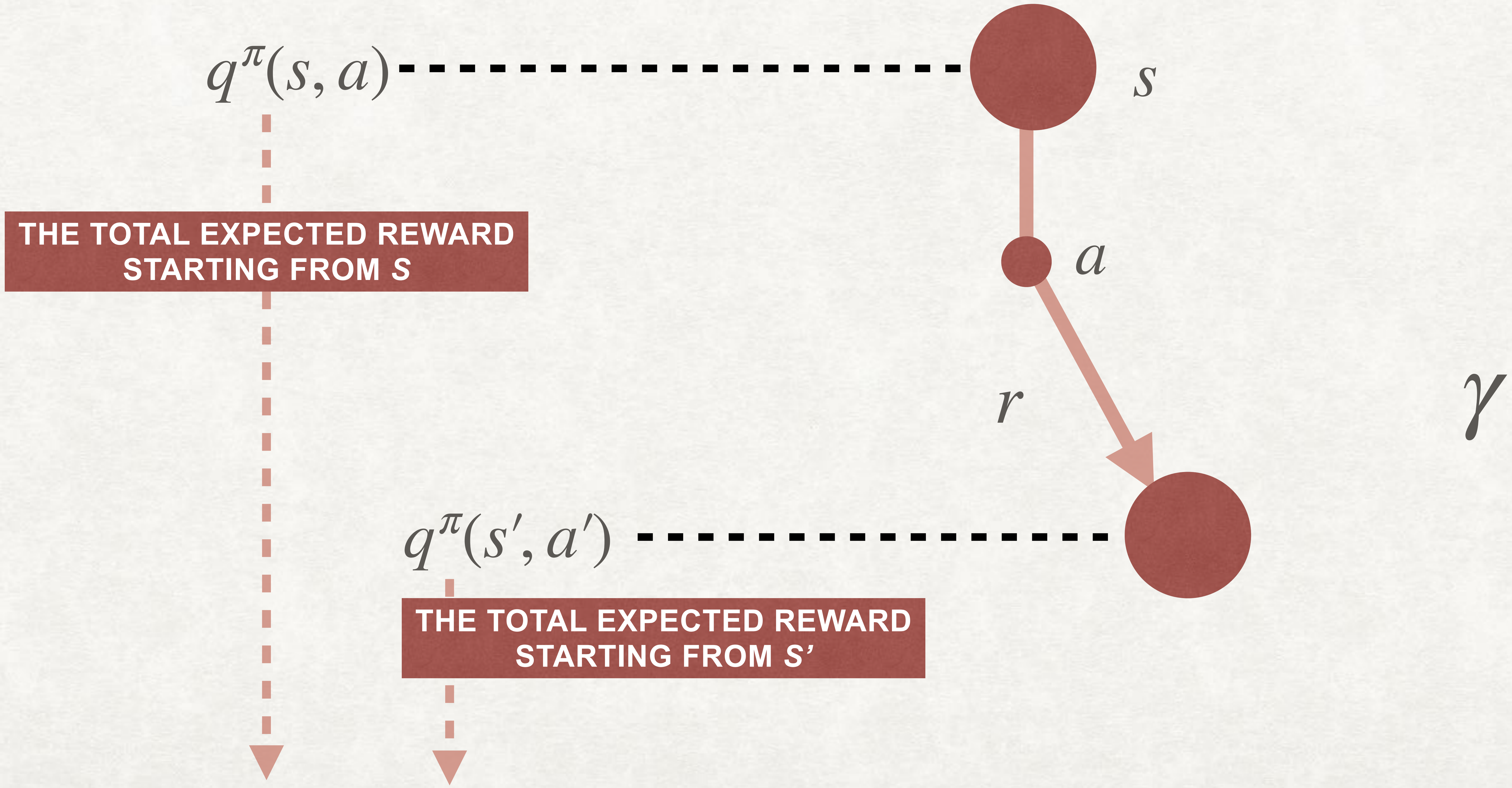
Next state-action

(Stochastic) immediate reward due to action a from state s

FUTURE REWARDS

ACTION-VALUE FUNCTIONS

$$q^{\pi}(s,a) = \mathbb{E}_{\pi}[r + \gamma\, q^{\pi}(s',a') \,|\, s,a]$$



FUTURE REWARDS

SARSA

$$q^{\pi}(s,a) = E_{\pi}[r + \gamma\, q^{\pi}(s',a') \,|\, s,a]$$

$$E_{\pi}[r + \gamma\, q^{\pi}(s',a') - q^{\pi}(s,a) \,|\, s,a] = 0$$

$$\mathscr{L}(s,a) = \frac{1}{2N}\sum_{i=1}^{N}\left(Q(s,a) - [r^{(i)} + \gamma Q(s^{'(i)}, a^{'(i)})]\right)^2$$

FUTURE REWARDS

SARSA

$$\mathscr{L}(s,a) = \frac{1}{2N}\sum_{i=1}^{N}\left(Q(s,a) - [r^{(i)} + \gamma Q(s^{'(i)}, a^{'(i)})]\right)^2$$

