COM 3240

REINFORCEMENT LEARNING


DALL·E

OPTIMISATION

GRADIENT DESCENT

Desirable: A system that performs a specific task

The system has parameters that we need to select appropriately (optimise) for that task

$$f(x_1, x_2, \ldots, x_n)$$

parameters

$$f$$

System's performance: maximise

System's error: minimise

OPTIMISATION

ARTIFICIAL NEURAL NETWORKS


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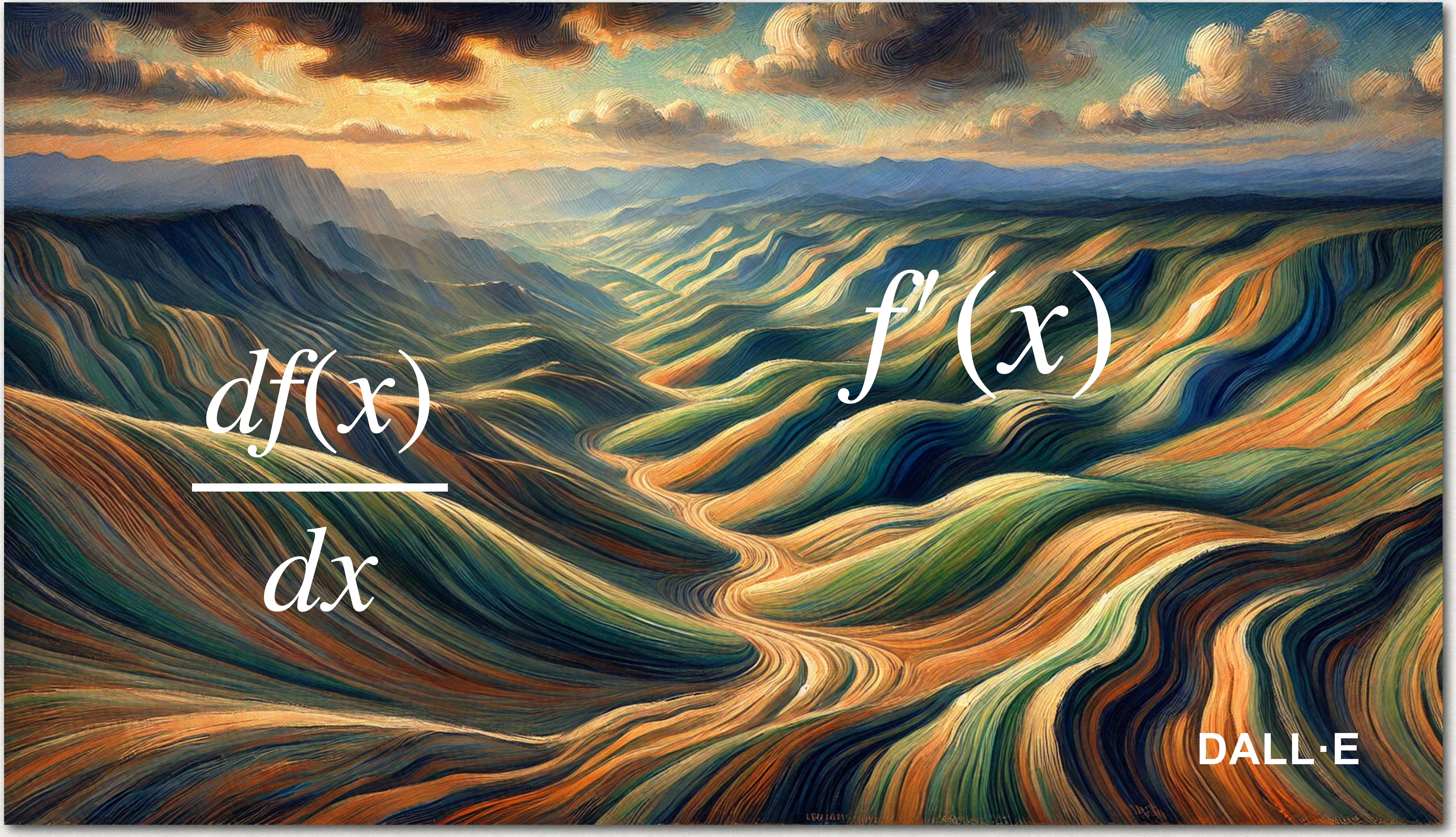
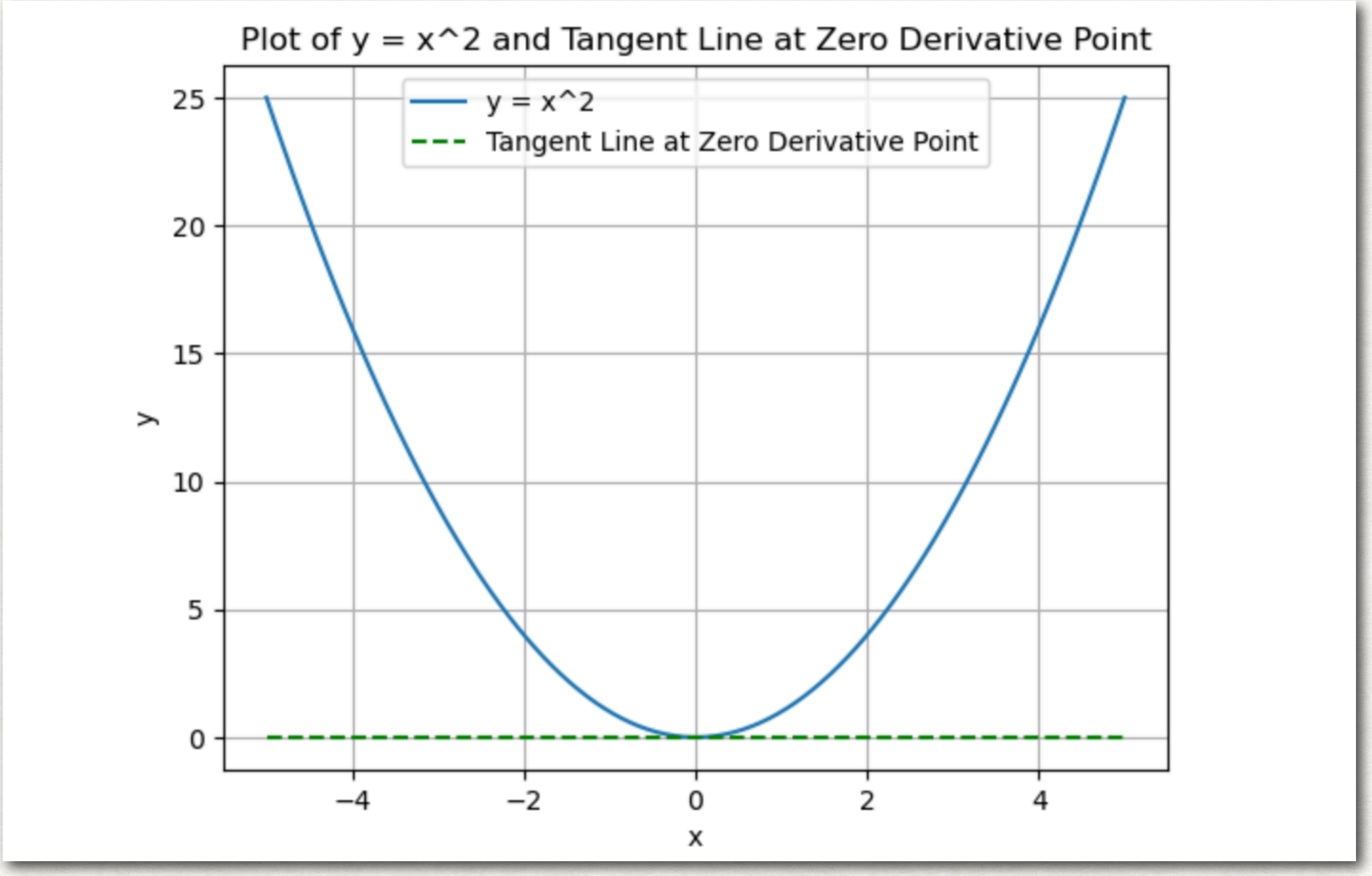
DERIVATIVES

