A First Principles Approach to Dark Physics

Paul Bird

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Abstract

Starting from very simple assumptions we derive values for the Cosmological Constant and produce an extended Einsteinian gravitational theory with deviations from GR in the outer galaxy regime. In the second part, by assuming the simplest U(3) family symmetry, we show how the previous results also agree with the mass scale of the known elementary particles.

This paper serves as a counter-argument to the ideas of multiverse or evolutionary universe hypothesis as an explanation of fundamental constants in that it demonstrates that the values do indeed satisfy a 'naturalness' criteria and lie within expected ranges.

We define the 'MoG Radius' which is the radius beyond which generic modified gravity theories should expect General Relativity to lose accuracy which is a useful formula for experimental confirmation.

1 Introduction

It is commonly held that there are two big 'mysteries' in Cosmology known colloquially as the Dark Energy and Dark Matter problems. The first relates to the problem of why the cosmological constant is so small, the second relates to why there is a deviation from the expected angular velocities of the stars at the outer edges of galaxies.

We shall show using very basic assumptions that these two problems have related simple solutions. Other authors have postulated explanations for these so-called 'coincidences' but so far, it seems, none has been convincing enough to persuade the wider community. In fact, some of the more popular proposals are 'non-explanations' relegating the values to the anthropic principle and the multiverse. [5]

In the second part we shall demonstrate how another big mystery, the 'smallness of the particle masses' which asks why particles such as the electron have such a small mass in relation to the 'natural' Planck mass scale and relates to the previous results. (This is also known as the 'hierarchy problem'.)

The aim of this paper is to convince the reader that not only are the smallness of the Cosmological Constant and particle masses not 'unnatural' but they are precisely at the order of magnitude that we should expect them to be in given some simple assumptions.

The arguments are broad in the sense that we are not particularly concerned with precise details of the Lagrangian - of which there are many candidates. We are interested in determining the magnitude of certain constants to within 1 or 2 orders of magnitude. Thus for example, the age and radius of the observable Universe are both about 10^{60} in Planck units up to 1 order of magnitude.

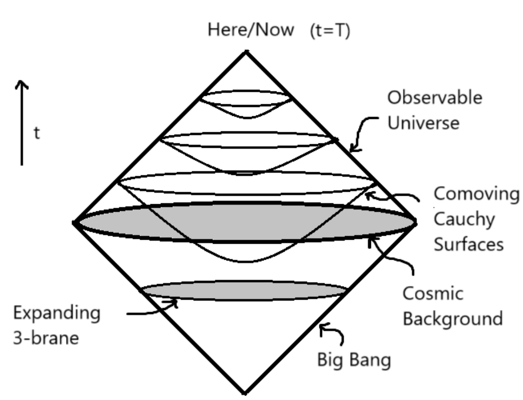
A holistic approach is taken to answer these questions, using both arguments from cosmology (General Relativity), and particle physics (Higgs sector).

2 A Conceptual Model

The 'Universe as a Membrane' model is a useful construction on which to pin our ideas and to derive equations.

Consider a 3 dimensional membrane growing, from a point, at the speed of light in some higher dimensional bulk space. For this to happen the membrane tension must drop according to $1/t$ where t is the bulk time coordinate (according to the principle that the approximate extent of a string/membrane is inversely proportional to it's tension). The membrane world-volume forms a solid cone in space-time. The boundary of this will feel no time due to moving at the speed of light and hence we can associate the whole boundary with the big bang (cosmological time 0). Nested hyperboloids inside this code represent subsequent co-moving time coordinates. The 'Observable Universe' is identified with the surface of the upper-cone with the observer at the apex. Although this is simply a change to 'Milne' coordinates it will help clarify further discussion.

This is a purely pragmatic model in that it will be shown to give correct predictions for physical observations. Whether this membrane actually 'exists' or how it came to be or have these particular properties such as decreasing tension is not something we shall concern ourselves with at this time. The actual physics will be the equations suggested by this model and their numerical predictions.



