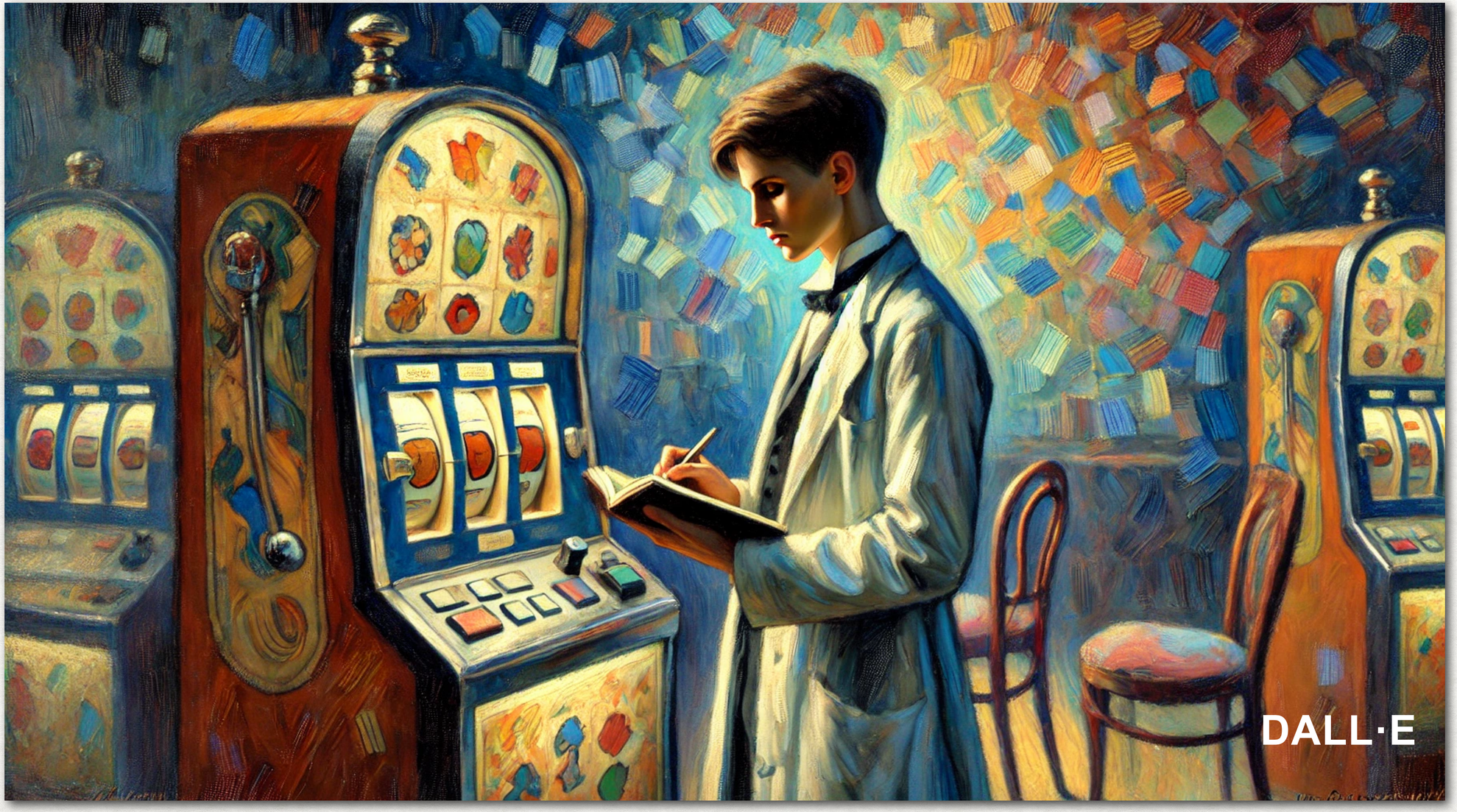
COM 3240

REINFORCEMENT LEARNING

IMMEDIATE REWARDS

BANDITS



IMMEDIATE REWARDS

BANDITS

To know which arm to play we need the expected reward for each of them.

Why expected reward?

The setting is probabilistic.

$$q^*(a) \doteq E[R \mid A_t = a]$$

We cannot calculate this quantity!

IMMEDIATE REWARDS

BANDITS

$$q^*(a) \doteq E[R \mid A_t = a]$$

$$Q(a) \approx q^*(a)$$

Idea: lets “track” the reward.

