

COM 3240

REINFORCEMENT LEARNING



DALL·E

FUTURE REWARDS

THE Q-LEARNING ALGORITHM

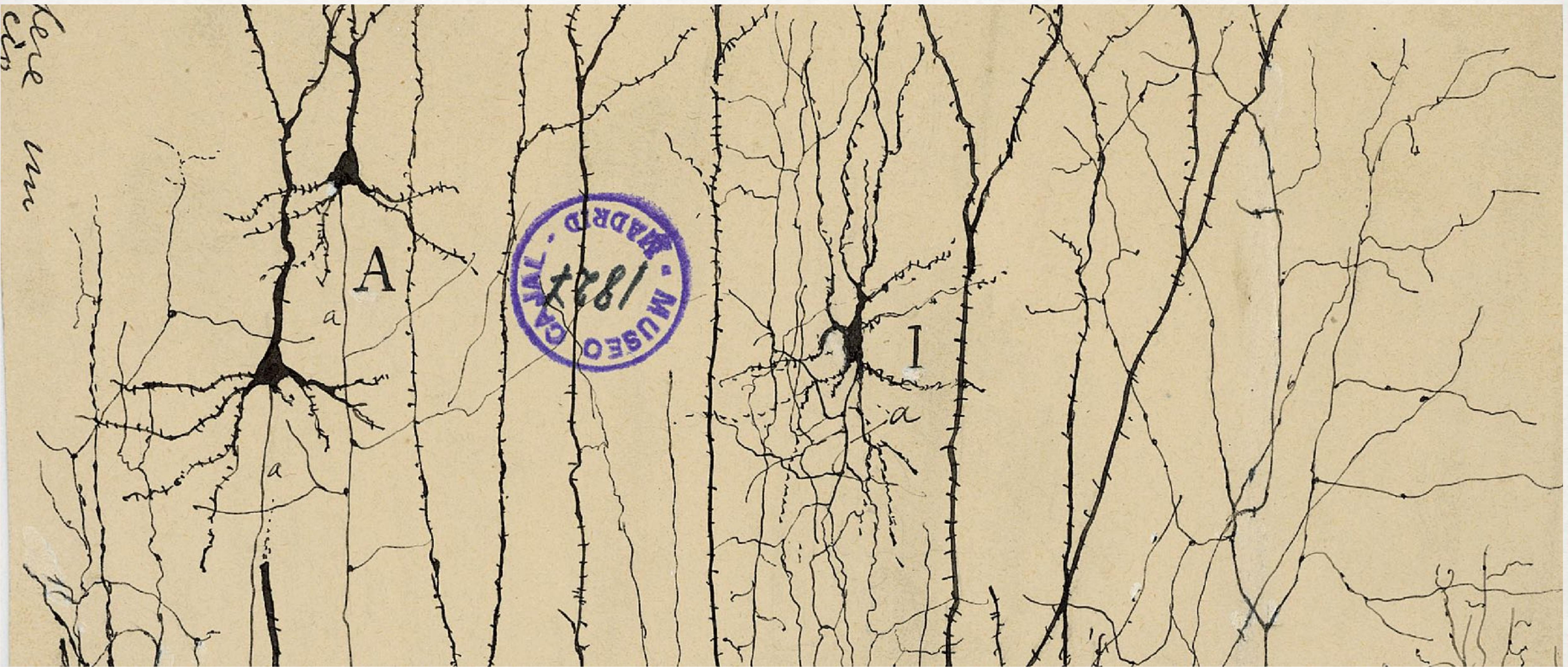
- 1. Initialise $Q(s, a)$ arbitrarily for all $s \in S$ and $a \in A(s)$.**
- 2. Repeat (for each episode):**
 - a. Initialise s .**
 - b. Repeat (for each step of episode):**
 - i. Choose an action a from s using a policy derived from Q (e.g., ε -greedy).**
 - ii. Take action a , observe reward r and next state s' .**
 - iii. $Q(s, a) \leftarrow Q(s, a) + \eta * [r + \gamma * \max_{a'} Q(s', a') - Q(s, a)]$.**
 - iv. $s \leftarrow s'$.**
 - c. until s is terminal.**

Off-policy

ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS

Q-VALUES TABLE

$Q(s_1, a_1)$	$Q(s_2, a_1)$		
$Q(s_1, a_2)$			



Ramon y Cahal

1mm³: 10,000 neurons 3km wires

ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS

SINGLE NEURON

$$\mathbf{y}^* = F^*(\mathbf{x})$$

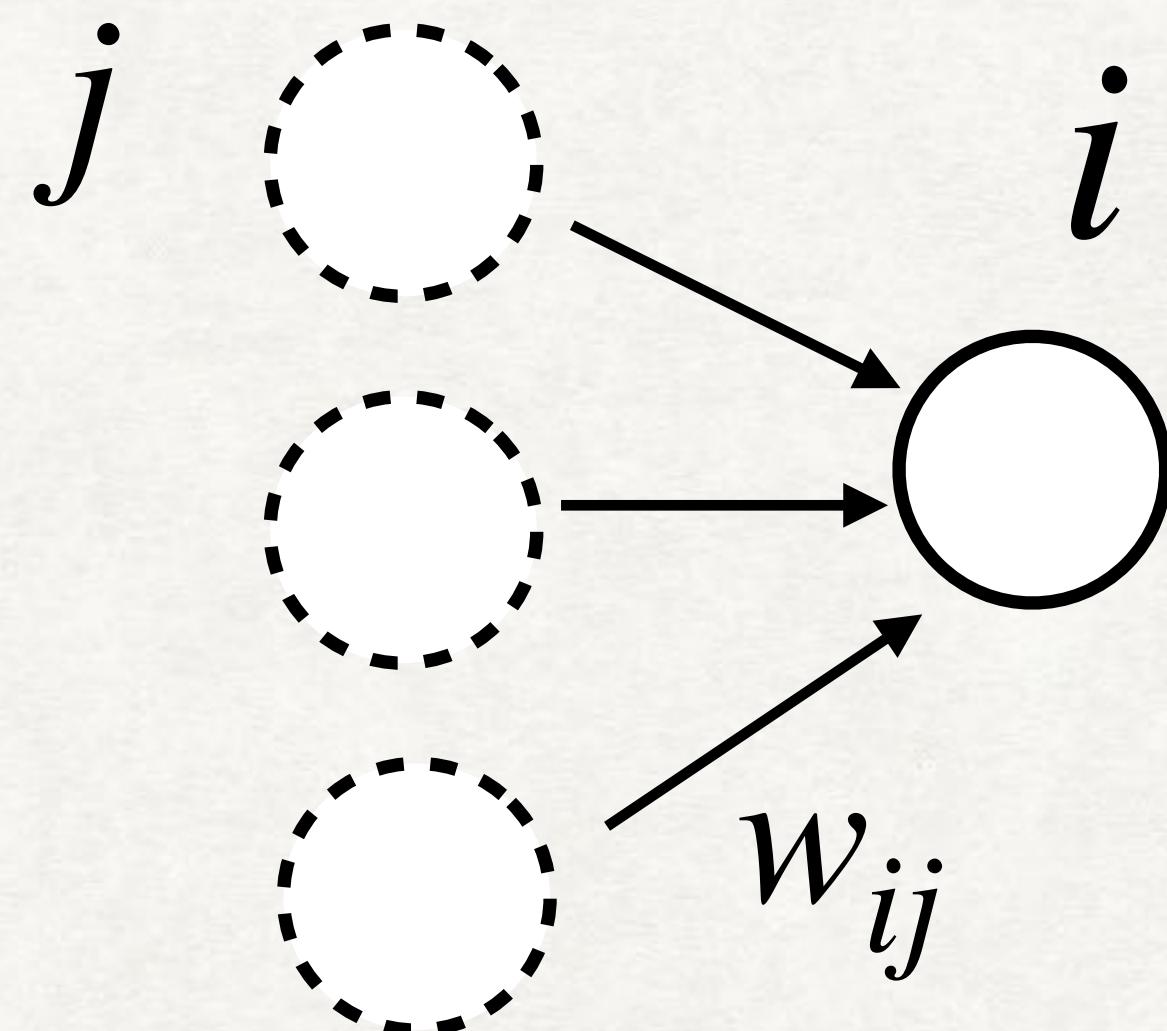
$$\mathbf{y} = F(\mathbf{x})$$

$$L = \frac{1}{2n} \sum_i \sum_{\mathbf{x}} (y_i^t(\mathbf{x}) - y_i(\mathbf{x}; \mathbf{W}))^2$$

Samples from the input and output space of the function.

$$\mathbf{x}^{(1)} \ \mathbf{x}^{(2)} \ \mathbf{x}^{(3)} \dots \mathbf{x}^{(n)}$$

$$\mathbf{y}^{(1)} \ \mathbf{y}^{(2)} \ \mathbf{y}^{(3)} \dots \mathbf{y}^{(n)}$$



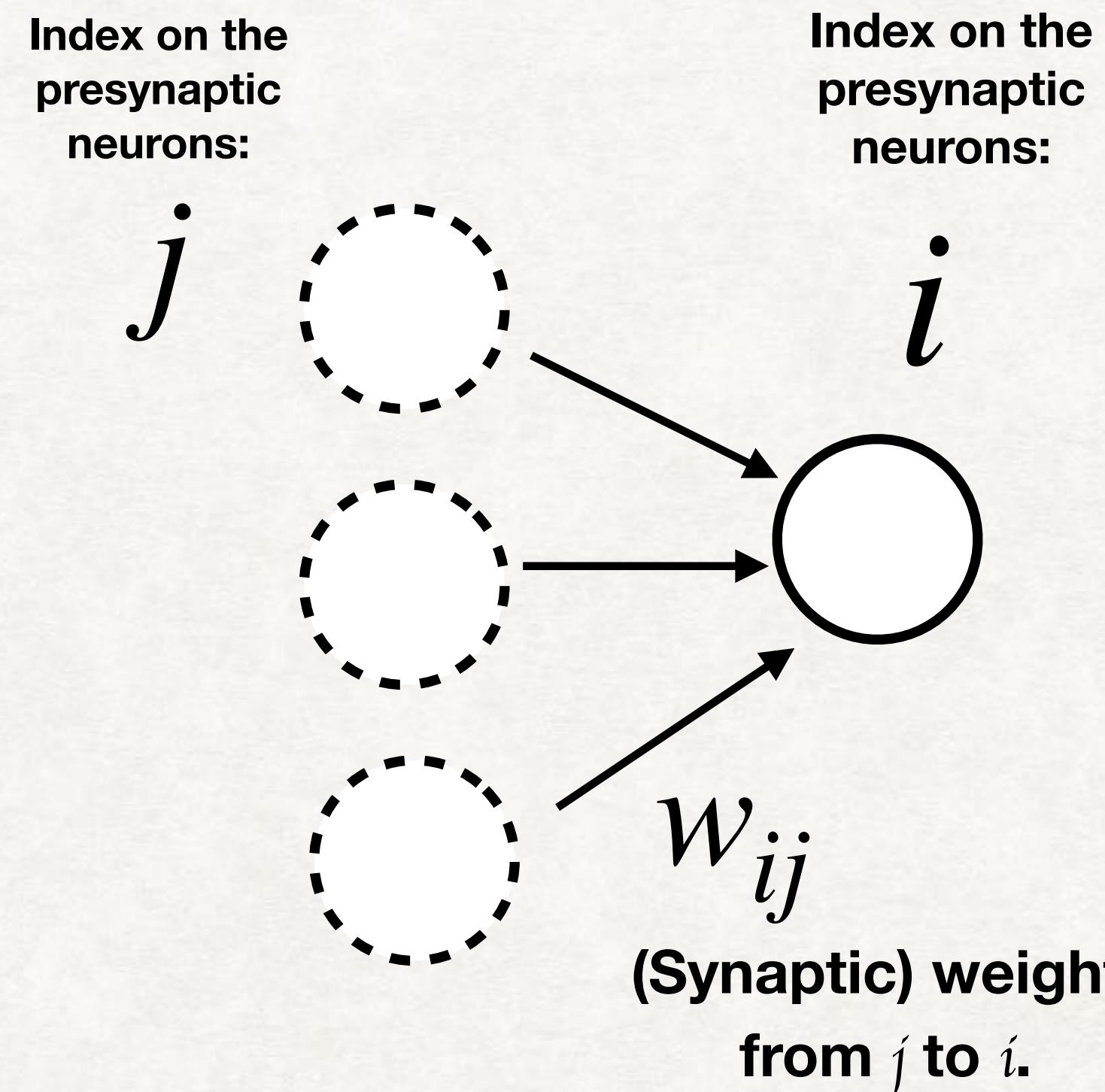
Find parameters (weights) that minimise the Loss function.

