

COM 3240

REINFORCEMENT LEARNING



“
**FUTURE ACTIONS DO NOT AFFECT
THE PAST.**

— *Common experience*

”



THE MONTY HALL PROBLEM

DALL·E

PROBABILISTIC NATURE OF REINFORCEMENT LEARNING

EXPECTATIONS



DALL·E

PROBABILITIES

LIKELIHOOD THAT AN EVENT WILL OCCUR

$$P(A)$$

$$0 \leq P(A) \leq 1$$

Sample space $S = \{\text{all possible outcomes}\}$

$$P(S) = 1$$



Example d4: Rolls 1 to 4

RPROBABILITIES

RANDOM VARIABLES

Variables that can assume different values,
each with a certain probability.



Example d4: Rolls 1 to 4

$$P(X = x)$$

$$P(X = 1) \quad P(X = 2) \quad P(X = 3) \quad P(X = 4)$$

PROBABILITIES

PROBABILITY DISTRIBUTION

Example d4: Rolls 1 to 4 with equal probability (if fair)

$$P(X = x) = \frac{1}{4}, \quad x \in \{1,2,3,4\}$$

uniform probability distribution

A probability distribution specifies how probabilities are distributed over the values of a random variable.



PROBABILITIES

EXPECTATION

Example d4: Rolls 1 to 4 with equal probability (if fair)



uniform probability distribution

$$E[X] = \sum_x x \cdot P(X = x)$$

$$E[X] = \frac{1}{4}(1 + 2 + 3 + 4) = 2.5$$

Expected Reward

PROBABILITIES

JOINT PROBABILITY

$$P(X = x, Y = y) = P(\{X = x\} \cap \{Y = y\})$$

$$P(X = x, Y = y) \geq 0$$



PROBABILITIES

JOINT PROBABILITY FOR INDEPENDENT VARIABLES

$$P(X = x, Y = y) = P(\{X = x\} \cap \{Y = y\})$$

$$= P(X = x) \cdot P(Y = y)$$



Dice rolls

PROBABILITIES

JOINT PROBABILITY DISTRIBUTION

$$P(X = x, Y = y) = P(\{X = x\} \cap \{Y = y\})$$

$$P(X = x, Y = y) \geq 0$$

Describes the probabilities of all possible combinations of outcomes for variables X and Y (e.g. in a table).

$$\sum_x \sum_y P(X = x, Y = y) = 1.$$



EXAMPLE

ROLLING TWO DICES

Dungeons & Dragons

To hit: roll 1d20

Advantage: roll two 1d20 dices

Assume known: probably p to hit with one roll

Calculate the probability to hit with advantage



Consider the joint probability of hitting your opponent when rolling two dices



joint probability distribution

First dice hits but not the second	$p(1-p)$
Second dice hits but not the first	$(1-p)p$
Both dices hit	$p \cdot p$
None hits	$(1-p) \cdot (1-p)$
$p_{hit} = p$	Sum 1