Q(a) tracks the rewards associated with action a.

$$R(A_t = a) \sim P(R \mid A_t = a)$$ **Sample**

IMMEDIATE REWARDS

BANDITS

We will do it through minimising a loss function.

Ingredients of the loss function:

$Q(a)$ $R(A_t = a)$

$$\left(Q(a) - R(A_t = a)\right)^2$$

$$E\left[\left(Q(a) - R(A_t = a)\right)^2\right]$$ **Expectation!**

IMMEDIATE REWARDS

BANDITS

$$L(a) = \frac{1}{2T_a} \sum_{t=1}^{T} \left(Q(a) - R\right)^2 \mathbf{1}_{A_t=a}$$

Replaced expectation with average.

$$\mathbf{1}_{A_t=a} = \begin{cases} 1 & \text{if } A_t = a \\ 0 & \text{otherwise.} \end{cases}$$

Indicator function

$$T(a) = \sum_{t=1}^{T} \mathbf{1}_{A_t=a}$$

R=R(t) is the reward at trial t

IMMEDIATE REWARDS

BANDITS

We can do two things (see optimisation section!).

1) Take the gradient and set it to 0 - stationary, convexity.

2) Use gradient descent.

One more thing!

$$L(a) = \frac{1}{2T_a} \sum_{t=1}^{T} \left(Q(a) - R\right)^2 \mathbf{1}_{A_t=a}$$

Use gradient descent, works for non-stationary environments!

POLICIES

THE EXPLORATION EXPLOITATION DILEMMA

POLICIES

THE EXPLORATION EXPLOITATION DILEMMA

While we exploit we are missing out on potentially better rewards.

While we explore we may not collect reward, or collect less reward.

POLICIES

THE EXPLORATION EXPLOITATION DILEMMA

Greedy $a_t = \underset{a}{\operatorname{argmax}}\ Q_t(a)$

Optimistic Greedy: initialise Q-values unrealistically high

Epsilon-Greedy : explore with probability epsilon, greedy otherwise

Softmax: $P(a) = \dfrac{e^{Q_t(a)/\tau}}{\sum_b e^{Q_t(b)/\tau}}$

What happens if *τ* grows very large or tends to 0?

IMMEDIATE REWARDS

BANDITS

- Initialise Q-values, and count T(a) for all actions a
- For each time step
  - Select an action a using policy
  - Increase the count T(a)
  - Take action a and observe reward R
  - Update Q (a)
    - For a stationary environment: $Q(a) = Q(a) + (R - Q(a))/T(a)$
    - For non-stationary environment: $Q(a) = Q(a) + \eta(R - Q(a))$

FUTURE REWARDS

CHESS, BACKGAMMON, MAZES

Stochastic

Deterministic

DALL·E

RL THROUGH THE LENS OF OPTIMISATION

REWARD MAXIMISATION

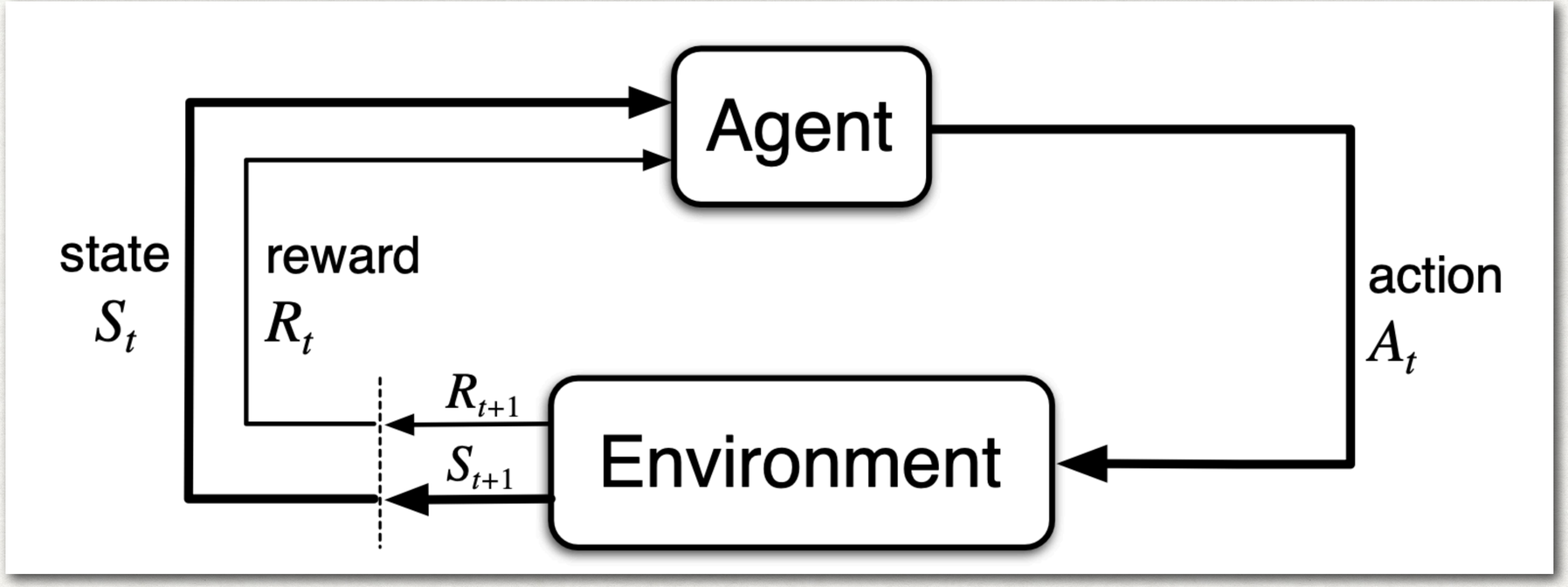
$$G_t = R_{t+1} + R_{t+2} + R_{t+3} + \ldots + R_{t+N}$$

