ΠΡΟΧΕΙΡΕΣ ΛΥΣΕΙΣ – Ψ.Ε.Σ.

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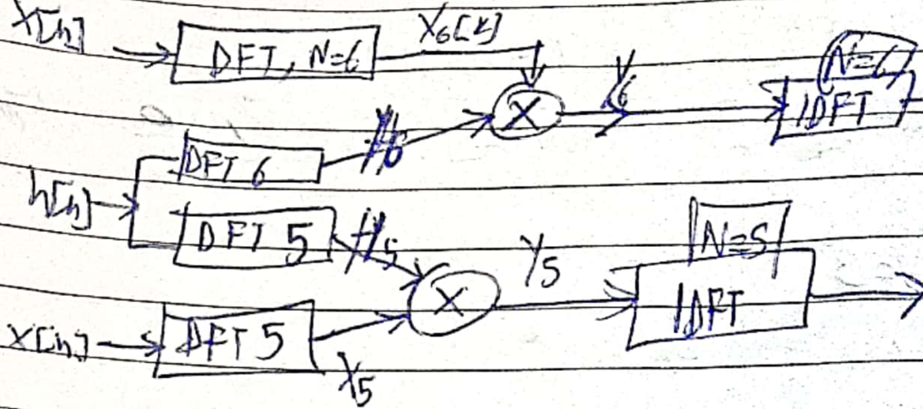
Χρονιές: 2022,2020,2019,2018,2017,2015,2009



$y_6[n] = \{3, 3, 3, 4, 4, 3\}$
$n = 0, \dots, 5$

$y_5[n] = \{4, 4, 4, 4, 4\}$
$n = 0, 1, \dots, 4$

$x[n] = 0 \ , \ n < 0, \ n \geq 4$

$h[n] = 0 \ , \ n < 0, \ n \geq 5$

$$y_N[n] = \sum_{r=-\infty}^{\infty} y[n - rN]$$

Εφόσον $x[n]$ & $h[n] = 0 \ \forall n < 0$
$\Rightarrow y[n] = 0 \ \forall n < 0$

Εφόσον $x[n] = 0$, $n \geq 4$ & $h[n] = 0$, $n \geq 5$ $\Rightarrow y[n] = 0 \ \forall n \geq 8$

$$y_6[n] = \sum_{r=-\infty}^{\infty} y[n - 6r]$$

$$y_5[n] = \sum_{r=-\infty}^{\infty} y[n - 5r]$$

$$y_6[0] = 3 = \sum_{r=-\infty}^{\infty} y[-6r] = \sum_{k=-\infty}^{\infty} y[6k] = y[0] + y[6]$$

$$y_5[0] = 4 = \sum_{r=-\infty}^{\infty} y[-5r] = \sum_{k=-\infty}^{\infty} y[5k] = y[0] + y[5]$$

$$y_6[1] = 3 = \sum_{r=-\infty}^{\infty} y[1 - 6r] = y[1] + y[7]$$

$$y_5[1] = 4 = \sum_{r=-\infty}^{\infty} y[1 - 5r] = y[1] + y[6]$$

$$y_6[2] = 3 = \sum_{r=-\infty}^{\infty} y[2 - 6r] = y[2] \Rightarrow \boxed{y[2] = 3}$$

$$y_5[2] = 4 = \sum_{r=-\infty}^{\infty} y[2 - 5r] = y[2] + y[7] \Rightarrow \boxed{y[7] = 1}$$

$$\Rightarrow y[1] = 2$$

$$y_6[3] = 4 = \sum_{r=-\infty}^{\infty} y[3 - 6r] = y[3] \Rightarrow \boxed{y[3] = 4} \Rightarrow y[6] = 2$$

$$y_5[3] = 4 = y[3]$$

$y_6[4] = 4 = y[4] \Rightarrow \boxed{y[4]=4}$

$y_6[5] = 3 = y[5] \Rightarrow \boxed{y[5]=3} \qquad \Rightarrow \boxed{y[0]=1}$

$y_5[5] = 4 = y[5] + y[0] \qquad \Rightarrow \boxed{y[6]=2}$

$\Rightarrow y[n] = \{1, 2, 3, 4, 4, 3, 2, 1\}$

$$h_a(t) = \mathcal{L}^{-1}\left\{ \frac{1/2}{s+a+jb} + \frac{1/2}{s+a-jb} \right\} = \left(\frac{1}{2}e^{-at}e^{-jbt} + \frac{1}{2}e^{-at}e^{jbt}\right)u(t)$$

$$h[n] = e^{-aTn}\cos(bTn)\,u[n] \qquad = \frac{1}{2}e^{-at}u(t)\left(2\cos bt\right)$$

$$= e^{-at}\cos bt\, u(t) \qquad \frac{1}{1+j\Omega}$$

$$H(z) = \left.\frac{1}{s^2+\sqrt{2}s+1}\right|_{s \leftarrow 2\frac{1-z^{-1}}{1+z^{-1}}}$$

$$= \frac{(1+z^{-1})^2}{4(1-z^{-1})^2 + \sqrt{2}\cdot 2\cdot(1-z^{-2}) + (1+z^{-1})^2} = \frac{\dots}{4(1-2z^{-1}+z^{-2})}$$

$$2\sqrt{2} - 2\sqrt{2}z^{-2}$$

$$1 + 2z^{-1} + z^{-2}$$

$$4+2\sqrt{2}+1+(2-8)z^{-1}+(4-2\sqrt{2}+1)z^{-2}$$

ΣΕΠΤ 22

Θ1]
- $\mathrm{Im}\{X(e^{j\omega})\} = \dfrac{X(e^{j\omega}) - X^*(e^{j\omega})}{2j}$
- $FT\{x^*[n]\} = \sum_{k=-\infty}^{\infty} x^*[n]e^{-j\omega n} = \left(\sum_{k=-\infty}^{\infty} x[n]e^{j\omega n}\right)^* = X^*(e^{-j\omega})$

$$FT\{x^*[-n]\} = X^*(e^{j\omega})$$

Άρα $\mathrm{Im}\{X(e^{j\omega})\} = \dfrac{X(e^{j\omega}) - X^*(e^{j\omega})}{2j} = 3\sin(2\omega) - 2\sin(3\omega)$

$$\frac{x[n]-x[-n]}{2j} = 3\cdot\frac{1}{2\pi}\int_{-\pi}^{\pi}\sin(2\omega)e^{j\omega n}d\omega$$
$$-2\cdot\frac{1}{2\pi}\int_{-\pi}^{\pi}\sin(3\omega)e^{j\omega n}d\omega$$

$$x[n]-x[-n] = \frac{3}{2\pi}\left\{\int_{-\pi}^{\pi}e^{j\omega(n+2)}d\omega - \int_{-\pi}^{\pi}e^{j\omega(n-2)}d\omega\right\}$$
$$-\frac{2}{2\pi}\left\{\int_{-\pi}^{\pi}e^{j\omega(n+3)}d\omega - \int_{-\pi}^{\pi}e^{j\omega(n-3)}d\omega\right\}$$

$$x[n]-x[-n] = \frac{3}{2\pi}\left\{2\pi\delta[n+2] - 2\pi\delta[n-2]\right\} - \frac{2}{2\pi}\left\{2\pi\delta[n+3] - 2\pi\delta[n-3]\right\}$$

$$= 3\delta[n+2] - 3\delta[n-2] - 2\delta[n+3] + 2\delta[n-3]$$

για $n=0$: $x[0]=1$

για $n>0$: $-n<0 \Rightarrow \begin{cases} x[-n]=0 \\ \delta[n+2],\delta[n+3]=0\end{cases} \Rightarrow x[n] = -3\delta[n-2] + 2\delta[n-3],\ n>0$

Άρα τελικά $\forall n\in\mathbb{Z}$ $x[n] = \delta[n] - 3\delta[n-2] + 2\delta[n-3]$

$$\int_{-\pi}^{\pi} e^{j\omega n}\, d\omega = \begin{cases} 2\pi & \text{αν } n=0 \\ \left[\dfrac{e^{j\omega n}}{jn}\right]_{-\pi}^{\pi} = \dfrac{e^{j\pi n} - e^{-j\pi n}}{jn} = \dfrac{2}{n}\sin(\pi n) = 0 & n \neq 0 \end{cases}$$

Επαλήθευση: $X(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n] e^{-jwn} = 1 - 3e^{-jw2} + 2e^{-jw3}$

$$X^*(e^{jw}) = 1 - 3e^{j2w} + 2e^{+j3w}$$

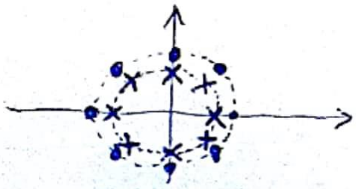
$$X(e^{jw}) - X^*(e^{jw}) = 3(e^{j2w} - e^{-j2w}) - 2(e^{j3w} - e^{-jw3})$$

$$\frac{X(e^{jw}) - X^*(e^{jw})}{2j} = 3\sin 2w - 2\sin(3w) \quad \square$$

Θ2) A) $H(z) = \frac{1-z^{-8}}{1-R^8 z^{-8}}, \ R<1 \Leftrightarrow H(z) = \frac{z^8-1}{z^8-R^8}, \ R<1$

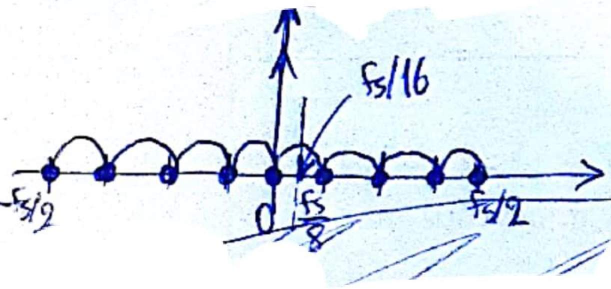
Μηδενικά: $z^8 - 1 = 0 \Leftrightarrow z_k = e^{j\frac{2\pi}{8}\cdot k}$ $\quad k=0,1,\dots,7$

Πόλοι: $z^8 - R^8 = 0 \Leftrightarrow z_m = |R| e^{j\frac{2\pi}{8}\cdot m}$ $\quad m=0,1,\dots,7$



B) $|H(e^{j2\pi f/f_s})|$ f_s-περιοδική → Μηδενίζει στα μηδενικά δηλ. όταν $\frac{f}{f_s} = \frac{k}{8}$, $k \in \mathbb{Z}$

