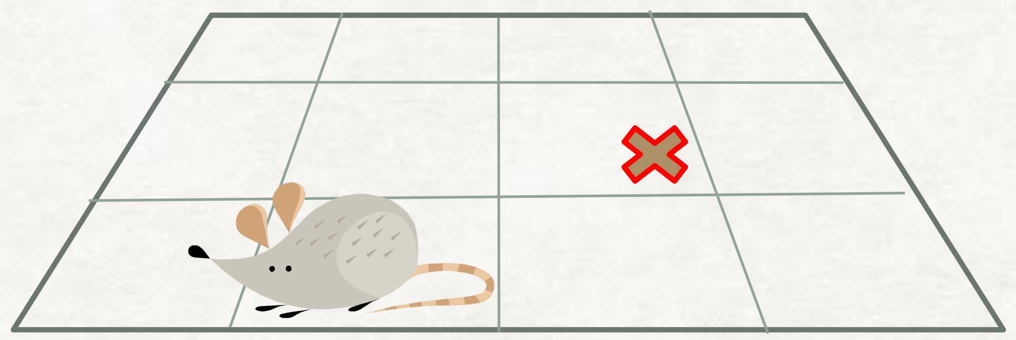
COM 3240

REINFORCEMENT LEARNING

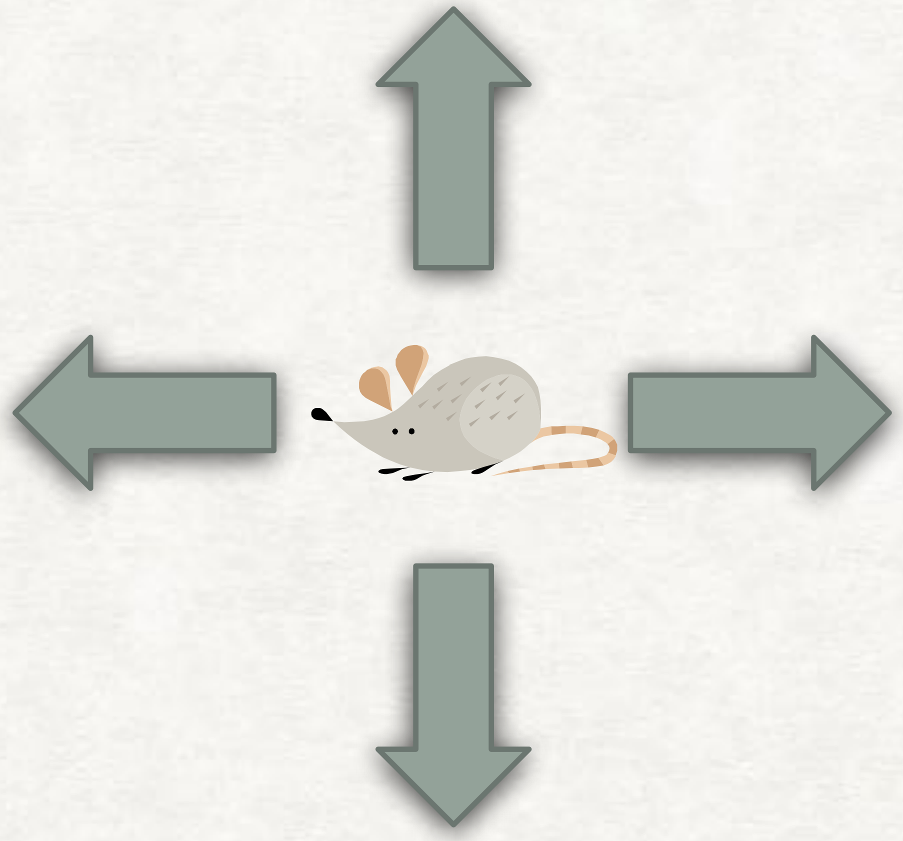
Create me a painting in the style of Dali with the title "waiting for the discounted expected returns"

FUTURE REWARDS

MARCOVIAN PROPERTY



Squares = State (in this case)



Actions

The future state depends ONLY on the current state and not on the sequence of events that preceded it.