$$G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \gamma^3 R_{t+4} + \ldots$$

$$0 \leq \gamma < 1$$

FUTURE REWARDS

MARKOV DECISION PROCESS

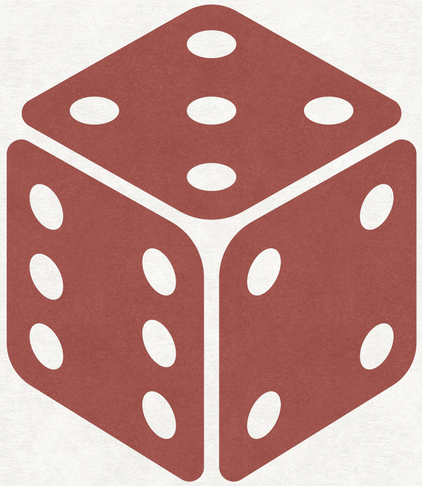


The agent-environment interaction from Sutton & Barto, 2020 - with permission

FUTURE REWARDS

MARKOVIAN PROPERTY

The future depends only on the present state and action taken, not on the past history!



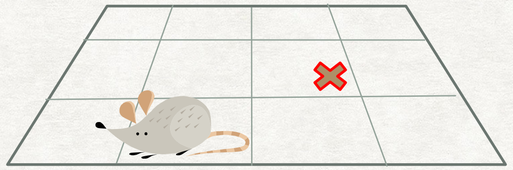
D6 dice: the probability of rolling e.g. 6, is 1/6 and it does NOT depend on the previous rolls.

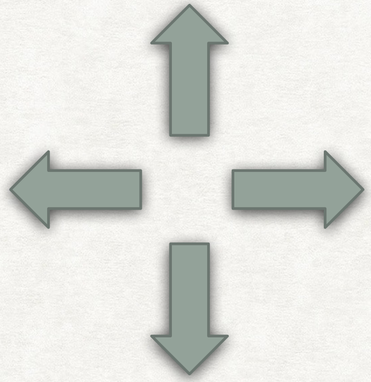
Not true in real life and in many practical scenarios, but there is a remedy.

FUTURE REWARDS

MARKOVIAN PROPERTY

The future depends only on the present state and action taken, not on the past history!





$$P(S_{t+1} \mid S_t, A_t, S_{t-1}, A_{t-1}, \ldots, S_0, A_0) = P(S_{t+1} \mid S_t, A_t)$$

FUTURE REWARDS

APPROACH

- We will see:
  - Scenarios of known environment, i.e. known transition probabilities and rewards.
  - We will be able to calculate the total expected rewards.
  - Next week we will study an algorithm that is able to estimate the rewards, so we can use it in “unknown” environments.
  - It will be again via a sensible loss function and gradient learning.