We start with the simplest diffeomorphism invariant Lagrangian-density which is the volume element for the membrane:

$$\mathcal{L} = \sqrt{-g}$$

where $g^{\mu\nu}(x)$ is the metric on the 3-brane. As in string field theory, higher derivative terms will occur as a power series in the inverse of the membrane tension $\frac{1}{T}$ where we are now setting T to be the age of the Universe in Planck units as we are interested in the effective Lagrangian from our perspective in time. Insisting on only diffeomorphism invariant terms we find that the generalised effective action must be of the approximate form:

$$\mathcal{L} = \sqrt{-g}(1 + T^2\mathcal{R} + T^4\mathcal{R}^2 + ...)$$

where $\mathcal{R}^n$ represents some contraction of n Riemann curvature tensors and it's covariant derivatives which contain $2n$ derivatives of g per term.

We now assign $\Lambda = \frac{1}{T^2}$ and write this as:

$$\mathcal{L} = \Lambda^{-1}\sqrt{-g}(\Lambda + \mathcal{R} + \frac{1}{\Lambda}\mathcal{R}^2 + ...)$$

The overall factor of Λ^{-1} has no effect classically.

We note, in passing, that this series is similar to that if we make the translation $x \rightarrow x + T$ (which is not a symmetry of a bounded Universe) and expand out $\sqrt{-g(x+T)}$ as a Taylor series. If x are geodesic coordinates we get a similar power series. This suggests an alternative description where the curvature terms result from a change in coordinates from the boundary to the interior. We also note that the series is also what we would expect from dimensional analysis. Thus there are many ways to arrive at the form of this series.

Comparing the first terms of this power series with the Einstein-Hilbert action we associate Λ with the Cosmological constant and find that this is proportional to the inverse square of the age of the Universe. This matches with observations as both values $\approx 10^{-122}$ in Planck units.

It may seem a bit strange to expand out a sequence in powers of Λ^{-1} as this is such a large number. Usually when we are augmenting General Relativity with higher curvature terms we want them to be small, not in the order of 10^{122}. But we shall see that continuing the power series to infinity really does produce only small changes to General Relativity in the known regimes.

So far we have made an argument for why the cosmological constant should have the value it does but at the expense of introducing an infinite number of other terms to the action. We shall next find that these are precisely what is needed in connection with the Dark Matter problem.

2.1 Conditions on the Field Equations

Let first review why the Einstein-Hilbert action has it's particular form. When Einstein was constructing his field equations, he had main two conditions in mind:

- *'Space tells matter how to move.'* Particles travel on geodesics in curved space-time.
- *'Matter tells space how to curve.'* It must reproduce Newton's inverse square to high accuracy in known regimes (outside the orbit of Mercury and up to the orbit of Pluto, for example).

The first can be considered a symmetry principle and cannot be changed without abandoning Relativity Theory. The second is purely physical and comes from fitting the theory to observations. It is sometimes erroneously taught that the Einstein-Hilbert action with cosmological constant is the unique solution to these conditions (plus some optional higher derivative terms with very small coefficients). This is true only if we restrict ourselves to the set of actions which are polynomial in the curvature tensors - which of course have the advantage of being mathematically simpler.

In modern times we can add three more conditions that we would like our gravitational field equations to have.

- *Accelerating Universe.* Has a cosmological constant term
- *Fits Galaxy rotation curves.* Deviates from the inverse square law in the outer reaches of galaxies
- *Quantizable* The action should be able to be written in terms of first derivatives of the metric only - if it has any hope of being quantized.[1]

A set of actions which obey all 5 of these conditions include those of the form: $\mathcal{L}_{grav} = \sqrt{-g}(\Lambda^{2n} + \mathcal{R}^{2n})^{1/(2n)}$ where $\mathcal{R}^2$ represents the Gauss-Bonnet term in order to satisfy the quantizable condition.

We can formally expand this Lagrangian out as a power series in Λ^{-1}. That this produces a power series is independent of our first argument which was primarily to deduce the value of Λ in terms of T.

3 The Dark Matter Problem

It has been observed that the stars in the outer reaches of many galaxies are orbiting 'too fast', as if the gravitational potential were falling off less steeply in that regime as a $1/r$ force law instead of a $1/r^2$ force law. The force being stronger than it should be, hence the stars can have more momentum without leaving orbit.

