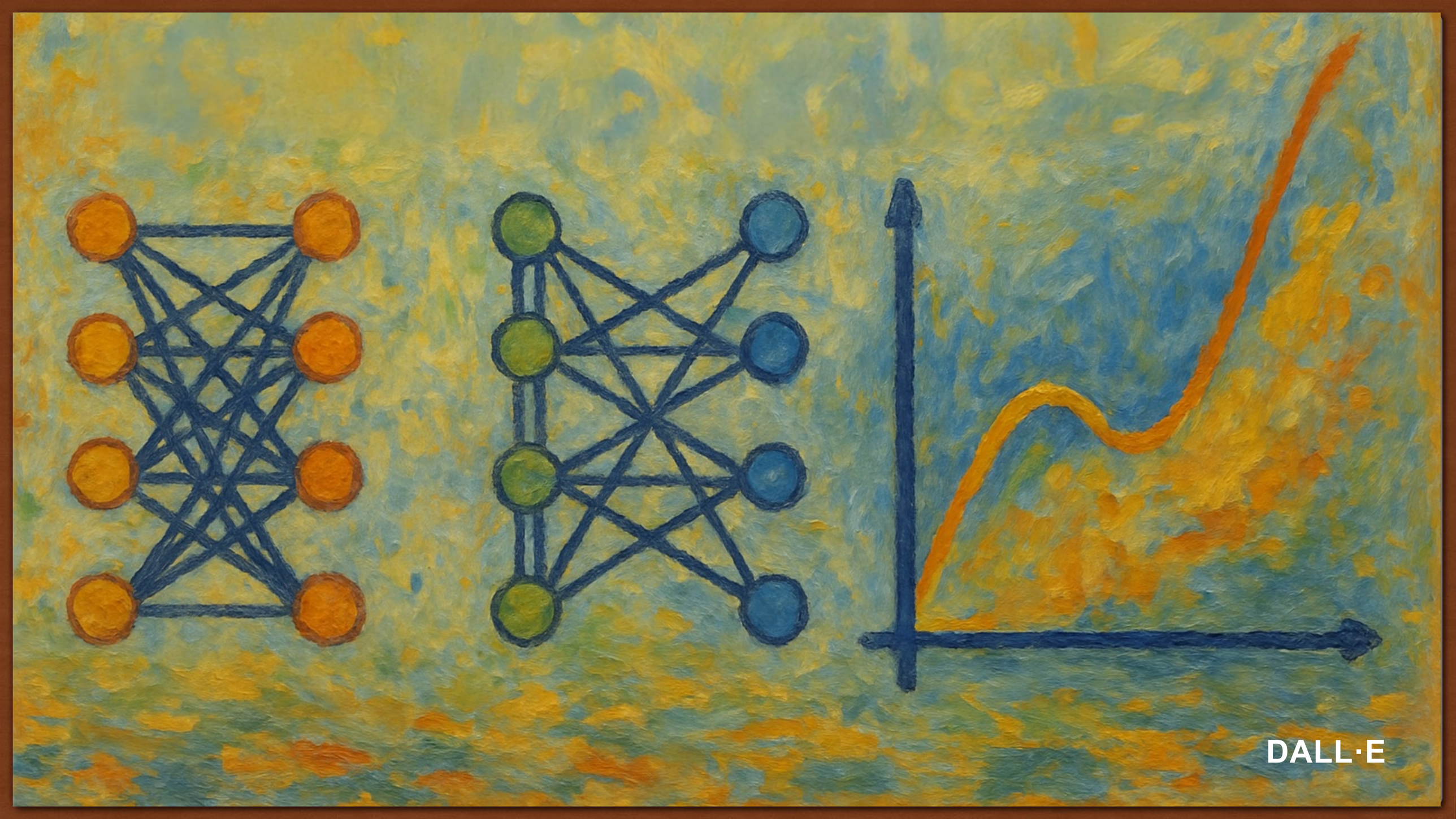
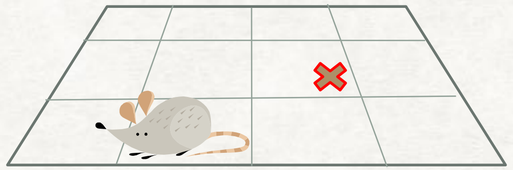
COM 3240

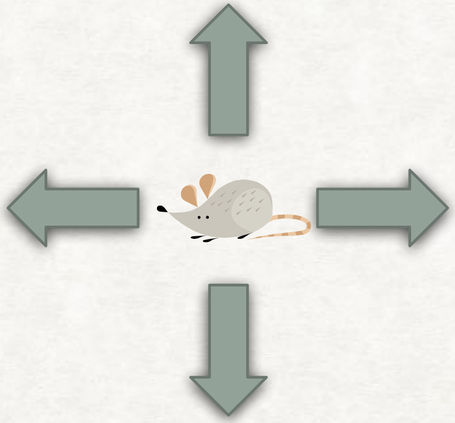
REINFORCEMENT LEARNING


DALL·E

FUTURE REWARDS

Q-VALUES





Squares = State (in this case)