$\Leftrightarrow f = k\frac{f_s}{8}$ άρα και για $k = \pm 4$ $f = \pm\frac{f_s}{2}$



$$\overline{H}(f) = H(e^{j2\pi f/f_s}) = \frac{e^{j2\pi\cdot\frac{8f}{f_s}} - 1}{e^{j2\pi\frac{8f}{f_s}} - R^8}$$

$\boxed{\forall n \in \mathbb{Z} \quad \overline{H}(n f_0) = 0}$

Ναι μπορεί να χρησιμοπ. εφόσον υπάρχει δυνατότητα ρύθμισης της f_s

$n\cdot\frac{f_s}{8} = n\cdot f_0$

$$H(e^{j2\pi\frac{f+f_s/8}{f_s}}) = H(e^{j2\pi f/f_s}) \quad \frac{f_s}{8}\text{-περιοδική}$$

γ) $H(e^{j2\pi/16}) = \frac{e^{j2\pi\cdot 8/16}-1}{e^{j2\pi 8/16}-R^8} = \frac{-2}{-1-R^8} = \frac{2}{1+R^8} \simeq 1,02$

δ) Έστω f_0 η συχνότητα του δικτύου. Αν επιλέξουμε $\boxed{f_s = 8f_0} \Leftrightarrow \frac{f_s}{8} = f_0$

$n\cdot f_0$ $\ n=1,2,\dots$ οι αρμονικές

Θ3) • $P(z) = X(z) - Q(z)$ (1)

• $(z^{-1}P(z)) \cdot z^{-1} + Q(z) = Y(z) \Leftrightarrow z^{-2}P(z) + Q(z) = Y(z)$ (2)

• $z^{-2}P(z) + X(z) = R(z)$

• $d_2 R(z) + d_1 \cdot z^{-1}P(z) = Q(z)$

$$\left.\begin{array}{l} z^{-2}P(z) + X(z) = R(z) \\ d_2 R(z) + d_1 \cdot z^{-1}P(z) = Q(z) \end{array}\right\} \Rightarrow Q(z) = d_2 X(z) + P(z)\{d_2 z^{-2} + d_1 z^{-1}\}$$

$$\overset{Q(z)=X(z)-P(z)}{\Longleftrightarrow} X(z)\{1 - d_2\} = P(z)\{1 + d_1 z^{-1} + d_2 z^{-2}\}$$

(1) + (2) $Y(z) + P(z) = X(z) + z^{-2}P(z)$

$$\Leftrightarrow Y(z) = X(z)\left\{\frac{(z^{-2}-1)\cdot(1-d_2)}{1 + d_1 z^{-1} + d_2 z^{-2}} + 1\right\}$$

$$\Leftrightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{(1-\cancel{d_2})z^{-2} + d_2 - \cancel{1} + \cancel{1} + d_1 z^{-1} + \cancel{d_2 z^{-2}}}{\dots}$$

$$= \frac{z^{-2} + d_1 z^{-1} + d_2}{d_2 z^{-2} + d_1 z^{-1} + 1} = \frac{(1-d_2)z^{-2} + d_2 - 1}{d_2 z^{-2} + d_1 z^{-1} + 1} + 1$$

$$= \frac{(d_2-1)z^2 - (d_2-1)}{z^2 + d_1 z + d_2} + 1 = \frac{d_2 z^2 + d_1 z + 1}{z^2 + d_1 z + d_2}$$

▸ Αν $d_2 = 1 \Rightarrow H(z) = 1 \Rightarrow h[n] = \delta[n]$ All-pass FIR φίλτρο τάξης 0

▸ Αν $d_2 \neq 1$

▸ Αν $d_2 = 0 \Rightarrow H(z) = \dfrac{d_1 z + 1}{z(z + d_1)}$

→ $d_1 = 0 \Rightarrow H(z) = \dfrac{1}{z^2}$ $2^{ης}$ τάξης FIR All pass

→ $d_1 \neq 0 \Rightarrow H(z) = d_1 \cdot \dfrac{z + 1/d_1}{z(z + d_1)}$

→ $d_1 = 1 \Rightarrow H(z) = \dfrac{1}{z}$ $1^{ης}$ τάξης FIR All pass

→ $d_1 \neq 1 \Rightarrow H(z) = \dfrac{z + 1/d_1}{z(z + d_1)}$ $1^{ης}$ τάξης IIR

▸ Αν $d_2 \neq 0$

$\Delta_{\mu} = d_1^2 - 4d_2$ Μηδενικά $z_m = \dfrac{-d_1 \pm \sqrt{\Delta}}{2d_2}$

$\Delta_{πόλων} = d_1^2 - 4d_2$ Πόλοι $z_p = \dfrac{-d_1 \pm \sqrt{\Delta}}{2} \neq z_m$ αφού $d_2 \neq 0$

Δ

⇒ μηδενικής τάξης φίλτρο (Αν $|z_p| > 1 \to$ μη πραγματοποιήσιμο = αιτιατό & ευσταθές)

ΦΕΒ 22

Θ2 • $x(t) = \frac{T_s}{2W} \cdot \frac{\sin 2\pi W t}{\pi t} \cdot \frac{\sin(2\pi 3Wt)}{\pi t}$

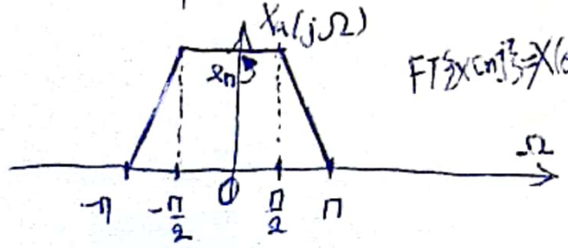
$$\Rightarrow x(nT_s) = \frac{\cancel{T_s}}{2W} \frac{\sin(2\pi n W/8W)}{\pi \cdot \frac{1}{\cancel{8W}} \cdot n} \cdot \frac{\sin(2\pi n\, 3W/8W)}{\pi \cdot \frac{1}{8W} \cdot n}$$

$$\Rightarrow x[n] = \frac{4}{\pi^2 n^2} \sin(\pi n/4) \cdot \sin(3\pi n/4)$$

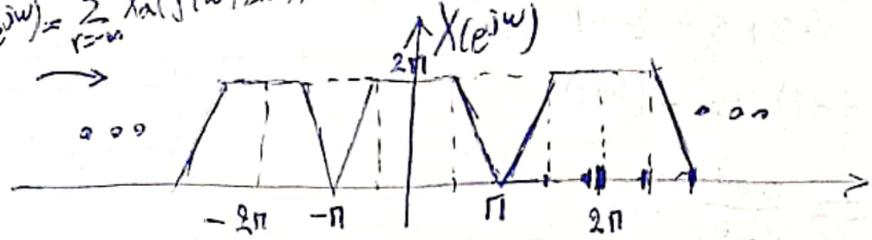
$$= \frac{3}{4} \cdot \frac{\sin(\pi n/4)}{\pi n/4} \cdot \frac{\sin(3\pi n/4)}{3\pi n/4} = \frac{3}{4} \cdot \left[\mathrm{sinc}(t/4) \cdot \mathrm{sinc}(3t/4)\right]\Big|_{t=n} \leftarrow (T=1)$$

• $x_a(t) = \frac{3}{4} \sin(t/4) \cdot \mathrm{sinc}(3t/4)$

• $X_a(j\Omega) = \frac{3}{4} \cdot \left(4 \cdot \Pi(\frac{\Omega}{\pi/2})\right) * \left(\frac{4}{3}\, \Pi(\frac{\Omega}{3\pi/2})\right) = 4\, \Pi(\frac{\Omega}{\pi/2}) * \Pi(\frac{\Omega}{3\pi/2}) = 4 \begin{cases} \pi/2 & , |\Omega| \le \frac{\pi}{2} \\ \pi - |\Omega| & , \frac{\pi}{2} < |\Omega| \le \pi \\ 0 & , \sim \end{cases}$



$FT\{x[n]\} = X(e^{jw}) = \sum_{r=-\infty}^{\infty} X_a(j(w+2\pi r))$



Θ3 Εφόσον $x[n] \in \mathbb{R}$ ο $2^{ος}$ πόλος της $X(z)$ θα είναι συζυγής του $1^{ου}$ δηλ.

$$X(z) = A \frac{z^2}{(z-z_1)(z-z_1^*)} = A \frac{z^2}{z^2 - 2 \cdot \mathrm{Re}\{z_1\} z + |z_1|^2}$$

$$\Leftrightarrow X(z) = A \frac{z^2}{z^2 - 2 \cdot \frac{1}{2} \cos\frac{\pi}{3} z + \frac{1}{4}} = \frac{Az^2}{z^2 - \frac{1}{2} z + \frac{1}{4}}$$

$$X(1) = \frac{8}{3} \Leftrightarrow A = \frac{8}{3} \cdot \left(1 - \frac{1}{2} + \frac{1}{4}\right) = \frac{8}{3} \cdot \frac{3}{4} = 2$$

$$\Rightarrow X(z) = \frac{2z^2}{(z - \frac{1}{2}e^{j\frac{\pi}{3}})(z - \frac{1}{2}e^{-j\frac{\pi}{3}})}$$

$|z_1| = |z_1^*| = \frac{1}{2}$

Πιθανές ROC
$|z| < |z_1| = \frac{1}{2}$
$|z| > |z_1| = \frac{1}{2}$

Άρα $ROC_{X(z)} = \{z \in \mathbb{C} : |z| > \frac{1}{2}\}$ ← αιτιατό σήμα (δεξιάς πλευράς), αλλιώς για $|z| < \frac{1}{2}$ θα 'ταν αντιαιτιατό

Θ4

a) $H(s) = A\dfrac{s^2+1}{(s+1)^2+4} = A\dfrac{s^2+1}{s^2+2s+5} \xrightarrow[s=\frac{z-1}{z+1}]{} H(z) = A\dfrac{(z-1)^2+(z+1)^2}{(z-1)^2+2(z-1)(z+1)+5(z+1)^2}$

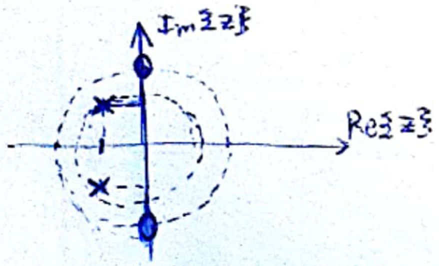
$\Leftrightarrow H(z) = A\dfrac{2(z^2+1)}{z^2-2z+1+2z^2-2+5z^2+10z+5}$

