Laboratory 4

Second Order Switched RLC Circuits

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EE101L

Intro. to Electronic Circuits Laboratory

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Introduction:

The purpose of this lab was to investigate the voltage and current relationships for second order RLC circuits. We were to observe the voltage response of the circuit using the Oscilloscope. This experiment was conducted in two circuits, the series RLC circuit and the parallel RLC circuit. Our goal was to find the values of resistance needed to make a critically damped response, over damped response, and underdamped response. In this lab, we need to keep in mind that the step voltage source had an internal resistance of 50 Ω.

Methods:

Part 1: Series RLC

As stated in the introduction, our goal was to find the values of resistance needed to make a critically damped response, over damped, and underdamped response. I calculated the theoretical values of the resistance before constructed circuits. Due to the limited resource of our lab, I used a capacitor with 1000pF capacitance and a inductor with 4.7µH inductance.

The following formulas are needed to determine the values of resistance.

$$\alpha = \frac{Rs}{2L} \qquad \omega = \frac{1}{\sqrt{(LC)}}$$

Overdamped: $\alpha^2 - \omega^2 > 0$ $\qquad \alpha^2 > \omega^2$

Critically damped: $\alpha^2 - \omega^2 = 0$ $\qquad \alpha^2 = \omega^2$

Underdamped : $\alpha^2 - \omega^2 < 0$ $\qquad \alpha^2 < \omega^2$

With the equations above, I calculated the values of resistance respectively, in figure 1.1:

Cases	Theoretical Resistance	Capacitance	Inductance
Overdamped	137 < R	1000pF	4.7µH

Cases	Theoretical Resistance	Capacitance	Inductance
Critically Damped	137 = R	1000pF	4.7µH
Underdamped	137>R	1000pF	4.7µH

Figure 1.1 theoretical values for resistance, given the capacitance and inductance

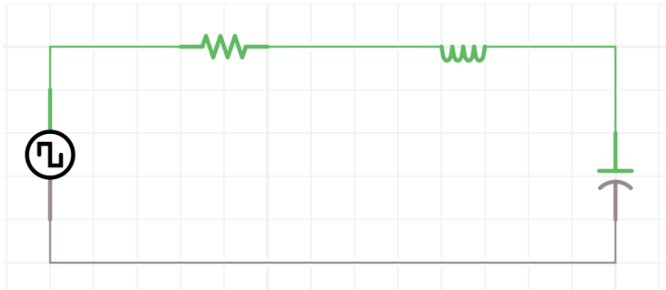


Figure 1.2 Operation reduced circuit, the values of inductor and capacitor are set. Change different resistors over time.

Using an Oscilloscope, we observed the folate response of three regions of operation across the circuits equivalent capacitor. The response data was plotted in a graph.

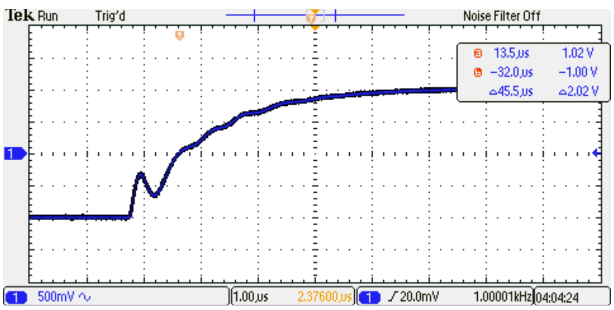


Figure 1.3 Experimental response from oscilloscope for over damped series.

I replaced the 137 Ω resistor with a 800Ω resistor. Above graph was what I got from the the oscilloscope. It oscillated a little bit, but it gets to equilibrium fast.

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4.7\cdot 10^{-6}\cdot 1\cdot 10^{-9}}} = 14.6\ \frac{Mrad}{s}$$

$$\alpha = \frac{Rs}{2L} = \frac{800}{2\cdot\left(4.7\cdot 10^{-6}\right)} = 85\ M\ N\ \frac{p}{s}$$

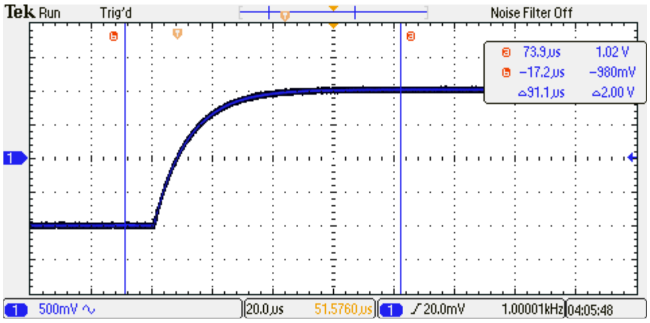


Figure 1.4 Experimental response from oscilloscope for critically damped series.

The response from the graph above is not oscillatory and it gets to equilibrium.

From the graph, we can see there is no cycle.