$$\Delta \mathbf{W}_{lk} \propto - \frac{\partial E}{\partial \mathbf{W}_{lk}}$$

$$\Delta \mathbf{W} \propto - \nabla_{\mathbf{W}} E$$

ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS

SINGLE NEURON



$$y = ax + b$$

$$h_i = \sum_j w_{ij}x_j + b_i$$

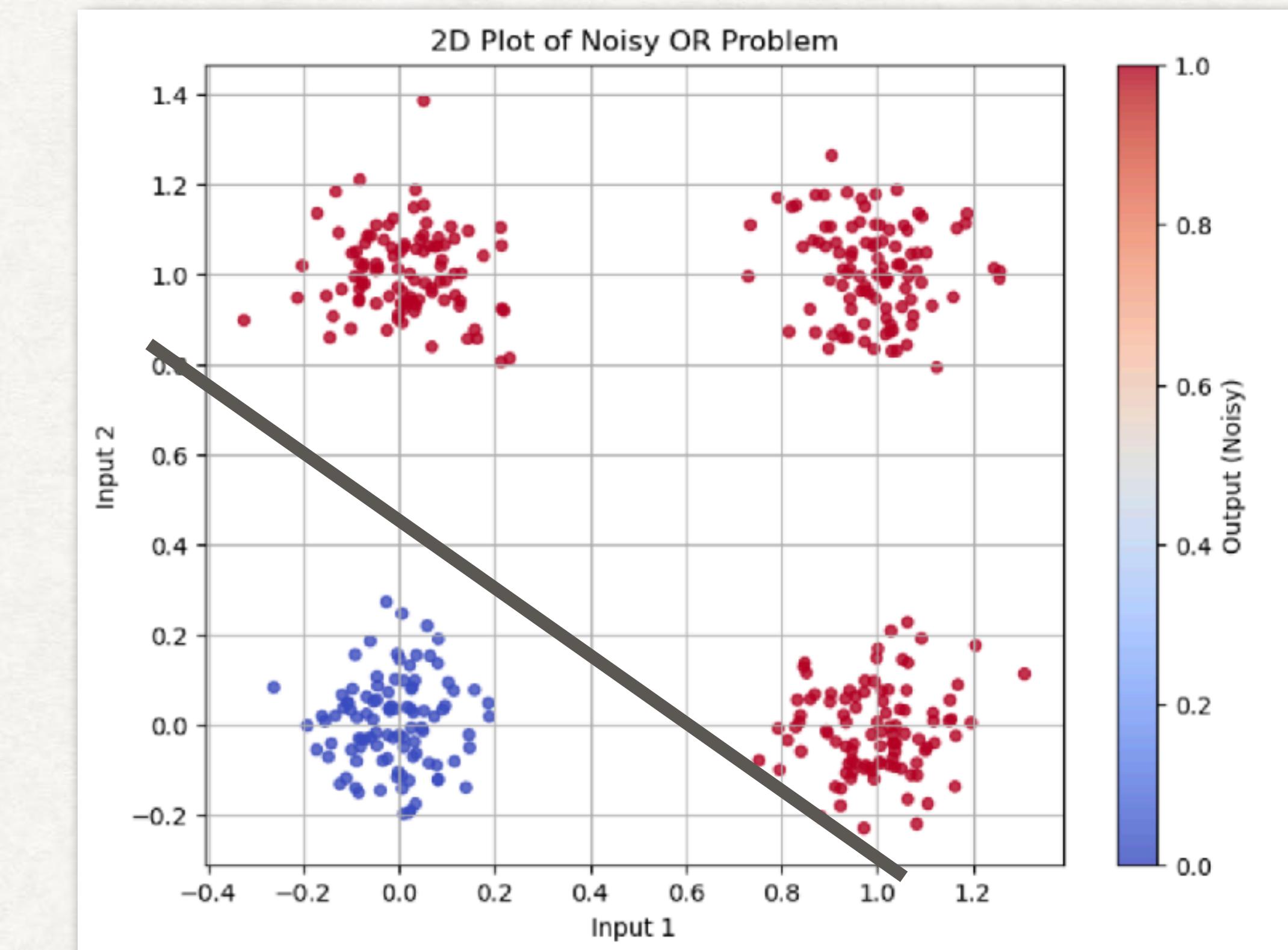
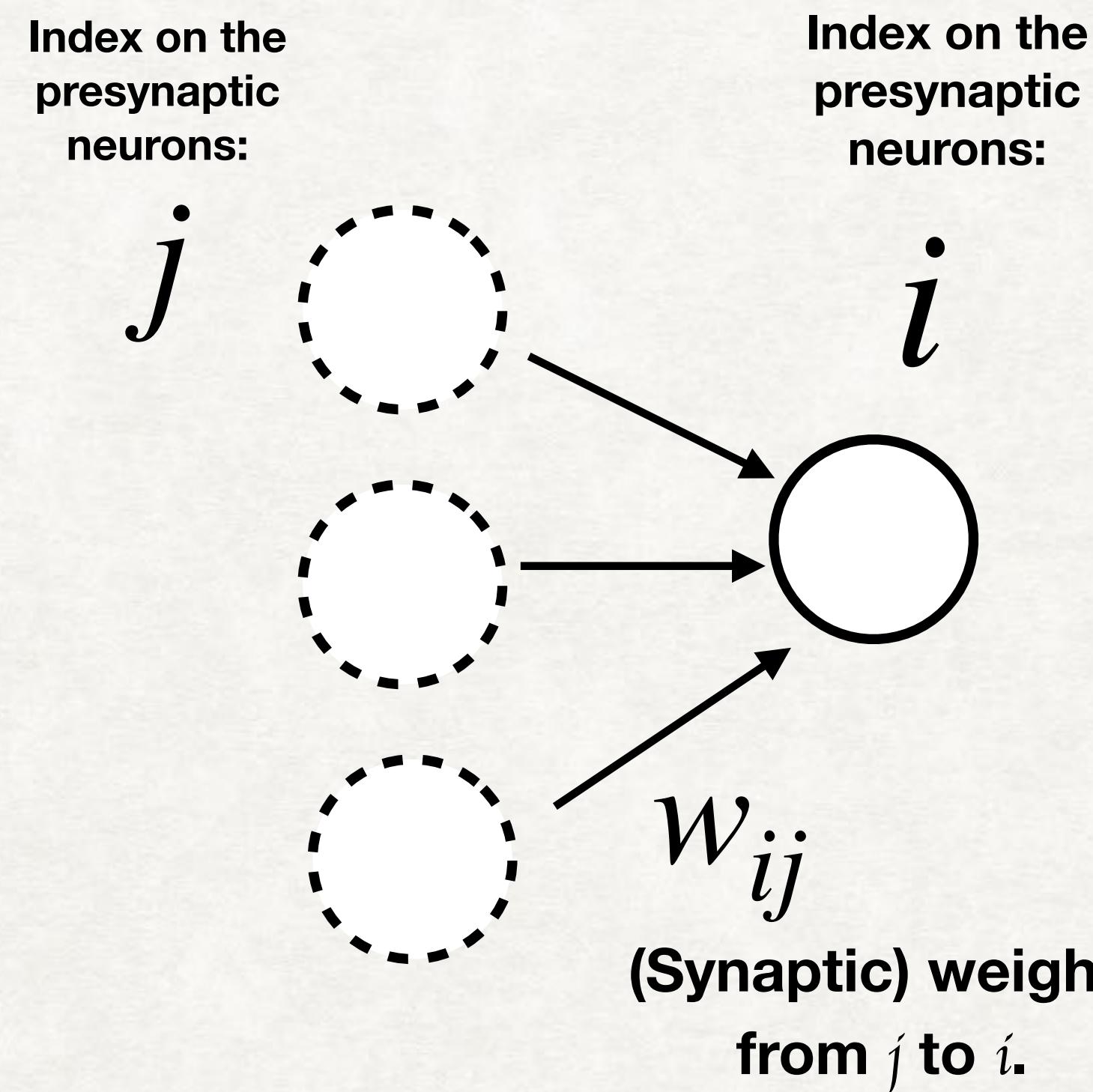
$$x_i = f(h_i)$$

Bias

Activation function f , sigmoidal or ReLU

ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS

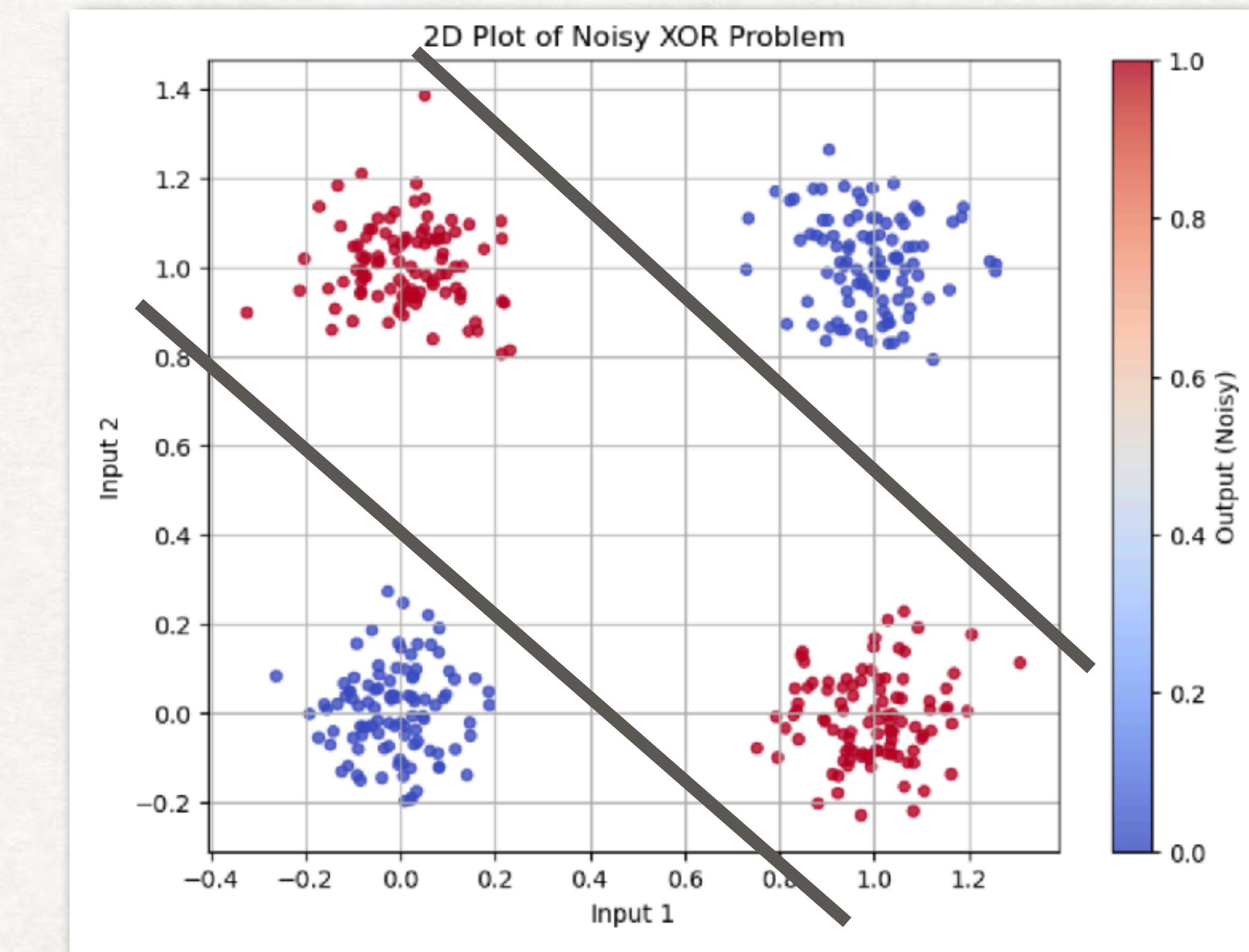
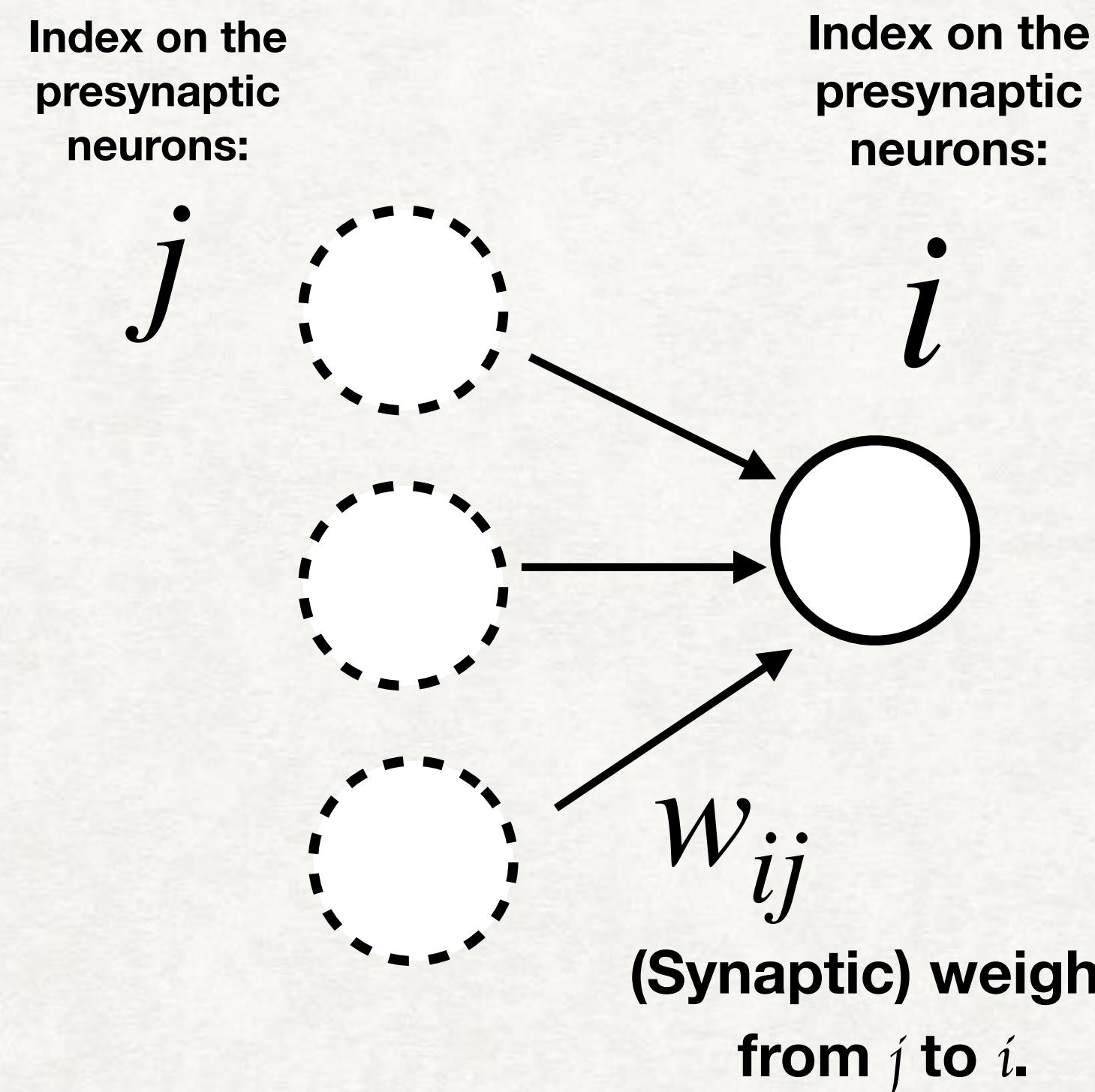
NON-LINEARLY SEPARABLE PROBLEMS



A single layer can solve very easy problems.