$\Leftrightarrow H(z) = 2A\dfrac{z^2+1}{8z^2+8z+4} = \dfrac{A}{2}\dfrac{z^2+1}{2z^2+2z+1}$

Μηδενικά: $z^2+1=0 \Leftrightarrow z=\pm j$

Πόλοι: $2z^2+2z+1=0 \rightsquigarrow \Delta = 4-8=-4 \Rightarrow z_{1,2} = \dfrac{-2\pm 2j}{4}$

$= -\dfrac{1}{2}\pm\dfrac{1}{2}j$



b) Το αναλογικό ήταν ευσταθές και αιτιατό (ώστε να είναι υλοποιήσιμο)
Άρα και το ψηφιακό θα είναι αιτιατό με Π.Σ. $|z| > |z_1| = |z_2| = \frac{\sqrt{2}}{2}$
$e^{j\omega} \in ROC \Rightarrow$ ευσταθές

c) $H(1) = 0{,}8 \Leftrightarrow \dfrac{A}{2}\cdot\dfrac{2}{5} = \dfrac{4}{5} \Leftrightarrow A=4$

$\Rightarrow H(z) = 2\dfrac{z^2+1}{2z^2+2z+1} \Leftrightarrow \dfrac{Y(z)}{X(z)} = 2\dfrac{1+z^{-2}}{2+2z^{-1}+z^{-2}}$

$\Leftrightarrow 2Y(z)+2z^{-1}Y(z)+z^{-2}Y(z) = 2X(z)+2z^{-2}X(z)$

$\Rightarrow 2y[n]+2y[n-1]+y[n-2] = 2x[n]+2x[n-2]$

$\Leftrightarrow y[n]+y[n-1]+\frac{1}{2}y[n-2] = x[n]+x[n-2]\ ,\ n\geq 0$

ΣΕΠΤ 19 | Θ1 | $h[n] = \left(\frac{1}{2}\right)^n u[n] + (-1/3)^n u[n-1]$

α) $h[n]$ αιτιατή $\Rightarrow$ αιτιατό σύστ.

$$\mathcal{E}_h = \sum_{n=-\infty}^{\infty} |h[n]|^2 = \left|\left(\frac{1}{2}\right)^0\right|^2 + \sum_{n=1}^{\infty} \left|\left(\frac{1}{2}\right)^n + (-1)^n\left(\frac{1}{3}\right)^n\right|^2$$

$$= 1 + \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{2n} + 2\left(-\frac{1}{2}\cdot\frac{1}{3}\right)^n + \left(\frac{1}{3}\right)^{2n}$$

$$= \sum_{n=0}^{\infty}\left(\frac{1}{4}\right)^n + 2\sum_{n=1}^{\infty}\left(-\frac{1}{6}\right)^n + \sum_{n=1}^{\infty}\left(\frac{1}{9}\right)^n$$

$$= \frac{1}{1-\frac{1}{4}} + 2\sum_{n=0}^{\infty}\left(-\frac{1}{6}\right)^n + \sum_{n=0}^{\infty}\left(\frac{1}{9}\right)^n - 1 - 2$$

$$= \frac{4}{3} + 2\frac{1}{1+\frac{1}{6}} + \frac{1}{1-\frac{1}{9}} - 3$$

$$= -\frac{5}{3} + \frac{12}{7} + \frac{9}{8} = \frac{197}{168} \simeq 1,1726$$

β) $H(z) = \frac{z}{z-\frac{1}{2}} - \frac{1}{3}\cdot\frac{z}{z+\frac{1}{3}}\cdot z^{-1}$, $|z| > \max\{|\frac{1}{2}|, |-\frac{1}{3}|\} = \frac{1}{2}$

$$= \frac{z}{z-\frac{1}{2}} - \frac{1}{3z+1} = \frac{z(3z+1) - (z-\frac{1}{2})}{(z-\frac{1}{2})(3z+1)} = \frac{3z^2 + \cancel{z} - \cancel{z} + \frac{1}{2}}{3z^2 + z - \frac{3}{2}z - \frac{1}{2}}$$

$$\Rightarrow H(z) = \frac{3z^2 + 1/2}{3z^2 - \frac{1}{2}z - \frac{1}{2}} = \frac{3 + \frac{1}{2}z^{-2}}{3 - \frac{1}{2}z^{-1} - \frac{1}{2}z^{-2}} = \frac{Y(z)}{X(z)}$$

$$\Rightarrow 3X(z) + \frac{1}{2}z^{-2}X(z) = 3Y(z) - \frac{1}{2}z^{-1}Y(z) - \frac{1}{2}z^{-2}Y(z)$$

$$\Rightarrow 3x[n] + \frac{1}{2}x[n-2] = 3y[n] - \frac{1}{2}y[n-1] - \frac{1}{2}y[n-2] \quad , n \geq 0$$

γ) $H(z) = \frac{3z^2 + \frac{1}{2}}{(z-\frac{1}{2})(3z+1)}$, $|z| > \frac{1}{2}$ $\quad e^{jw} \in ROC_H \Rightarrow H(e^{jw}) = \frac{3e^{2jw} + \frac{1}{2}}{(e^{jw}-\frac{1}{2})(3e^{jw}+1)}$

Θ2 | $H(z) = \frac{z+0,5}{z^2-2,5z+1} = \frac{z+0,5}{(z-2)(z-0,5)}$

πιθανές ROC: $|z|>2$, $0,5<|z|<2$, $|z|<0,5$

θέλω $e^{jw} \in ROC$ άρα ROC: $|z| \in (0,5, 2)$

$$= \frac{A}{z-2} + \frac{B}{z-0,5} \quad , A = \left.\frac{z+0,5}{z-0,5}\right|_{z=2} = \frac{2,5}{1,5} = \frac{5}{3} \quad \text{και } B = \left.\frac{z+0,5}{z-2}\right|_{z=0,5} = \frac{1}{-1,5} = -\frac{2}{3}$$

Άρα $H(z) = \frac{5}{3} z^{-1} \frac{z}{z-2} - \frac{2}{3} z^{-1} \frac{z}{z-\frac{1}{2}}$, $\frac{1}{2} < |z| < 2$

$$\Rightarrow h[n] = -\frac{5}{3} \cdot 2^{n-1} u[-n] - \frac{2}{3} \left(\frac{1}{2}\right)^{n-1} u[n-1]$$

B) $x[n] = 2\cos(\pi n/6)$ ~~$\to y[n] =$~~

$\omega_0 = \pi/6$

$$H(e^{j\omega_0}) = \frac{e^{j\frac{\pi}{6}} + \frac{1}{2}}{e^{j\frac{\pi}{3}} - 2{,}5 e^{j\frac{\pi}{6}} + 1} = 1{,}89421 \angle 2{,}96887 \text{ rad}$$

$$\Rightarrow y[n] = 2 \cdot |H(e^{j\frac{\pi}{6}})| \cdot \cos(\pi n/6 + \angle H(e^{j\frac{\pi}{6}}))$$

$$\simeq 3{,}7884 \cos(\pi n/6 + 2{,}96887)$$

Θέμα 3

$$\left(X(z) + \frac{1}{2} Q(z)\right) \cdot \left(z^{-1} + \frac{1}{2}\right) = Y(z)$$

$$\left(X(z) + \frac{1}{2} Q(z)\right) \cdot z^{-1} = Q(z)$$

$$\Leftrightarrow z^{-1} X(z) = Q(z)\left[1 - \frac{1}{2} z^{-1}\right]$$

$$\Leftrightarrow Q(z) = \frac{z^{-1} X(z)}{1 - \frac{1}{2} z^{-1}}$$

Άρα $\left(X(z) + \frac{1}{2} \frac{z^{-1} X(z)}{1 - \frac{1}{2} z^{-1}}\right)\left(z^{-1} + \frac{1}{2}\right) = Y(z)$

$$\Leftrightarrow X(z)\left[\frac{1 - \cancel{\frac{1}{2} z^{-1}} + \cancel{\frac{1}{2} z^{-1}}}{1 - \frac{1}{2} z^{-1}}\right]\left[z^{-1} + \frac{1}{2}\right] = Y(z)$$

$$\Leftrightarrow X(z) z^{-1} + \frac{1}{2} X(z) = Y(z) - \frac{1}{2} z^{-1} Y(z)$$

$$\Leftrightarrow x[n-1] + \frac{1}{2} x[n] = y[n] - \frac{1}{2} y[n-1]$$

Π.Σ. $|z| > \frac{1}{2}$

Υποθέτοντας αιτιατό σύστ.

$$H(z) = \frac{z^{-1} + \frac{1}{2}}{1 - \frac{1}{2} z^{-1}} = \frac{1 + z/2}{z - \frac{1}{2}} = \frac{1}{2} \frac{z+2}{z - \frac{1}{2}}$$

$$h[n] = \frac{1}{2} \cdot \left(\frac{1}{2}\right)^n u[n] + \left(\frac{1}{2}\right)^{n-1} u[n-1]$$

ΦΕΒ 20 | Θ2 | $H(z)=\frac{1}{1-\frac{1}{3}z^{-1}} = H_0(z^2)+z^{-1}H_1(z^2)$

$$= \frac{z}{z-\frac{1}{3}} = \frac{z(z+\frac{1}{3})}{z^2-\frac{1}{9}} = \frac{z^2}{z^2-\frac{1}{9}} + \frac{1}{3}z^{-1}\frac{z^2}{z^2-\frac{1}{9}}$$

$\Rightarrow H_0(z)=\frac{z}{z-\frac{1}{9}}$, $H_1(z)=\frac{1}{3}\frac{z}{z-\frac{1}{9}}$

Θ4 | $w[n+1]=aw[n]+x[n] \Rightarrow zW(z)=aW(z)+X(z)$

$\Rightarrow (z-a)W(z)=X(z)$

$\Rightarrow W(z)=\frac{X(z)}{z-a}$

$y[n]=w[n]+x[n] \Rightarrow Y(z)=W(z)+X(z)=X(z)\left[1+\frac{1}{z-a}\right]$

$\Rightarrow \frac{Y(z)}{X(z)} = \frac{z-a+1}{z-a}$ Π.Σ. αν $a\neq 0$ $|z|>|a|$ ή $|z|<|a|$; αν $a=0$ $\Rightarrow z\neq 0 \Rightarrow e^{j\omega}\in ROC \Rightarrow$ ευσταθές