$$\frac{\partial \mathscr{L}(Q(s,a))}{\partial Q(s,a)} = \frac{1}{N}\sum_{i=1}^{N}\left(Q(s,a) - [r^{(i)} + \gamma Q(s^{'(i)}, a^{'(i)})]\right)$$

$$\Delta Q(s,a) = -\eta\frac{1}{N}\sum_{i=1}^{N}\left(Q(s,a) - [r^{(i)} + \gamma Q(s^{'(i)}, a^{'(i)})]\right)$$

FUTURE REWARDS

SARSA

$$\Delta Q(s,a) = -\eta \frac{1}{N} \sum_{i=1}^{N} \left(Q(s,a) - [r^{(i)} + \gamma Q(s'^{(i)}, a'^{(i)})] \right)$$

N=1 (online learning):

$$\Delta Q(s,a) = -\eta \left(Q(s,a) - [r + \gamma Q(s',a')] \right)$$

$$\Delta Q(s,a) = \eta \left(r + \gamma Q(s',a') - Q(s,a) \right)$$

FUTURE REWARDS

SARSA: REWARD - “ANTICIPATED REWARD”

$$\Delta Q = \eta \left(\underbrace{r + \gamma\, Q(s', a')}_{\text{What I “actually” get}} - \underbrace{Q(s, a)}_{\text{Anticipated reward}} \right)$$