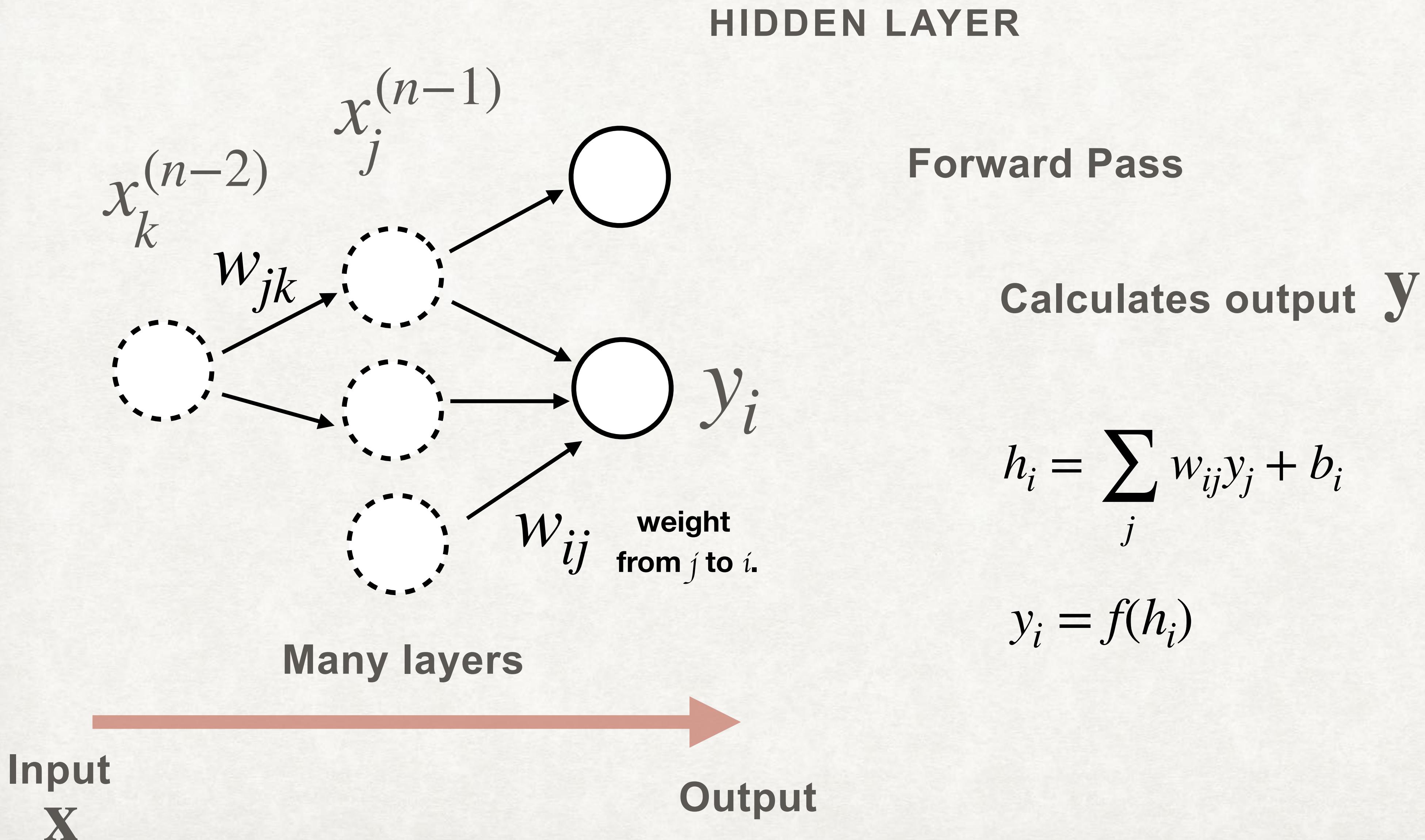
ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS

NON-LINEARLY SEPARABLE PROBLEMS

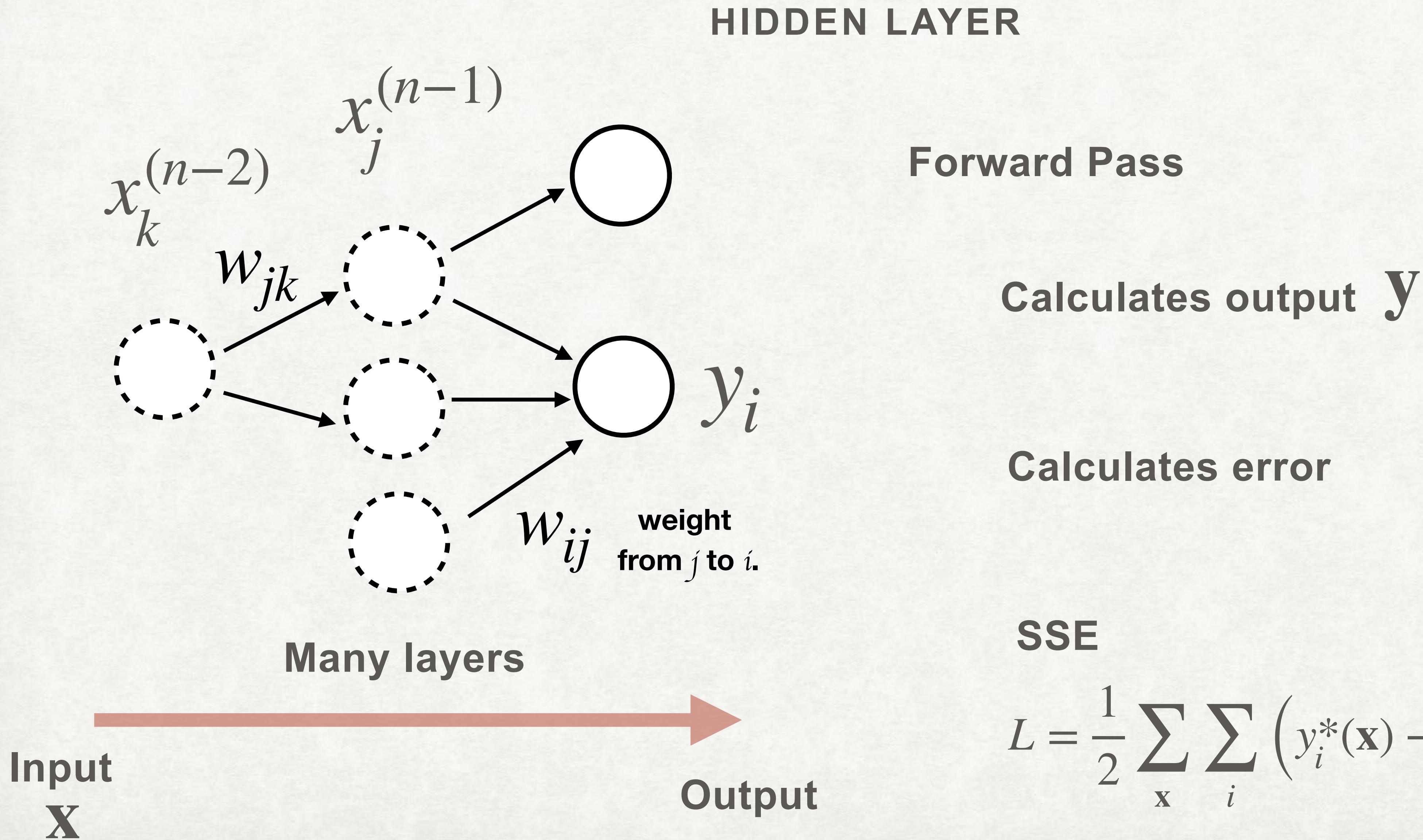


A single layer of neurons cannot learn such problems

ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS

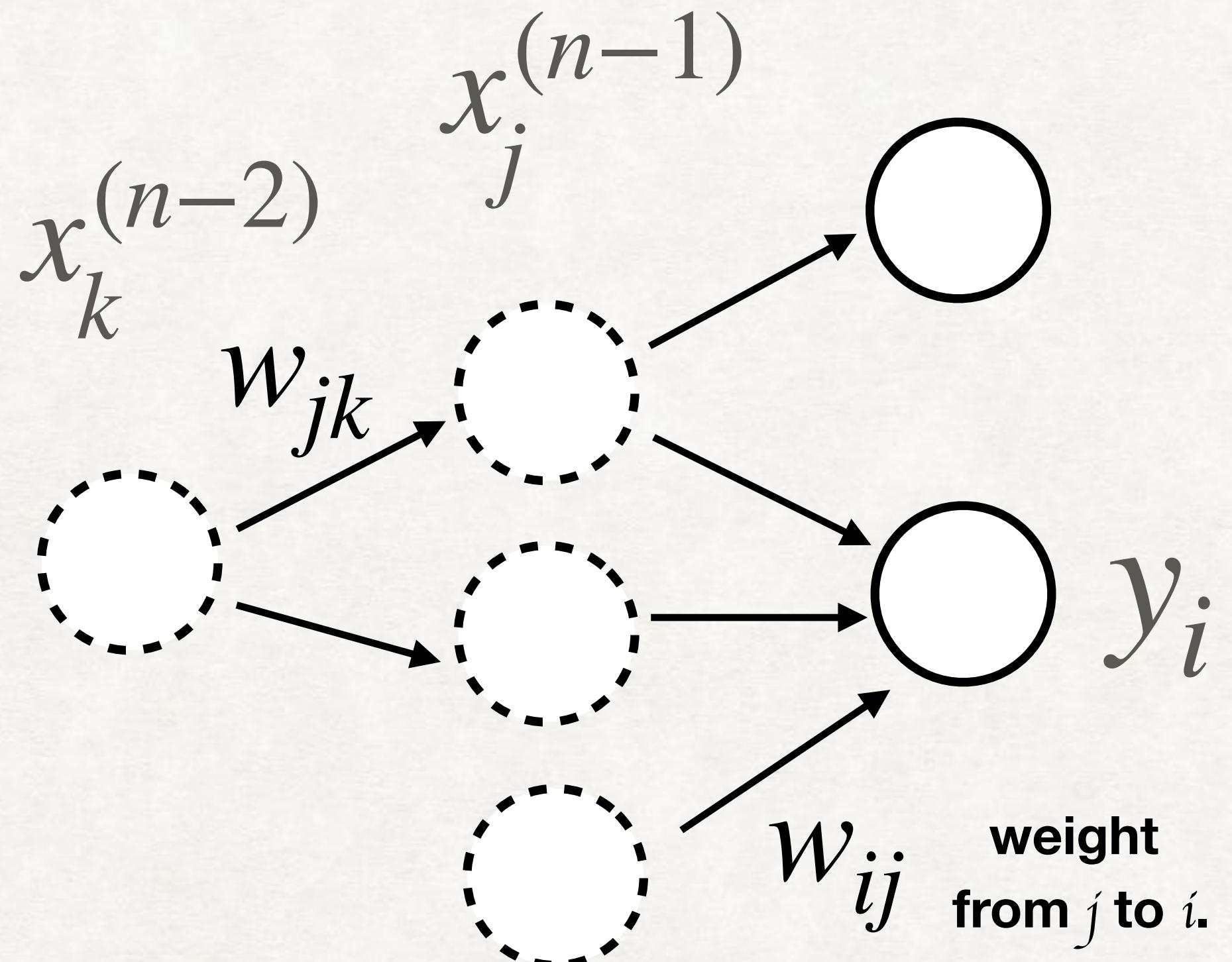


ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS



ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS

HIDDEN LAYER



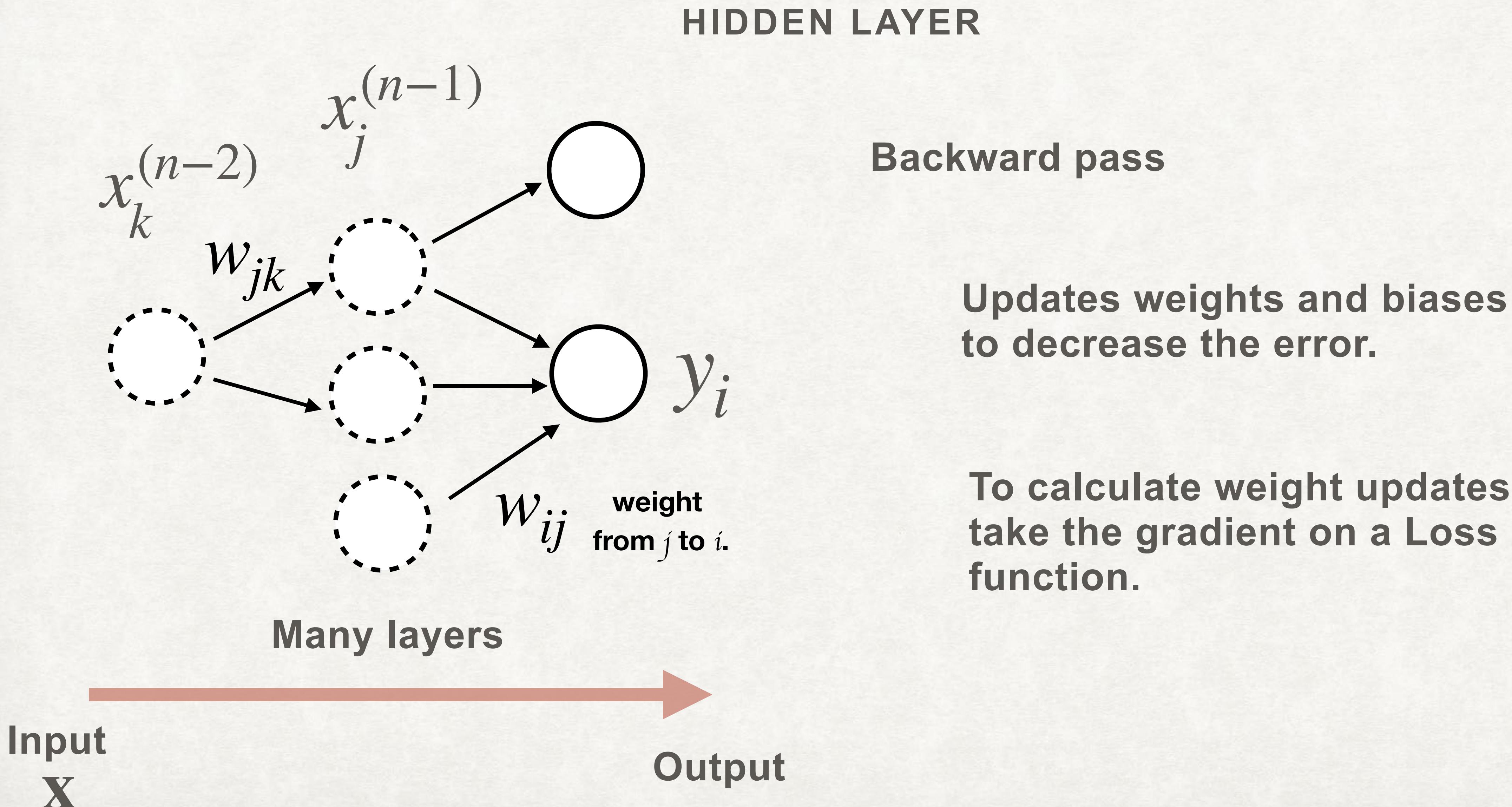
SSE

$$L = \sum_{\mathbf{x}} L_o$$

$$L_o = \frac{1}{2} \sum_i \left(y_i^*(\mathbf{x}) - y_i(\mathbf{x}; \mathbf{W}) \right)^2$$