OPTIMISATION - GRADIENT METHODS



DERIVATIVES

OPTIMISATION - GRADIENT METHODS



Slope

Error (Loss) function - Machine Learning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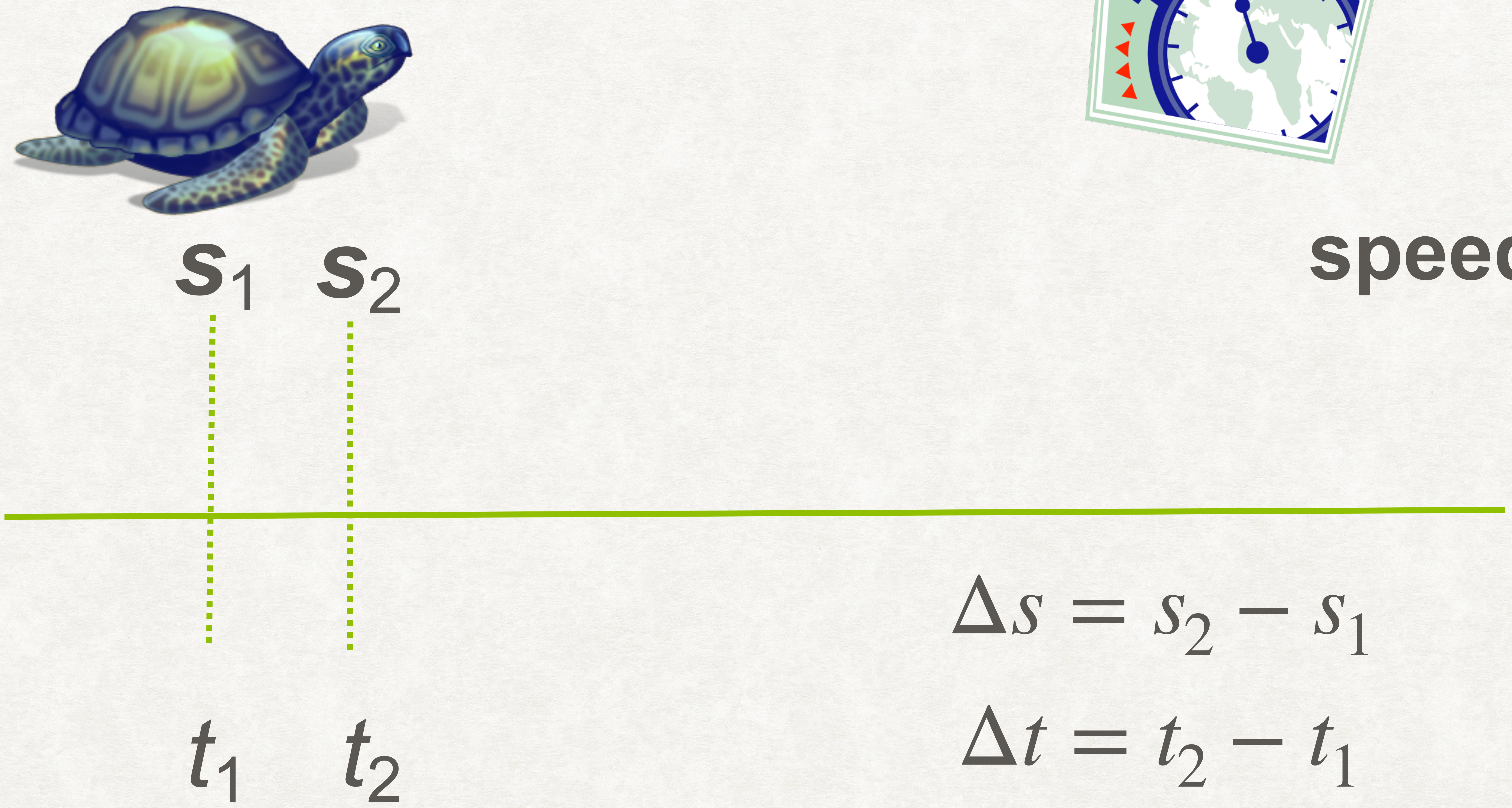
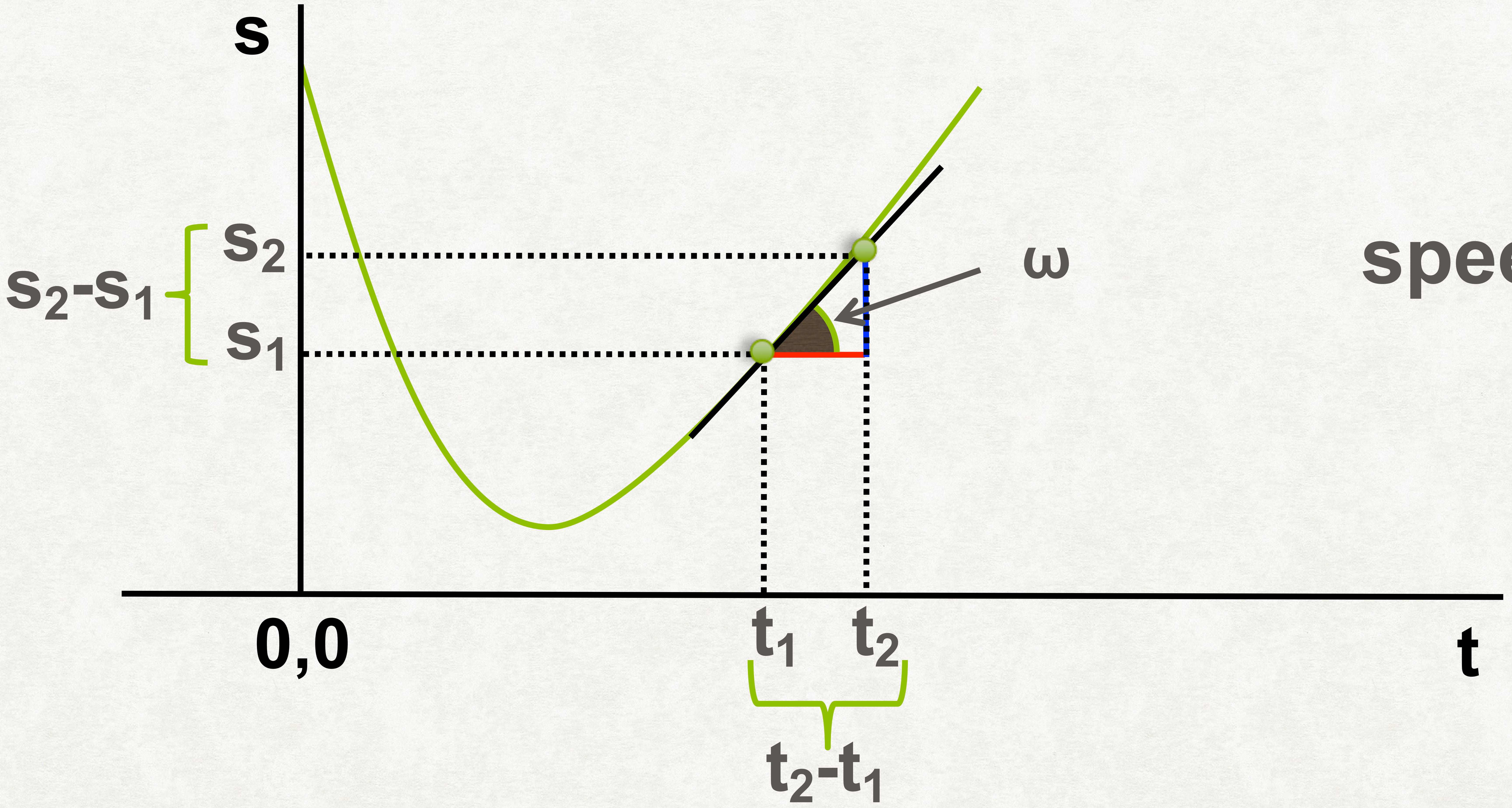