FUTURE REWARDS

MARKOV DECISION PROCESS

2 dials {0,1}

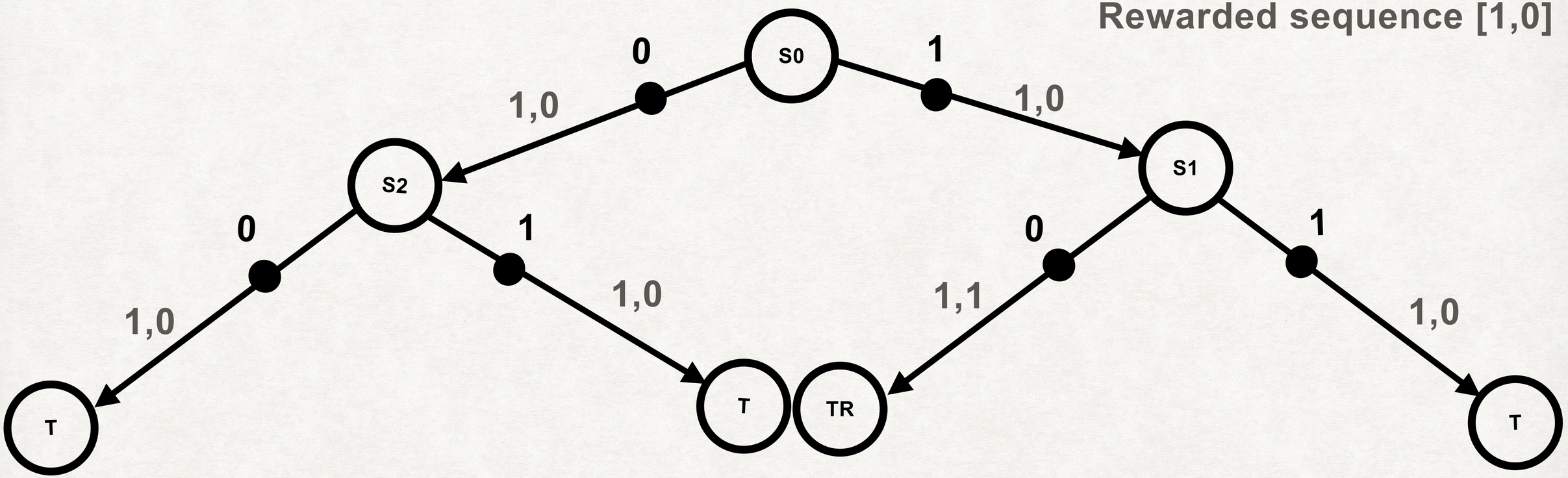
2 digits code

Example of code [1,0]

FUTURE REWARDS

DETERMINISTIC EXAMPLE

Rewarded sequence [1,0]

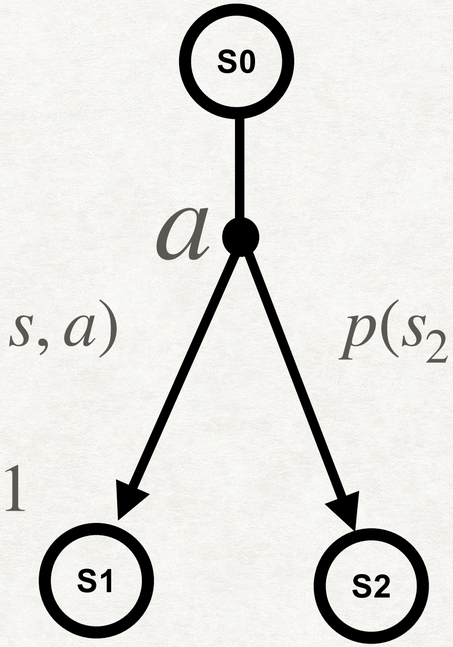


Dots correspond to actions (turning the dial to 0 or 1).
Number pairs correspond to transition probability, reward (here 0 or 1).

Deterministic setup, all probabilities are 1.