FUTURE REWARDS

TOTAL RETURN

$$G_t = R_{t+1} + R_{t+2} + R_{t+3} + \ldots + R_T$$

$$G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \ldots$$

FUTURE REWARDS

TOTAL RETURN

$$0 \leq \gamma < 1$$

$$G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \ldots$$

$$G_t = R_{t+1} + \gamma G_{t+1}$$

FUTURE REWARDS

BELLMAN "EXPECTATION" EQUATION FOR STATE-ACTION-VALUES

$$q^\pi(s,a) = \mathbb{E}_\pi[R_{t+1} + \gamma\, q^\pi(s',a') \,|\, S_t = s, A_t = a]$$

Simplify notation: $q^\pi(s,a) = \mathbb{E}_\pi[r + \gamma\, q^\pi(s',a') \,|\, s,a]$

Current state-action

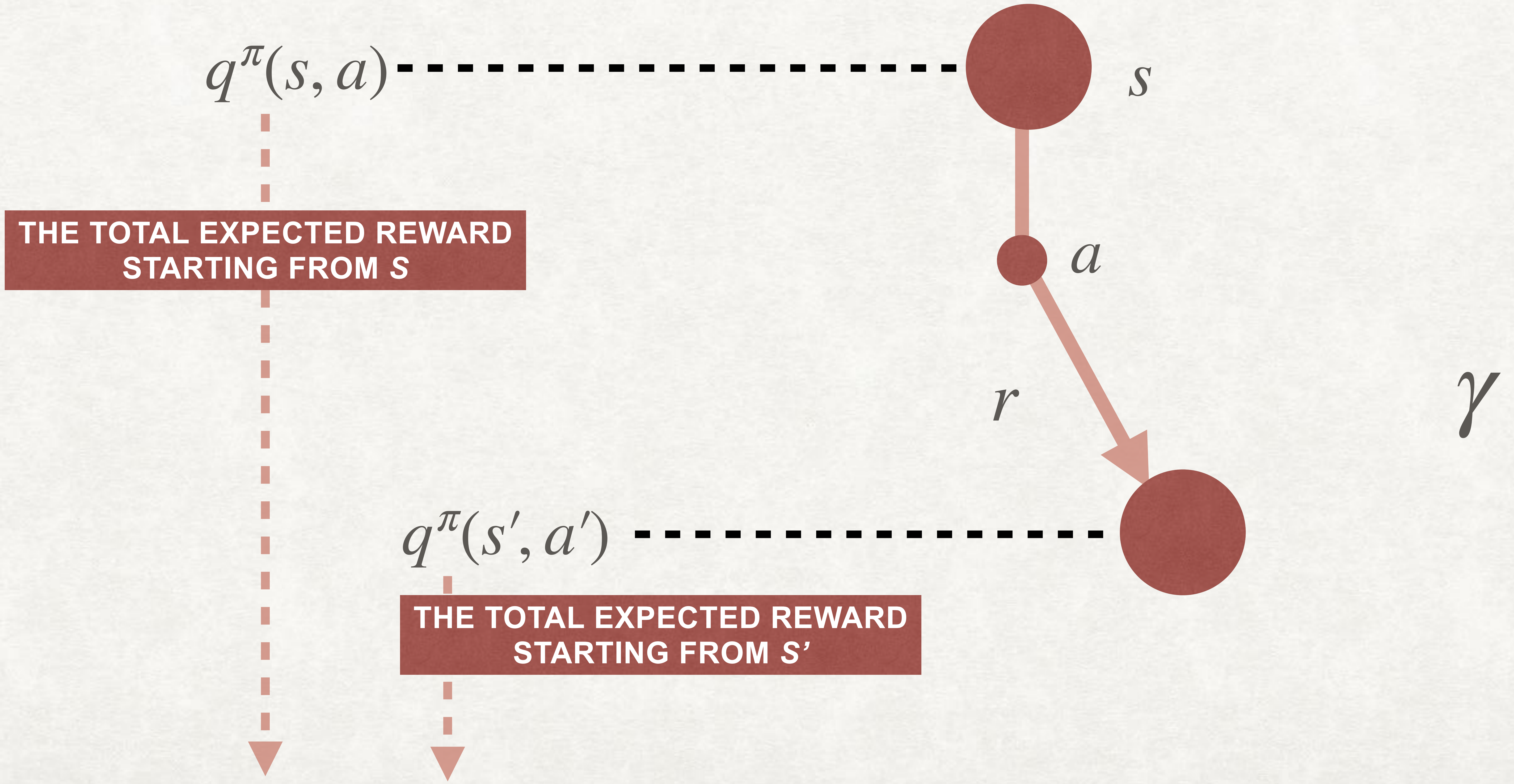
Next state-action

(Stochastic) immediate reward due to action a from state s

FUTURE REWARDS

ACTION-VALUE FUNCTIONS

$$q^\pi(s,a) = \mathbb{E}_\pi[r + \gamma\, q^\pi(s',a') \,|\, s,a]$$



FUTURE REWARDS

BELLMAN OPTIMALITY “EXPECTATION” EQUATION FOR STATE-ACTION-VALUES

$$q^*(s, a) = \max_{\pi} q^{\pi}(s, a)$$

One policy better or equal than any other

Greedy policy *

$$q^{\pi}(s, a) = \mathbb{E}_{\pi}[r + \gamma\, q^{\pi}(s', a') \,|\, s, a] \qquad \pi = *$$

$$q^*(s, a) = \mathbb{E}[r + \gamma \max_{a'} q^*(s', a') \mid s, a]$$

IMMEDIATE REWARDS - A REMINDER

BANDITS

$$q^*(a) \doteq E[R \mid A_t = a]$$

Q-value : estimate of the expected return

$$Q(a) \approx q^*(a)$$

In RL we obtain an estimate via sampling

$$R(A_t = a) \sim P(R \mid A_t = a)$$

Desirable: $E[\left(Q(a) - R(A_t = a)\right)^2] \approx 0$

IMMEDIATE REWARDS - A REMINDER

BANDITS

Minimise: $$\frac{E[\left(Q(a) - R(A_t = a)\right)^2]}{2}$$

