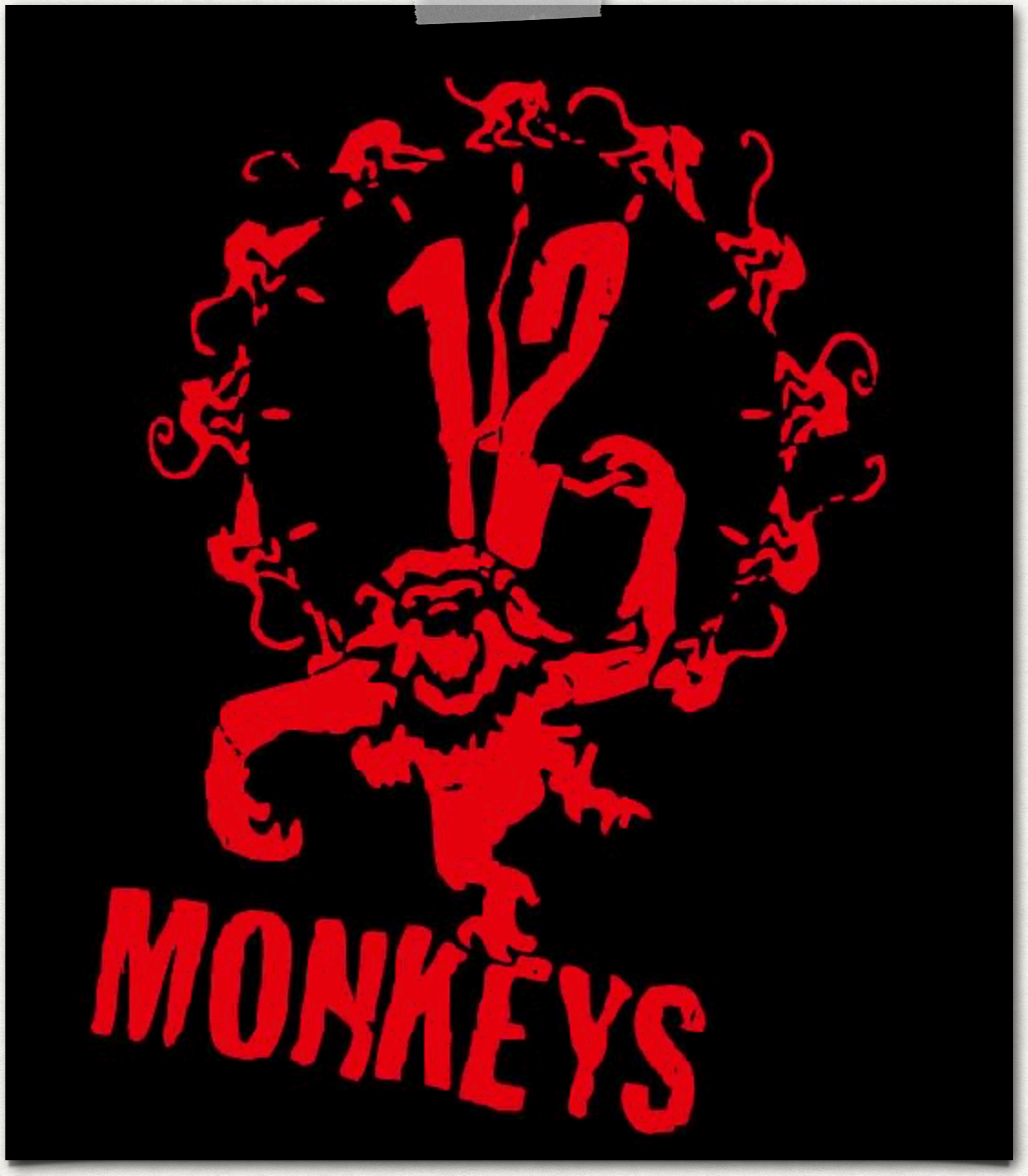
COM 3240

REINFORCEMENT LEARNING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MONKEYS

“

FUTURE ACTIONS DO NOT AFFECT THE PAST.

— Common experience

”


THE MONTY HALL PROBLEM
DALL·E

PROBABILISTIC NATURE OF REINFORCEMENT LEARNING

EXPECTATIONS

PROBABILITIES

LIKELIHOOD THAT AN EVENT WILL OCCUR

$$P(A)$$

$$0 \leq P(A) \leq 1$$

Example d4: Rolls 1 to 4

Sample space $S = \{$**all possible outcomes**$\}$

$$P(S) = 1$$

RPROBABILITIES

RANDOM VARIABLES

Variables that can assume different values, each with a certain probability.