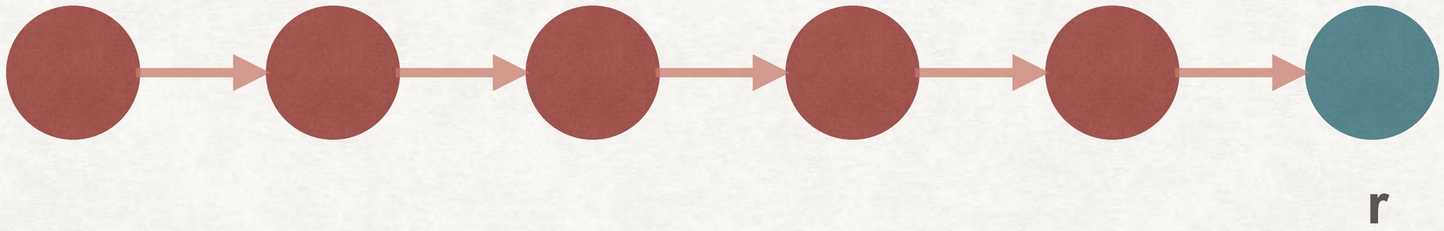
Actions

Q-values stored in a table

FUTURE REWARDS

SARSA

$$\Delta Q = \eta \left(r + \gamma \, Q(s', a') - Q(s, a) \right)$$



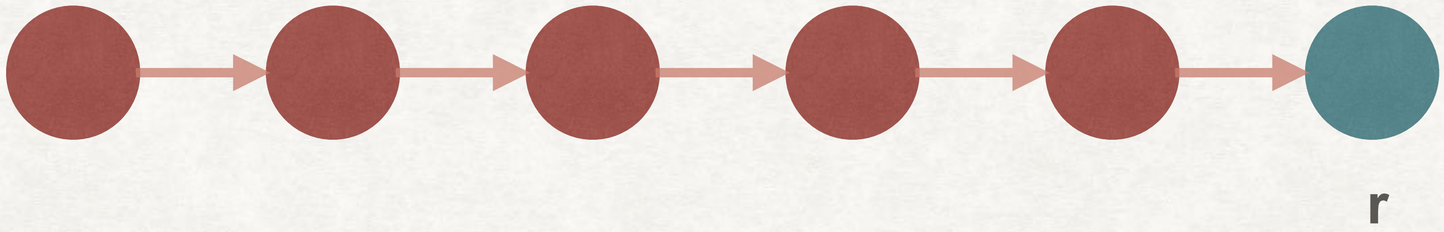
A robot walks this corridor and finds reward.

How many Q-values will be updated each time?

FUTURE REWARDS

SARSA

$$\Delta Q = \eta \left(r + \gamma \, Q(s', a') - Q(s, a) \right)$$

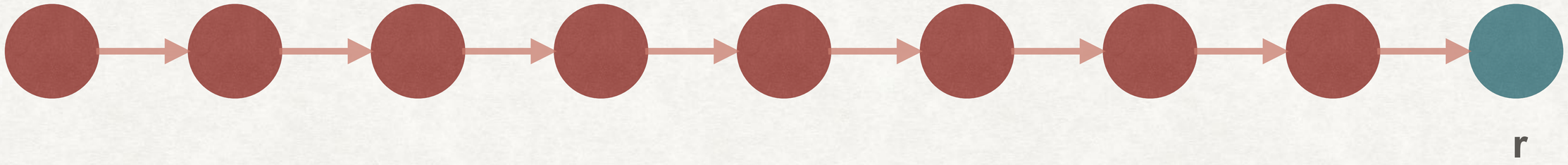


How large has to be the table storing the Q values?

FUTURE REWARDS

SARSA

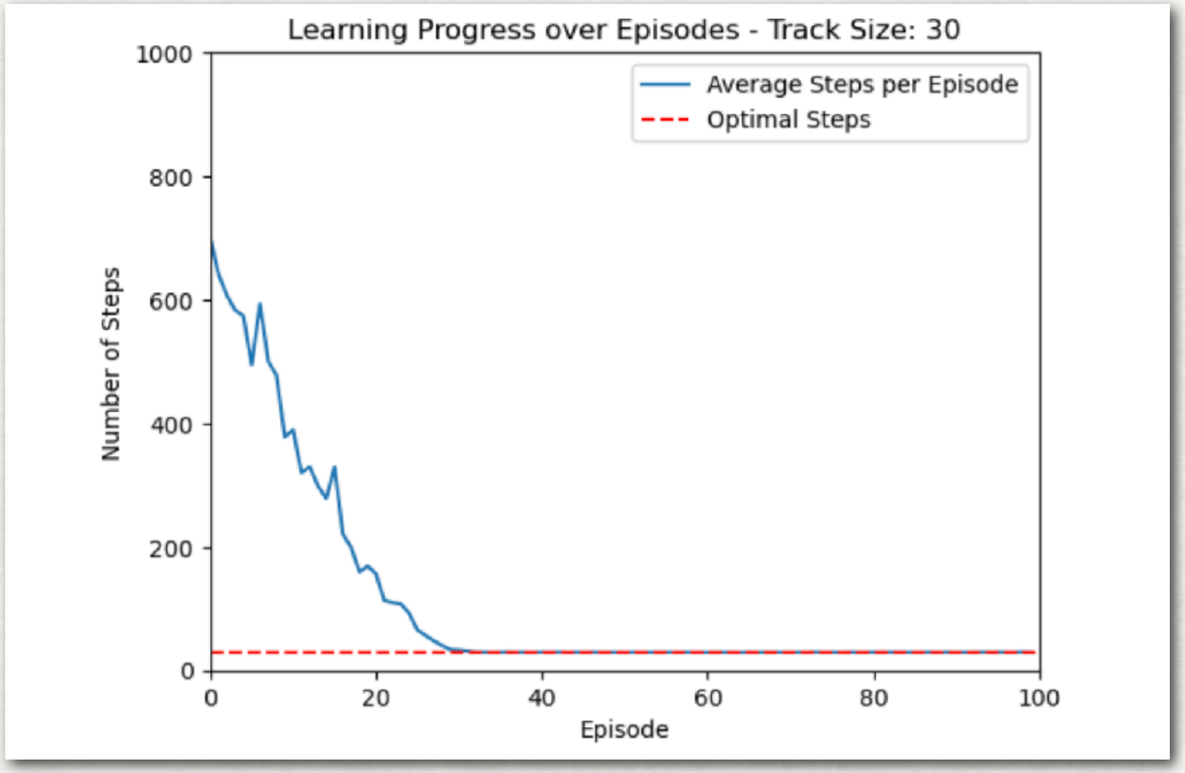
$$\Delta Q = \eta \left(r + \gamma \, Q(s', a') - Q(s, a) \right)$$