$$L(a) = \frac{1}{2T_a} \sum_{t=1}^{T} \left(Q(a) - R\right)^2 \mathbf{1}_{A_t=a}$$

$$\mathbf{1}_{A_t=a} = \begin{cases} 1 & \textbf{if } A_t = a \\ 0 & \textbf{otherwise.} \end{cases}$$

Indicator function

$$T(a) = \sum_{t=1}^{T} \mathbf{1}_{A_t=a}$$

R=R(t) is the reward at trial t

IMMEDIATE REWARDS

BATCH AND ONLINE UPDATES

$$\Delta Q(a) = -\eta \frac{dL(a)}{dQ} = -\eta \cancel{\sum_{t=1}^{T}} \left(Q(a) - R\right) \mathbf{1}_{A_t=a}$$

Note a: action, α: learning rate!

We can suppress the sum. This suggests we now update our estimate after each trial (sample) known as online learning. We then write:

$$\Delta Q(a) = -\eta \left(Q(a) - R\right) \mathbf{1}_{A_t=a}$$

i.e. for trial 1 to T, if action a is taken, we update its Q value by:

$$Q(a) = Q(a) - \eta \left(Q(a) - R\right)$$

FUTURE REWARDS

SARSA

$$q^{\pi}(s, a) = E_{\pi}[r + \gamma\, q^{\pi}(s', a') \,|\, s, a]$$

$$E_{\pi}[r + \gamma\, q^{\pi}(s', a') - q^{\pi}(s, a) \,|\, s, a] = 0$$

$$\mathcal{L}(s, a) = \frac{1}{2N} \sum_{i=1}^{N} \left(Q(s, a) - [r^{(i)} + \gamma Q(s^{'(i)}, a^{'(i)})] \right)^2$$

FUTURE REWARDS

SARSA

$$\mathscr{L}(s,a) = \frac{1}{2N}\sum_{i=1}^{N}\left(Q(s,a) - [r^{(i)} + \gamma Q(s^{'(i)}, a^{'(i)})]\right)^2$$

$$\frac{\partial \mathscr{L}(Q(s,a))}{\partial Q(s,a)} = \frac{1}{N}\sum_{i=1}^{N}\left(Q(s,a) - [r + \gamma Q(s', a')]\right)$$

$$\Delta Q(s,a) = -\eta\frac{1}{N}\sum_{i=1}^{N}\left(Q(s,a) - [r + \gamma Q(s', a')]\right)$$

FUTURE REWARDS

SARSA

$$\Delta Q(s,a) = -\eta \frac{1}{N} \sum_{i=1}^{N} \left(Q(s,a) - [r + \gamma Q(s',a')] \right)$$