speed: $\dfrac{\Delta s}{\Delta t}$

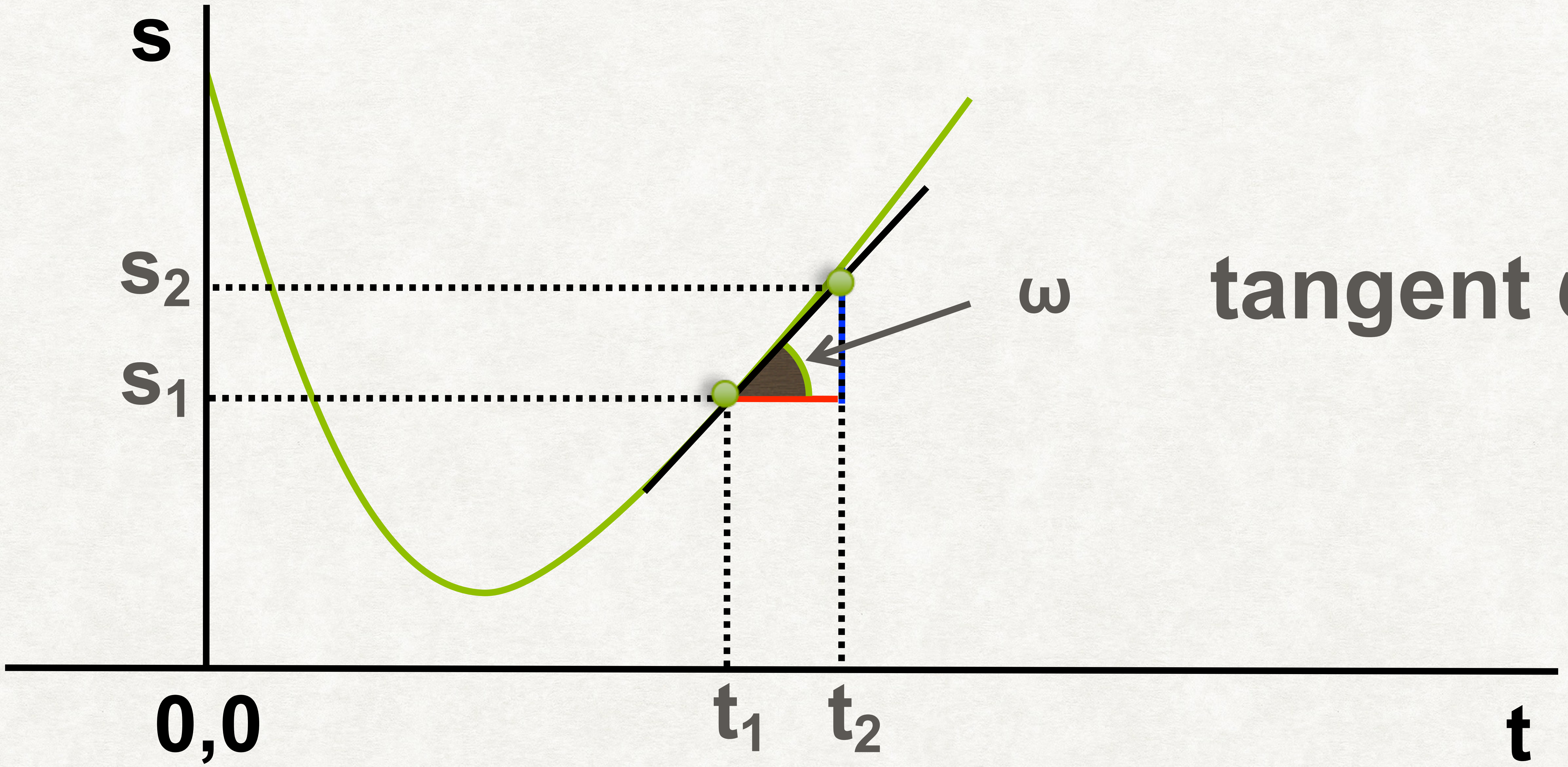
$$\Delta s = s_2 - s_1$$

$$\Delta t = t_2 - t_1$$



speed: $\frac{\Delta s}{\Delta t}$

$$\frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{s(t + \Delta t) - s(t)}{\Delta t}$$



tangent ω: $\frac{\Delta s}{\Delta t}$

DERIVATIVES

USE THE DEFINITION TO CALCULATE DERIVATIVES OF FUNCTIONS

$$\frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{s(t + \Delta t) - s(t)}{\Delta t}$$

More generally:

$$f'(x) = \frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

What is the effect of incrementing x by a small amount, Δx, on the value of f(x)?

$f(x) = 7$ $\quad$ $f(x) = 7x$ $\quad$ $f(x) = 7x^2$

DERIVATIVES IN PRACTICE

MEMORISE BASIC FUNCTIONS AND USE RULES

Power function:	$f(x) = x^n$	$f'(x) = nx^{n-1}$
Exponential function:	$f(x) = e^x$	$f'(x) = e^x$
Natural logarithm:	$f(x) = \ln(x)$	$f'(x) = \dfrac{1}{x}$

DERIVATIVES - RULES

ADDITION

$$\frac{d}{dx}\Big(f(x) + g(x)\Big) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