Example d4: Rolls 1 to 4

$$P(X = x)$$

$$P(X = 1) \quad P(X = 2) \quad P(X = 3) \quad P(X = 4)$$

PROBABILITIES

PROBABILITY DISTRIBUTION

Example d4: Rolls 1 to 4 with equal probability (if fair)

uniform probability distribution

$$P(X = x) = \frac{1}{4}, \quad x \in \{1,2,3,4\}$$

A probability distribution specifies how probabilities are distributed over the values of a random variable.

PROBABILITIES

EXPECTATION

Example d4: Rolls 1 to 4 with equal probability (if fair)

uniform probability distribution

$$E[X] = \sum_x x \cdot P(X = x)$$

$$E[X] = \frac{1}{4}(1 + 2 + 3 + 4) = 2.5$$

Expected Reward

PROBABILITIES

JOINT PROBABILITY

$$P(X = x, Y = y) = P(\{X = x\} \cap \{Y = y\})$$

$$P(X = x, Y = y) \geq 0$$

PROBABILITIES

JOINT PROBABILITY FOR INDEPENDENT VARIABLES

$$P(X = x, Y = y) = P(\{X = x\} \cap \{Y = y\})$$

$$= P(X = x) \cdot P(Y = y)$$

Dice rolls

PROBABILITIES

JOINT PROBABILITY DISTRIBUTION

$$P(X = x, Y = y) = P(\{X = x\} \cap \{Y = y\})$$

$$P(X = x, Y = y) \geq 0$$

Describes the probabilities of all possible combinations of outcomes for variables X and Y (e.g. in a table).

$$\sum_x \sum_y P(X = x, Y = y) = 1.$$

EXAMPLE

ROLLING TWO DICES

Dungeons & Dragons

To hit: roll 1d20

Advantage: roll two 1d20 dices

Assume known: probably p to hit with one roll

Calculate the probability to hit with advantage

Consider the joint probability of hitting your opponent when rolling two dices

First dice hits but not the second	p(1-p)
Second dice hits but not the first	(1-p)p
Both dices hit	p·p
None hits	(1-p)·(1-p)

$$p_{hit} = p$$

Sum 1

joint probability distribution

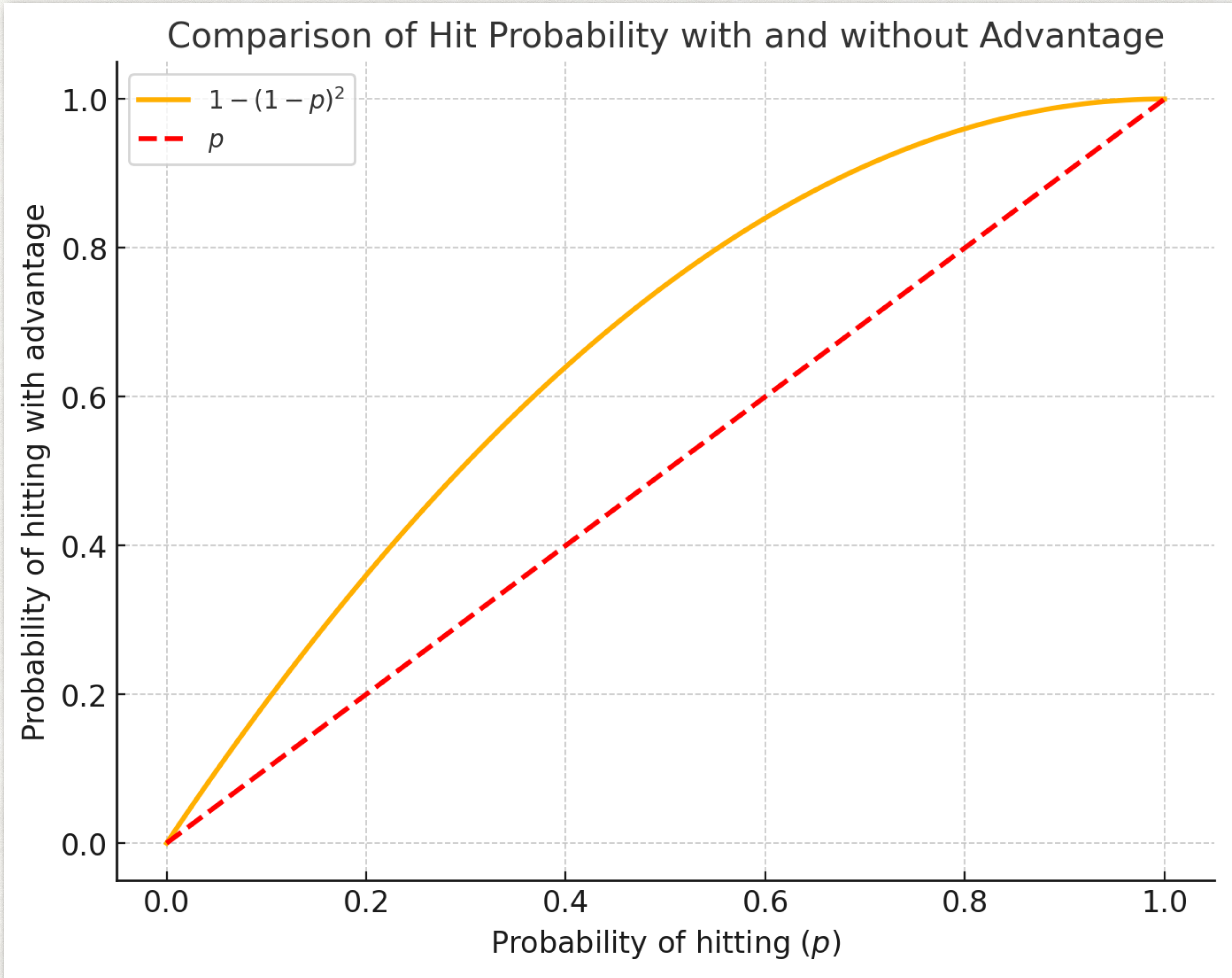
Consider the joint probability of hitting your opponent when rolling two dices