N=1 (online learning):

$$\Delta Q(s,a) = -\eta \left(Q(s,a) - [r + \gamma Q(s',a')] \right)$$

$$\Delta Q(s,a) = \eta \left(r + \gamma Q(s',a') - Q(s,a) \right)$$

FUTURE REWARDS

SARSA: REWARD - “ANTICIPATED REWARD”

$$\Delta Q = \eta \left(\underbrace{r + \gamma\, Q(s', a')}_{\text{What I “actually” get}} - \underbrace{Q(s, a)}_{\text{Anticipated reward}} \right)$$

FUTURE REWARDS

POLICIES

Greedy $a = \underset{a}{\operatorname{argmax}}\ Q(s,a)$

Optimistic Greedy: initialise Q-values unrealistically high

Epsilon-Greedy : explore with probability epsilon, greedy otherwise

Softmax: $P(a) = \dfrac{e^{Q(s,a)/\tau}}{\sum_b e^{Q(s,b)/\tau}}$

FUTURE REWARDS

THE SARSA ALGORITHM

1. Initialise Q(s, a) arbitrarily for all $s \in S$ and $a \in A(s)$.
2. Repeat (for each episode):
   a. Initialise s.
   b. Choose an action a from s using a policy derived from Q (e.g., ε-greedy).
   c. Repeat (for each step of episode):
      i. Take action a, observe reward r and next state s'.
      ii. Choose a' from s' using policy derived from Q (e.g., ε-greedy).
      iii. $Q(s, a) \leftarrow Q(s, a) + \eta * [r + \gamma * Q(s', a') - Q(s, a)]$.
      iv. $s \leftarrow s'$; $a \leftarrow a'$.
   d. until s is terminal.

On-policy

FUTURE REWARDS

THE SARSA ALGORITHM

Former head of COM

In 1994, Gavin Rummery and Mahesan Niranjan published a paper titled "Online Q-Learning using Connectionist Systems," in which they introduced an algorithm they called at the time "Modified Connectionist Q-Learning." In 1996, Singh and Sutton dubbed this algorithm Sarsa because of the quintuple of events that the algorithm uses: (S_t, A_t, R_{t+1}, S_{t+1}, A_{t+1}). People often like knowing where these names come from as you will soon see, RL researchers can get pretty creative with these names.

https://livebook.manning.com/concept/reinforcement-learning/this-algorithm

FUTURE REWARDS

THE SARSA ALGORITHM

Right after obtaining his Ph.D. in 1995, Gavin became a programmer and later a lead programmer for the company responsible for the series of the Tomb Raider games. Gavin has had a very successful career as a game developer.

Mahesan, who became Gavin’s Ph.D. supervisor after the unexpected death of Gavin’s original supervisor, followed a more traditional academic career holding lecturer and professor roles ever since his Ph.D. graduation in 1990.

https://livebook.manning.com/concept/reinforcement-learning/this-algorithm

FUTURE REWARDS

Q-LEARNING

$$q*(s,a) = \max_{\pi} q^{\pi}(s,a)$$

$$q*(s,a) = E[R_{t+1} + \gamma \max_{a'} q*(s',a') \mid S_t = s, A_t = a]$$

SARSA

$$\Delta Q(s,a) = \eta \left(r + \gamma Q(s',a') - Q(s,a) \right)$$

Q-Learning

$$\Delta Q(s,a) = \eta(r + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

FUTURE REWARDS

Q-LEARNING

$$\Delta Q = \eta \left(\underbrace{r + \gamma \max_{a'} Q(s', a')}_{\text{What I “actually” get}} - \underbrace{Q(s, a)}_{\text{Anticipated reward}} \right)$$