Αν το σύστημα είναι αιτιατό πρέπει $|a|<1$ ώστε ROC : $|z|>|a|$

Αν είναι αντιαιτιατό $\Rightarrow$ ΠΣ $|z|<|a|$ — οπότε $|e^{j\omega}|=1>|a| \Rightarrow e^{j\omega}\in ROC \Rightarrow$ ευσταθές

πρέπει $|a|>1$ ώστε

$|e^{j\omega}|=1<|a| \Rightarrow e^{j\omega}\in ROC \Rightarrow$ ευσταθές

Θ3 |

a) Για $x[n]=\delta[n] \Rightarrow y[n]=h[n]$

$h[n]=\delta[n]+2\delta[n-1]+2\delta[n-2]+\delta[n-3] \Rightarrow$ FIR αιτιατό

$H(z)=1+2z^{-1}+2z^{-2}+z^{-3} = 1+\frac{2}{z}+\frac{2}{z^2}+\frac{1}{z^3}$

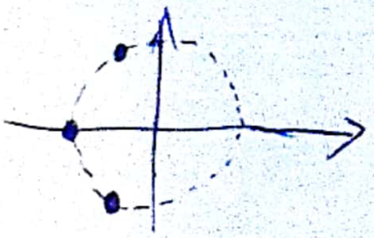
$= \frac{z^3+2z^2+2z+1}{z^3}$ Π.Σ. $|z|>0$

Μηδενικά : $z^3+2z^2+2z+1=0 \Leftrightarrow z(z^2+2z+1)+z+1=0$

$\Leftrightarrow (z+1)[z(z+1)+1]=0$

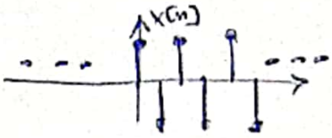
$\Leftrightarrow z=-1$ ή $z^2+z+1=0$ $\Delta=-3$

$\Leftrightarrow z=-1$ ή $z_{1,2}=\frac{-1\pm j\sqrt{3}}{2}$



B) $y[n]=h[n]*x[n]=(-1)^n+2(-1)^{n-1}+2(-1)^{n-2}+(-1)^{n-3} = 3[(-1)^n-(-1)^n]=0$

Θ3 γ)



Υψίσυχνη είσοδος (εναλλασσόμενη ακολουθία) $\Rightarrow y[n]=0 \Rightarrow$ αποκόπτει τελείως την έξοδο

$\Rightarrow$ σίγουρα όχι υψιπερατό

Θ5

α) $h_a(t) = ILT\left\{\frac{1}{(s+1)(s+2)}\right\}$

$\frac{1}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$ $\quad A = \left.\frac{1}{s+2}\right|_{s=-1} = 1$ $\quad B = \left.\frac{1}{s+1}\right|_{s=-2} = -1$

$= ILT\left\{\frac{1}{s+1} - \frac{1}{s+2}\right\}$

$= \left(e^{-t} - e^{-2t}\right) \cdot u(t)$

$$T h_a(nT) = T\left(e^{-nT} - e^{-2nT}\right) u(nT)$$

$\Rightarrow h[n] = T\, e^{-nT}\left(1 - e^{-nT}\right) u[n]$, αιτιατή $\Rightarrow$ αιτιατό σύστ.

$$H(z) = T\left[\mathcal{Z}\{e^{-nT} u[n]\} - \mathcal{Z}\{e^{-2nT} u[n]\}\right]$$

$$= T \cdot \frac{z}{z-e^{-T}} - T\frac{z}{z-e^{-2T}}$$

$T>0 \Rightarrow -T<0 \Rightarrow e^{-T}<e^{0} \Rightarrow \boxed{0<e^{-T}<1}$

όμοιως $e^{-2T}<e^{0} \Rightarrow \boxed{0<e^{-2T}<1}$

$$= T\left[\frac{z(z-e^{-2T}) - z(z-e^{-T})}{(z-e^{-T})(z-e^{-2T})}\right]$$

$$= T\,\frac{\cancel{z^2} - ze^{-2T} - \cancel{z^2} + ze^{-T}}{(z-e^{-T})(z-e^{-2T})}$$

$e^{-T} = e^{-2T} \Leftrightarrow T = 2T \Leftrightarrow \boxed{T=0}$ ΑΤΟΠΟ

$$= T\,\frac{z(e^{-T}-e^{-2T})}{(z-e^{-T})(z-e^{-2T})}$$

Επειδή $h[n]$ αιτιατή $\Rightarrow$ αιτιατό σύστ.

β) Πόλοι e^{-T}, e^{-2T}

Μηδενικά $z=0$

Επιπλέον $0<T<2T$

$\Rightarrow -2T<-T<0$

$\Rightarrow 0<e^{-2T}<e^{-T}<1$

Άρα Π.Σ. $|z|>e^{-T}$ $\quad |e^{j\omega}|=1>e^{-T} \Rightarrow e^{j\omega}\in ROC \Rightarrow$ ευσταθές γ) $|H(e^{j\pi})| \leq 0{,}1 \Rightarrow 2\left|H_a\left(\frac{j\pi}{T}\right)\right| \leq 0{,}1$

$$\Rightarrow \frac{20}{\left|\frac{j\pi}{T}+1\right|\left|\frac{j\pi}{T}+2\right|} \leq 1$$

$$\Leftrightarrow \left|1-\left(\frac{\pi}{T}\right)^2+2+j\frac{3\pi}{T}\right| \geq 20$$

$$\Leftrightarrow \left(2-\left(\frac{\pi}{T}\right)^2\right)^2+\left(\frac{3\pi}{T}\right)^2 \geq 400$$

$$\Leftrightarrow 4-4\left(\frac{\pi}{T}\right)^2+\left(\frac{\pi}{T}\right)^4+9\left(\frac{\pi}{T}\right)^2 \geq 400$$

$$\Leftrightarrow \left(\frac{\pi}{T}\right)^4+5\left(\frac{\pi}{T}\right)^2-396 \geq 0$$

$$\Leftrightarrow \left[\left(\frac{\pi}{T}\right)^2-17{,}556\right]\left[\left(\frac{\pi}{T}\right)^2+22{,}556\right] \geq 0$$

$\left(\frac{\pi}{T}\right)^2+22{,}556>0$

$$\Leftrightarrow \left(\frac{\pi}{T}\right)^2 \geq 17{,}556$$

$$\Leftrightarrow \frac{\pi^2}{17{,}556} \geq T^2$$ Άρα πρέπει $T \in \left(0, \frac{3}{4}\right]$

$$\Leftrightarrow T \leq \frac{\pi}{\sqrt{17{,}556}} \simeq 0{,}75$$

$\boxed{\text{ΙΟΥΝ 19}}$ $1/T = 80$ Μονοτονη αποκρ. πλατους $\Rightarrow$ Butterworth

$\Omega_c = 20\pi$ rad/s $\rightarrow -3dB$ $\longrightarrow$ $\omega_c = \Omega_c \cdot T = 20\pi \cdot \frac{1}{80} = \frac{\pi}{4}$ rad

$\Omega_r = 30\pi$ rad/s $\rightarrow 10dB$ $\quad \omega_r = \Omega_r \cdot T = 30\pi \cdot \frac{1}{80} = \frac{3\pi}{8}$ rad

Prewarping $\Omega_c' = \frac{2}{T}\tan\left(\frac{\omega_c}{2}\right) = 66{,}27$ rad/s

$\Omega_r' = \frac{2}{T}\tan\left(\frac{\omega_r}{2}\right) = 106{,}91$ rad/s

Θέλουμε $-10\log\left(1+\left(\frac{\Omega_r'}{\Omega_c'}\right)^{2n}\right) \leq -10$ // γραμμικά

$$\Leftrightarrow \left(\frac{\Omega_r'}{\Omega_c'}\right)^{2n} \geq 10-1 \Leftrightarrow \cancel{\left(\frac{\Omega_c'}{\Omega_r'}\right)^{2n} \leq \frac{1}{9}}$$

$$\cancel{\Leftrightarrow 2n \log\frac{\Omega_r'}{\Omega_c'} \geq \log 9}$$

$$\Leftrightarrow n \geq \frac{\log 9}{2\log \Omega_r'/\Omega_c'} = 2{,}297 \Rightarrow \boxed{n_{min}=3}$$

$$H_3(s) = \frac{1}{(s+1)(s^2+s+1)} \Rightarrow H(s) = H_3(s)\Big|_{s \leftarrow \frac{s}{\Omega_c'}} = \frac{s}{66{,}27}$$

$$H(z) = H(s)\Big|_{s \leftarrow 160\frac{1-z^{-1}}{1+z^{-1}}}$$

$$= H_3(s)\Big|_{s\leftarrow \frac{160}{66{,}27}\frac{1-z^{-1}}{1+z^{-1}}} = 2{,}414\,\frac{1-z^{-1}}{1+z^{-1}}$$

$$= \frac{1}{\left[2{,}414\frac{(1-z^{-1})}{1+z^{-1}}+1\right]\left[2{,}414^2\frac{(1-z^{-1})^2}{(1+z^{-1})^2}+\frac{1-z^{-1}}{1+z^{-1}}+1\right]}$$

$$= \frac{(1+z^{-1})^3}{\left[3{,}414+z^{-1}(1-2{,}414)\right]\left[2{,}414^2(1-z^{-1})^2+(1-z^{-1})(1+z^{-1})+(1+z^{-1})^2\right]}$$

Θ3

$$\frac{x[n]+x[-n]}{2} = \frac{1}{2\pi}\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} e^{j\omega n}\,d\omega = \begin{cases} \frac{1}{2\pi}\cdot\frac{2\pi}{3} & n=0 \\ \frac{1}{2\pi}\cdot\frac{e^{j\frac{\pi}{3}n}-e^{-j\frac{\pi}{3}n}}{jn} & n\neq 0\end{cases}$$

$$= \begin{cases}\frac{1}{3}, & n=0\\ \frac{1}{\pi n}\cdot \sin\left(\frac{\pi}{3}n\right), & n\neq 0\end{cases}$$