FUTURE REWARDS

POLICIES

Greedy $a = \underset{a}{\operatorname{argmax}}\ Q(s, a)$

Optimistic Greedy: initialise Q-values unrealistically high

Epsilon-Greedy : explore with probability epsilon, greedy otherwise

Softmax: $P(a) = \dfrac{e^{Q(s,a)/\tau}}{\sum_b e^{Q(s,b)/\tau}}$

FUTURE REWARDS

THE SARSA ALGORITHM

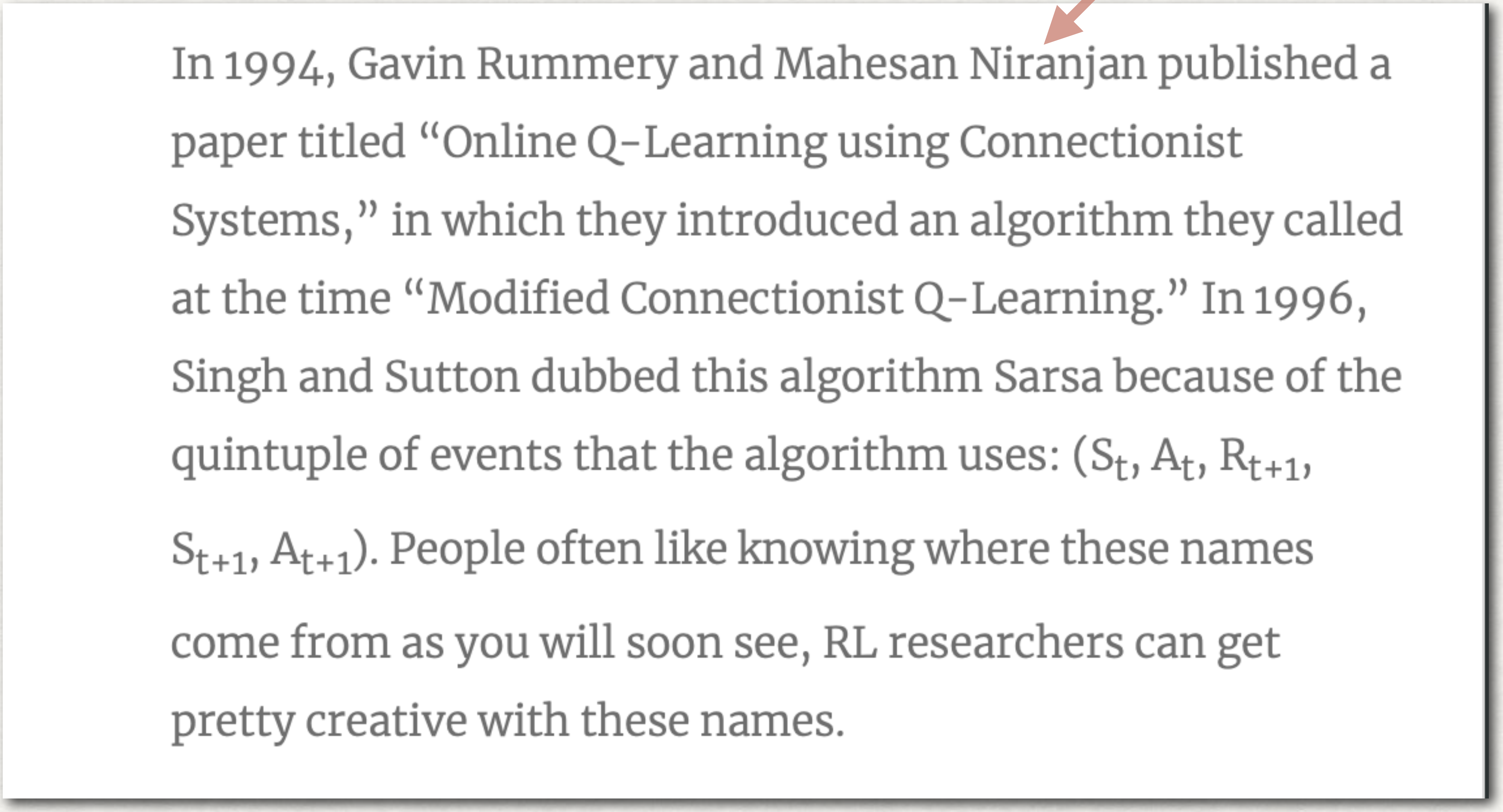
1. Initialise Q(s, a) arbitrarily for all $s \in S$ and $a \in A(s)$.
2. Repeat (for each episode):
   a. Initialise s.
   b. Choose an action a from s using a policy derived from Q (e.g., ε-greedy).
   c. Repeat (for each step of episode):
      i. Take action a, observe reward r and next state s'.
      ii. Choose a' from s' using policy derived from Q (e.g., ε-greedy).
      iii. $Q(s, a) \leftarrow Q(s, a) + \eta * [r + \gamma * Q(s', a') - Q(s, a)]$.
      iv. $s \leftarrow s'$; $a \leftarrow a'$.
   d. until s is terminal.

