
Meta-Learning of Compositional Task Distributions in Humans and Machines

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Abstract

1 Modern machine learning systems struggle with sample efficiency and are usually
2 trained with enormous amounts of data for each task. This is in sharp contrast with
3 humans, who often learn with very little data. In recent years, meta-learning, in
4 which one trains on a family of tasks (i.e. a task distribution), has emerged as an
5 approach to improving the sample complexity of machine learning systems and to
6 closing the gap between human and machine learning. However, in this paper, we
7 argue that current meta-learning approaches still differ significantly from human
8 learning. We argue that humans learn over tasks by constructing compositional
9 generative models and using these to generalize, whereas current meta-learning
10 methods are biased toward the use of simpler statistical patterns. To highlight this
11 difference, we construct a new meta-reinforcement learning task with a composi-
12 tional task distribution. We also introduce a novel approach to constructing a “null
13 task distribution” with the same statistical complexity as the compositional distri-
14 bution but without explicit compositionality. We train a standard meta-learning
15 agent, a recurrent network trained with model-free reinforcement learning, and
16 compare it with human performance across the two task distributions. We find
17 that humans do better in the compositional task distribution whereas the agent
18 does better in the non-compositional null task distribution – despite comparable
19 statistical complexity. This work highlights a particular difference between hu-
20 man learning and current meta-learning models, introduces a task that displays
21 this difference, and paves the way for future work on human-like meta-learning.

1 Introduction

23 A contributing factor towards humans’ incredible ability to learn efficiently from very few exam-
24 ples is their compositional inductive bias (Lake et al., 2017). Such compositional representations
25 allow generalization far outside the training regime to more complex environments via recursive
26 combination of simpler building blocks (Kemp & Tenenbaum, 2008; Lake et al., 2015; Schulz et al.,
27 2017). Meta-learning has become a popular approach to learn inductive biases that enable machine
28 learning systems learn from fewer examples (Hospedales et al., 2020). In meta-learning, a model
29 is trained on a family of tasks to learn an inductive bias that improves its ability to learn a new
30 task from the same task distribution using fewer examples. In this work, we directly examine the
31 extent to which such approaches are able to learn an inductive bias mentioned above: the ability
32 to acquire and use compositional structure. Previous work demonstrates that some compositionality
33 can be meta-learned (Lake, 2019). However, we argue that simply doing well on a compositional
34 task distribution does not necessarily indicate meta-learning compositional structure, because an
35 agent can be exploiting statistical patterns across tasks that are consequences of the compositional
36 structure instead of learning the underlying compositional structure. We provide a direct way to
37 disentangle these two. In particular, we introduce a grid-based task which contains embedded com-

positional structure, as well as a new method for generating control “null” tasks that are matched for statistical complexity but do not have compositional structure. These matched tasks allow us to disentangle an agent that meta-learns compositionality vs statistical patterns. We directly compare the performance of a meta-learning algorithm with human performance on these two tasks. Just as every vanilla learning algorithm has its own inductive bias in learning a single task, meta-learners themselves have inductive biases (which we term *meta-inductive bias*) influencing what inductive biases they learn from their cross-task experience. In this paper, we show that standard meta-learners do not learn a compositional bias despite direct training on compositional task distributions. Since a large swath of real-world tasks contain compositional structure, this bias is objectively valuable – not simply a way to be more human-like. This work highlights that this valuable inductive bias cannot easily be learned by standard meta-learners.

2 Embedding compositionality in a task distribution

In this work, we define a broad family of task distributions that contain abstract compositional structure. Previous work on such datasets (Lake & Baroni, 2018; Johnson et al., 2017) focuses primarily on language. Here we instead directly consider the domain of structure learning. This is a fundamental tenet of human cognition and has been linked to how humans learn quickly in novel environments (Tenenbaum et al., 2011; Mark et al., 2020). Structure learning is required in a vast range of domains: from planning (understanding an interrelated sequence of steps for cooking), category learning (the hierarchical organization of biological species), to social inference (understanding a chain of command at the workplace, or social cliques in a high school). A task distribution based on structure learning can therefore be embedded into several domains relevant for machine learning.