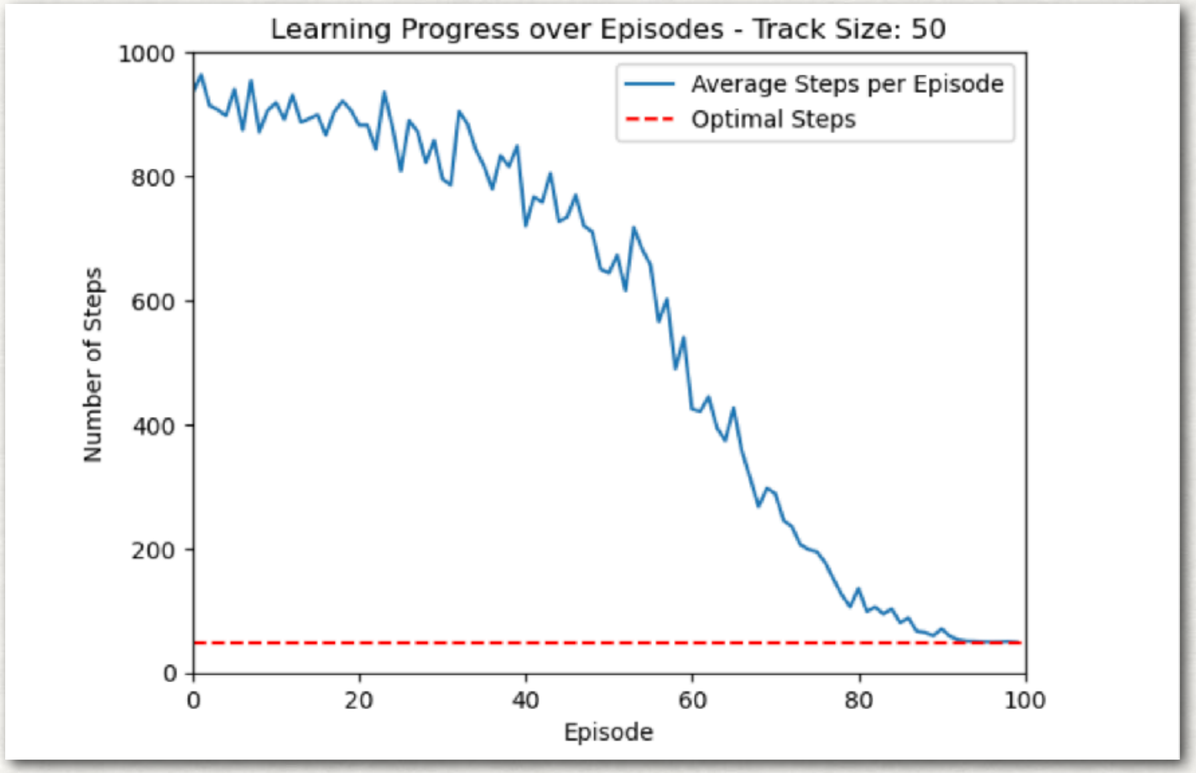


We increase the number of states. How many Q-values we need to store?