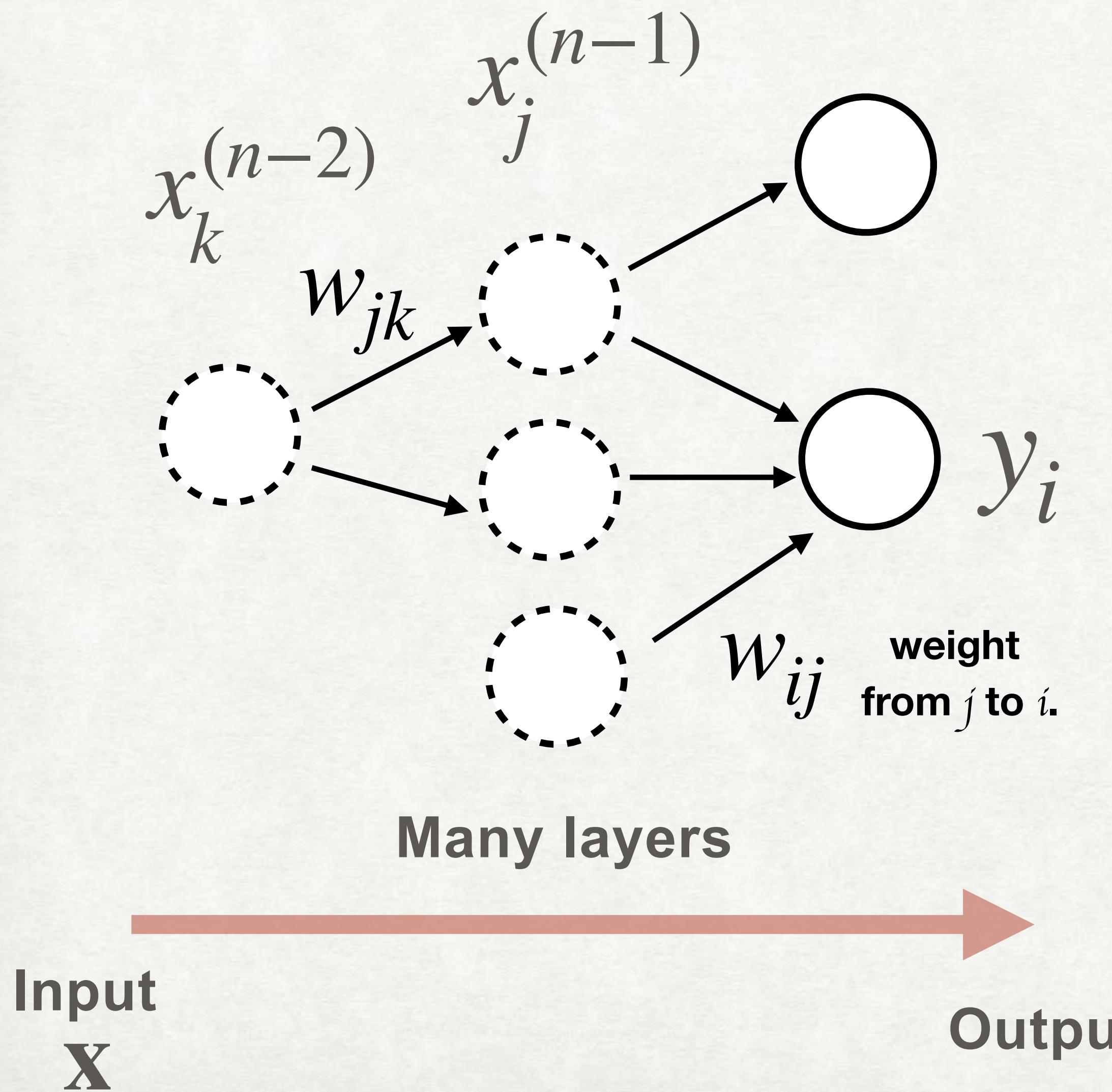
We suppress the sum to operate online.

ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS



ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS

HIDDEN LAYER



$$L = \frac{1}{2} \sum_{\mathbf{x}} \sum_i \left(y_i^*(\mathbf{x}) - y_i(\mathbf{x}; \mathbf{W}) \right)^2 = \sum_{\mathbf{x}} L_o$$

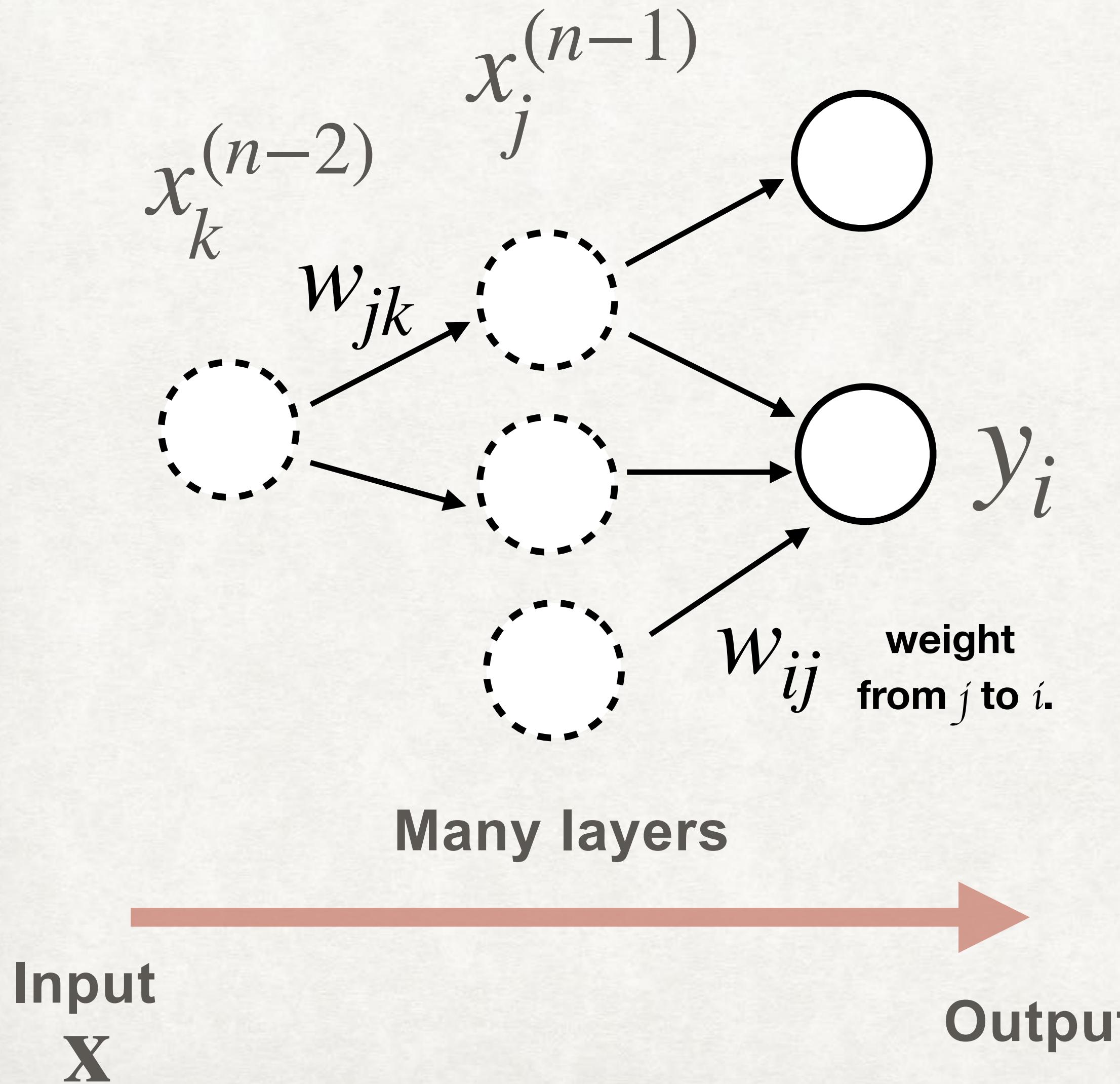
$$\Delta w_{ij} = -\eta \frac{\partial L}{\partial w_{ij}} = -\eta \sum_{\mathbf{x}} \frac{\partial L_o}{\partial w_{ij}}$$

$$\Delta w_{jk} = -\eta \frac{\partial L}{\partial w_{jk}} = -\eta \sum_{\mathbf{x}} \frac{\partial L_o}{\partial w_{jk}}$$

Similarly for biases

ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS

HIDDEN LAYER



$$L = \frac{1}{2} \sum_{\mathbf{x}} \sum_i (y_i^* - y_i)^2 = \sum_{\mathbf{x}} L_o$$

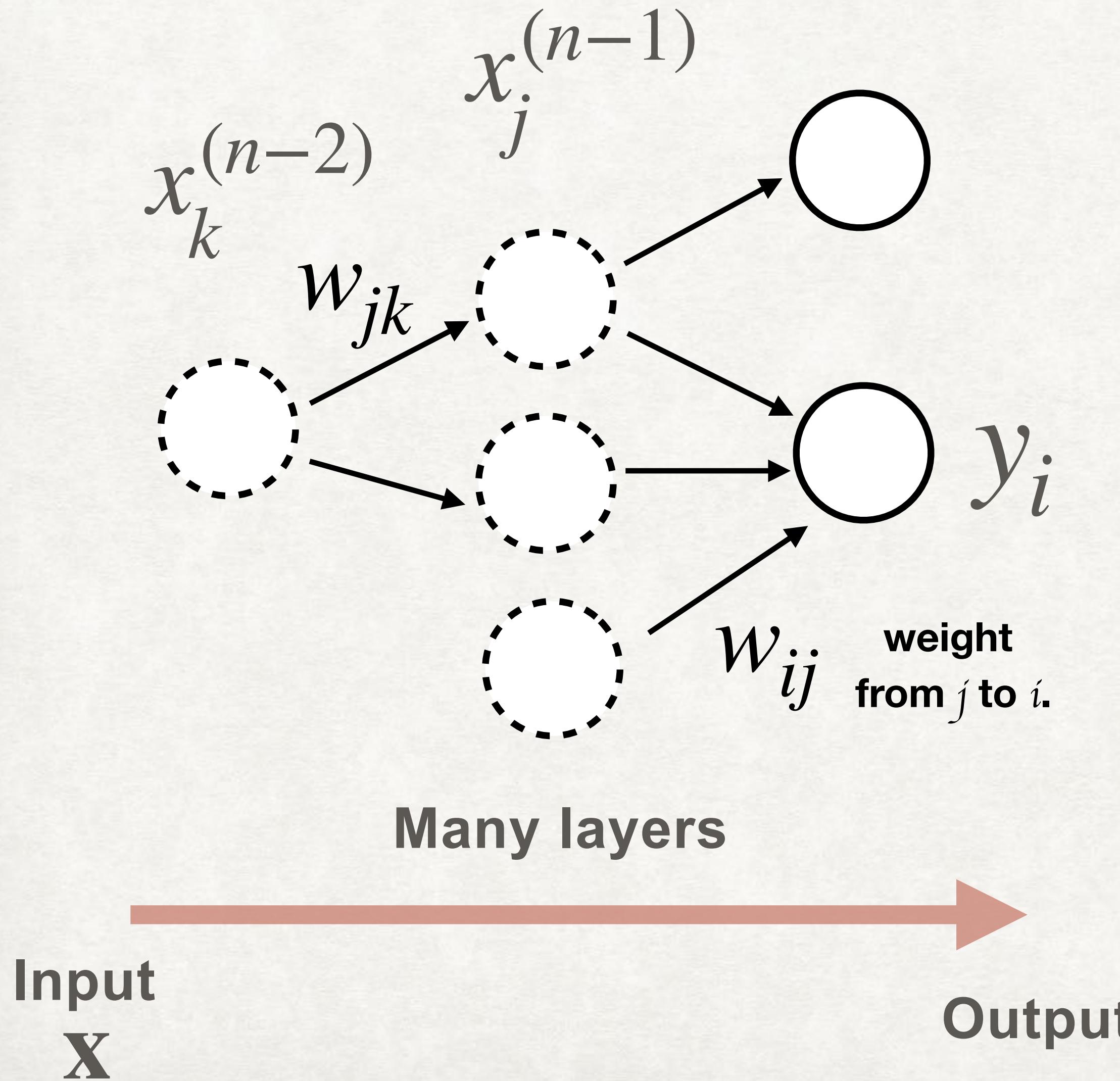
$$\Delta w_{ij} = -\eta \frac{\partial L}{\partial w_{ij}}$$

$$\Delta w_{ij} = -\eta \sum_{\mathbf{x}} \frac{\partial L_o}{\partial y_i} \frac{\partial y_i}{\partial w_{ij}}$$

We have done it previously
(delta rule).

ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS

HIDDEN LAYER



$$L = \frac{1}{2} \sum_{\mathbf{x}} \sum_i (y_i^* - y_i)^2 = \sum_{\mathbf{x}} L_o$$

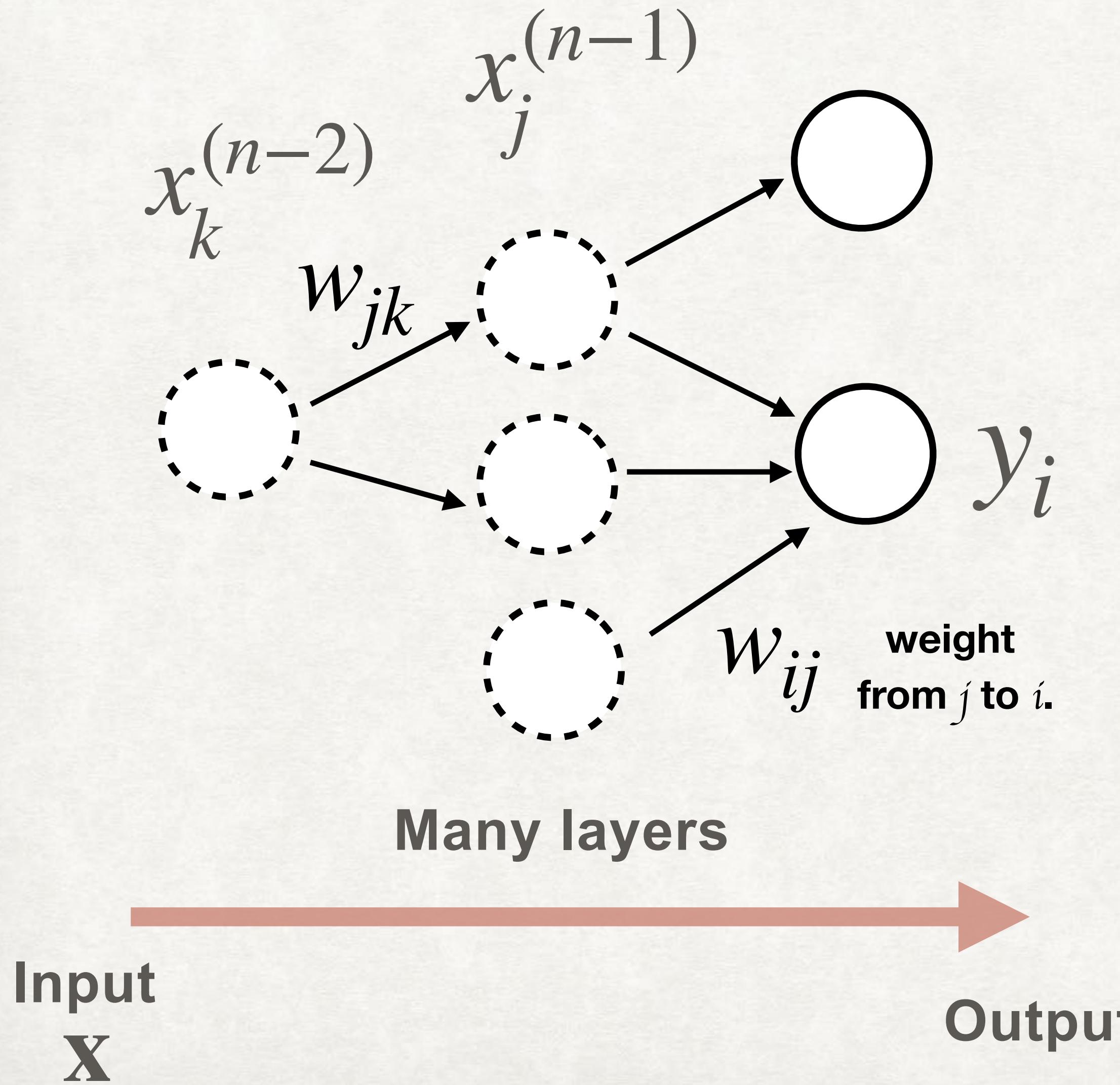
$$\Delta w_{ij} = -\eta \frac{\partial L}{\partial w_{ij}}$$

$$\Delta w_{ij} = -\eta \sum_{\mathbf{x}} \frac{\partial L_o}{\partial y_i} \frac{\partial y_i}{\partial w_{ij}}$$

$$\Delta w_{ij} = \eta \sum_{\mathbf{x}} (y_i^* - y_i) f_i^{(n)} x_j^{(n-1)}$$

ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS

HIDDEN LAYER



$$L = \frac{1}{2} \sum_{\mathbf{x}} \sum_i (y_i^* - y_i)^2 = \sum_{\mathbf{x}} L_o$$