FUTURE REWARDS

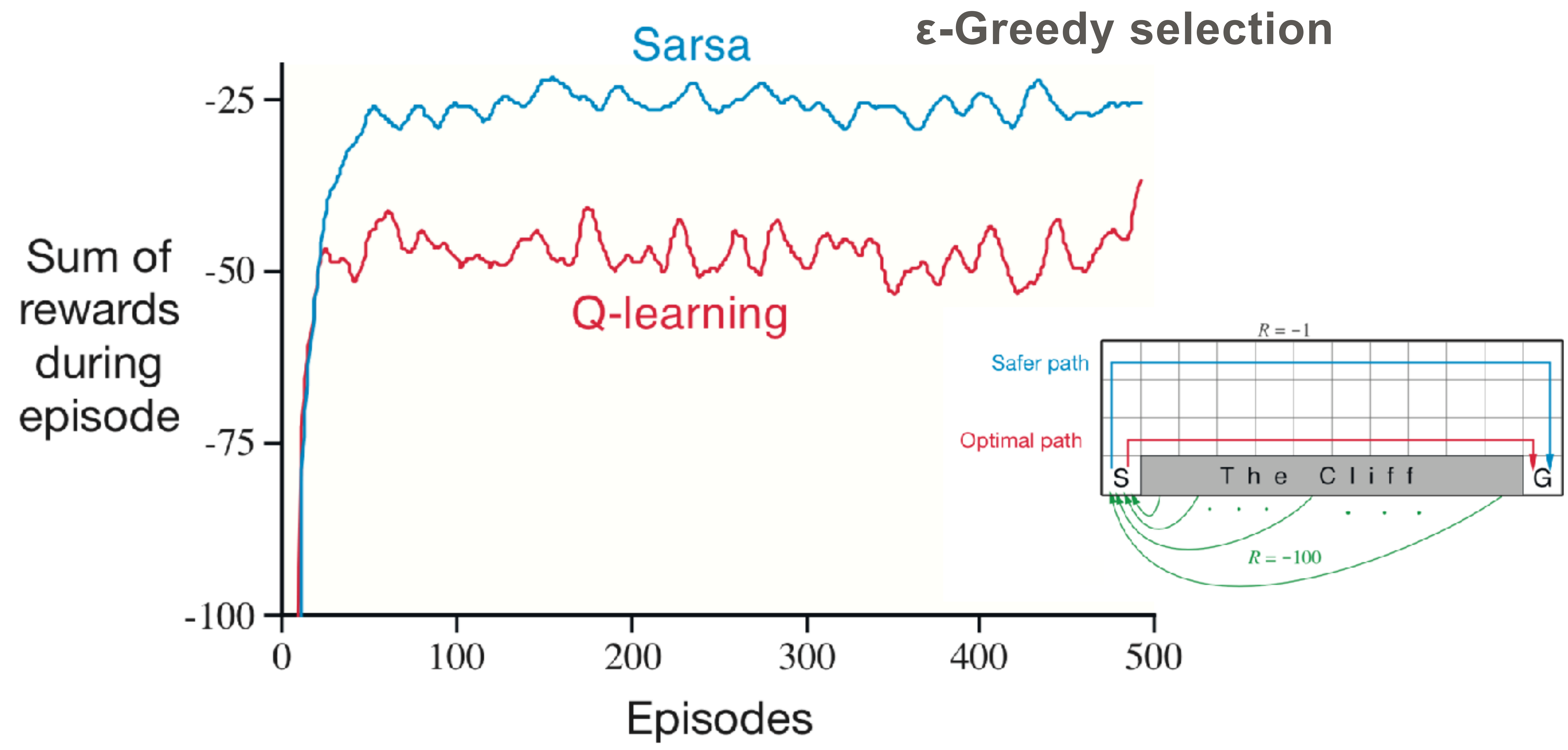
THE Q-LEARNING ALGORITHM

1. Initialise Q(s, a) arbitrarily for all $s \in S$ and $a \in A(s)$.
2. Repeat (for each episode):
   a. Initialise s.
   b. Repeat (for each step of episode):
      i. Choose an action a from s using a policy derived from Q (e.g., ε-greedy).
      ii. Take action a, observe reward r and next state s'.
      iii. $Q(s, a) \leftarrow Q(s, a) + \eta * [r + \gamma * \max_{a'} Q(s', a') - Q(s, a)]$.
      iv. $s \leftarrow s'$.
   c. until s is terminal.