Consider the joint probability of hitting your opponent when rolling two dices

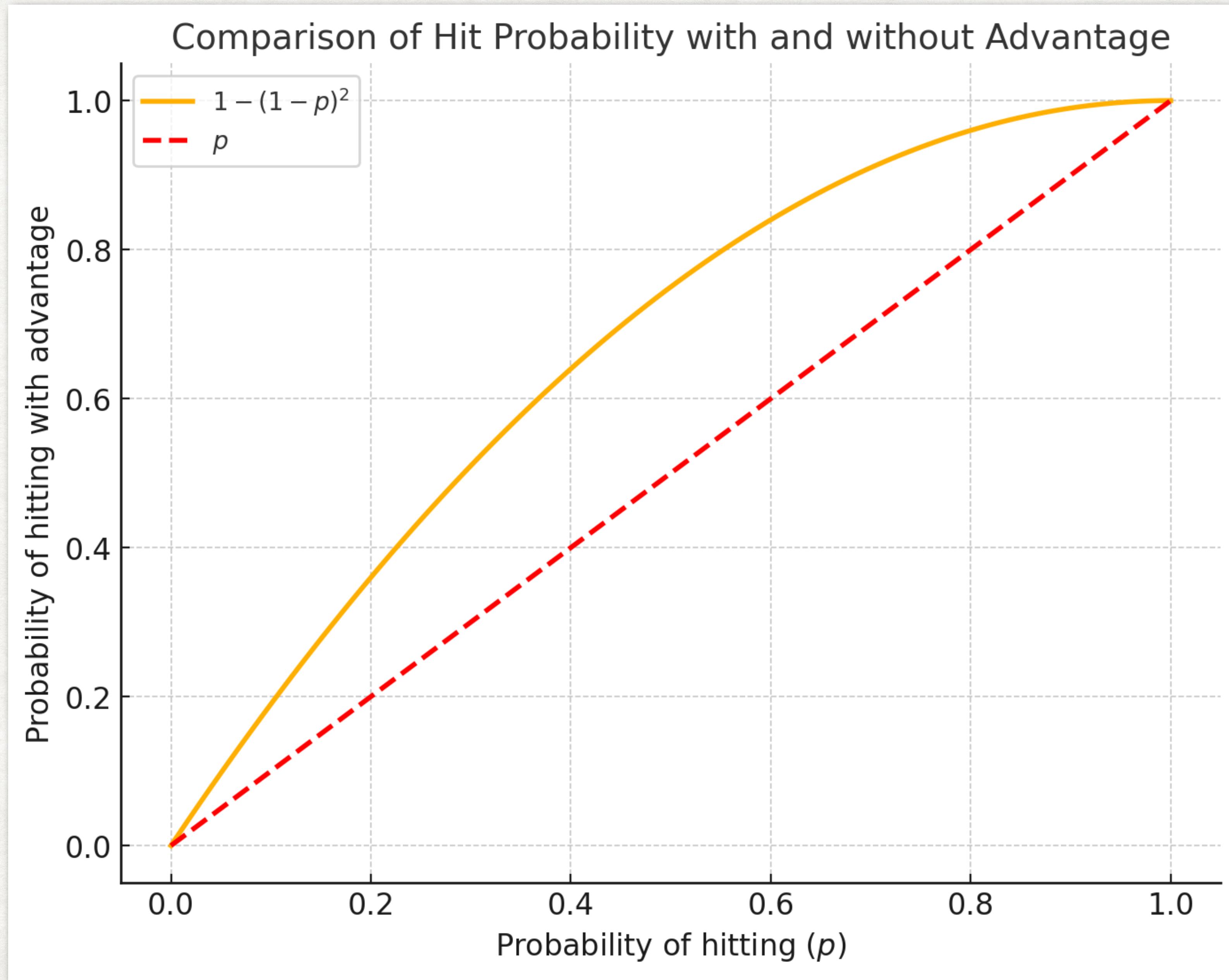


At least one dice hits = 1 - none of the dices hits

$$P_{hit-advantage} = 1 - (1 - p)^2 = p(2 - p)$$

$$P_{hit} = p$$

joint probability distribution



PROBABILITIES

PROPERTIES OF EXPECTATION

$$E[aX + bY] = aE[X] + bE[Y]$$

Linearity

$$E[XY] = E[X] \cdot E[Y]$$

Expectation of products for independent variables

$$E[c] = c$$

PROBABILITIES

VARIANCE

Definition:

$$\mathbf{Var}(X) = E[(X - E[X])^2]$$

$$\mathbf{Var}(X) = E[X^2] - (E[X])^2$$

Easy to show, using the properties of expectation

BURNING HANDS

1st level evocation

Casting Time: 1 action

Range: Self (15-foot cone)

Target: Self (15-foot cone)

Components: V S

Duration: Instantaneous

Classes: Sorcerer, Wizard

As you hold your hands with thumbs touching and fingers spread, a thin sheet of flames shoots forth from your outstretched fingertips. Each creature in a 15-foot cone must make a Dexterity saving throw. A creature takes $3d6$ fire damage on a failed save, or half as much damage on a successful one. The fire ignites any flammable objects in the area that aren't being worn or carried.

At Higher Levels: When you cast this spell using a spell slot of 2nd level or higher, the damage increases by $1d6$ for each slot level above 1st.

MAGIC MISSILE

1st level evocation

Casting Time: 1 action

Range: 120 feet

Target: A creature of your choice that you can see within range

Components: V S

Duration: Instantaneous

Classes: Sorcerer, Wizard

You create three glowing darts of magical force. Each dart hits a creature of your choice that you can see within range. A dart deals $1d4 + 1$ force damage to its target. The darts all strike simultaneously, and you can direct them to hit one creature or several.

At Higher Levels: When you cast this spell using a spell slot of 2nd level or higher, the spell creates one more dart for each slot above 1st.