FUTURE REWARDS

STOCHASTIC EXAMPLE



$p(s_1 \mid s, a)$

$p(s_2 \mid s, a)$

$p(s_1 \mid s, a) + p(s_2 \mid s, a) = 1$

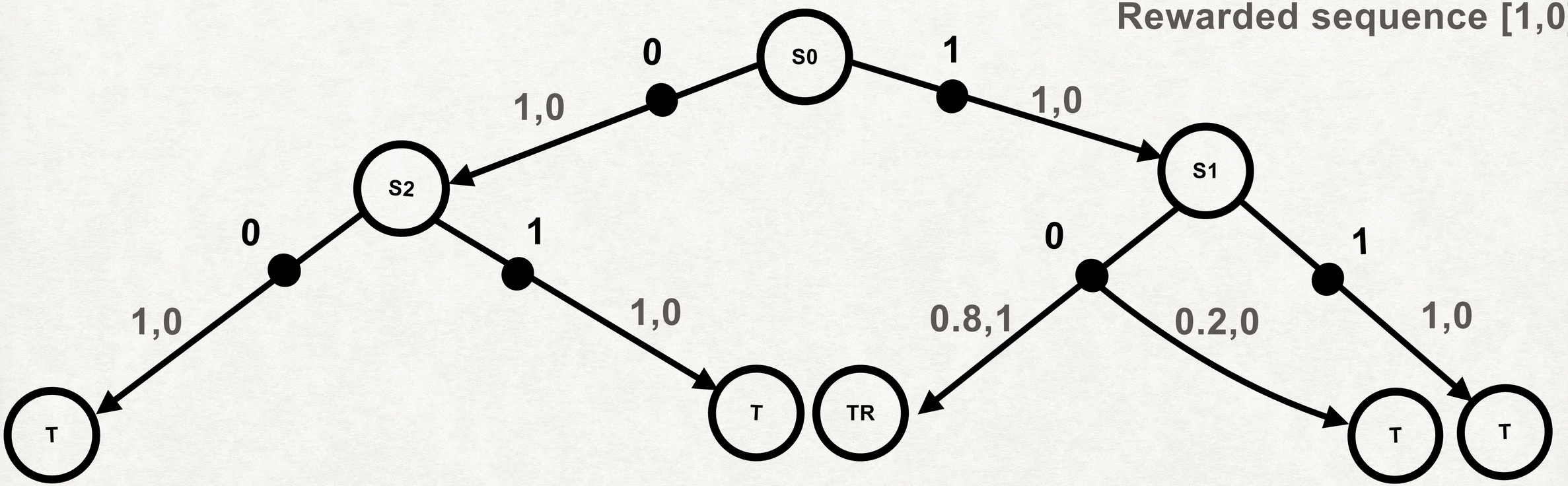
$$\sum_{s'} p(s' \mid S_t = s, A_t = a) = 1$$

$$s' \in \{s_1, s_2\ , \ldots\}$$

FUTURE REWARDS

STOCHASTIC EXAMPLE

Rewarded sequence [1,0]



Dots correspond to actions (turning the dial to 0 or 1).
Number pairs correspond to transition probability, reward (here 0 or 1).

Probabilistic setup, at state S1 the safe sometimes jams instead of opening.

FUTURE REWARDS

STATE AUGMENTATION

FUTURE REWARD

EXPECTED RETURN

$$G_t = R_{t+1} + R_{t+2} + R_{t+3} + \ldots + R_T$$

$$G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \ldots$$

FUTURE REWARD

EXPECTED RETURN

$$0 \leq \gamma < 1$$

$$G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \dots$$

Agent behaviour will be different depending on gamma values.