On-policy

FUTURE REWARDS

THE SARSA ALGORITHM

Former head of COM



In 1994, Gavin Rummery and Mahesan Niranjan published a paper titled "Online Q-Learning using Connectionist Systems," in which they introduced an algorithm they called at the time "Modified Connectionist Q-Learning." In 1996, Singh and Sutton dubbed this algorithm Sarsa because of the quintuple of events that the algorithm uses: (S_t, A_t, R_{t+1}, S_{t+1}, A_{t+1}). People often like knowing where these names come from as you will soon see, RL researchers can get pretty creative with these names.

https://livebook.manning.com/concept/reinforcement-learning/this-algorithm

FUTURE REWARDS

THE SARSA ALGORITHM

Right after obtaining his Ph.D. in 1995, Gavin became a programmer and later a lead programmer for the company responsible for the series of the Tomb Raider games. Gavin has had a very successful career as a game developer.

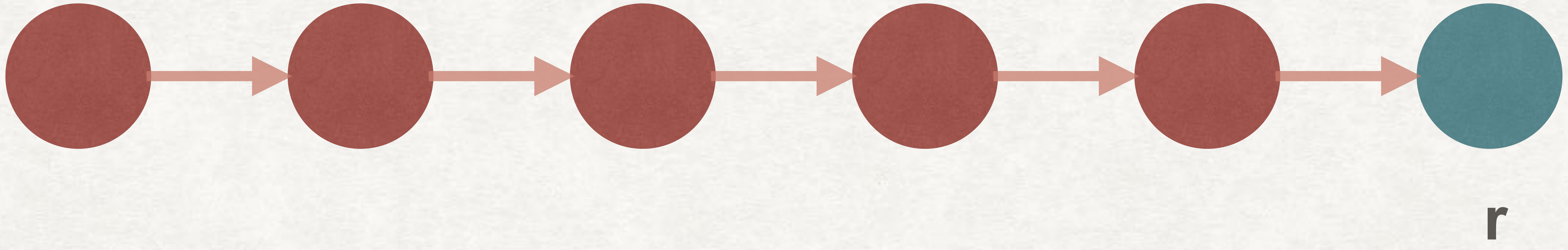
Mahesan, who became Gavin's Ph.D. supervisor after the unexpected death of Gavin's original supervisor, followed a more traditional academic career holding lecturer and professor roles ever since his Ph.D. graduation in 1990.

https://livebook.manning.com/concept/reinforcement-learning/this-algorithm

FUTURE REWARDS

SARSA

$$\Delta Q = \eta \left(r + \gamma\, Q(s', a') - Q(s, a) \right)$$



A robot walks this corridor and finds reward. How many Q-values will be updated?

FUTURE REWARDS

ELIGIBILITY TRACE

Design an ant leaving a trace of pheromone.

DALL·E

FUTURE REWARDS

ELIGIBILITY TRACE

We mark the action pair that were visited/used:
$e(s, a)$ initialised at 0.

Update eligibility trace for the most recent state action pair:
$e(s, a) \leftarrow e(s, a) + 1$

Update $Q(s, a) \leftarrow Q(s, a) + \eta * [r + \gamma * Q(s', a') - Q(s, a)] * e(s, a)$

Decay eligibility trace for all states:
$e(s, a) \leftarrow \gamma\lambda * e(s, a)$

FUTURE REWARDS

SARSA: TABULAR ELIGIBILITY TRACE

1. Initialise $Q(s, a)$ arbitrarily for all $s \in S$ and $a \in A(s)$.
2. Initialise eligibility traces $e(s, a)$ for all s, a to zeros.
3. Repeat (for each episode):
   a. Initialise s, eligibility traces $e(s, a)$ for all s, a to zeros.
   b. Choose an action a from s using policy derived from Q (e.g., ε-greedy).
   c. Repeat (for each step of episode):
      i. Take action a, observe reward r, and next state s'.
      ii. Choose a' from s' using policy derived from Q (e.g., ε-greedy).
      iii. For the visited s, a:
         - Update delta: $\delta \leftarrow r + \gamma * Q(s', a') - Q(s, a)$
         - Update eligibility trace: $e(s, a) \leftarrow e(s, a) + 1$
      iv. For all states:
         - Update $Q(s, a) \leftarrow Q(s, a) + \eta * \delta * e(s, a)$
         - Decay eligibility trace: $e(s, a) \leftarrow \gamma\lambda * e(s, a)$
      v. $s \leftarrow s'$; $a \leftarrow a'$.
   until s is terminal.