We know that:

$$\alpha^2 = \omega^2 \qquad \omega d = \sqrt{\omega 0^2 - \alpha} = 0$$

so that:

$$fd = \frac{\omega d}{2\pi} = 0 \qquad Td = \frac{1}{fd}$$

Also, the period of Herizan oscillation would be infinite.

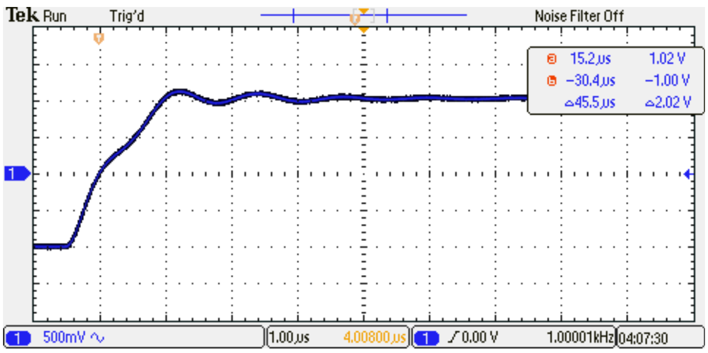


Figure 1.5 Experimental response from oscilloscope for under damped series.

To obtain an underdamped response, I removed the resistor and not put any resistor considering the internal resistance is 50 Ω. In this case, the oscillation appears and the amplitude of the response decrease exponentially. This is not an ideal underdamped graph because according to the following calculation there should be 5 cycles.

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4.7\cdot10^{-6}\cdot1\cdot10^{-9}}} = 14.6\ \frac{M\text{rad}}{s} \qquad \omega d = \sqrt{\omega^2 - \alpha^2} = 13.7M\ \frac{\text{rad}}{s}$$

$$\alpha = \frac{Rs}{2L} = \frac{50}{2\cdot(4.7\cdot10^{-6})} = 5.3\ \frac{MNp}{s} \qquad fd = \frac{\omega d}{2\pi} = 2.16M\ Hertz$$

Question: As the never frequency alpha increases, underdamped resonate frequency omega zero decreases and period would be decrease at the same time?

Proof:

$$\alpha = \frac{Rs}{2L} \qquad \omega d = \sqrt{\omega^2 - \alpha^2}$$

Since omega zero does not change, omega d would decrease when alpha increase.

We also know that :

$$\omega d = 2\pi fd$$

The period of Herzian oscillation, $Td = \frac{1}{fd}$

also increases. Additionally, comparing figure 1.5 and 1.3, the graphs of over damped and underdamped, we can see that over damped response has higher period of oscillation.

Part 2: Parallel RLC

The section repeats what I have done in the previous section, but this time with a parallel circuit reading voltage across the inductor. All formulas will look the same except this time our alpha values will be different.

We need a current source in parallel with resistor built into the circuit. Since the signal generator acts like a voltage source, I connected the resistor in series with it and use the source transformation, as in figure 2.1 shown.

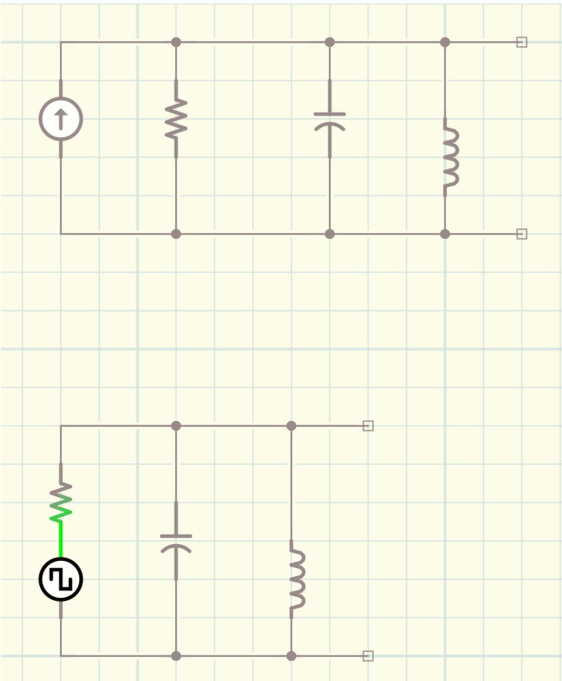


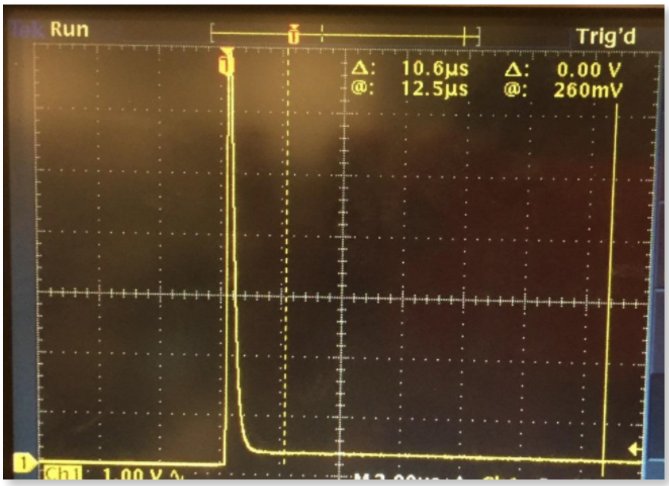
figure 2.1 source transformation of parallel RLC circuit

1. Critically damped response,

$$\alpha = \omega \qquad \frac{1}{2RpC} = \frac{1}{\sqrt{LC}}$$

$$Rp = \frac{\sqrt{L}}{2\sqrt{C}} = \frac{\sqrt{10 \cdot 10^{-6}}}{2\sqrt{1000 \cdot 10^{-9}}} = 50$$

We know that the signal generator has a 50Ω internal resistance. I did not build any resistor in the circuit.