1000

FUTURE REWARDS

EXPECTED RETURN

$$G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \dots$$

$$G_t = R_{t+1} + \gamma \left(R_{t+2} + \gamma R_{t+3} + \dots \right)$$

$$G_{t+1} = R_{t+2} + \gamma R_{t+3} + \gamma^2 R_{t+4} + \dots$$

$$G_t = R_{t+1} + \gamma G_{t+1}$$ **Recursive!**

FUTURE REWARDS

ACTION-VALUE FUNCTIONS

$$G_t = R_{t+1} + \gamma G_{t+1}$$

$$q^{\pi}(s, a) = \mathbb{E}_{\pi}[G_t \,|\, S_t = s, A_t = a]$$

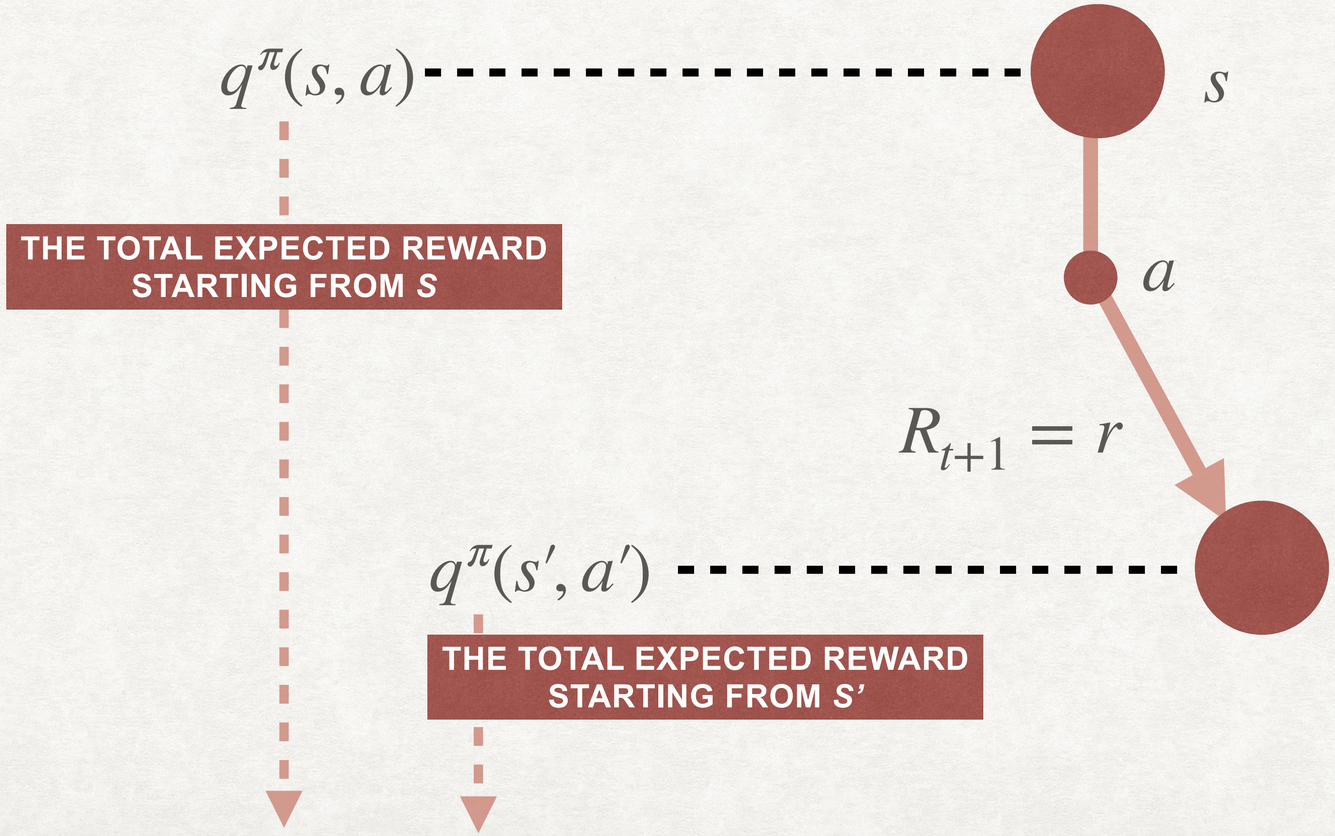
$$q^{\pi}(s, a) = \mathbb{E}_{\pi}[R_{t+1} + \gamma \, q^{\pi}(s', a') \,|\, S_t = s, A_t = a]$$

The recursive form of total returns is also followed by the expected reward.

FUTURE REWARDS

ACTION-VALUE FUNCTIONS

$$q^{\pi}(s,a) = \mathbb{E}_{\pi}[R_{t+1} + \gamma\, q^{\pi}(s',a') \,|\, S_t = s, A_t = a]$$



γ

FUTURE REWARDS

OPTIMALITY & BELLMAN EQUATION FOR ACTION-VALUES

$$q^{\pi}(s,a) = \mathbb{E}_{\pi}[R_{t+1} + \gamma\, q^{\pi}(s',a') \,|\, S_t = s, A_t = a]$$

$$q^{*}(s,a) = \max_{\pi} q^{\pi}(s,a)$$

$$q^{*}(s,a) = \mathbb{E}[R_{t+1} + \gamma \max_{a'} q^{*}(s',a') \mid S_t = s, A_t = a]$$

FUTURE RETURNS

STATE VALUE FUNCTIONS

$$v(s) = \mathbb{E}[G_t \,|\, S_t = s]$$

$$v^*(s) = \max_a q^*(s, a)$$

SUMMARY

FUTURE REWARDS

- Total return and its recurrent form.
- Expected reward: action-value functions q(s,a).
- Optimal policy.
- Optimal value function q* under optimal policy π*.
- “Greedy” selection of q.

THANK YOU!