Kemp & Tenenbaum (2008) provide a model for how people infer such structure. They present a probabilistic context-free graph grammar that produces a space of possible structures, over which humans do inference. A grammar consists of a start symbol S , terminal and non-terminal symbols Σ and V , as well as a set of production rules R . Different structural forms arise from recursively applying these production rules. This framework allows us to specify abstract structures (via the grammar) and to produce various instantiations of this abstract structure (via the noisy generation process), naturally producing different families of task distributions.

We consider three structures: chains, trees, and loops. These exist in the real world across multiple domains. Chains describe objects on a one-dimensional spectrum, like people on the left-right political spectrum. Trees describe objects organized in hierarchies, like evolutionary trees. Loops describe cycles, like the four seasons. Here we embed these structures into a grid-based task.

Exploration on a grid is an extensively studied problem in machine learning, particularly in reinforcement learning. Further, it is also a task that is easy for humans to perform on online crowdsourcing platforms – but not trivially so. This allows us to directly compare human and machine performance on the same task. Fig. 1 displays the symbols of the grammar we use and the production rules that give rise to grids of different structural forms.

2.1 A task to test structure learning

Here we describe the specific task built atop this embedding of structural forms. We use a tile revealing task on the grid. Humans as well as agents are shown a 7x7 grid of tiles, which are initially white except for one red tile. This red tile is the initial start tile in the grid’s generative process (see Fig. 1). Clicking white tiles reveal them to be either red or blue. The episode finishes when the agent reveals all the red tiles. There is a reward for each red tile revealed, and a penalty for every blue tile revealed. The goal therefore is to reveal all the red tiles while revealing as few blue tiles as possible. The particular configuration of the red tiles defines a single task. The distribution of tasks for meta-learning is defined by the grammar from which these structures are sampled. Here, we randomly sampled from a uniform mixture of chains, trees, and loops as defined in Fig. 1.

2.2 A statistically equivalent non-compositional task

Previous approaches to testing compositionality in machine-learned representations (Lake & Baroni, 2018; Dasgupta et al., 2018) have relied on examining average performance on held-out examples

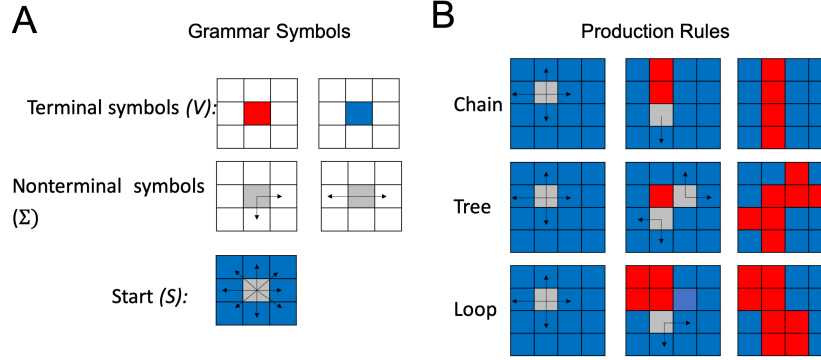


Figure 1: **Generative Grammar** (A) Grammar symbols and (B) production rules. A board is formed by beginning with the start symbol and recursively applying production rules until only terminal symbols (red and blue tiles) are left. Each production rule either adds a non-terminal symbol (from first column to second) or a terminal symbol (from second column to third) with 0.5 probability.

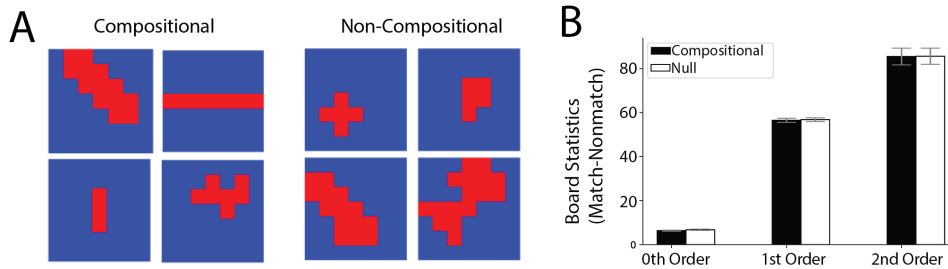


Figure 2: **Comparing compositional and null task distributions** (A) Example compositional and null distribution boards. Compositional boards are distinctly either chain, trees, or loops while null boards have similar statistical properties but don't necessarily obey the recursive rules used to generate compositional boards. (B) Ising statistics across the two task distributions. Error bars are 95% non-parametric bootstrap confidence intervals across different boards of the respective distribution.