FUTURE REWARDS

BELLMAN OPTIMALITY “EXPECTATION” EQUATION FOR STATE-ACTION-VALUES

$$q^*(s, a) = \max_{\pi} q^{\pi}(s, a)$$

One policy better or equal than any other

Greedy policy *

$$q^{\pi}(s, a) = \mathbb{E}_{\pi}[r + \gamma\, q^{\pi}(s', a') \,|\, s, a]$$

$$\pi = *$$

$$q^*(s, a) = \mathbb{E}[r + \gamma \max_{a'} q^*(s', a') \mid s, a]$$

FUTURE REWARDS

Q-LEARNING

$$q^*(s,a) = \max_{\pi} q^{\pi}(s,a)$$

$$q^*(s,a) = E[R_{t+1} + \gamma \max_{a'} q^*(s',a') \mid S_t = s, A_t = a]$$

SARSA

$$\Delta Q(s,a) = \eta \left(r + \gamma Q(s',a') - Q(s,a) \right)$$

Q-Learning

$$\Delta Q(s,a) = \eta(r + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

FUTURE REWARDS

Q-LEARNING

$$\Delta Q = \eta \left(\underbrace{r + \gamma \max_{a'} Q(s', a')}_{\textit{What I “actually” get}} - \underbrace{Q(s, a)}_{\textit{Anticipated reward}} \right)$$