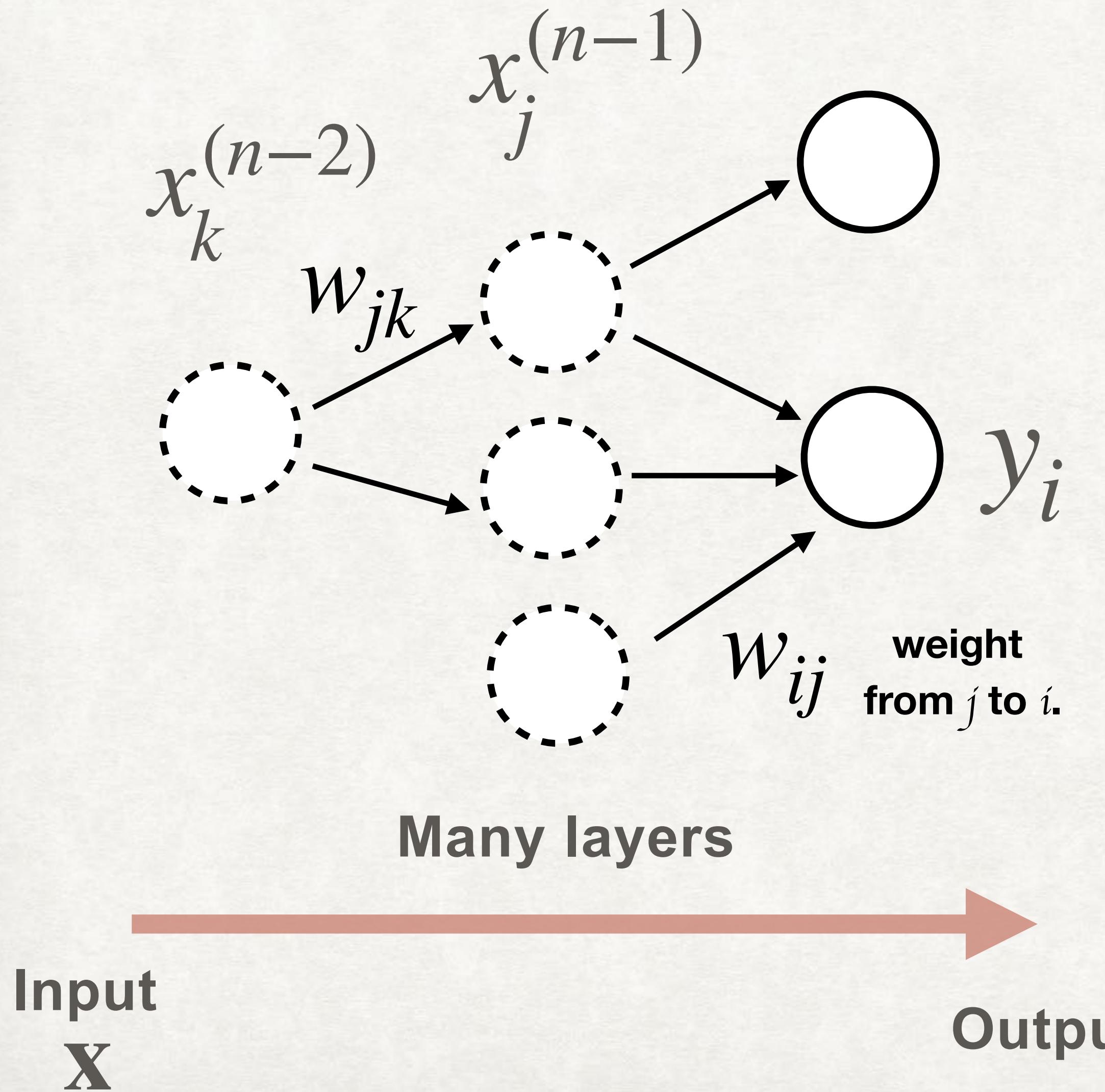
$$\Delta w_{jk} = -\eta \frac{\partial L}{\partial w_{jk}} = -\eta \sum_{\mathbf{x}} \frac{\partial L_o}{\partial w_{jk}}$$

$$\Delta w_{jk} = -\eta \sum_{\mathbf{x}} \frac{\partial L_o}{\partial x_j^{(n-1)}} \frac{\partial x_j^{(n-1)}}{\partial w_{jk}}$$

It is safe to write like this because the weight affects just this specific neuron directly.

ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS

HIDDEN LAYER



$$L = \frac{1}{2} \sum_{\mathbf{x}} \sum_i (y_i^* - y_i)^2 = \sum_{\mathbf{x}} L_o$$

$$\Delta w_{jk} = -\eta \sum_{\mathbf{x}} \frac{\partial L_o}{\partial x_j^{(n-1)}} \frac{\partial x_j^{(n-1)}}{\partial w_{jk}}$$

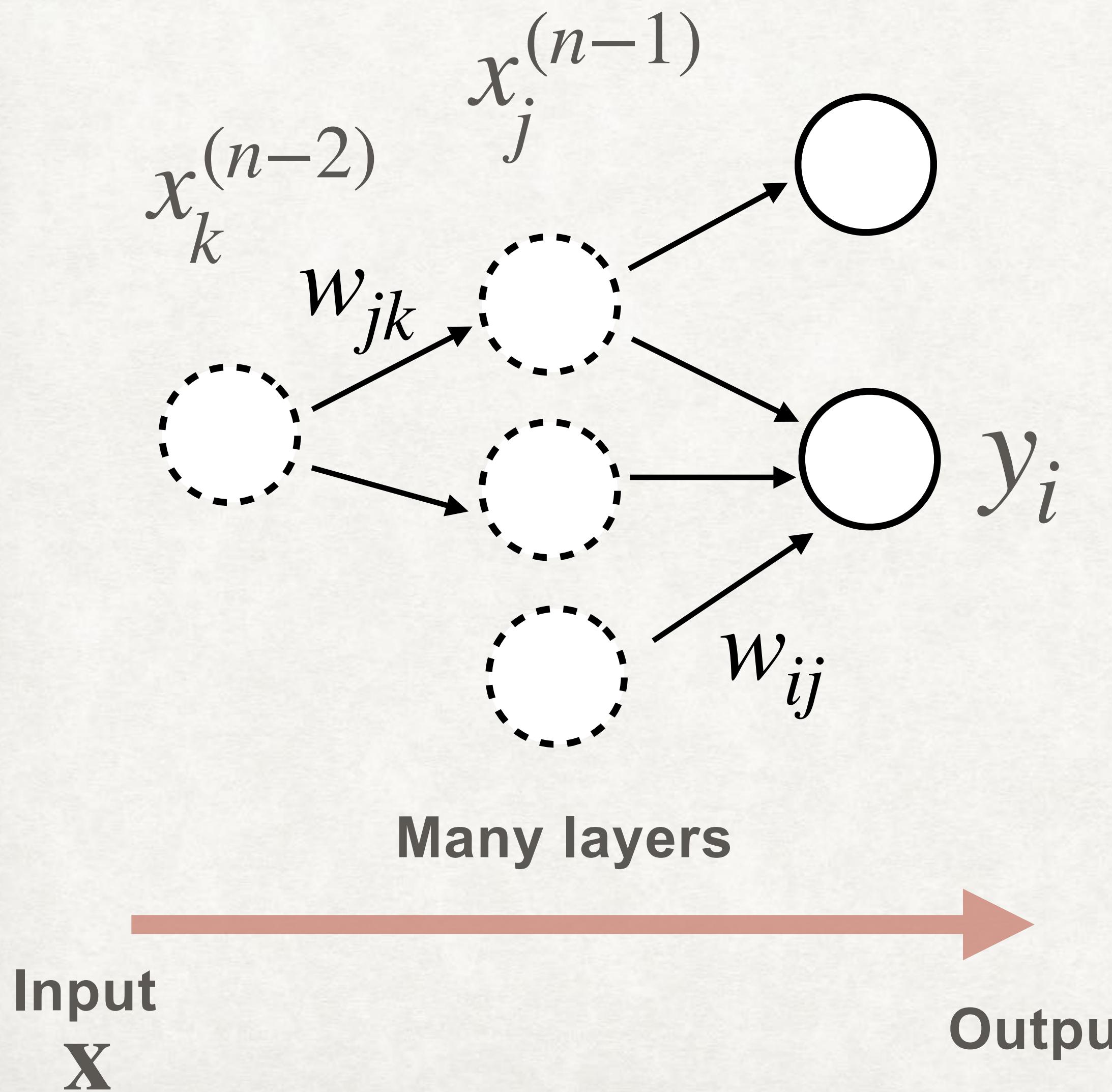
We calculate the two terms separately:

$$\frac{\partial L_o}{\partial x_j^{(n-1)}}$$

$$\frac{\partial x_j^{(n-1)}}{\partial w_{jk}}$$

ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS

HIDDEN LAYER



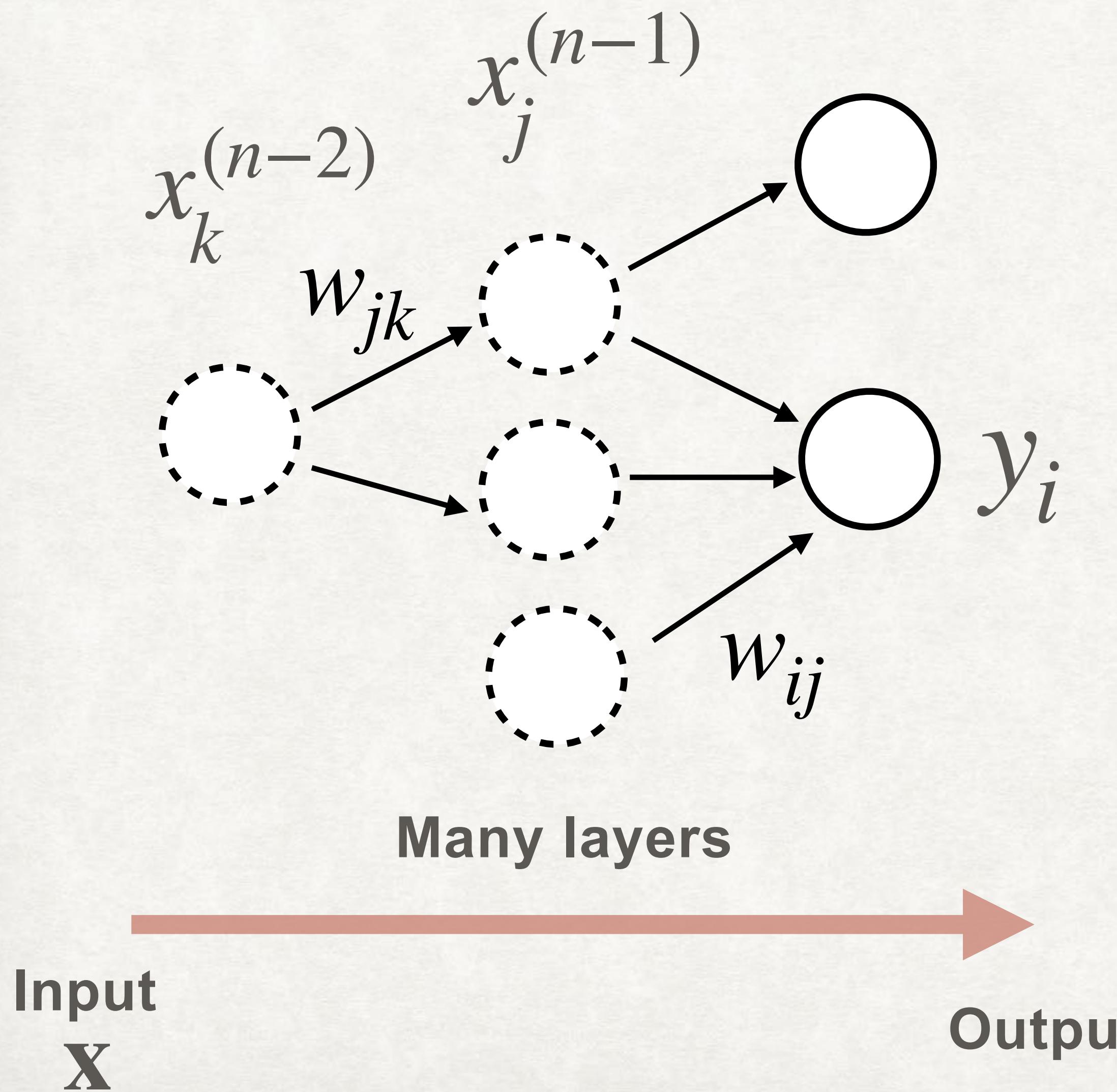
$$L_o = \frac{1}{2} \sum_i (y_i^* - y_i)^2$$

$$\frac{\partial L_o}{\partial x_j^{(n-1)}} = \frac{1}{2} \sum_i \frac{\partial (y_i^* - y_i)^2}{\partial x_j^{(n-1)}}$$

$$= \frac{1}{2} \sum_i \frac{\partial (y_i^* - y_i)^2}{\partial y_i} \frac{\partial y_i}{\partial x_j^{(n-1)}}$$

ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS

HIDDEN LAYER



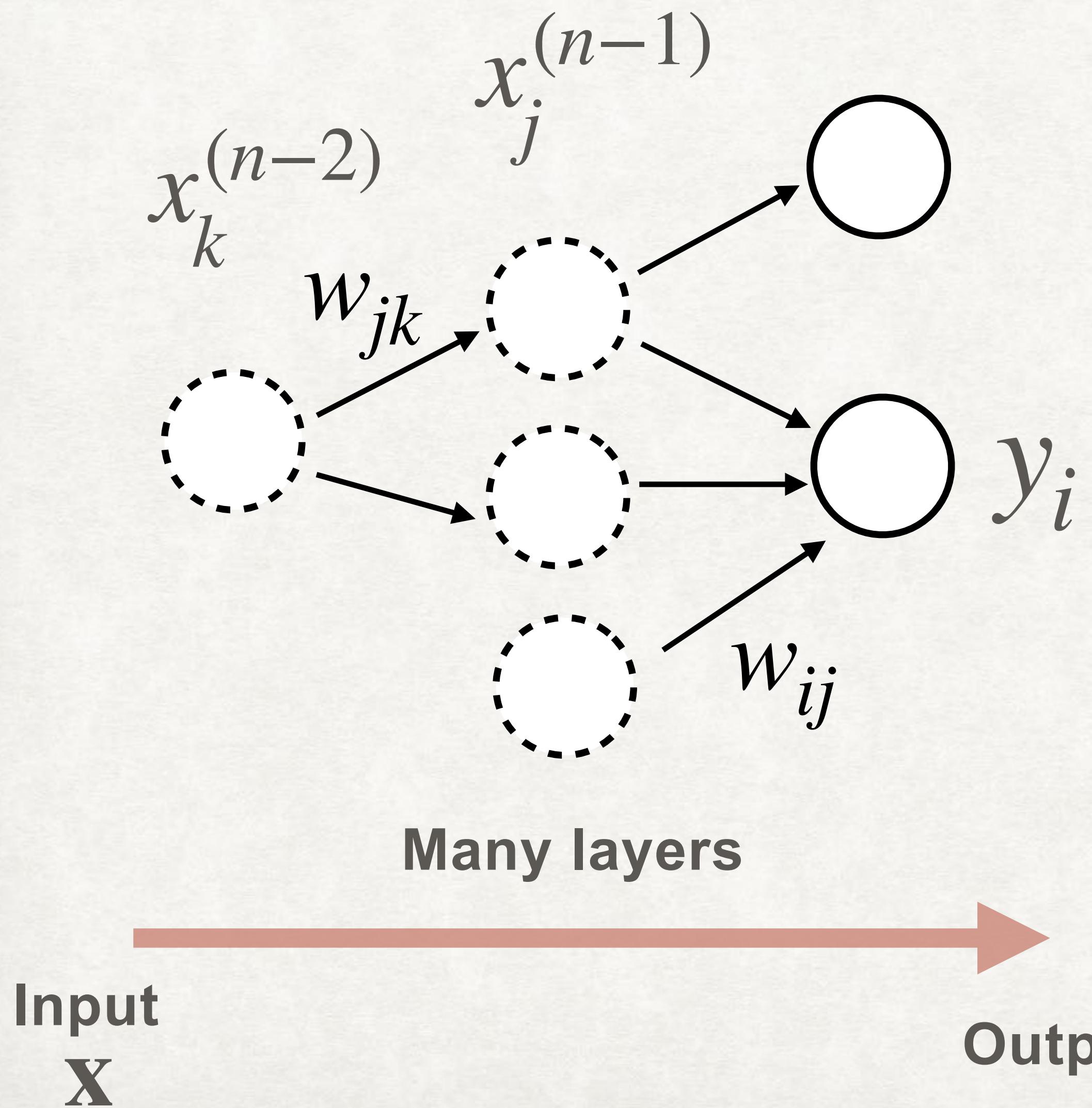
$$L_o = \frac{1}{2} \sum_i (y_i^* - y_i)^2$$

$$\frac{\partial L_o}{\partial x_j^{(n-1)}} = \frac{1}{2} \sum_i \frac{\partial (y_i^* - y_i)^2}{\partial x_j^{(n-1)}}$$

$$= - \sum_i (y_i^* - y_i) \frac{\partial y_i}{\partial x_j^{(n-1)}}$$

ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS

HIDDEN LAYER

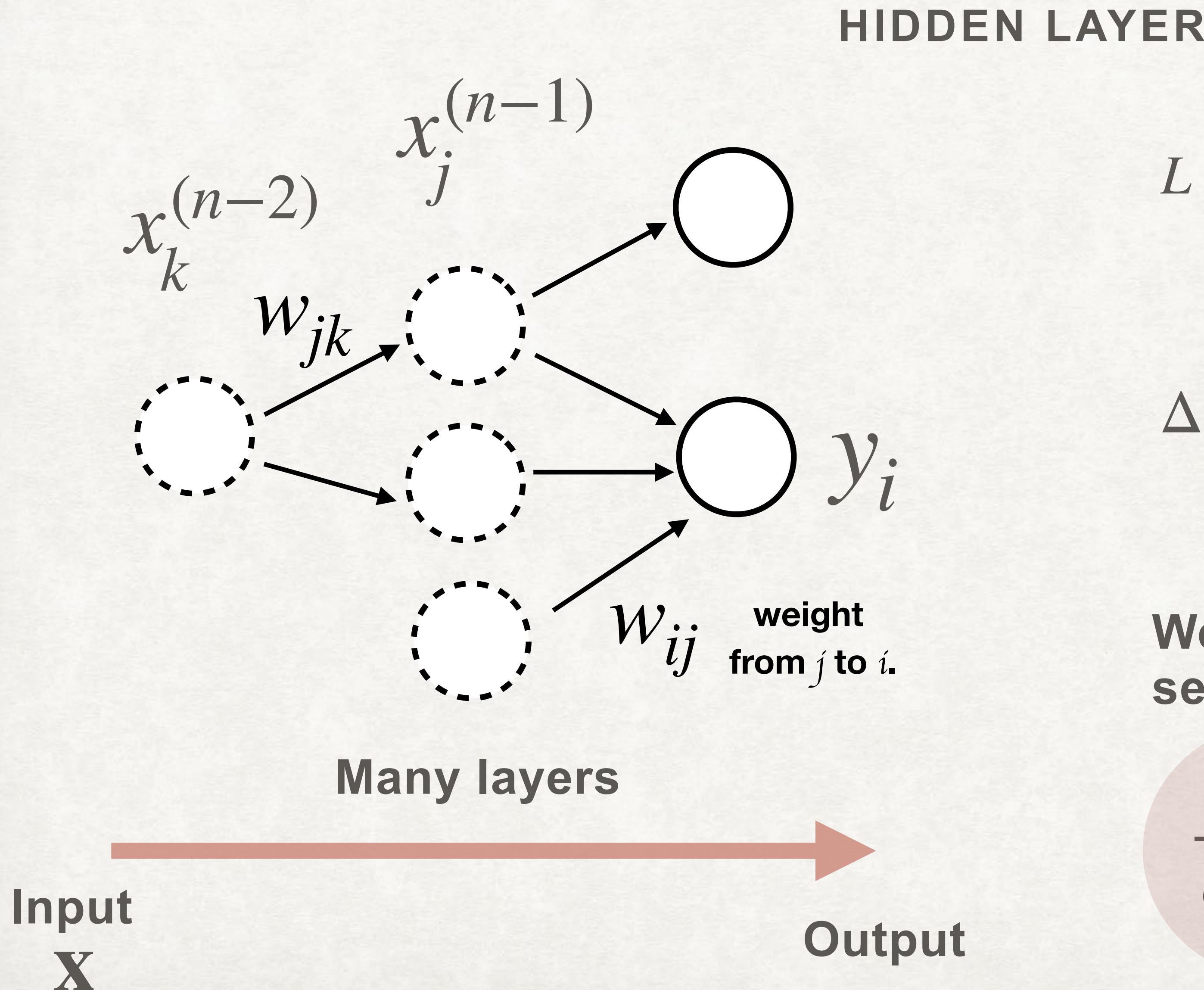


$$\frac{\partial y_i}{\partial x_j^{(n-1)}} \quad y_i = f(\sum_j w_{ij} x_j^{(n-1)})$$

$$\frac{\partial y_i}{\partial x_j^{(n-1)}} = \frac{\partial f\left(\sum_j w_{ij} x_j^{(n-1)}\right)}{\partial x_j^{(n-1)}} = f'_i w_{ij}$$

$$\frac{\partial L}{\partial x_j^{(n-1)}} = - \sum_i \sum_{\mathbf{x}} (y_i^* - y_i) f_i'^{(n)} w_{ij}$$

ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS



$$L = \frac{1}{2} \sum_{\mathbf{x}} \sum_i (y_i^* - y_i)^2 = \sum_{\mathbf{x}} L_o$$

$$\Delta w_{jk} = -\eta \sum_{\mathbf{x}} \frac{\partial L_o}{\partial x_j^{(n-1)}} \frac{\partial x_j^{(n-1)}}{\partial w_{jk}}$$

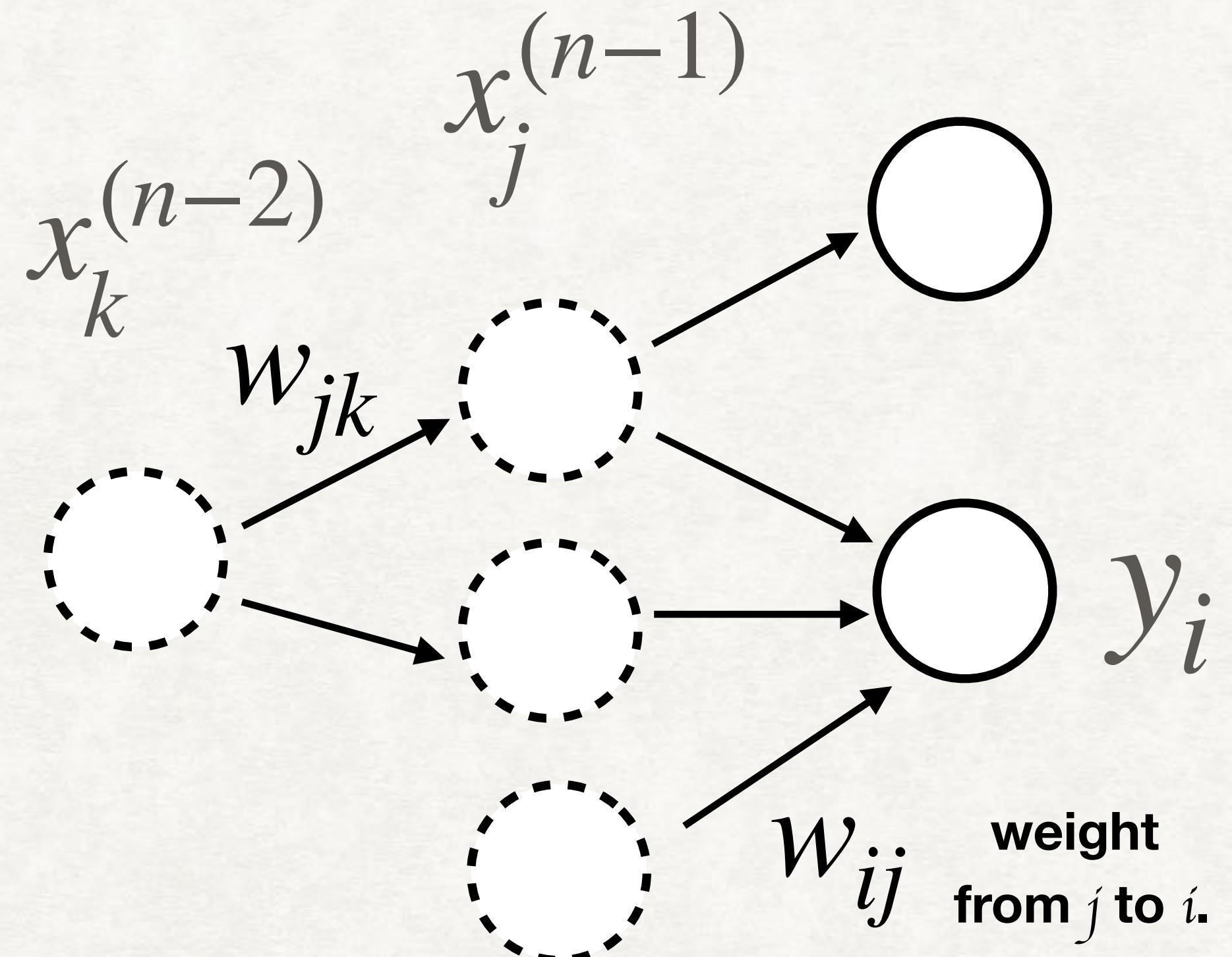
We calculate the two terms separately:

$$\frac{\partial L}{\partial x_j^{(n-1)}} \quad \checkmark$$

$$\frac{\partial x_j^{(n-1)}}{\partial w_{jk}}$$

ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS

HIDDEN LAYER



weight
from j to i .

$$\frac{\partial x_j^{(n-1)}}{\partial w_{jk}} \quad x_j^{(n-1)} = f\left(\sum_j w_{jk} x_k^{(n-2)}\right)$$
$$\frac{\partial x_j^{(n-1)}}{\partial w_{jk}} = \frac{\partial f\left(\sum_k w_{jk} x_k^{(n-2)}\right)}{\partial w_{jk}} = f'_j(x_k^{(n-2)})$$