89 from compositionally structured task distributions. However, we argue that this often confounds
90 whether a system has truly acquired compositional structure or whether it is relying on statistical
91 structure that comes about as a consequence of compositional rules.

92 To directly examine whether compositionality is a factor in how humans and meta-learning agents
93 perform this task, we need a control task distribution that is similar in statistical complexity but is not
94 explicitly compositional. To this end, we trained a fully connected neural network (3 layers, 49 units
95 each) to learn the conditional distribution of each tile given the all other tiles on the compositional
96 boards. Note that these conditional distributions contain all the relevant statistical information about
97 the boards. We do this by training on an objective inspired by masked language models like BERT
98 (Devlin et al., 2018). The network was given a compositional board with a random tile masked out
99 and trained to reproduce the entire board including the randomly masked tile. The loss was binary
100 cross entropy between the predicted and actual masked tiles. The network was trained on all possible
101 compositional boards for 10^4 epochs, training accuracy was $\sim 99\%$.

102 We then sampled boards from these conditionals with Gibbs sampling. We started with a grid where
103 each tile is randomly set to red or blue with probability 0.5. We then masked out a tile and ran the
104 grid through the network to get the conditional probability of the tile being red given the other tiles,
105 turning the tile red with that probability. We repeated this by masking each tile in the 7×7 grid (in
106 a random order) to complete a single Gibbs sweep, and repeat this whole Gibbs sweep 20 times to
107 generate a single sample. We refer to the distribution of boards generated this way as the null task
108 distribution. Fig. 2 shows example compositional and null distribution grids.

While the statistical structure looks similar, the non-compositional null boards shown could not have been generated by the grammar in Fig. 1. The conditional distributions for the two distributions are similar by design, we further quantify statistical similarity using Ising statistics (Zhang, 2007). We compared the 0th order, 1st order, and 2nd order effects defined as follows. The 0th order statistic corresponds to the number of red minus number of blue tiles. The 1st order statistic counts the number of agreeing neighbours (vertically or horizontally adjacent) minus the disagreeing ones, where agreeing means being of the same color. The 2nd order statistic is the number of triples (tile+its neighbor+its neighbor’s neighbor) that agree, minus those that don’t. Fig. 2b shows that the two distributions are not significantly different in terms of the Ising statistics measured ($p > 0.05$ for all three orders).

3 Experiments

We analyze and compare the performance of standard meta-learners and human learning on our tile-revealing task. We test them on boards that are sampled from the generative grammar and contain explicit compositional structure, as well as on boards that are matched for statistical complexity, but are sampled from a null distribution that does not contain explicit compositional structure. Comparing performance across these two task distributions allows us to pinpoint the role of compositionality as distinct from the statistical patterns that arise as a downstream consequence of compositional rules.

3.1 Methods

Meta-Reinforcement Learning Agent Following previous work in meta-reinforcement learning (Wang et al., 2016; Duan et al., 2016) we use an LSTM meta-learner that takes the full board as input, passes it through 2 fully connected layers (49 units each) and feeds that, along with the previous action and reward, to 120 LSTM units. It is trained with a linear learning rate schedule and 0.9 discount. The reward function was: +1 for revealing red tiles, -1 for blue tiles, +10 for the last red tile, and -2 for choosing an already revealed tile. The agent was trained using Advantage Actor Critic (A2C) (Stable baselines package ?). The agent was trained for 10^6 episodes. We performed a hyperparameter sweep (value function loss coefficient, entropy loss coefficient, learning rate) using a held-out validation set for evaluation (see Appendix). The selected model’s performance was evaluated on held-out test grids. We trained different agents in the same way on the compositional and null task distributions, with separate hyperparameter sweeps for each.

Human Experiment We crowdsourced human performance on our task using Prolific (www.prolific.co) for a compensation of \$1.50. Participants were shown the 7x7 grid on their web browser and used mouse-clicks to reveal tiles. Each participant was randomly assigned to the compositional or null task distribution, 25 participants in each. Each participant was directly evaluated on the test set of grids for the models (24 grids from their assigned task distribution in randomized order). This is a key difference between the human and agent tasks – humans did not receive training on a task distribution. While we are examining whether agents can *meta-learn* an inductive bias toward compositionality (by training on compositional task distributions), we assume that humans already have this bias from pre-experimental experience. Since participants had to reveal all red tiles to move on to the next grid, they were implicitly incentivized to be efficient (clicking as few blue tiles as possible) in order to finish the task quickly. We found that this was adequate to get good performance. A reward structure similar to that given to agents was displayed as the number of points accrued, but did not translate to monetary reward.