FUTURE REWARDS

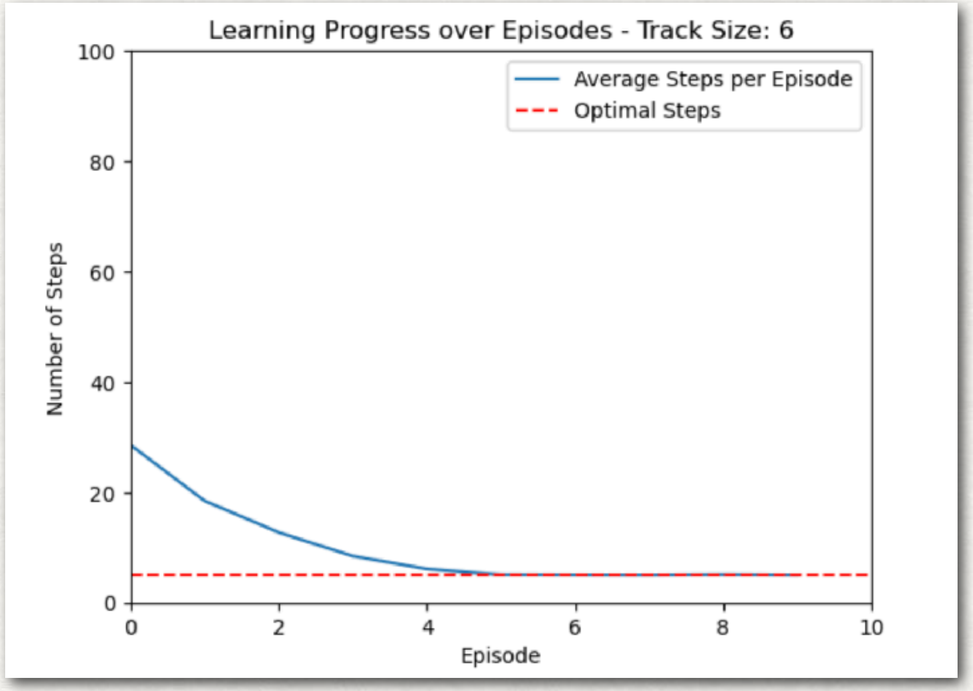
SARSA





FUTURE REWARDS

SARSA



Backward Forward

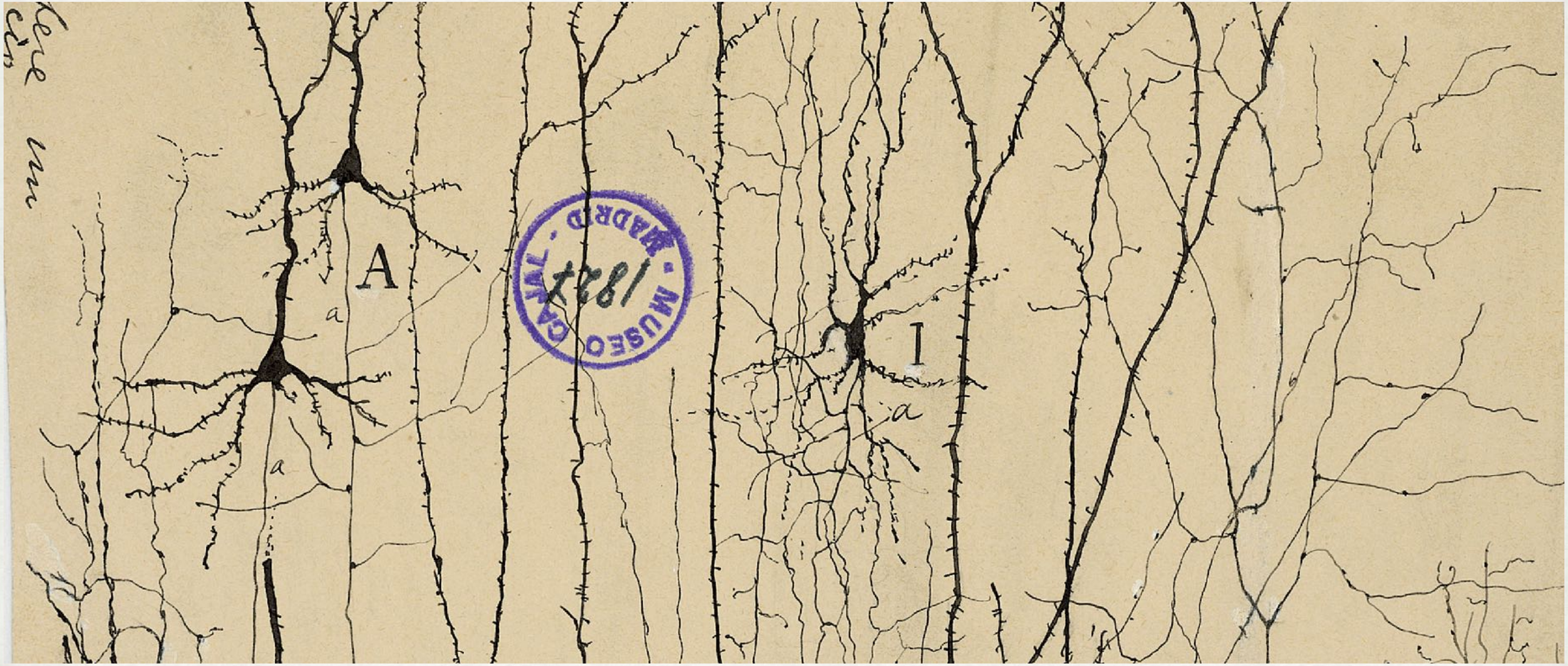
```
[0.000, 0.002, ]
[0.000, 0.012, ]
[0.000, 0.069, ]
[0.000, 0.261, ]
[0.000, 0.651, ]
[0.000, 0.000, ]
```

Has the algorithm converged?