Many layers

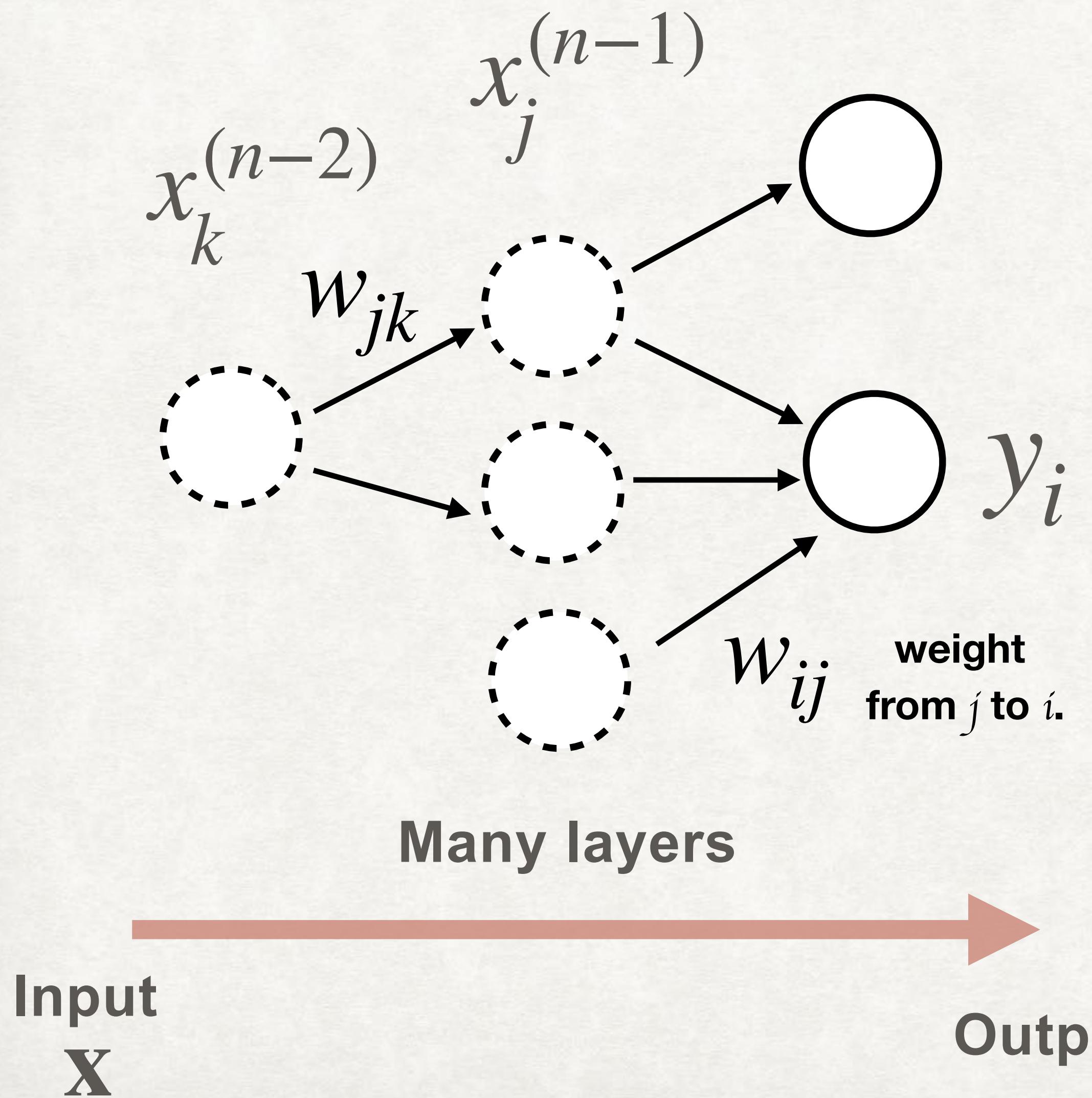
Input
 \mathbf{x}

Output

Let us combine the results.

ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS

HIDDEN LAYER



$$L = \frac{1}{2} \sum_{\mathbf{x}} \sum_i (y_i^* - y_i)^2$$

$$\Delta w_{jk} = -\eta \frac{\partial L}{\partial x_j^{(n-1)}} \frac{\partial x_j^{(n-1)}}{\partial w_{jk}}$$

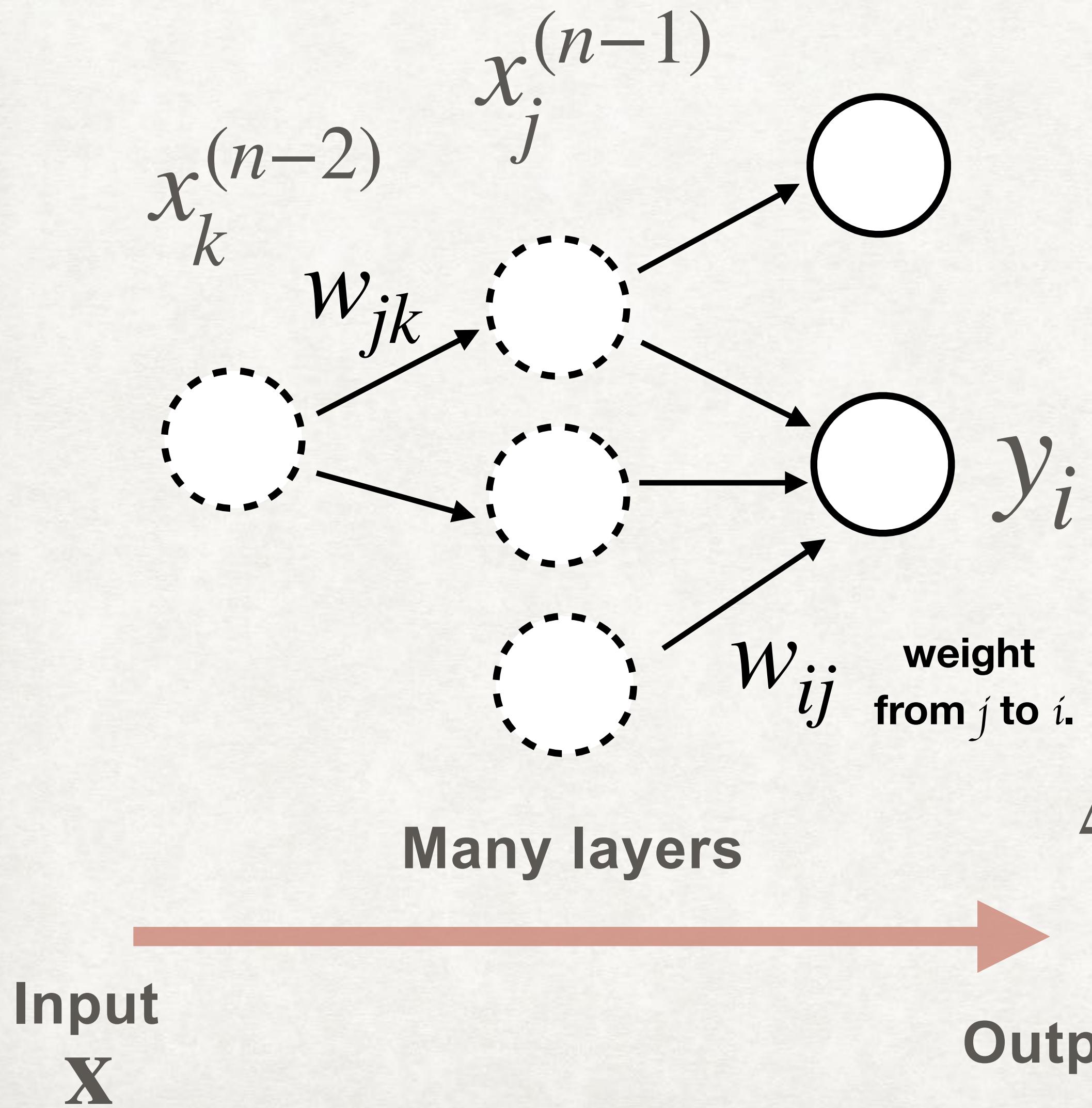
We will calculate the two terms separately:

$$\frac{\partial L}{\partial x_j^{(n-1)}}$$

$$\frac{\partial x_j^{(n-1)}}{\partial w_{jk}}$$

ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS

HIDDEN LAYER



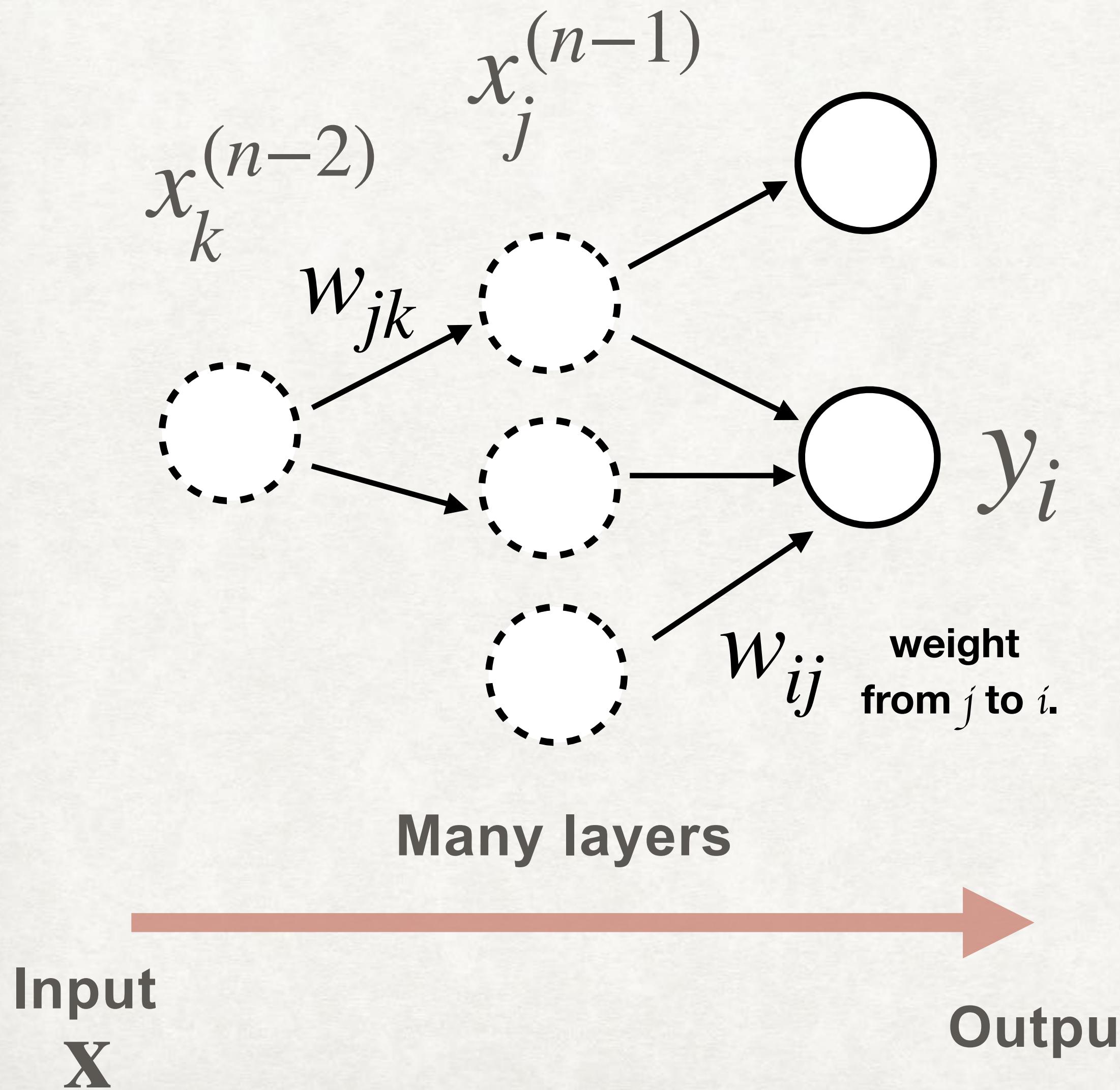
$$L = \frac{1}{2} \sum_{\mathbf{x}} \sum_i (y_i^* - y_i)^2 = \sum_{\mathbf{x}} L_o$$

$$\Delta w_{jk} = -\eta \sum_{\mathbf{x}} \frac{\partial L_o}{\partial x_j^{(n-1)}} \frac{\partial x_j^{(n-1)}}{\partial w_{jk}}$$

$$\Delta w_{jk} = \eta \sum_{\mathbf{x}} \sum_i \left(\frac{(y_i^* - y_i) f_i'^{(n)} w_{ij}}{f_j'^{(n-1)} x_k^{(n-2)}} \right) \delta_i^{(n)}$$

ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS

HIDDEN LAYER



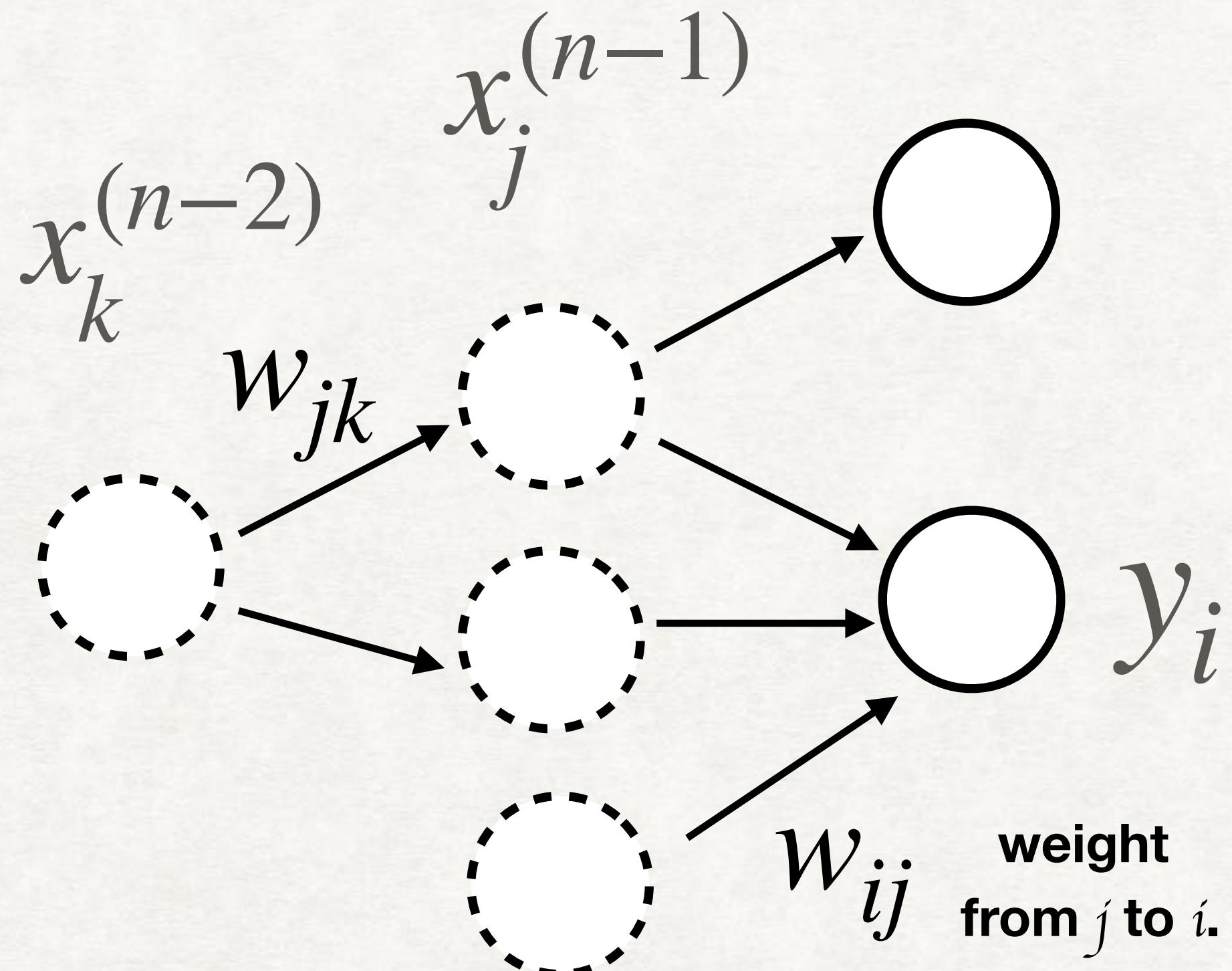
$$L = \frac{1}{2} \sum_{\mathbf{x}} \sum_i (y_i^* - y_i)^2 = \sum_{\mathbf{x}} L_o$$

$$\Delta w_{jk} = -\eta \sum_{\mathbf{x}} \frac{\partial L_o}{\partial x_j^{(n-1)}} \frac{\partial x_j^{(n-1)}}{\partial w_{jk}}$$

$$\Delta w_{jk} = \eta \sum_{\mathbf{x}} \sum_i \frac{\delta_i^{(n)} w_{ij} f_j^{(n-1)} x_k^{(n-2)}}{\delta_j^{(n-1)}}$$

ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS

HIDDEN LAYER



Many layers

Input X → Output

$$L = \frac{1}{2} \sum_{\mathbf{x}} \sum_i (y_i^* - y_i)^2$$

$$\delta_i^{(n)} = (y_i^* - y_i) f_i^{(n)}$$

$$\delta_j^{(n-1)} = \sum_i \delta_i^{(n)} w_{ij} f_j^{(n-1)}$$

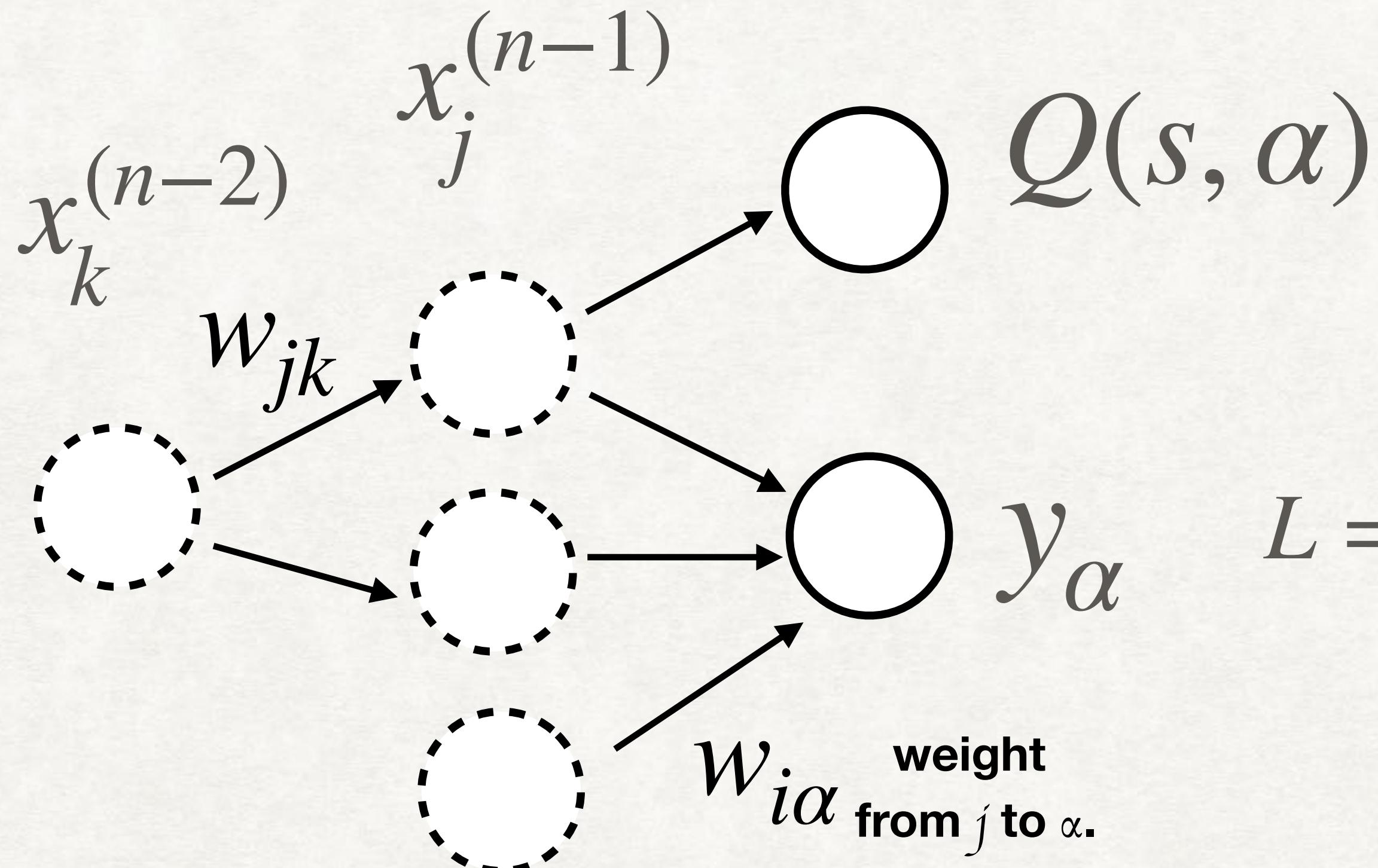
$$\Delta w_{ij} = \eta \sum_{\mathbf{x}} \delta_i^{(n)} x_j^{(n-1)}$$

$$\Delta w_{jk} = \eta \sum_{\mathbf{x}} \delta_j^{(n-1)} x_k^{(n-2)}$$

For arbitrary number of layers.

ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS

REPLACING THE Q-VALUE TABLE



One output neuron per action

$$L = \frac{1}{2} \left(r + \gamma \max_{a'} Q(s', a') - Q(s, a) \right)^2$$

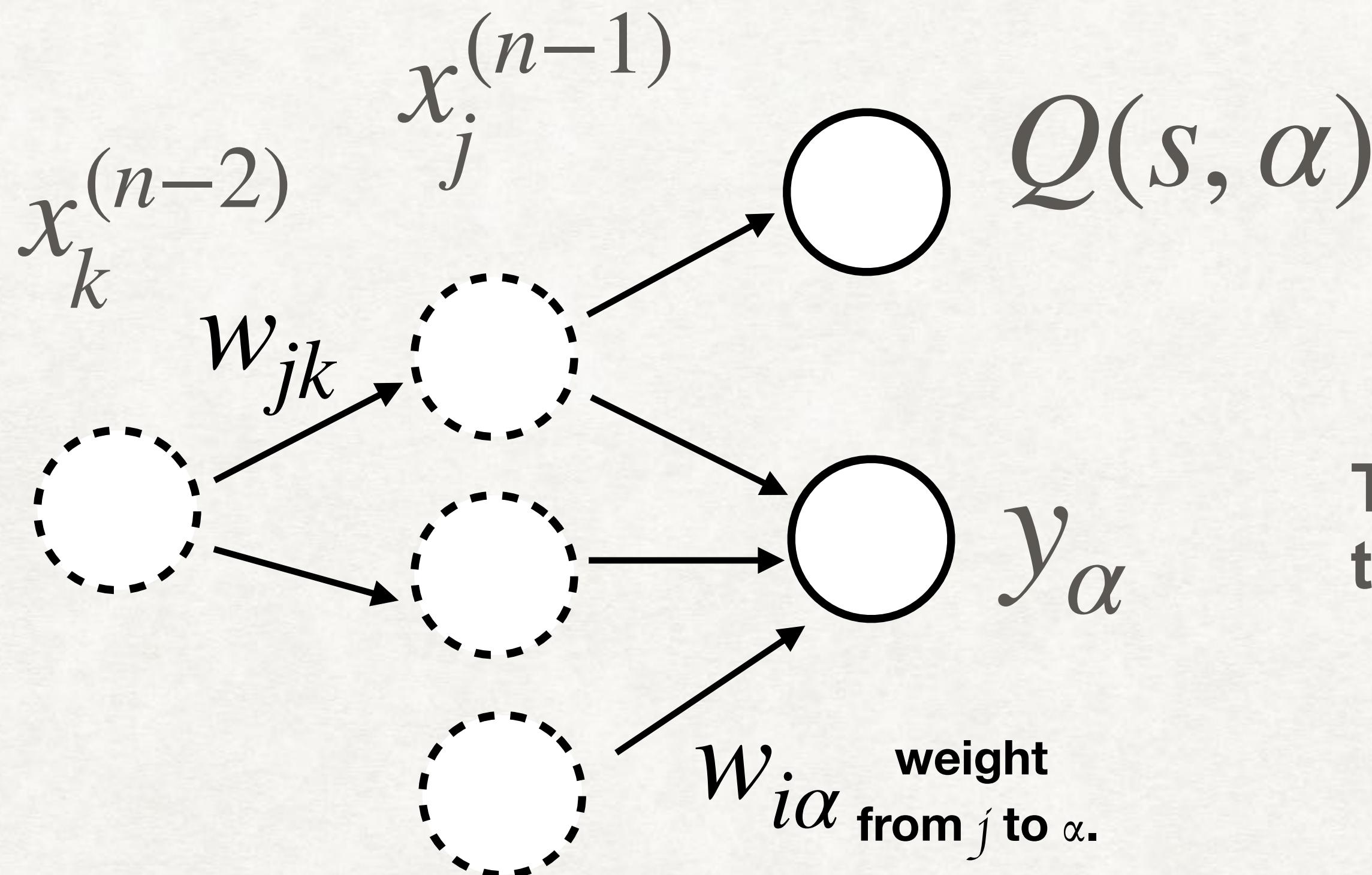
Online setting

Input=State s

$$y_\alpha^*(s) = r + \gamma \max_{a'} Q(s', a')$$

ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS

REPLACING THE Q-VALUE TABLE



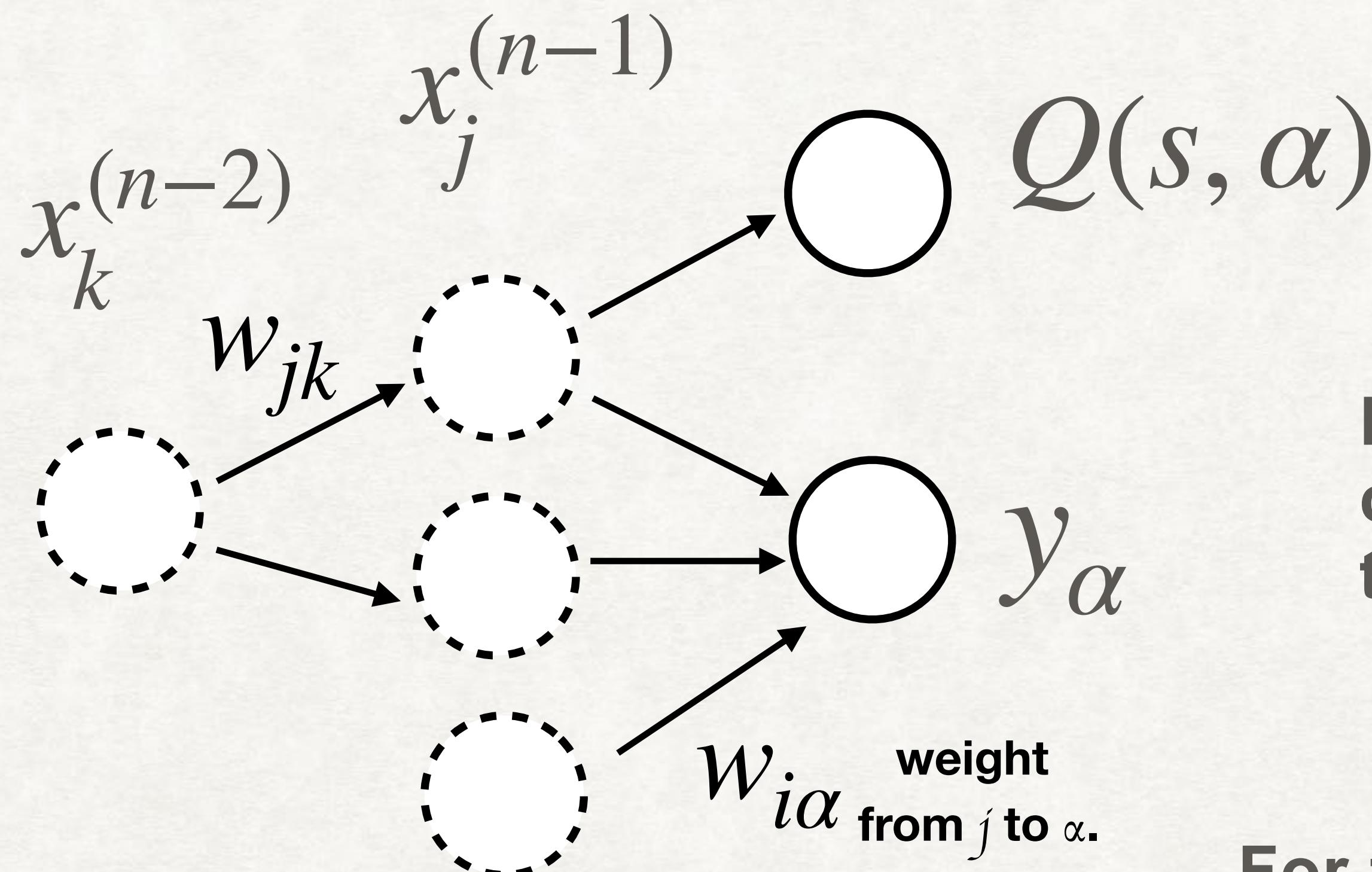
The reward collected is only relevant to the action taken!

Input=State s

$$y_\alpha^*(s) = r + \gamma \max_{a'} Q(s', a')$$

ARTIFICIAL NEURAL NETWORKS AS FUNCTION APPROXIMATIONS

REPLACING THE Q-VALUE TABLE



Trick: I construct my target into following way.

For the actions not taken the corresponding output neuron has target its true output. No changes!

For the action taken the target is set as:

$$y_\alpha^*(s) = r + \gamma \max_{a'} Q(s', a')$$

A painting depicting three board games: chess, checkers, and backgammon. The chess board is on the left, showing white pieces on a light-colored board. The checkers board is in the center, with black and white pieces on a light board. The backgammon board is on the right, with dark and light pieces on a dark board. The painting uses a palette knife or impasto technique, creating a textured, expressive surface.

DALL·E

THANK YOU!