At least one dice hits = 1 - none of the dices hits

$$p_{hit-advantage} = 1 - (1 - p)^2 = p(2 - p)$$

$$p_{hit} = p$$

joint probability distribution



Comparison of Hit Probability with and without Advantage
$1-(1-p)^2$
p
Probability of hitting with advantage
Probability of hitting (p)
0.0
0.2
0.4
0.6
0.8
1.0

PROBABILITIES

PROPERTIES OF EXPECTATION

$$E[aX + bY] = aE[X] + bE[Y]$$

Linearity

$$E[XY] = E[X] \cdot E[Y]$$

Expectation of products for independent variables

$$E[c] = c$$

PROBABILITIES

VARIANCE

Definition: $$\textbf{Var}(X) = E\left[(X - E[X])^2\right]$$

$$\textbf{Var}(X) = E[X^2] - (E[X])^2$$

Easy to show, using the properties of expectation

Burning Hands

1st level evocation

Casting Time: 1 action
Range: Self (15-foot cone)
Target: Self (15-foot cone)
Components: V S
Duration: Instantaneous
Classes: Sorcerer, Wizard

As you hold your hands with thumbs touching and fingers spread, a thin sheet of flames shoots forth from your outstretched fingertips. Each creature in a 15-foot cone must make a Dexterity saving throw. A creature takes 3d6 fire damage on a failed save, or half as much damage on a successful one. The fire ignites any flammable objects in the area that aren't being worn or carried.

At Higher Levels: When you cast this spell using a spell slot of 2nd level or higher, the damage increases by 1d6 for each slot level above 1st.