LIMITATION OF TABULAR IMPLEMENTATIONS

LEARNING THE VALUE OF ONE STATE DOES NOT INFORM OTHER STATES

```
[0.000, 0.002, ]
[0.000, 0.012, ]
[0.000, 0.069, ]
[0.000, 0.261, ]
[0.000, 0.651, ]
[0.000, 0.000, ]
```



Ramon y Cahal

1mm³ : 10,000 neurons 3km wires

ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS

SINGLE NEURON

$$\mathbf{y}^* = F^*(\mathbf{x})$$

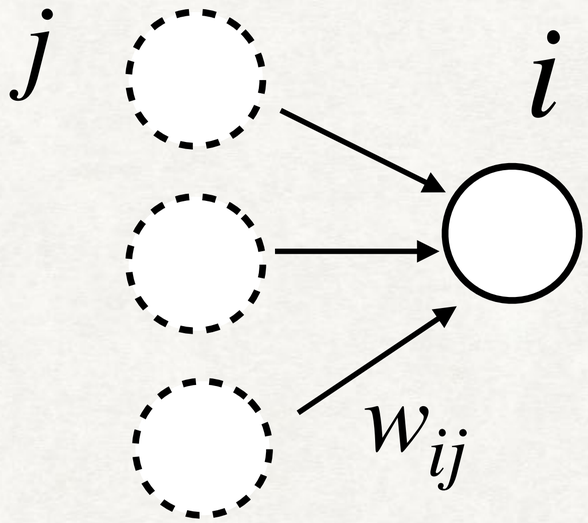
$$\mathbf{y} = F(\mathbf{x})$$

$$L = \frac{1}{2n} \sum_i \sum_{\mathbf{x}} \left(y_i^t(\mathbf{x}) - y_i(\mathbf{x}; \mathbf{W}) \right)^2$$

Samples from the input and output space of the function.

$$\mathbf{x}^{(1)}\ \mathbf{x}^{(2)}\ \mathbf{x}^{(3)} \ldots \mathbf{x}^{(n)}$$

$$\mathbf{y}^{(1)}\ \mathbf{y}^{(2)}\ \mathbf{y}^{(3)} \ldots \mathbf{y}^{(n)}$$



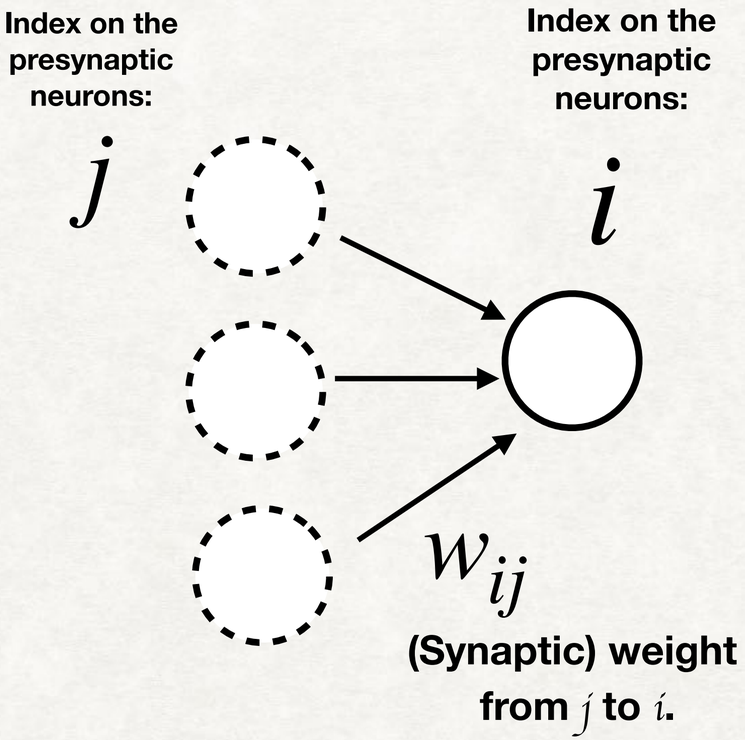
Find parameters (weights) that minimise the Loss function.