FUTURE REWARDS

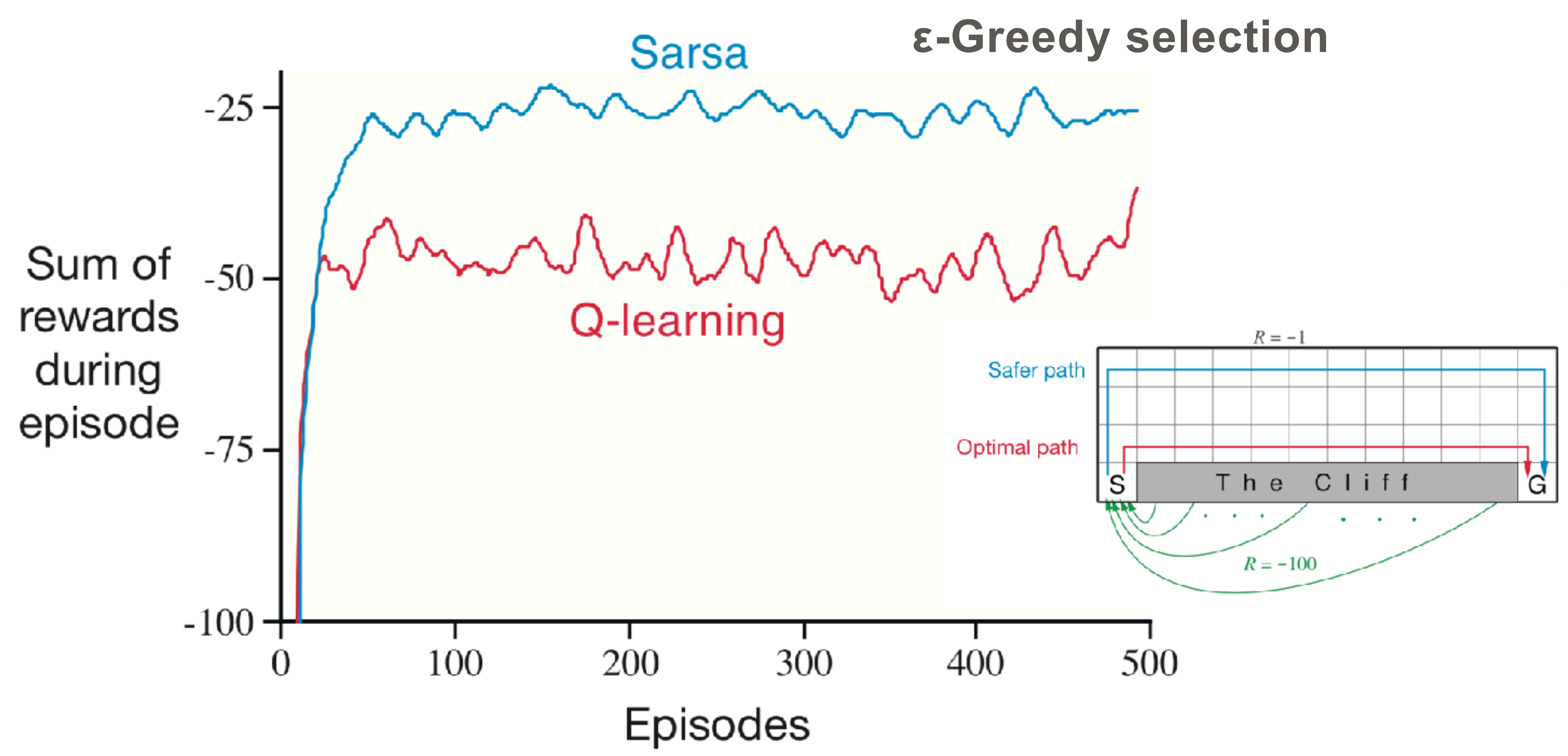
THE Q-LEARNING ALGORITHM

1. Initialise Q(s, a) arbitrarily for all $s \in S$ and $a \in A(s)$.
2. Repeat (for each episode):
   a. Initialise s.
   b. Repeat (for each step of episode):
      i. Choose an action a from s using a policy derived from Q (e.g., ε-greedy).
      ii. Take action a, observe reward r and next state s'.
      iii. $Q(s, a) \leftarrow Q(s, a) + \eta * [r + \gamma * \max_{a'} Q(s', a') - Q(s, a)]$.
      iv. $s \leftarrow s'$.
   c. until s is terminal.

Off-policy

FUTURE REWARDS

SARSA VS Q-LEARNING



From Sutton & Barto, 2020 - with permission.

FUTURE REWARDS

TABULAR ELIGIBILITY TRACE

1. Initialise Q(s, a) arbitrarily for all s ∈ S and a ∈ A(s).
2. Repeat (for each episode):
   a. Initialise s and eligibility traces e(s, a) for all s, a to zeros.
   b. Repeat (for each step of episode):
      i. Choose an action a from s using policy derived from Q (e.g., ε-greedy).
      ii. Take action a, observe reward r, and next state s'.
      iii. For the selected s, a:
         - Update delta: δ ← r + γ * max_a' Q(s', a') - Q(s, a)
         - Update eligibility trace: e(s, a) ← e(s, a) + 1
      iv. For all s,a
         -Update Q(s, a) ← Q(s, a) + η * δ * e(s, a)
         -Decay eligibility trace: e(s, a) ← γλ * e(s, a)
      v. s ← s'.
   until s is terminal.

FUTURE REWARDS

TEMPORAL DIFFERENCE LEARNING

- By approximating the action-value q by an estimate Q and by writing an error function based on a form of the Bellman equation we derived two Temporal Difference algorithms:
  - SARSA (State Action Reward State Action) : “safe”
  - Q-Learning : optimal
- An eligibility trace can speed up the performance of the algorithms by propagating to the pathway taken information about the reward.

THANK YOU!