Off-policy

FUTURE REWARDS

SARSA VS Q-LEARNING

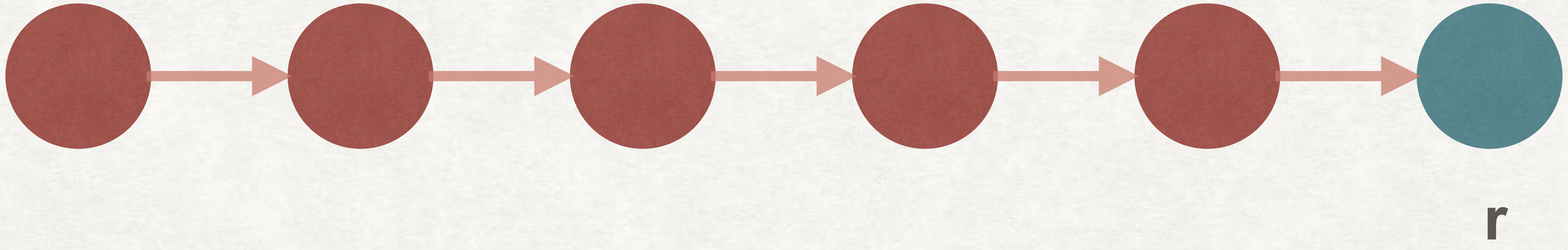


From Sutton & Barto, 2020 - with permission.

FUTURE REWARDS

SARSA

$$\Delta Q = \eta \left(r + \gamma\, Q(s', a') - Q(s, a) \right)$$



A robot walks this corridor and finds reward. How many Q-values will be updated?

FUTURE REWARDS

ELIGIBILITY TRACE

Design an ant leaving a trace of pheromone.

DALL·E

FUTURE REWARDS

ELIGIBILITY TRACE

We mark that action pair that were visited/used:
$e(s, a)$

Update eligibility trace for the most recent state action pair:
$e(s, a) \leftarrow e(s, a) + 1$

Decay eligibility trace for all other states:
$e(s, a) \leftarrow \gamma\lambda * e(s, a)$

FUTURE REWARDS

SARSA: TABULAR ELIGIBILITY TRACE

1. Initialise $Q(s, a)$ arbitrarily for all $s \in S$ and $a \in A(s)$.
2. Initialise eligibility traces $e(s, a)$ for all s, a to zeros.
3. Repeat (for each episode):
    a. Initialise s, eligibility traces $e(s, a)$ for all s, a to zeros.
    b. Choose an action a from s using policy derived from Q (e.g., ε-greedy).
    c. Repeat (for each step of episode):
        i. Take action a, observe reward r, and next state s'.
        ii. Choose a' from s' using policy derived from Q (e.g., ε-greedy).
        iii. For the visited s, a:
            - Update delta: $\delta \leftarrow r + \gamma * Q(s', a') - Q(s, a)$
            - Update eligibility trace: $e(s, a) \leftarrow e(s, a) + 1$
        iv. For all states:
            - Update $Q(s, a) \leftarrow Q(s, a) + \eta * \delta * e(s, a)$
            - Decay eligibility trace: $e(s, a) \leftarrow \gamma\lambda * e(s, a)$
        v. $s \leftarrow s'$; $a \leftarrow a'$.
    until s is terminal.

FUTURE REWARDS

TABULAR ELIGIBILITY TRACE

1. Initialise $Q(s, a)$ arbitrarily for all $s \in S$ and $a \in A(s)$.
2. Repeat (for each episode):
    a. Initialise s and eligibility traces $e(s, a)$ for all s, a to zeros.
    b. Repeat (for each step of episode):
        i. Choose an action a from s using policy derived from Q (e.g., ε-greedy).
        ii. Take action a, observe reward r, and next state s'.
        iii. For the selected s, a:
            - Update delta: $\delta \leftarrow r + \gamma * \max\_a' Q(s', a') - Q(s, a)$
            - Update eligibility trace: $e(s, a) \leftarrow e(s, a) + 1$
        iv. For all s,a
            -Update $Q(s, a) \leftarrow Q(s, a) + \eta * \delta * e(s, a)$
            -Decay eligibility trace: $e(s, a) \leftarrow \gamma\lambda * e(s, a)$
        v. $s \leftarrow s'$.
    until s is terminal.

FUTURE REWARDS

TEMPORAL DIFFERENCE LEARNING

- By approximating the action-value q by an estimate Q and by writing an error function based on a form of the Bellman equation we derived two Temporal Difference algorithms:
  - SARSA (State Action Reward State Action) : “safe”
  - Q-Learning : optimal
- An eligibility trace can speed up the performance of the algorithms by propagating to the pathway taken information about the reward.

THANK YOU!