Evaluation Unless specified otherwise, performance is evaluated as the number of blue tiles revealed before all red tiles are revealed (lower is better). All error bars are 95% non-parametric bootstrap confidence intervals calculated across agents / participants. Non-overlapping confidence intervals indicate differences that are statistically significant. We also provide non-parametric bootstrap p-values when comparing differences across different samples.

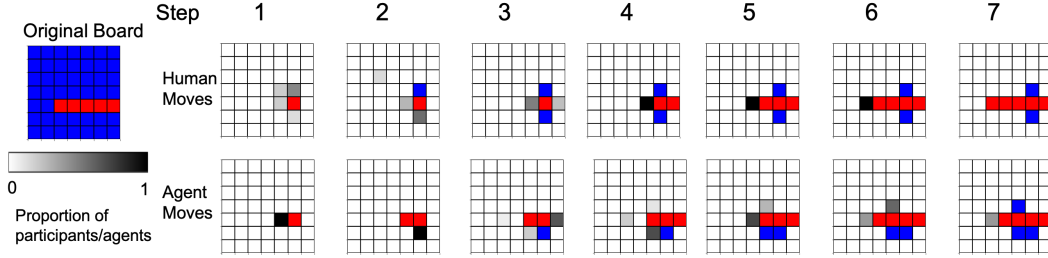


Figure 3: **Human and agent policies on the task.** Red/blue indicate already revealed tiles while grayscale indicate what proportion of humans or agents (conditioned on the moves so far) revealed that tile in the next step. In this chain example, once humans figure out that the board is a chain structural form (step 5), they get perfect performance by choosing tiles along the chain’s production direction, while agents still choose other blue tiles.

3.2 Results

In this section, we demonstrate that humans have a clear bias toward compositional distributions even while directly controlling for statistical complexity. We compare human performance with that of a meta-learning agent, and consider the role of a compositional inductive biases in explaining these differences.

Comparing human and agent performance: We start with an example that highlights the difference between human and agent policies on this task (Fig. 3). In this chain example, once humans figure out that the board is a chain structural form, they never deviate from the chain’s production direction while agents do. This indicates that humans are learning the generative rules of the chain form and using these rule to determine their actions, while the agent is using simpler statistical patterns that do not have strict rules.

We now consider various ways to quantify this difference. First, we see that humans do better overall on both the null and compositional distributions (Fig. 4; $p < 0.0001$ for both task distributions). This is despite, unlike the agents, no direct experience with this task. This indicates that humans have useful inductive biases from pre-experimental experience that are valuable in this task (Dubey et al., 2018). We discuss the role of these inductive biases in the following sections. The meta-learner has had extensive experience with each task distribution, and had the chance to pick up the inductive biases relevant for this task. Persistent differences in performance indicate that standard meta-learners differ from humans in the kinds of inductive biases they learn (i.e. in their meta-inductive biases).

Inductive bias toward compositionality. First, we note that humans perform better on the compositional versus the null distribution (Fig. 4a), whereas the agent does better on the null task distribution than on the compositional tasks. This reflects a significant difference between their performance. We hypothesized that humans perform well on the compositional task distribution by first inferring what kind of structure they are in, and then following the production rules for that structure. Since such structure does not reliably exist in the null distribution, they cannot learn, infer, and use it. Further, we hypothesized that the agents learn statistical patterns instead.

Fig. 3 supports this intuition but here we look to quantify it further. If a system uses a compositional generative representation, we would expect success rate (rate of choosing red tiles) to be low in the beginning of a trial while figuring out what the structure underlying this trial is. Conversely, we would expect a higher success rate towards the end while following inferred production rules to reveal red tiles. To test this hypothesis, we split human and agent behavior in each trial into the first and last half, and examine success rate in each Fig. (4b and c). For the compositional distribution, we find that humans have a higher success rate in the second half, providing support for our hypothesis. In contrast, we find that agent success rate does not increase over a trial, and in fact decreases. We also find that humans do not show increasing success rate in the null task distribution while agents do, providing further evidence for our hypothesis.