https://roll20.net/compendium/dnd5e/Burning Hands#content

Magic Missile

1st level evocation

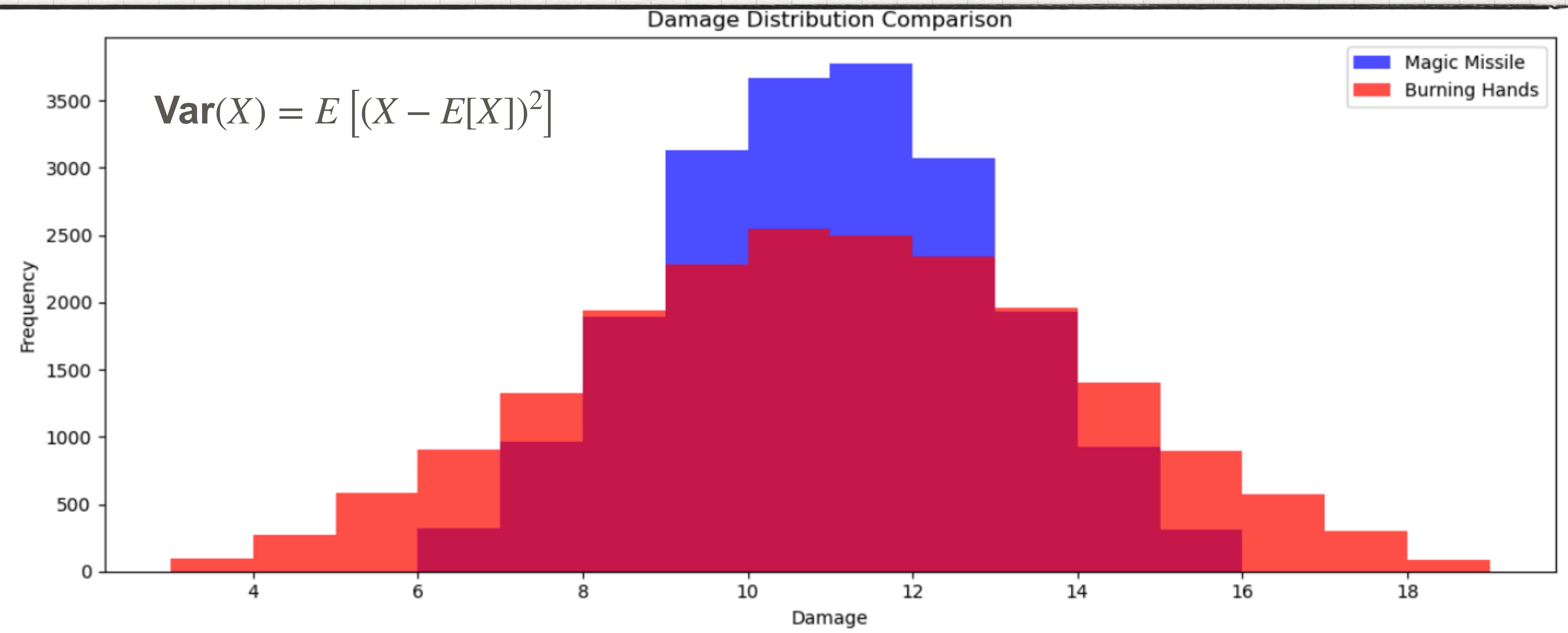
Casting Time: 1 action
Range: 120 feet
Target: A creature of your choice that you can see within range
Components: V S
Duration: Instantaneous
Classes: Sorcerer, Wizard
You create three glowing darts of magical force. Each dart hits a creature of your choice that you can see within range. A dart deals 1d4 + 1 force damage to its target. The darts all strike simultaneously, and you can direct them to hit one creature or several.
At Higher Levels: When you cast this spell using a spell slot of 2nd level or higher, the spell creates one more dart for each slot above 1st.

https://roll20.net/compendium/dnd5e/Magic Missile#content

Calculate the expected (average) damage

Burning hands: 3d6

Magic missile: 3 x (1d4+1)

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Magic Missile – Mean Damage: 10.493, Variance: 3.782051
Burning Hands – Mean Damage: 10.5193, Variance: 8.74442751
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MEAN AND VARIANCE

CONSIDERATIONS FOR THE REWARD

Option 1: low reward with high probability (low variance)

Option 2: high reward with low probability (high variance)