για $n=0$: $\frac{2x[0]}{2} = \frac{1}{3} \Rightarrow x[0] = \frac{1}{3}$

για $n>0 \Rightarrow -n<0 \Rightarrow x[-n]=0$ (ως αιτιατό)

$$\Rightarrow x[n] = \frac{2}{\pi n}\sin\left(\frac{\pi}{3}n\right),\ n>0$$

Άρα $\boxed{x[n] = \frac{1}{3}\delta[n] + \frac{2}{\pi n}\sin\left(\frac{\pi}{3}n\right)u[n-1]}$

$\underline{04}$ $y[n] - 2,5\,y[n-1] + y[n-2] = x[n-1]\ ,\ n \geq 0$

$$\Rightarrow Y(z) - 2,5\left(z^{-1}Y(z) + \cancel{y(-1)}^{\,0} + z\,y(-2)\right) + z^{-2}Y(z) + z^{-1}\cancel{y(-1)}^{\,0} + y(-2) = z^{-1}X(z)$$

$$\Rightarrow Y(z)\left[1 - 2,5z^{-1} + z^{-2}\right] = z^{-1}X(z) + 2,5z - 1$$

$$X(z) = 1 - 2z^{-1}$$

$$\Rightarrow Y(z) = \frac{z^{-1}(1-2z^{-1})}{1-2,5z^{-1}+z^{-2}} + \frac{2,5z-1}{1-2,5z^{-1}+z^{-2}}\ ,\ |z|>2 \quad \text{(αιτιατό συστ.)}$$

$$= \frac{z(z-2)}{z^2-2,5z+1} + \frac{z^2(2,5z-1)}{z^2-2,5z+1}$$

$$\begin{array}{r|l} 5z^3 - 2z^2 & 2z^2-5z+2 \\ \hline -5z^3 + \frac{25}{2}z^2 - 5z & \frac{5}{2}z + \frac{21}{4} \\ \frac{21}{2}z^2 - 5z & \\ -\frac{21}{2}z^2 - \frac{105}{4}z - \frac{21}{2} & \\ \hline -\frac{125}{4}z - \frac{21}{2} & \end{array}$$

$$= \frac{z\cancel{(z-2)}}{\cancel{(z-2)}(z-\frac{1}{2})} + \frac{z^2(2,5z-1)}{(z-2)(z-\frac{1}{2})}$$

$$= \frac{z}{z-\frac{1}{2}} + \frac{5z^3-2z^2}{2z^2-5z+2}$$

$$= \frac{z}{z-\frac{1}{2}} + \frac{5}{2}z + \frac{21}{4} - \frac{\frac{125}{4}z + \frac{21}{2}}{2(z-2)(z-\frac{1}{2})} \rightarrow \frac{A}{z-2} + \frac{B}{z-\frac{1}{2}}$$

όπου $A = \left.\frac{\frac{125}{4}z + \frac{1}{2}}{2(z-\frac{1}{2})}\right|_{z=2} = \frac{63}{2\cdot\frac{3}{2}} = \frac{63}{3}$

και $B = \left.\frac{\frac{125}{4}z + \frac{1}{2}}{2(z-2)}\right|_{z=\frac{1}{2}} = -\frac{43}{8}$

$$= \frac{z}{z-\frac{1}{2}} + \frac{5}{2}z + \frac{21}{4} - \frac{63}{3}z^{-1}\frac{z}{z-2} + \frac{43}{8}z^{-1}\frac{z}{z-\frac{1}{2}}\ ,\ |z|>2$$

$$\Rightarrow y[n] = \left(\frac{1}{2}\right)^n u[n] + \frac{5}{2}\delta[n+1] + \frac{21}{4}\delta[n] - \frac{63}{3}\cdot 2^{n-1}u[n-1] + \frac{43}{8}\left(\frac{1}{2}\right)^{n-1}u[n-1]$$

[P19]

Θ1 ΓΑΧΜ, ευσταθές $\quad z^{-1}Y(z) - \frac{10}{3}Y(z) + zY(z) = X(z)$

$$\Leftrightarrow \left(z^{-1} - \frac{10}{3} + z\right)Y(z) = X(z)$$

$$\Leftrightarrow \frac{Y(z)}{X(z)} = \frac{z}{z^2 - \frac{10}{3}z + 1} = \frac{z}{(z-3)\left(z-\frac{1}{3}\right)}, \quad \text{Π.Σ.} \ |z| \in \left(\tfrac{1}{3}, 3\right)$$

$|e^{j\omega}| = |1| \in$ Π.Σ. λόγω ευστάθειας

$$\Rightarrow H(z) = \frac{A}{z-3} + \frac{B}{z-\frac{1}{3}} \qquad A = \left.\frac{z}{z-\frac{1}{3}}\right|_{z=3} = \frac{3}{8/3} = \frac{9}{8}$$

$$B = \left.\frac{z}{z-3}\right|_{z=\frac{1}{3}} = \frac{1/3}{-8/3} = -\frac{1}{8}$$

$$\Rightarrow H(z) = \frac{9}{8}z^{-1}\frac{z}{z-3} - \frac{1}{8}z^{-1}\frac{z}{z-\frac{1}{3}}, \quad |z| \in \left(\tfrac{1}{3}, 3\right)$$

$$\cancel{= \frac{9}{8}3^{n-1}u[n-1] - \frac{1}{8}\left(\frac{1}{3}\right)^{n-1}u[n-1] = \frac{3}{8}\left(3^n u[n-1] - \left(\frac{1}{3}\right)^n u[n-1]\right)}$$

$$= \frac{9}{8}\cdot(-1)\cdot 3^{n-1}\cdot u[-(n-1)-1] - \frac{1}{8}\left(\frac{1}{3}\right)^{n-1}u[n-1]$$

$$= -\frac{3}{8}\,3^n u[-n] - \frac{3}{8}\left(\frac{1}{3}\right)^n u[n-1]$$

$$1 - \frac{N(z)}{P(z)} \qquad \frac{P(z)-N(z)}{P(z)}$$

Θ2 a) $H(z) \quad e^{j\omega} \in ROC$

$\Rightarrow \exists\, H(e^{j\omega})$

$Y(z) = 1 - H(z) \qquad$ Π.Σ. $Y(z) \supseteq$ Π.Σ. $H(z)$

$\Rightarrow \exists\, Y(e^{j\omega}) = 1 - H(e^{j\omega})$

Έστω ότι για ω κοντά σε $(2k+1)\cdot\pi$, $k \in \mathbb{Z}$ έχουμε $|H(e^{j\omega})| \ll 1$

δηλαδή χαμηλοπερατό το $H(e^{j\omega})$

τότε για ω κοντά σε $(2k+1)\cdot\pi$, $k \in \mathbb{Z}$ έχουμε

$$\underbrace{1 + |H(e^{j\omega})|}_{\simeq 1} \geq |Y(e^{j\omega})| \geq \left|1 - |H(e^{j\omega})|\right| \simeq 1 \quad \Rightarrow \boxed{|Y(e^{j\omega})| \simeq 1}$$

Άρα περνάν οι υψηλές συχνότητες

$\Downarrow$

υψιπερατό

~~β) Ομοίως έχουμε για ω κοντά σε $(2k+1)\cdot\pi$, $k \in \mathbb{Z}$~~

$\cancel{|H(e^{j\omega})| \ll 1 \qquad |Y(e^{j\omega})| = \frac{1}{|H(e^{j\omega})|} \gg 1}$ ~~συντονισμός στις υψηλές συχνότητες αφενός μεν περνάν αφετέρου δε ενισχύονται υπερβολικά~~

B) $Y(z) = \frac{1}{H(z)}$ Αν $H(z) = \frac{N(z)}{P(z)}$ οπου N, P πολυώνυμα του z

$= \frac{P(z)}{N(z)}$ τα μηδενικά γίναν πόλοι κ οι πόλοι μηδενικά $\Longrightarrow$ δεν μπορούμε να είμαστε βέβαιοι για ευσταθεία

Επιπλέον [illegible] παρατηρούμε ότι για συχνότητες ω κοντά στα $(2k+1)\pi$, $k\in\mathbb{Z}$ έχουμε $|H(e^{j\omega})| \ll 1$ αλλά και $|Y(e^{j\omega})| = \frac{1}{|H(e^{j\omega})|} \gg 1 \rightsquigarrow$ πιθανώς αστάθεια

ειδικά στο πλησιάζει κοντά στη ~~[illegible]~~ απόκριση.

Θ3 $H(s) = A \frac{N(s)}{[(s+2)^2+4](s+3)}$

$H(z) = H(s)|_{s\leftarrow \frac{z-1}{T}} \Rightarrow sT = z-1 \Rightarrow z = sT+1$

Αν ο πόλος είναι $p = a + jb$ τότε ~~[illegible]~~ επειδή έχω αιτιατό σύστημα ~~[illegible]~~ για ευστάθεια θα πρέπει ~~[illegible]~~ του. έκας πόλος ~~[illegible]~~ να βρίσκεται πάνω ή έξω απ' τον μοναδιαίο κύκλο τότε $|z| \geq \max\{|a|, |b|\} \geq 1$

$|1+T(a+jb)| \geq 1 \Leftrightarrow (1+aT)^2 + (bT)^2 \geq 1$

$\Leftrightarrow 1 + 2aT + (aT)^2 + (bT)^2 \geq 1$

$\overset{T>0}{\Leftrightarrow} 2a + (a^2+b^2)T \geq 1$

$\Leftrightarrow T \geq \frac{-2a}{a^2+b^2}$ $\quad \frac{1}{2}, \frac{2}{3}$

αρκεί λοιπόν $T \geq \frac{1}{2}$

Θ4 $\Omega_c = j$ $\quad \Omega_1 = 2\pi \cdot 400 = 800\pi$ rad/s, $\Omega_2 = 2\pi \cdot \cancel{10^3}\, 990$ rad/s $\Big\} \rightarrow$ $w_1 = \Omega_1 \cdot T = 0{,}4\pi$ rad, $w_2 = \Omega_2 \cdot T = [illegible]\pi$ rad $\Big\} \xrightarrow{\text{Prewarping}}$

$\Omega_1' = \frac{2}{T}\tan\left(\frac{w_1}{2}\right) = 2906{,}17$ rad/s

$\Omega_2' = \frac{2}{T}\tan\left(\frac{w_2}{2}\right) \rightarrow$ δεν ορίζεται $\simeq 254626{,}96$