$$\Delta \mathbf{W}_{lk} \propto - \frac{\partial E}{\partial \mathbf{W}_{lk}}$$

$$\Delta \mathbf{W} \propto - \nabla_{\mathbf{W}} E$$

ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS

SINGLE NEURON



$$y = ax + b$$

$$h_i = \sum_j w_{ij} x_j + b_i$$

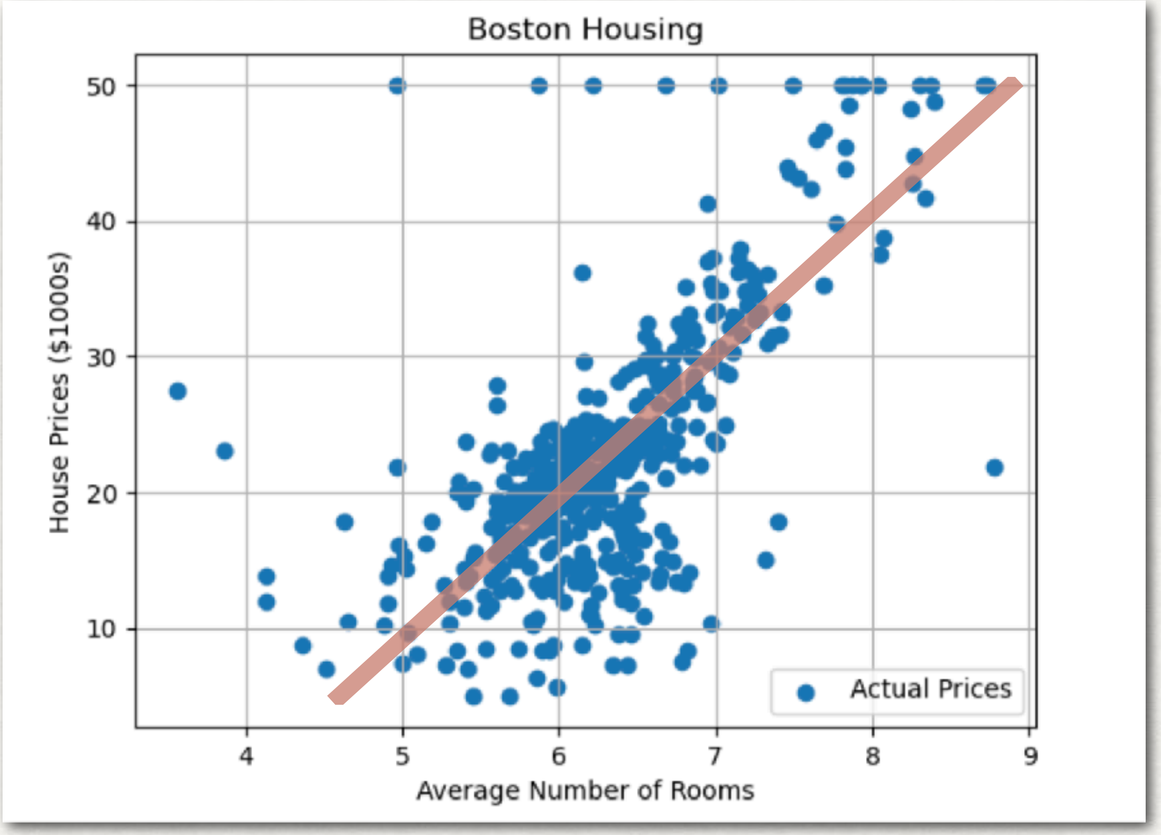
Bias

$$x_i = f(h_i)$$

Activation function *f*, sigmoidal or ReLU

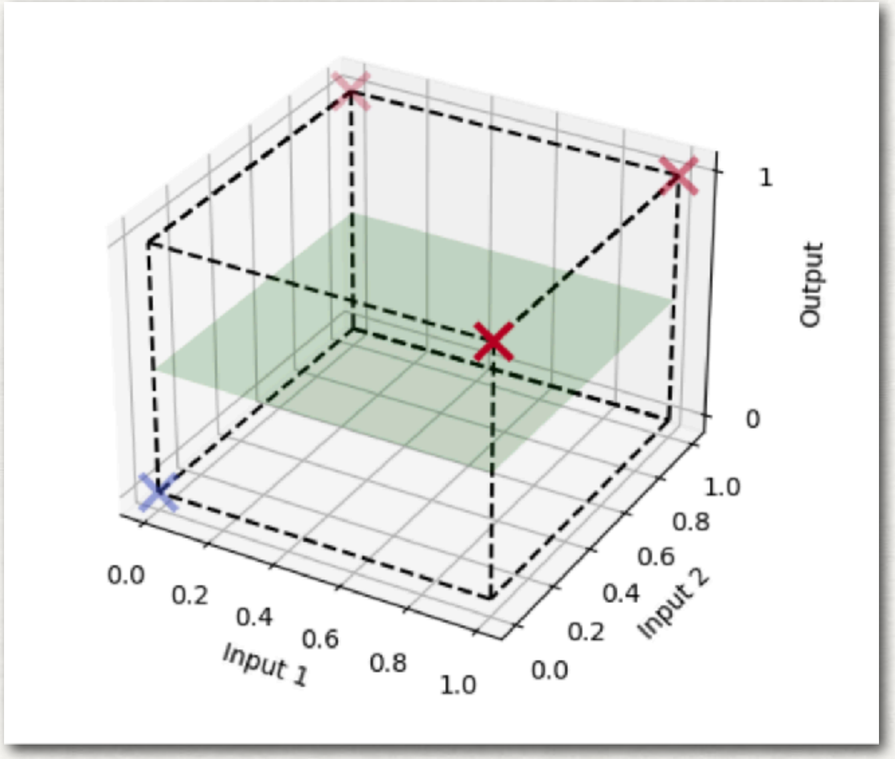
ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS

LINEAR MODEL



ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS

LINEAR SEPARABILITY



ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS

GRADIENT DESCENT

$$L = \frac{1}{2n} \sum_{i} \sum_{\mathbf{x}} \left(y_i^t(\mathbf{x}) - y_i(\mathbf{x}; \mathbf{W}) \right)^2$$

$$\Delta w_{ij} = -\eta \frac{\partial L}{\partial w_{ij}}$$

ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS

GRADIENT DESCENT

$$L = \frac{1}{2n} \sum_{i} \sum_{\mathbf{x}} \left(y_i^t(\mathbf{x}) - y_i(\mathbf{x}; \mathbf{W}) \right)^2$$

$$\frac{\partial L}{\partial w_{ij}} = -\frac{1}{n} \sum_{\mathbf{x}} \left(y_i^t(\mathbf{x}) - y_i(\mathbf{x}) \right) \frac{\partial y_i(\mathbf{x})}{\partial w_{ij}}$$

ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS

GRADIENT DESCENT

$$L = \frac{1}{2n} \sum_{i} \sum_{\mathbf{x}} \left(y_i^t(\mathbf{x}) - y_i(\mathbf{x}; \mathbf{W}) \right)^2$$

$$\frac{\partial L}{\partial w_{ij}} = -\frac{1}{n} \sum_{\mathbf{x}} \left(y_i^t(\mathbf{x}) - y_i(\mathbf{x}) \right) \frac{\partial y_i(\mathbf{x})}{\partial w_{ij}}$$

$$y_i(\mathbf{x}) = f\left(\sum_{j} w_{ij} x_j + b_i \right)$$

ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS

GRADIENT DESCENT

$$L = \frac{1}{2n} \sum_i \sum_{\mathbf{x}} \left(y_i^t(\mathbf{x}) - y_i(\mathbf{x}; \mathbf{W}) \right)^2$$

$$\frac{\partial L}{\partial w_{ij}} = -\frac{1}{n} \sum_{\mathbf{x}} \left(y_i^t(\mathbf{x}) - y_i(\mathbf{x}) \right) \frac{\partial y_i(\mathbf{x})}{\partial w_{ij}}$$

$$\Delta w_{ij} = -\eta \frac{1}{n} \sum_{\mathbf{x}} \left(y_i(\mathbf{x}) - y_i^t(\mathbf{x}) \right) y_i(\mathbf{x})(1 - y_i(\mathbf{x}))x_j$$

$$\Delta b_i = -\eta \frac{1}{n} \sum_{\mathbf{x}} \left(y_i(\mathbf{x}) - y_i^t(\mathbf{x}) \right) y_i(\mathbf{x})(1 - y_i(\mathbf{x}))$$

f: sigmoid

ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS

GRADIENT DESCENT

$$L = \frac{1}{2n} \sum_{i} \sum_{\mathbf{x}} \left(y_i^t(\mathbf{x}) - y_i(\mathbf{x}; \mathbf{W}) \right)^2$$

$$\frac{\partial L}{\partial w_{ij}} = -\frac{1}{n} \sum_{\mathbf{x}} \left(y_i^t(\mathbf{x}) - y_i(\mathbf{x}) \right) \frac{\partial y_i(\mathbf{x})}{\partial w_{ij}}$$

$$\Delta w_{ij} = \eta \, \frac{1}{n} \sum_{\mathbf{x}} \left(y_i^t(\mathbf{x}) - y_i(\mathbf{x}) \right) \cdot \mathscr{H}\left(\sum_{j} w_{ij} x_j + b_i \right) \cdot x_j$$

$$\Delta b_i = \eta \, \frac{1}{n} \sum_{\mathbf{x}} \left(y_i^t(\mathbf{x}) - y_i(\mathbf{x}) \right) \cdot \mathscr{H}\left(\sum_{j} w_{ij} x_j + b_i \right)$$

f: ReLU

ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS

EFFICIENT IMPLEMENTATION

$$y_i(\mathbf{x}) = f\left(\sum_j w_{ij} x_j + b_i\right)$$

```
X = ...  # Input data matrix, shape (batch_size, input_dim)
y_t = ...  # Target values matrix, shape (batch_size, output_dim)
y = ...  # Computed outputs matrix, shape (batch_size, output_dim)
W = ...  # Weight matrix, shape (input_dim, output_dim)
b = ...  # Bias vector, shape (output_dim,)
learning_rate = ...  # Learning rate, a scalar
```

Always check DIMENSIONS

ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS

EFFICIENT IMPLEMENTATION

$$y_i(\mathbf{x}) = f\left(\sum_j w_{ij}x_j + b_i\right)$$

Code Snippet:

```
# Compute the linear transformation using matrix multiplication
linear_output =  X @ W + b

# X( batch_size  × input_dim), W (input_dim × output_dim), b (output_dim,)
# Resulting matrix:(batch_size × output_dim)
```

ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS

EFFICIENT IMPLEMENTATION

$$y_i(\mathbf{x}) = f\left(\sum_j w_{ij}x_j + b_i\right)$$

Code Snippet:

```
# Compute the linear transformation using matrix multiplication
linear_output =  X @ W + b

# Apply activation function element-wise
y_i = activation_function(linear_output)
```

ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS

EFFICIENT IMPLEMENTATION

$$\Delta w_{ij} = \eta \sum_{\mathbf{x}} \left(y_i^t(\mathbf{x}) - y_i(\mathbf{x}) \right) y_i(\mathbf{x})(1 - y_i(\mathbf{x}))x_j,$$

Code Snippet:

```
error_term = y_t - y  # (batch_size × output_dim)
activation_derivative = y * (1 - y)
error_times_derivative = error_term * activation_derivative

# Compute updates for weights

update_weights = learning_rate * (X.T @ error_times_derivative) / X.shape[0]
```

f: sigmoid

ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS

EFFICIENT IMPLEMENTATION

$$\Delta w_{ij} = \eta \sum_{\mathbf{x}} \left(y_i^t(\mathbf{x}) - y_i(\mathbf{x}) \right) y_i(\mathbf{x})(1 - y_i(\mathbf{x}))x_j,$$

Code Snippet:

```
# X.T: (input_dim × batch_size)
# error_times_derivative: (batch_size × output_dim)
# Resulting matrix: (input_dim × output_dim), matching shape of W

# Compute updates for weights

update_weights = learning_rate * (X.T @ error_times_derivative) / X.shape[0]
```

f: sigmoid

ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS

EFFICIENT IMPLEMENTATION

$$\Delta b_i = \eta \sum_{\mathbf{x}} \left(y_i^t(\mathbf{x}) - y_i(\mathbf{x}) \right) y_i(\mathbf{x})(1 - y_i(\mathbf{x}))$$