Potential issue

PROBABILITIES

CONDITIONAL EXPECTATIONS

$$E[X \mid Y = y] = \sum_x x \cdot P(X = x \mid Y = y)$$

$$E[X] = E[E[X \mid Y]]$$

Law of Total Expectation

$$E[aX + bY \mid Z] = aE[X \mid Z] + bE[Y \mid Z]$$

Linearity

$$E[X \mid Y] = E[X]$$

Independence

PROBABILITIES

PROPERTIES OF CONDITIONAL EXPECTATIONS

$$E[X \mid Y = y] = \sum_x x \cdot P(X = x \mid Y = y)$$

$$E[X] = E[E[X \mid Y]]$$

Law of Total Expectation

$$E[aX + bY \mid Z] = aE[X \mid Z] + bE[Y \mid Z]$$

Linearity

$$E[X \mid Y] = E[X]$$

Independence


THE MONTY HALL PROBLEM
DALL·E


DALL·E

PROBABILITIES

MARKOVIANITY

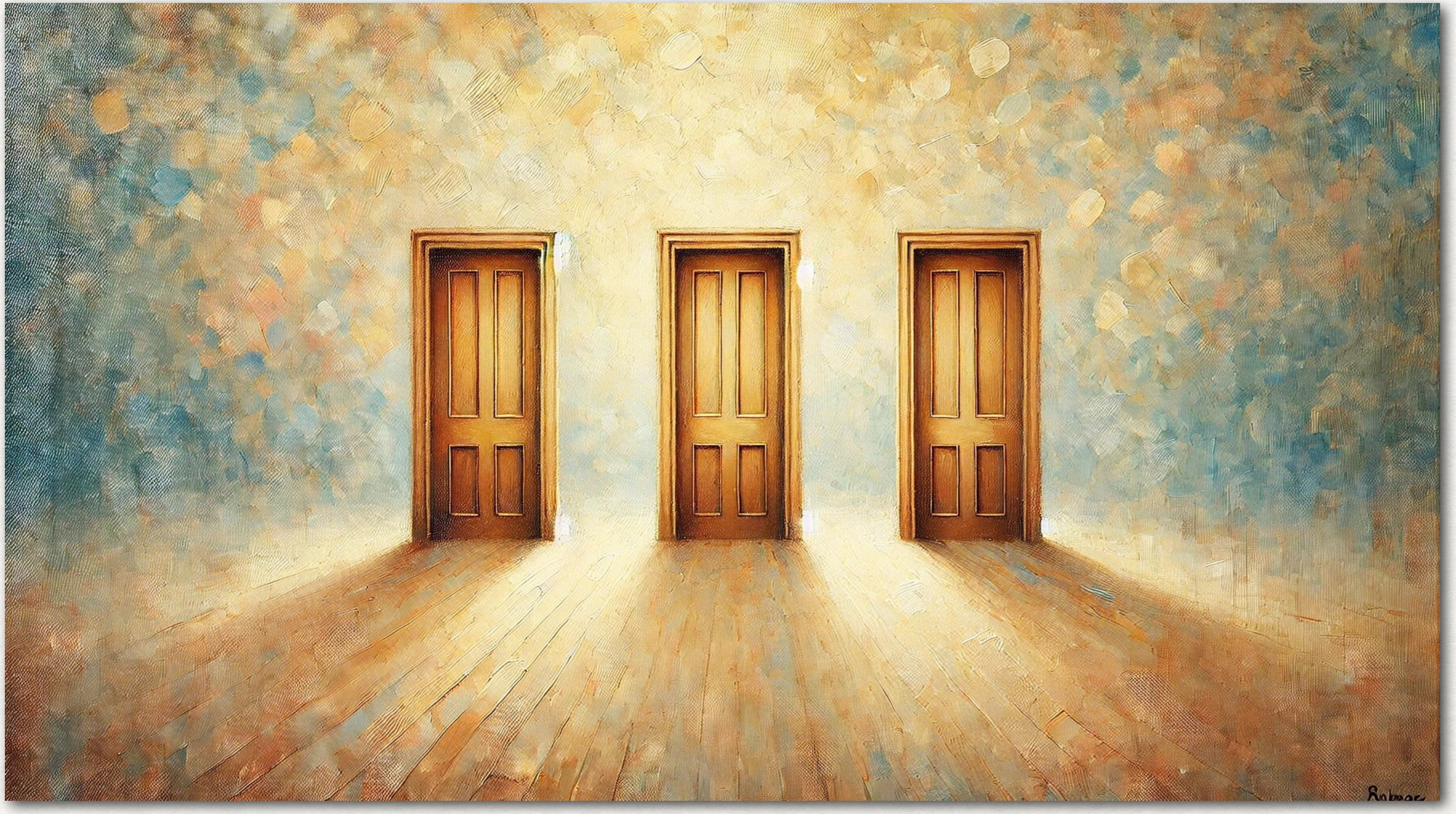
The future state of a system depends only on its present state and not on its past history.

$$P(X_{t+1} \mid X_t, X_{t-1}, \ldots, X_0) = P(X_{t+1} \mid X_t)$$

Conditional probability of transitioning in the next state

QUESTION

IS THE MONTY HALL PROBLEM MARKOVIAN? (HOST'S PERSPECTIVE)



PROBABILITIES

SUMMARY

- Random (or stochastic) variables
- Probability distribution
- Expectation
- Joint probability and joint probability distribution distribution
- Properties of expectation and conditions expectation
- Variance

THANK YOU!