έστω $10\log|H(\Omega)|^2 = f(\Omega)$

θέλω για $\Omega > 10^3 \cdot 2\pi$ rad/s

$f(\Omega) \leq -20$dB

Αν θέσω $\Omega_2 = 2\pi \cdot 999$ rad/s

τότε για $\Omega > 2\pi \cdot 10^3 > \Omega_2$

$\overset{f\downarrow}{\Rightarrow} f(\Omega) < f(\Omega_2)$ $\Big\} \Rightarrow$

Άρα θα απαιτήσω $f(\Omega_2) < -20$ dB

- $-10\log\left(1+\left(\frac{\Omega_1'}{\Omega_c'}\right)^{2n}\right)=-2 \Leftrightarrow \left(\frac{\Omega_1'}{\Omega_c'}\right)^{2n}=10^{0,2}-1 \Leftrightarrow (\Omega_c')^{2n}=\frac{(\Omega_1)^{2n}}{10^{0,2}-1}$ (1)

- $-10\log\left(1+\left(\frac{\Omega_2'}{\Omega_c'}\right)^{2n}\right)\leq -20 \Leftrightarrow \left(\frac{\Omega_2'}{\Omega_c'}\right)^{2n} \geq 10^{2}-1=99$

$$\Leftrightarrow \left(\frac{\Omega_2'}{\Omega_1'}\right)^{2n} \geq \frac{99}{10^{0,2}-1}$$

$$\Leftrightarrow n \geq \frac{1}{2}\,\frac{\log\left(99/(10^{0,2}-1)\right)}{\log(\Omega_2'/\Omega_1')} \simeq 0,57$$

$$\Rightarrow n_{min}=1$$

- $H_p(s)=\frac{1}{s+1}$

- (1) $\Rightarrow \Omega_c'=\frac{\Omega_1}{\sqrt{10^{0,2}-1}}=3799,99$ rad/s

Θ5
- $x(t)=\sin(2\pi t)u(t)$ $\quad f_s=4\,Hz \Rightarrow T_s=\frac{1}{4}$

$x(nT_s)=\sin\left(\frac{2\pi n}{4}\right)u(n/4)$

$x[n]=\sin\left(\frac{\pi n}{2}\right)u[n]$

- $\left\{\left[X(z)-\frac{1}{2}Y(z)\right]\cdot z^{-1}+Y(z)\right\}(-4)+X(z)=Y(z)$

$$\Leftrightarrow \left[z^{-1}X(z)-\frac{1}{2}z^{-1}Y(z)+Y(z)\right](-4)+X(z)=Y(z)$$

$$\Leftrightarrow -4z^{-1}X(z)+2z^{-1}Y(z)-4Y(z)+X(z)=Y(z)$$

$$\Leftrightarrow (1-4z^{-1})X(z)=Y(z)\left[5-2z^{-1}\right]$$

$Y(z)/X(z)=H(z)$

$$\Leftrightarrow H(z)=\frac{1-4z^{-1}}{5-2z^{-1}}=\frac{z-4}{5z-2}$$

Πόλος $z=2/5$
Αιτιατό $\Rightarrow$ Περιοχή σύγκλ.
$|z|>\frac{2}{5}$

$$h[n]=Z^{-1}\left\{\frac{1}{5}\frac{z}{z-\frac{2}{5}}-\frac{4}{5}z^{-1}\frac{z}{z-\frac{2}{5}}\right\}$$

$$=\frac{1}{5}u[n]\cdot\left(\frac{2}{5}\right)^{n}-\frac{4}{5}u[n-1]\cdot\left(\frac{2}{5}\right)^{n-1}=\begin{cases}0 & \text{για } n<0\\ \frac{1}{5} & \text{για } n=0\\ \left(\frac{2}{5}\right)^{n}\left[\frac{1}{5}-\frac{4}{\cancel{5}}\frac{\cancel{5}}{2}\right]=\left(\frac{2}{5}\right)^{n}\left(-\frac{9}{5}\right) & n=1,2,\ldots\end{cases}$$

$$=\delta[n]-\frac{9}{5}\cdot\left(\frac{2}{5}\right)^{n}u[n-1]$$

$$y[n] = x[n] * h[n] = \sin\left(\frac{\pi n}{2}\right)\cdot u[n] - \frac{9}{5}\underbrace{\left\{\sin\left(\frac{\pi n}{2}\right)u[n]\right\} * \left\{\left(\frac{2}{5}\right)^n u[n-1]\right\}}$$

$$\left(\frac{2}{5}\right)^n \sum_{k=-\infty}^{\infty} \sin\left(\frac{\pi k}{2}\right) u[k] \left(\frac{2}{5}\right)^{-k} \cdot u[n-k-1]$$

$$\left(\frac{2}{5}\right)^n \cdot \underbrace{\left[\sum_{k=0}^{n-1} \left(\frac{2}{5}\right)^{-k} \sin\left(\frac{\pi k}{2}\right)\right]}_{\Sigma_1} u[n-1]$$

$$\Sigma_1 = \frac{1}{2j}\underbrace{\sum_{k=0}^{n-1}\left(\frac{5}{2}e^{j\frac{\pi}{2}}\right)^k}_{\Sigma_2} - \frac{1}{2j}\underbrace{\sum_{k=0}^{n-1}\left(\frac{5}{2}e^{-j\frac{\pi}{2}}\right)^k}_{\Sigma_3}$$

$$\Sigma_2 = \sum_{k=0}^{n-1}\left(\frac{5}{2}e^{j\frac{\pi}{2}}\right)^k = \frac{\left(\frac{5}{2}e^{j\frac{\pi}{2}}\right)^n - 1}{\frac{5}{2}e^{j\frac{\pi}{2}} - 1}$$

$$\Sigma_3 = \sum_{k=0}^{n-1}\left(\frac{5}{2}e^{-j\frac{\pi}{2}}\right)^k = \frac{\left(\frac{5}{2}e^{-j\frac{\pi}{2}}\right)^n - 1}{\frac{5}{2}e^{-j\frac{\pi}{2}} - 1}$$

Τελικά $\left(\frac{2}{5}\right)^n \cdot \Sigma_1 = \frac{1}{2j}\left\{\frac{e^{j\frac{\pi}{2}n} - \left(\frac{2}{5}\right)^n}{\frac{5}{2}e^{j\frac{\pi}{2}} - 1} - \frac{e^{-j\frac{\pi}{2}n} - \left(\frac{2}{5}\right)^n}{\frac{5}{2}e^{j\frac{\pi}{2}} - 1}\right\}$

για $n \to \infty$ $u[n-1] = 1$

$$\left(\frac{2}{5}\right)^n \to 0 \Rightarrow \underbrace{\left(\frac{2}{5}\right)^n \Sigma_1 u[n]}_{A} = \frac{e^{j\frac{\pi}{2}n}\left(\frac{5}{2}e^{-j\frac{\pi}{2}} - 1\right) - e^{-j\frac{\pi}{2}n}\left(\frac{5}{2}e^{j\frac{\pi}{2}} - 1\right)}{2j \cdot \underbrace{\left|\frac{5}{2}e^{j\frac{\pi}{2}} - 1\right|^2}_{B}}$$

$$A = \frac{1}{B}\left\{-\sin\left(\frac{\pi}{2}n\right) + \frac{5}{2}\sin\left(\frac{\pi}{2}n - \frac{\pi}{2}\right)\right\}$$

Τελικά στη μόνιμη κατάσταση $n \to \infty$

$$y_{ss}[n] = \sin\left(\frac{\pi}{2}n\right)\left[1 - \frac{1}{B}\right] + \frac{5}{2B}\sin\left(\frac{\pi}{2}n - \frac{\pi}{2}\right)$$

ΣΕΠΤ. 18 Θ1

$$Y(z) = 0{,}5 z^{-1} Y(z) + 0{,}5 z^{-1} z \cancel{y(-1)}^{\to 0}$$
$$- 0{,}25 z^{-2} Y(z) - 0{,}25 z^{-2} z \cancel{y(-1)}^{0} - 0{,}25 z^{-2} z^2 \cancel{y(-2)}^{\to 0}$$
$$+ z^{-1} X(z) + z^{-1} \cancel{x(-1)}^{\to 0}$$

$$\Rightarrow Y(z)\left[1 - 0{,}5 z^{-1} + 0{,}25 z^{-2}\right] = z^{-1} X(z) \quad \Big| \; X(z) = 1 + z^{-1}.$$

$$\cancel{\Rightarrow Y(z)} \Rightarrow H(z) = \frac{z^{-1}}{1 - 0{,}5 z^{-1} + 0{,}25 z^{-2}}$$

$$\cancel{Y(z)}$$

$$= \frac{z}{z^2 - \frac{1}{2} z + \frac{1}{4}} \qquad a = \frac{1}{2} \quad , |z| > 1/2$$

$$\Delta = \frac{1}{4} - 1 = -\frac{3}{4} \qquad \sqrt{\frac{1}{16} + \frac{3}{16}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

$$\frac{\frac{1}{2} \pm \frac{\sqrt{3}}{2}}{2} = \frac{1}{4} \pm \frac{\sqrt{3}}{4} \qquad + 2a \cos w_0 = \frac{1}{2} \rightarrow \sin w_0 = \frac{\sqrt{3}}{2}$$

$$H(z) = \frac{4}{\sqrt{3}} \; \frac{\frac{1}{2} z \cdot \frac{\sqrt{3}}{2}}{z^2 - 2 \cdot \frac{1}{2} z \cdot \frac{1}{2} + \left(\frac{1}{2}\right)^2}$$

$$h[n] = \frac{4}{\sqrt{3}} \cdot u[n] \cdot \left(\frac{1}{2}\right)^n \sin\left(\frac{\sqrt{3}}{2} n\right)$$

$$x[n] = \delta[n] + \delta[n-1]$$

$$y[n] = x[n] * h[n] = \frac{4}{\sqrt{3}} u[n] \left(\frac{1}{2}\right)^n \sin\left(\frac{\sqrt{3}}{2} n\right)$$
$$+ \frac{4}{\sqrt{3}} u[n-1] \left(\frac{1}{2}\right)^{n-1} \sin\left(\frac{\sqrt{3}}{2} n - \frac{\sqrt{3}}{2}\right)$$