$$h(x) = x + x^2$$

DERIVATIVES - RULES

MULTIPLICATION

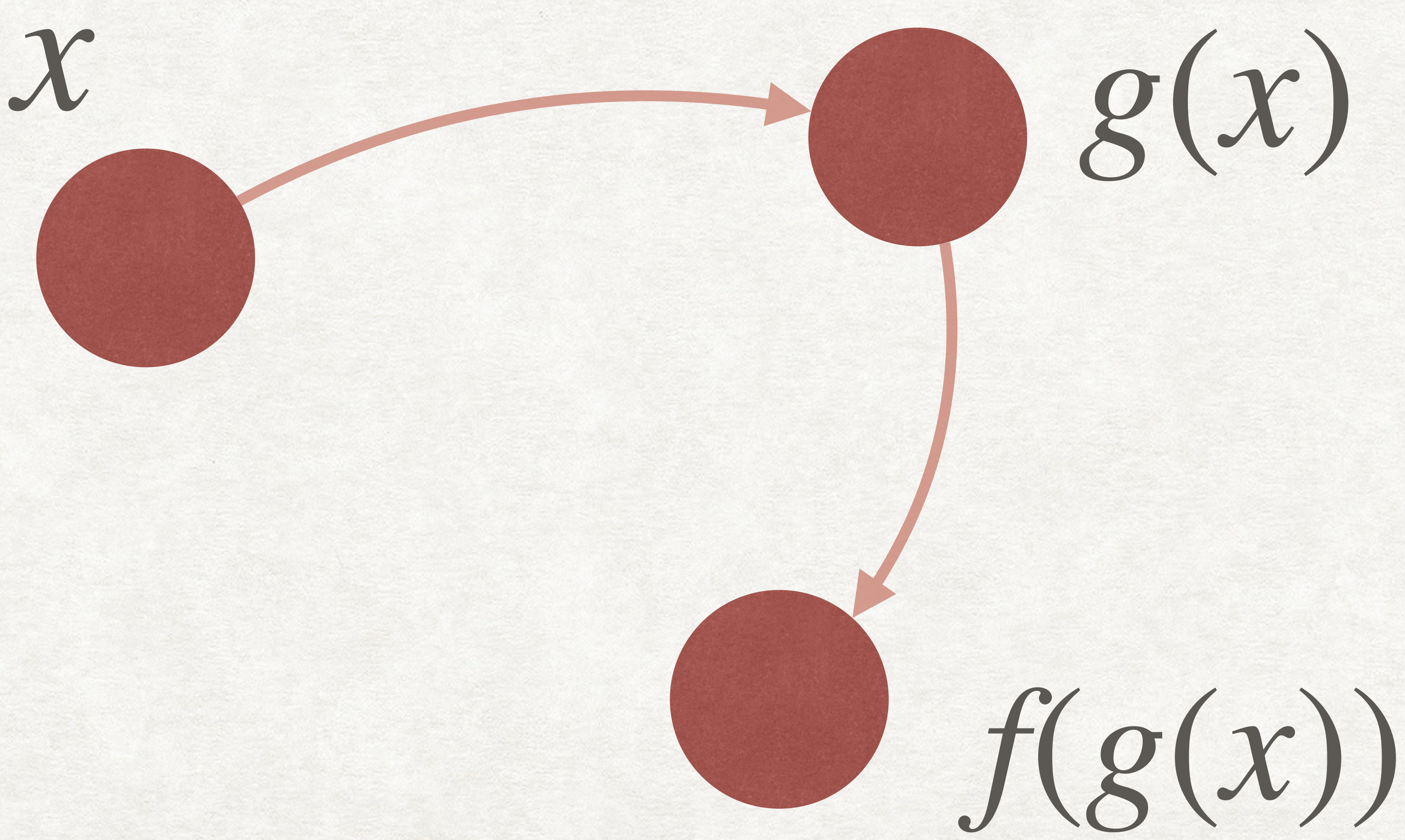
$$\frac{d}{dx}\left(f(x) \cdot g(x)\right) = \frac{df(x)}{dx} \cdot g(x) + f(x) \cdot \frac{dg(x)}{dx}$$

$$h(x) = x \cdot ln(x)$$

DERIVATIVES - RULES

THE CHAIN RULE

$$\frac{df\left(g(x)\right)}{dx} = \frac{df\left(g(x)\right)}{dg(x)} \cdot \frac{dg(x)}{dx}$$



$$h(x) = (x - c)^2$$

MOTIVATION FOR PARTIAL DERIVATIVES

LIGHTS AND BUTTONS PUZZLE

$$f(x, y) = x^2 y + y^2$$

A more general way to write the same:

$$f(x_1, x_2) = x_1^2 x_2 + x_2^2$$

How do we estimate the gradient?

PARTIAL DERIVATIVES

AND THE NABLA NOTATION

$$f(x_1, x_2, \ldots)$$

$$\frac{\partial f}{\partial x_1} = \lim_{\Delta x_1 \to 0} \frac{f(x_1 + \Delta x_1, x_2, \ldots) - f(x_1, x_2, \ldots)}{\Delta x_1}$$

$$\frac{\partial f}{\partial x_2} = \lim_{\Delta x_2 \to 0} \frac{f(x_1, x_2 + \Delta x_2, \ldots) - f(x_1, x_2, \ldots)}{\Delta x_2}$$

$$\nabla f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2} \right)$$

PARTIAL DERIVATIVES

AND THE NABLA NOTATION

One variable at a time, treat others as constants!