Code Snippet:

```
# Compute updates for biases
# Sum over the batch dimension (axis=0) to get individual updates for each output neuron - in case there are more than one neurons

update_biases = learning_rate * np.sum(error_times_derivative, axis=0) / X.shape[0]
```

f: sigmoid

ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS

TRAINING, VALIDATION AND TESTING

parameters

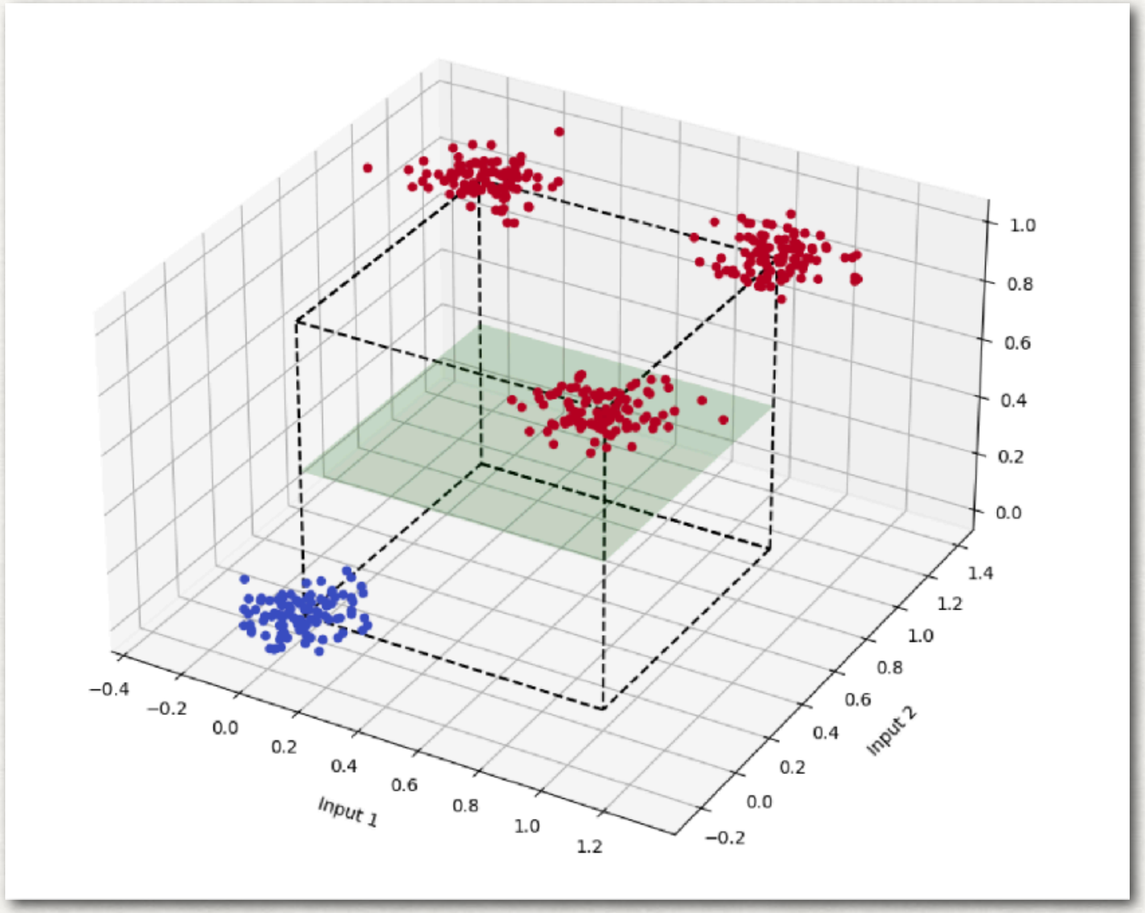
hyper-parameters

generalisation

Never check test data to optimise hyper parameters!
It may lead to overfitting.

ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS

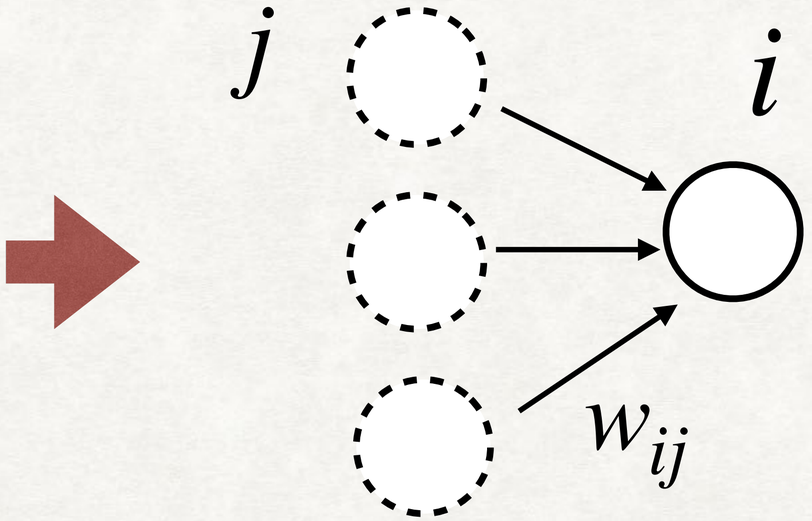
EXAMPLE



REPLACING TABLES WITH ANNS

LEARNING THE VALUE OF ONE STATE INFORMS OTHER STATES

```
[0.000, 0.002, ]
[0.000, 0.012, ]
[0.000, 0.069, ]
[0.000, 0.261, ]
[0.000, 0.651, ]
[0.000, 0.000, ]
```



THANK YOU!