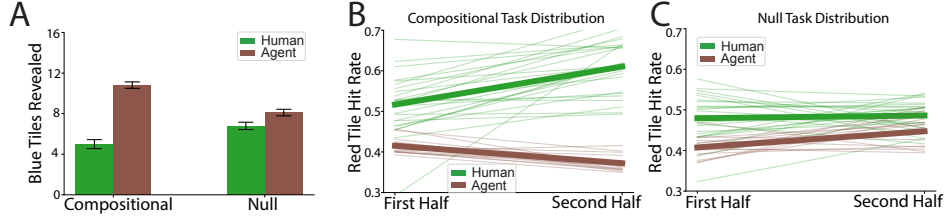


Figure 4: **Comparing human and agent performance** (A) Humans do better at the compositional task than the null ($p < 0.0001$), while agents do better at null ($p < 0.0001$). (B) Humans have a higher success rate revealing red tiles in the second half of a trial for the compositional task ($p < 0.0001$), agents do not. Transparent line represents individual human/agent average over trials, thick lines represent average over humans/agents. (C) Humans do not improve their success rates during a trial in the null task while agents do ($p = 0.0014$).

4 Discussion

Compositionality is a crucial inductive bias for human intelligence as it allows for the generation of an infinite number of concepts from a finite, relatively small, set of primitives (Fodor et al., 1988; Lake et al., 2017). We developed a compositionally structured task distribution for meta-learning using explicit generative grammars (Kemp & Tenenbaum, 2008). Previous work on meta-learning compositionality primarily utilizes performance on a task distribution that incorporates compositionality as a basis for deciding whether an architecture has correctly meta-learned a compositional inductive bias (Lake, 2019). However, it is possible for meta-learning systems to achieve performance on such tasks through statistical patterns instead of through learning compositional rules. To show this, we developed a way to create a task distribution with comparable statistical complexity to the compositional, rule-based distribution but is not explicitly compositional. This control distribution allows us to disentangle statistical pattern matching from rule-based compositionality. Our method uses a neural network to learn conditional distributions and generate Gibbs samples. This approach is similar to masked language modelling (Devlin et al., 2018), and our findings—that this procedure generates statistically similar but non-compositional distributions, that are in fact *easier* for downstream networks to learn than the true compositional distribution—are also relevant to understanding the representations learned by these systems more broadly (Rogers et al., 2020).

In our experiments, we show that humans have a clear bias toward compositionally structured domains, while directly controlling for statistical complexity. Further, we find that a common meta-learning agent, a recurrent network trained with model-free reinforcement learning (Wang et al., 2016; Duan et al., 2016), finds the non-compositional distribution easier to learn than the compositional one. This is in direct contrast with human behavior, indicating that meta-learning agents do not learn the human-like inductive bias towards compositionality.

While previous work has demonstrated that some aspects of compositionality can be meta-learned with training on extensive data (Lake, 2019), our focus here is to demonstrate that compositional structure remains *difficult* for these systems – and that they prefer other statistical features when possible. In other words, they do not have a meta-inductive bias toward learning compositionality as an inductive bias. This highlights the importance of endowing artificial systems with this bias. Meta-learning with graph neural networks (Battaglia et al., 2018) or neurosymbolic approaches (Ellis et al., 2020) is promising. An exciting direction for future work is to examine a range of approaches to learning compositional representations with the tools we set forth in this paper, and using the resulting insights to move toward closing the gap between human and machine intelligence.

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277 **A Appendix**

278 **A.1 Hyperparameter Details for Reinforcement Learning**

279 We did a hyperparameter search for the following: value function coefficient, entropy coefficient,
280 and learning rate. In particular, we evaluated each set of hyperparameters on a separate validation
281 set, selected the model with the highest performing set, and re-trained the model to be evaluated
282 on a previously-unseen test set of boards. Note that the final test set is not seen by the model un-
283 til the last, final evaluation step. The different learning rates evaluated were: Searches were ran
284 independently for both task distributions (compositional and null). The final selected hyperparam-
285 eters for both task distribution were: value function coefficient=0.0006747109316677081, entropy
286 coefficient=0.0006747109316677081, learning rate=0.0023483181861598565.