Θ2 $\frac{1}{T} = 6 \cdot 10^4$ Hz

$$\left.\begin{aligned} \Omega_1 &= 15 \cdot 2\pi \cdot 10^3 \text{ rad/s} \\ \Omega_2 &= 30 \cdot 2\pi \cdot 10^3 \text{ rad/s} \end{aligned}\right\} \xrightarrow{\text{μετατροπή σε ψηφιακές}} \begin{aligned} w_1 &= \Omega_1 T = 3\pi \cdot 10^4 / (6 \cdot 10^4) = \frac{\pi}{2} \text{ rad} \\ w_2 &= \Omega_2 T = 6\pi \cdot 10^4 / (6 \cdot 10^4) = \pi \text{ rad} \end{aligned}$$

$$\Omega_1' = \frac{2}{T} \tan\left(\frac{w_1}{2}\right) = 12 \cdot 10^4 \text{ rad/s}$$

$$\Omega_2' = \frac{2}{T} \tan\left(\frac{w_2}{2}\right)$$

Ορίζω $f(\Omega) = 10 \log |H(\Omega)|^2 = -10 \log \left(1 + \left(\frac{\Omega}{\Omega_c}\right)^2\right)$

$f(\Omega) \downarrow$ θέλουμε για $\Omega >$ ~~Ω_2~~ $f(\Omega) < -10$dB ~~$\Rightarrow f(\Omega) < -10$dB~~ για $\Rightarrow$ πιο αυστηρή προδιαγραφή $\Omega > 0{,}99\,\Omega_2$ $f(\Omega) < -10$dB

~~Άρα αν $\Omega_3 < \Omega_2$ και $f(\Omega_3) < -10$dB~~ και ισχύει

~~έχω για $\Omega > \Omega_3 > \Omega_2 \Rightarrow f(\Omega) < f(\Omega_3) = -10$dB~~ άρα ~~[illegible]~~

Butterworth άρα $f(\Omega) = -10\log\left(1+\left(\frac{\Omega}{\Omega_c}\right)^{2n}\right)$

Θέλαμε $f(\Omega = 2\pi\cdot 15\cdot 10^3) = -2\,dB$

και $f(\Omega \geq \underbrace{2\pi\cdot 30\cdot 10^3}_{\Omega_2}) \leq -10\,dB$

~~[illegible]~~ πιο αυστηρά για κάθε $\Omega > 0{,}99\,\Omega_2$ $f(\Omega) \leq -10\,dB$

$$\left.\begin{array}{l}\Omega_1 = 3\pi\cdot 10^4 \text{ rad/s} \\ \Omega_3 = 0{,}99\,\Omega_2 = 0{,}99\cdot 6\pi\cdot 10^4 \text{ rad/s}\end{array}\right\} \Rightarrow \begin{array}{l} w_1 = \Omega_1 T = 3\pi\cdot 10^4/(6\cdot 10^4) = \frac{\pi}{2}\text{ rad} \\ w_2 = \Omega_3 T = 6\pi\cdot 10^4/(6\cdot 10^4)\cdot 0{,}99 \\ \quad = 0{,}99\,\pi \text{ rad}\end{array}$$

- $\Omega_1' = \frac{2}{T}\tan\left(\frac{w_1}{2}\right) = 12\cdot 10^4$ rad/s
- $\Omega_2' = \frac{2}{T}\tan\left(\frac{w_2}{2}\right) = 7'638'908{,}91$ rad/s

θέλαμε $\left(\frac{\Omega_1'}{\Omega_c'}\right)^{2n} = 10^{0,2} - 1 \Leftrightarrow \frac{1}{(\Omega_c')^{2n}} = \frac{10^{0,2}-1}{(\Omega_1')^{2n}}$

Επιπλέον $f(\Omega_2') \leq -10\,dB \Rightarrow -10\log\left(1+\left(\frac{\Omega_2'}{\Omega_c'}\right)^{2n}\right) \leq -10$

$$\Rightarrow \log\left(1+\left(\frac{\Omega_2'}{\Omega_c'}\right)^{2n}\right) \geq 1$$

$$\Rightarrow \left(\frac{\Omega_2'}{\Omega_c'}\right)^{2n} \geq 10-1 = 9 \Rightarrow \left(\frac{\Omega_2'}{\Omega_1'}\right)^{2n} \geq \frac{9}{10^{0,2}-1}$$

$$\Rightarrow n \geq \frac{1}{2}\log\left(\frac{9}{10^{0,2}-1}\right)\Big/\log(\Omega_2'/\Omega_1') \simeq 0{,}33$$

$$\Rightarrow n_{min} = 1 \Rightarrow H_p(s) = \frac{1}{s+1} \Rightarrow H(s) = H_p(s)\Big|_{s \leftarrow \frac{s}{\Omega_c'}}$$

$$\Rightarrow H(z) = H(s)\Big|_{s \leftarrow \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}} = \ldots$$

Θ4

a) ΓΑΚΜ
$h[n] = 0 \;\forall n < 0$ $\Big\}\Rightarrow$ αιτιατό

$H(z) = \frac{z}{z-\frac{1}{2}} + \frac{z}{z+\frac{1}{3}}$ με $|z| > \max\{|\tfrac{1}{2}|, |-\tfrac{1}{3}|\} = \frac{1}{2}$

$e^{jw} \in ROC_H \Rightarrow$ ευσταθής

για πεπερασμένης καθολικής απόκρισης

β) Η $H(z)$ όπως στο α)

Επειδή $e^{j\omega} \in ROC \Rightarrow H(e^{j\omega}) = \frac{e^{j\omega}}{e^{j\omega}-\frac{1}{2}} + \frac{e^{j\omega}}{e^{j\omega}+\frac{1}{3}}$

γ) $\frac{Y(z)}{X(z)} = \frac{z(2z-1/6)}{(z-\frac{1}{2})(z-\frac{1}{3})} = \frac{2z^2-1/6\,z}{z^2-\frac{5}{6}z+\frac{1}{6}} = \frac{2-1/6\,z^{-1}}{1-\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2}}$

$\Rightarrow Y(z) - \frac{5}{6}z^{-1}Y(z) + \frac{1}{6}z^{-2}Y(z) = 2X(z) - \frac{1}{6}z^{-1}X(z)$

$\Rightarrow y[n] - \frac{5}{6}y[n-1] + \frac{1}{6}y[n-2] = 2x[n] - \frac{1}{6}x[n-1],\ n \geq 0$

δ) $E_h = \sum_{k=-\infty}^{\infty} |h[k]|^2 = \sum_{k=0}^{\infty} \left|\left(\frac{1}{2}\right)^k + \left(\frac{1}{3}\right)^k\right|^2 = \sum_{k=0}^{\infty} \left(\frac{1}{4}\right)^k + 2\left(-\frac{1}{6}\right)^k + \left(\frac{1}{9}\right)^k$

$= \sum_{k=0}^{\infty}\left(\frac{1}{4}\right)^k + 2\sum_{k=0}^{\infty}\left(-\frac{1}{6}\right)^k + \sum_{k=0}^{\infty}\left(\frac{1}{9}\right)^k$

$= \frac{1}{1-\frac{1}{4}} + 2\frac{1}{1+\frac{1}{6}} + \frac{1}{1-\frac{1}{9}} = \frac{701}{168}$ J

~~2017 Z[n]~~

$\boxed{\Theta 1}$ $y[n] - 0{,}75y[n-1] + 0{,}125y[n-2] = x[n],\ n \geq 0$

$\Rightarrow Y(z) - 0{,}75z^{-1}Y(z) - 0{,}75z^{-1}z\,y(-1) + 0{,}125z^{-2}Y(z) = X(z)$

$+ 0{,}125z^{-2}z\,y(-1)$

$+ 0{,}125z^{-2}z^2 y(-2)$

$\sum_{n=0}^{\infty} y[n]z^{-n}$

$\Rightarrow Y(z)[1-0{,}75z^{-1}+0{,}125z^{-2}] = X(z) + 0{,}75 - 0{,}125z^{-1}$

$\Rightarrow Y(z) = \frac{X(z)}{1-0{,}75z^{-1}+0{,}125z^{-2}} + \frac{0{,}75 - 0{,}125z^{-1}}{1-0{,}75z^{-1}+0{,}125z^{-2}}$

για αρχ συνθ. 0 $H(z) = \frac{1}{1-0{,}75z^{-1}+0{,}125z^{-2}} = \frac{z^2}{z^2-\frac{3}{4}z+\frac{1}{8}}$

$= 1 + \frac{\frac{3}{4}z-\frac{1}{8}}{z^2-\frac{3}{4}z+\frac{1}{8}}$

$\frac{\frac{3}{4}z-\frac{1}{8}}{z^2-\frac{3}{4}z+\frac{1}{8}} = \frac{A}{z-\frac{1}{2}} + \frac{B}{z-\frac{1}{4}}$, $A = \frac{\frac{3}{4}\cdot\frac{1}{2}-\frac{1}{8}}{\frac{1}{2}-\frac{1}{4}} = \frac{\frac{1}{4}}{\frac{1}{4}} = 1$

$B = \left.\frac{\frac{3}{4}z-\frac{1}{8}}{z-\frac{1}{2}}\right|_{z=\frac{1}{4}}$

$= \frac{\frac{3}{16}-\frac{2}{16}}{-1/4} = \frac{1/16}{-1/4} = -\frac{1}{4}$

Θ3 Οι πόλοι της $H(z)$ ~~...~~ δίνονται από την απεικόνιση των πόλων της $H(s)$ ~~...~~ μέσω της σχέσης $s = \frac{z-1}{T}$ και αντιστοιχία με τις οπισθόδρομες διαφορές

Ως εκ τούτου προκύπτει ισοδύναμα $sT + 1 = z$

(Το διακριτό σύστημα παραμένει αιτιατό αφού οι νέες είσοδοι παράγονται από παλιές)

Έτσι για να έχουμε ευστάθεια λόγω αιτιότητας πρέπει $|z| < 1$ $\forall z \in \{p_i \cdot T + 1,\ i = 1, 2, \dots, M\}$