PARTIAL DERIVATIVES

AND THE NABLA NOTATION

$$f(x_1, x_2) = x_1^2 x_2 + x_2^2$$

$$\frac{\partial f}{\partial x_1} = \frac{\partial}{\partial x_1}(x_1^2 x_2 + x_2^2) = 2x_1 x_2$$

$$\frac{\partial f}{\partial x_2} = \frac{\partial}{\partial x_2}(x_1^2 x_2 + x_2^2) = x_1^2 + 2x_2$$

INTRODUCE (PARTIAL) DIFFERENTIAL EQUATIONS

EVERYTHING CHANGES

$$F\left(x, y, \frac{dy}{dx}, \frac{d^2y}{dx^2}, \ldots\right) = 0$$

Unknown: $y(x)$

$$F\left(x_1, x_2, \ldots, x_n, u, \frac{\partial u}{\partial x_1}, \frac{\partial u}{\partial x_2}, \ \ldots, \frac{\partial u}{\partial x_n}\right) = 0$$

$u(x_1, x_2, \ldots)$

DIFFERENTIAL EQUATIONS

RELAXATION

$$\tau \frac{dx(t)}{dt} = - x(t)$$

$x(t)$: **here it is a function of time $f(t) = x(t)$; not a variable!**

τ : **time constant, if it is large, you relax slowly (you have a long memory:)**

DIFFERENTIAL EQUATIONS

LEAKY INTEGRATOR

$$\tau \frac{dx(t)}{dt} = -x(t) + s(t)$$

$x(t)$: **a function**

$s(t)$: **sample, input**

t : **variable**

DEFINITION OF DERIVATIVES

AND ITS APPLICATION TO DIFFERENTIAL EQUATIONS

$$\frac{dx(t)}{dt} = \lim_{\Delta t \to 0} \frac{x(t + \Delta t) - x(t)}{\Delta t}$$

Approximate the derivative as difference.

$$\tau \frac{dx(t)}{dt} = -x(t) + s(t)$$

You need one initial point and a time step.

e.g., $x(0), \Delta t = 0.1$ **(seconds)**

Its discrete form is known as “exponential moving average”

DEFINITION OF DERIVATIVES

AND ITS APPLICATION TO DIFFERENTIAL EQUATIONS

$$\frac{dx(t)}{dt} \approx \frac{x(t+\Delta t) - x(t)}{\Delta t}$$

Approximate the derivative as difference.

$$\tau \frac{x(t+\Delta t) - x(t)}{\Delta t} = -x(t) + s(t)$$

You need one initial point and a time step.

e.g., $x(0), \Delta t = 0.1$ **(seconds)**

Its discrete form is known as "exponential moving average"

DEFINITION OF DERIVATIVES

AND ITS APPLICATION TO DIFFERENTIAL EQUATIONS

$$x(t + \Delta t) = x(t) + \frac{\Delta t}{\tau}\Big(-x(t) + s(t)\Big)$$

Only unknown!

Euler's method: I can start from, e.g. time 0 (sec) and iteratively calculate the value of x at any time

DEFINITION OF DERIVATIVES

AND ITS APPLICATION TO DIFFERENTIAL EQUATIONS

$$x(t + \Delta t) = x(t)\left(1 - \frac{\Delta t}{\tau}\right) + \frac{\Delta t}{\tau} s(t)$$

Useful equation for "forgetful" averaging, filtering high frequencies (noise)

DEFINITION OF DERIVATIVES

AND ITS APPLICATION TO DIFFERENTIAL EQUATIONS

$$\tau \frac{d\mathbf{w}(\mathbf{t})}{dt} = -\frac{\partial \mathscr{L}(\mathbf{t})}{\partial \mathbf{w}(\mathbf{t})}$$

$$\Delta \mathbf{w}(\mathbf{t}) = -\eta \frac{\partial \mathscr{L}(\mathbf{t})}{\partial \mathbf{w}(\mathbf{t})}$$

$\eta \approx \Delta t/\tau$: **learning rate**

SUMMARY

PREREQUISITES

- **Concept of Optimisation, Gradient Learning**
- **Derivatives & partial derivatives**
- **Differential equations, Euler’s method & leaky integrators**

THANK YOU!