From this graph, the response gets to equilibrium as soon as possible. There was no cycle in the graph. We know that:

$$\alpha^2 = \omega^2$$

$$\omega d = \sqrt{\omega 0^2 - \alpha} = 0$$

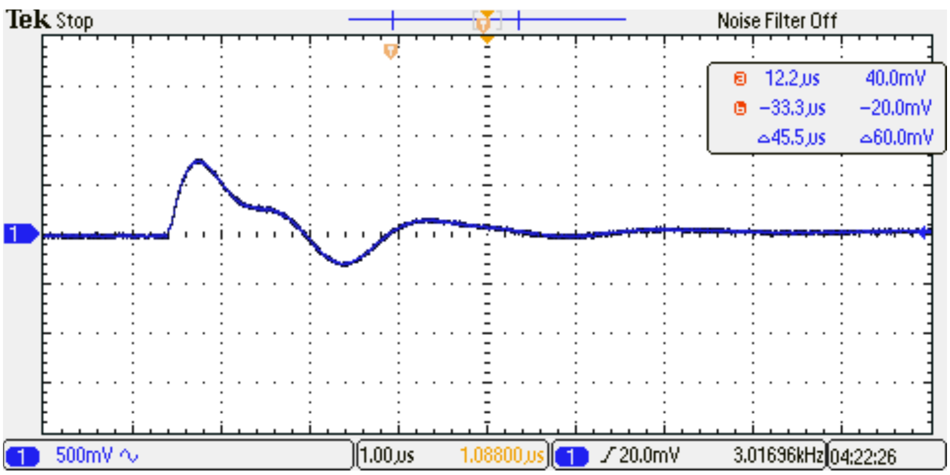
and $fd = \frac{\omega d}{2\pi} = 0$ $\quad Td = \frac{1}{fd}$

The period of Herizian oscillation would be infinite because fd approaches to zero.

2. Under damped response, $\alpha < \omega$

so $\frac{1}{2RpC} < \frac{1}{\sqrt{LC}}$

In this case, we need a resistance higher than 50. I used 3kΩ.



In this graph, we could see it is an under damped response because the oscillation appears and the amplitude of the response decrease exponentially.

Now we have:

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(4.7\cdot 10^{-6})(1\cdot 10^{-9})}} = 14.6 \ \frac{M\mathrm{rad}}{s}$$

$$f= \frac{\omega}{2\pi}= \frac{14.6\cdot10^{6}}{2\pi}= 22.9\ \frac{M\mathrm{rad}}{s}$$

$$\alpha = \frac{1}{2RpC}= \frac{1}{2(3k)(0.001\cdot10^{-6})}= 0.16\ M\ N\ \frac{p}{s}$$

$$\omega d= \sqrt{\omega^{2}-\alpha^{2}}= 14.6\ \frac{Mrad}{s}$$

$$fd = \frac{\omega d}{2\pi}= 2.32\ M\ Hertz$$

and the period of Herzian Oscillation Td = 1/fd = 43.103 ns

3. Over damped response $\alpha > \omega$

so $$\frac{1}{2RpC}>\frac{1}{\sqrt{LC}}$$

In order to make Rp less than 50 ohms, I connected a 51 ohms in parallel with the signal generator.

Therefore:

$$Rp=\frac{50\cdot51}{50+51}=25\ \Omega$$

$$\omega=\frac{1}{\sqrt{LC}}= \frac{1}{\sqrt{(4.7\cdot10^{-6})(1\cdot10^{-9})}}= 14.6\ \frac{M\mathrm{rad}}{s}$$

$$\alpha= \frac{1}{2Rp\mathrm{C}}= \frac{1}{2\cdot25\cdot(4.7\cdot10^{-6})}=365\ \frac{MNp}{s}$$

$$fd=\frac{\omega d}{2\pi}<0, Td = \frac{1}{fd}$$

The period of Herzian oscillation would be infinite. Below is the graph I got from the oscilloscope.

Conclusion:

There are three different phases of series and parallel RLC circuits: critically damped, over damped and underdamped. The phases are determined by the resistance of the circuit and either the capacitance in a parallel RLC circuit or the inductance in a series RLC circuit. The results from this lab went off a little bit. My guess of reason is man facts. For example, we don't know the precise values of inductor and capacitor. With this lab, I have experimentally test the equations applying on them and also three phases of operation for each of them.