• πόλος και $n \geq 0$

Άρα $\forall k \in \{1, 2, \dots, M\}$ πρέπει

$$|p_k T + 1| < 1$$

αν $p_k = a_k + j b_k$ με $a_k < 0$ λόγω ευστάθειας αναλογικού

$$\Leftrightarrow |a_k T + 1 + j b_k T| < 1$$

$\Rightarrow a_k^2 + b_k^2 \neq 0$

$$\Leftrightarrow (a_k T + 1)^2 + (b_k T)^2 < 1$$

$$\Leftrightarrow (a_k T)^2 + 2 a_k T + 1 + (b_k T)^2 < 1$$

$$\overset{T > 0}{\Leftrightarrow} (a_k^2 + b_k^2) T < -2 a_k$$

$$\overset{a_k < 0}{\Leftrightarrow} T < \frac{-2 a_k}{a_k^2 + b_k^2} > 0$$

Αν $j \in \{1, \dots, M\}$ τ.ω. $\mathrm{Re}\{p_j\} \geq a_k$ $\forall k \in \{1, \dots, M\}$

$\Rightarrow \max_k \{p_k\} \geq a_k$

$\Rightarrow -\max_k \{p_k\} \leq a_k$

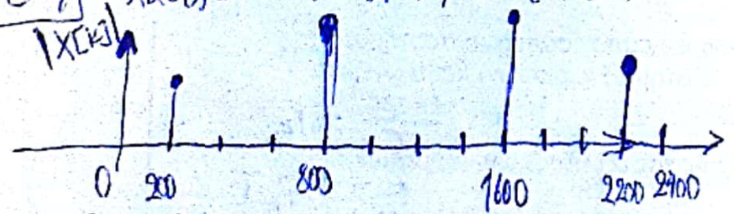
Δηλ. $\forall k : T < \frac{-2 a_k}{a_k^2 + b_k^2}$

$$\frac{-2 a_k}{a_k^2 + b_k^2} = \frac{-2 \mathrm{Re}\{p_k\}}{|p_k|^2} \geq \frac{-2A}{|p_k|^2} \geq \frac{-2A}{B}$$

άρα για $T < \frac{-2A}{B}$ εξασφαλίζω ευστάθεια

Θ4 $x_a(t) = \sin(2\pi f_1 t) + \sin(2\pi f_2 t)$

$N = 2400$
$F_s = 360 \text{Hz}$



$k \Leftrightarrow \frac{2\pi}{N} \cdot k = \omega_k \Leftrightarrow \frac{\omega_k / T_s}{2\pi} = f_k$

$$f_k = \frac{k}{N} \cdot F_s$$

Υποθέτω $f_2 > f_1$ χωρίς βλάβη γενικότητας

Υποθέτοντας $F_s > 2 f_2 > 2 f_1$

$$\Rightarrow \bullet\ f_1 = \frac{200}{2400} \cdot 360 = 30 \text{ Hz}$$

$$\bullet\ f_2 = \frac{800}{2400} \cdot 360 = 120 \text{ Hz}$$

• $H(z) = \dfrac{a + bz + cz^2}{c + bz + az^2}$ • $\Delta = b^2 - 4ac$ $\boxed{\Theta 3 \quad \Phi EB \ 17}$

Εφόσον ευσταθές $\Rightarrow e^{jw} \in ROC_H \Rightarrow H(e^{jw}) = \dfrac{a + be^{jw} + ce^{j2w}}{c + be^{jw} + ae^{j2w}}$

$$|H(e^{jw})|^2 = \frac{|a + be^{jw} + ce^{j2w}|^2}{|c + be^{jw} + ae^{j2w}|^2} = \frac{(a + be^{jw} + ce^{j2w})(a + be^{-jw} + ce^{-j2w})}{(c + be^{jw} + ae^{j2w})(c + be^{-jw} + ae^{-j2w})}$$

$$= \frac{a^2 + ab e^{-jw} + ace^{-j2w} + bae^{jw} + b^2 + bce^{-jw} + cae^{j2w} + cbe^{jw} + c^2}{c^2 + cbe^{-jw} + cae^{-j2w} + cbe^{jw} + b^2 + bae^{-jw} + ace^{j2w} + abe^{jw} + a^2}$$

$$= \frac{a^2 + b^2 + c^2 + 2ab\cos w + 2ac\cos(2w) + 2bc\cos(w)}{a^2 + b^2 + c^2 + 2ab\cos w + 2ac\cos(2w) + 2bc\cos(w)} = 1$$

$\Rightarrow \boxed{|H(e^{jw})| = 1} \longrightarrow$ All pass system

$a \neq 0$

$z_{m_{1,2}} = \dfrac{-b \pm \sqrt{\Delta}}{2a}$ ή $z_{m_1} = 0 \ \binom{a=0}{c=0}$ ή $\begin{cases} z_{m_1} = 0 \\ z_{m_2} = -\frac{b}{c} \end{cases} \binom{a=0}{c \neq 0}$

$c \neq 0$

$z_{p_{1,2}} = \dfrac{-b \pm \sqrt{\Delta}}{2c}$ ή $z_{p_1} = 0 \ (a, c = 0)$ ή $z_{p_1} = 0$, $z_{p_2} = -\dfrac{b}{a} \ (a \neq 0, c = 0)$

• $a, c = 0 \Rightarrow$ All pass

• $a \neq 0, \ c = 0 \Rightarrow z_{m_{1,2}} = \dfrac{-b \pm |b|}{2a} = \dfrac{-b \pm b}{2a} = \begin{cases} 0 \\ -\frac{b}{a} \end{cases}$

$z_{p_1} = 0$
$z_{p_2} = -b/a$

All pass

• $a = 0, \ c \neq 0 \Rightarrow z_{m_1} = 0$
$z_{m_2} = -b/c$

$z_{p_{1,2}} = \dfrac{-b \pm b}{2c} = \begin{cases} 0 \\ -\frac{b}{c} \end{cases}$ All pass

• $a \neq 0, \ c \neq 0 \Rightarrow a z_{m_1} = c z_{p_1}$, $a z_{m_2} = c z_{p_2}$ $\quad z_{p_1} = \frac{a}{c} \cdot z_{m_1}$ $\quad z_{m_1} = \frac{c}{a} z_{p_1} = \frac{c}{a} \cdot re^{j\theta}$

$\boxed{\Sigma E\Pi T \ 15}$ $\underline{\Theta 1}$ $\quad Y[k] = \sum_{n=0}^{N/2 - 1} \left(x[n] + x[n + \tfrac{N}{2}]\right) W_{\frac{N}{2}}^{nk}$

• $W_{\frac{N}{2}} = e^{-j\frac{2\pi}{N/2}} = \left(e^{-j\frac{2\pi}{N}}\right)^2 = W_N^2$

$$= \sum_{n=0}^{\frac{N}{2}-1} x[n] W_N^{2nk} + \sum_{n=0}^{\frac{N}{2}-1} x[n + \tfrac{N}{2}] W_N^{2nk}$$

$$= -\text{''}- + \sum_{m=\frac{N}{2}}^{N-1} x[m] W_N^{2(m - \frac{N}{2})k}$$

$$= -\text{''}- + \left(\sum_{n=\frac{N}{2}}^{N-1} x[n] W_N^{2nk}\right) \quad \left[W_N^{-Nk} = e^{+j2\pi k} = 1\right]$$

$$= \sum_{n=0}^{N-1} x[n] W_N^{2nk} = X[2k]$$

• $X[k] = \sum_{n=0}^{N-1} x[n] W_N^{nk}$

• $m = 2n$ $\quad \begin{matrix} n=0 \\ m=0 \end{matrix} \Big| \begin{matrix} n = \frac{N}{2}-1 \\ m = N-2 \end{matrix}$

• $X[2k] = \sum_{n=0}^{N-1} x[n] W_N^{2nk}$

ΦΕΒ 15 | Θ1

$$H(z) = \frac{z^2}{(z - re^{j\omega_0})(z - re^{-j\omega_0})} \qquad 0<r<1$$

$z=0$ διπλός μηδενικό
$z = re^{\pm j\omega_0}$ συζυγείς μιγ. πόλοι

Εφόσον αιτιατό
ROC: $z \in \mathbb{C}: |z| > |re^{\pm j\omega_0}| = r$

$$H(z) = \frac{z^2 - rz\cos\omega_0}{- - -} + \frac{\cos\omega_0}{\sin\omega_0} \frac{rz\sin\omega_0}{- - -}$$

$$h[n] = (r^n \cos\omega_0 n)u[n] + \frac{\cos\omega_0}{\sin\omega_0} \cdot (r^n \sin(\omega_0 n))u[n]$$

(Εφόσον $\sin\omega_0 \neq 0$
αν $\omega_0 = k\pi \Rightarrow \cos\omega_0 = \pm 1$

$$H(z) = \frac{z^2}{(z \pm r)^2}$$

$$= (-z)\frac{z}{(z \pm r)^2} \cdot (-1)$$

$$\lim_{\omega_0 \to k\pi} \frac{\sin(\omega_0 n)}{\sin\omega_0}$$

$$= \lim_{\omega_0 \to k\pi} \frac{n\cos(\omega_0 n)}{\cos\omega_0}$$

$$= n\frac{(-1)^{kn}}{(-1)^{k}} = n(-1)^{k(n-1)}$$

$\omega_0 = 0$
$(1+n)r^n u[n]$

$\boxed{Θ2 \mid ΦΕΒ\ 2009}$

$$r_l(n) = \{ r_{l0}, r_{l1}, \dots, r_{l26} \}$$

$$r(n) = \{ r_0, \dots, r_{19} \}$$

Ισχύει $r(n) = \sum_{k=-\infty} r_l(n + k \cdot 20)$

Δηλ. $r_0 = r_{l0} + r_{l20}$

$r_1 = r_{l1} + r_{l21}$

$r_2 = r_{l2} + r_{l22}$

$r_6 = r_{l6} + r_{l26}$

$r_7 = r_{l7}$

$r_8 = r_{l8}$

$\vdots$

$r_{19} = r_{l19}$

Ταυτίζονται από $n=7$ ως $n=19$

$$\frac{\cancel{v}z - v - s_i z - s_i}{z+1}$$

Αν $m > n$ $\Rightarrow$ ασταθές

~~$H(z) = A(z - z_{m1}$~~ $\quad s = \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}} = \frac{2}{T}\frac{z-1}{z+1} = v\frac{z-1}{z+1}$

$$\bullet\ H(s) = A\frac{(s-s_1)(s-s_2)\cdots(s-s_m)}{(s-\lambda_1)(s-\lambda_2)\cdots(s-\lambda_n)}$$

$$= A\frac{\left(v\frac{z-1}{z+1} - s_i\right)}{\left(v\frac{z-1}{z+1} - \lambda_i\right)} = A$$

$$vz - v - s_i z - s_i = 0$$