Calculate the expected damage

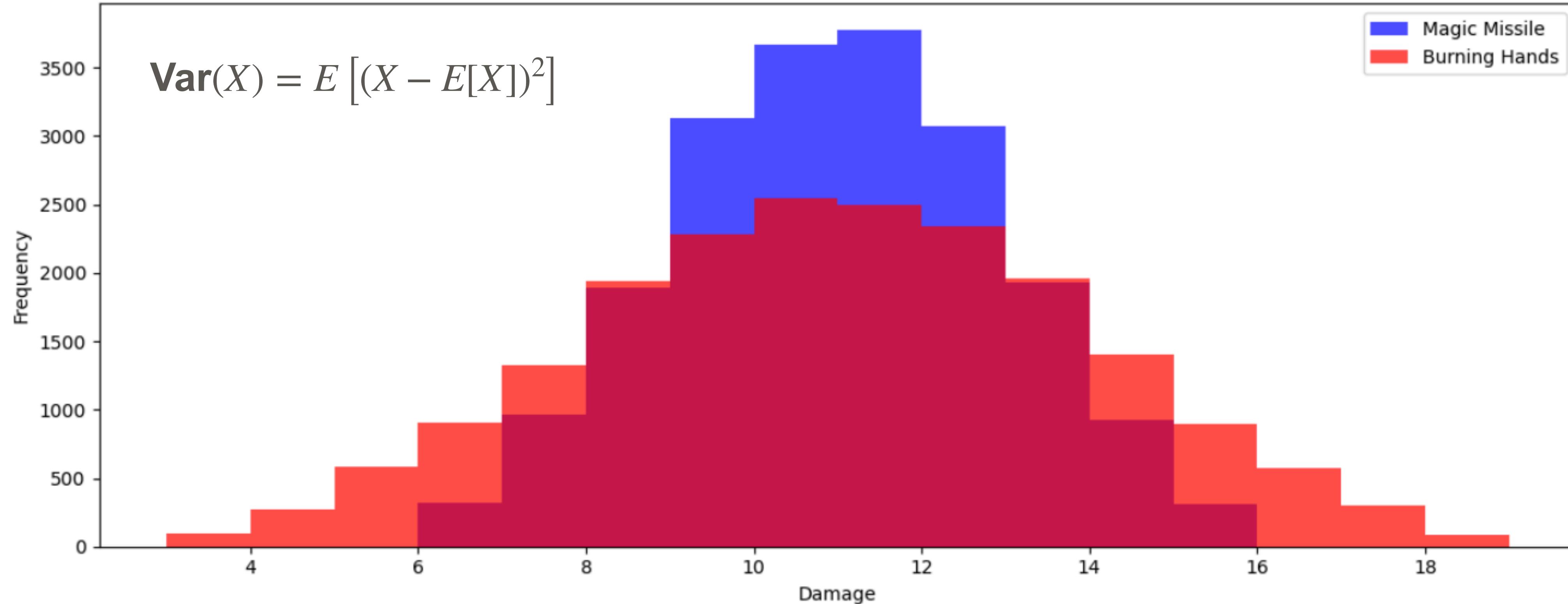
Burning hands: 3d6
(assume full damage, one creature)



Magic missile: 3 x (1d4+1)



Damage Distribution Comparison



Magic Missile – Mean Damage: 10.493, Variance: 3.782051

Burning Hands – Mean Damage: 10.5193, Variance: 8.74442751

MEAN AND VARIANCE CONSIDERATIONS FOR THE REWARD

Option 1: low reward with high probability (low variance)

Option 2: high reward with low probability (high variance)

Potential issue



PROBABILITIES

CONDITIONAL PROBABILITIES

$$P(X = x | Y = y) = \frac{P(X = x, Y = y)}{P(Y = y)}$$



DALL·E

X: sum of two d6 dices

Y: roll of the first dice

$$P(X = 4) = 3/36 = 1/12$$

$$P(Y = 1) = 1/6$$

$$P(X = 4 | Y = 1) = \frac{P(X = 4, Y = 1)}{P(Y = 1)} = \frac{1/36}{1/6} = 1/6$$

PROBABILITIES

PROPERTIES OF CONDITIONAL EXPECTATIONS

$$E_X[X \mid Y = y] = \sum_x x \cdot P(X = x \mid Y = y)$$

$$E[X] = E_Y[E_X[X \mid Y]] \quad \text{Law of Total Expectation}$$

$$E[aX + bY \mid Z] = aE[X \mid Z] + bE[Y \mid Z] \quad \text{Linearity}$$

$$E[X \mid Y] = E[X] \quad \text{Independence}$$

THE MONTY HALL PROBLEM

DALL·E

A perspective painting of a long hallway with many doors. The hallway is lined with doors on both sides, leading towards a bright light at the end. The walls are light blue, and the floor is made of tiles. The ceiling is high and has a cloudy sky painted on it.

DALL·E

PROBABILITIES

SUMMARY

- Random (or stochastic) variables
- Probability distribution
- Expectation
- Joint probability and joint probability distribution distribution
- Properties of expectation and conditions expectation
- Variance

THANK YOU!