There are two main categories of explanation for this which we might call the *bosonic* and the *fermionic*. The fermionic explanation is that there may be a shell of invisible matter surrounding the galaxy. The bosonic explanation is

[1]It can be noted that the Einstein-Hilbert action also has this special property by using integration by parts and discarding surface terms, but general $f(R)$ Lagrangians will not have this property.

that the difference is caused by some interaction with force particles (either a modified gravitational potential or some new force).

We are going to proceed with the presumption that at least part of the explanation is gravitational in nature.

For our model we will use one of the set of Lagrangians we found which satisfies our 5 conditions. The simplest being the following:

$$\mathcal{L} = \sqrt{-g}\sqrt{\Lambda^2 + \mathcal{R}^2} + \mathcal{L}_{matter}$$

With $\mathcal{R}^2$ representing the Gauss-Bonnet term $\mathcal{R}^2 = R^{abcd}R_{abcd} - 4R^{ab}R_{ab} + R^2$. This is the simplest contraction of Riemann tensors that contains no second derivatives of the metric. Symbolically $\mathcal{R}^2 = g^{-6}(\partial g)^4$.

If we were not concerned with the quantizable condition then we could have considered the slightly more general form $\mathcal{L}_{grav2} = \sqrt{-g}\sqrt{\Lambda^2 + c_0\Lambda R + \mathcal{R}^2}$ with the coefficient c_0 of order unity, R the scalar curvature and $\mathcal{R}^2$ now allowed to by some more general contraction of two curvature tensors. Such actions have been considered before [1], as well as more general $f(R)$ and $f(R, \mathcal{G})$ theories.

An alternative way to incorporate the matter terms would be to include it as $\sqrt{-g}\sqrt{\Lambda^2 + \mathcal{R}^2 + \mathcal{L}^2_{matter}}$. This would make a difference for cosmological models, but for our purposes, where we are concerned with the gravitational field in empty space, it doesn't affect our results.

3.1 Approximation with Newtonian Potential

Let us take the Newtonian approximation of the $\mathcal{L}_{grav}$ Lagrangian as a function of Λ:

$$\mathcal{L}_\Lambda[\phi] = \sqrt{\Lambda^2 + (\partial_\mu\phi\partial^\mu\phi)^2} + \rho\phi$$

We have also incorporated the source term inside the square root, which is just one way we could include the matter-density term. Mainly we are concerned with cases outside massive objects where $\rho(r) = 0$. Varying the action w.r.t ϕ gives:

$$\nabla^\mu\left(f_\Lambda[\phi]\nabla_\mu\phi(x)\right) = f_\Lambda[\phi]\rho(x)$$

where $f_\Lambda[\phi]$ is defined by:

$$\frac{1}{f_\Lambda[\phi]} = \frac{\mathcal{L}_\Lambda[\phi]}{\mathcal{L}_0[\phi]} = \sqrt{1 + \frac{\Lambda^2}{(\partial^\mu\phi\partial_\mu\phi)^2}}$$

which just becomes $\nabla^2\phi = \rho$ when $\Lambda \to 0$ since $f_0[\phi] = 1$.

The potential equation in polar coordinates is given by:

$$\left(\frac{\partial}{\partial r} + \frac{2}{r}\right)\left(\left(1 + \Lambda^2\left(\frac{\partial\phi(r)}{\partial r}\right)^{-4}\right)^{-1/2}\frac{\partial}{\partial r}\phi(r)\right) = \delta(r)$$

We can solve this as a series in $\beta = \frac{\Lambda r^4}{m^2}$

$$\phi(r) = c + \frac{m}{r}\left(1 - \frac{1}{14}\beta^2 + \frac{3}{40}\beta^4 + \mathcal{O}(\beta^6)\right)$$

We can solve this exactly for the derivative of the potential $\phi'(r)$ and end up with this much more complicated modification of the inverse square law:

$$\phi'(r) = \frac{m}{r^2}\left(\frac{1}{3} + \frac{1}{3}\left(1 + 2b^2 + 2b\sqrt{b^2+1}\right)^{1/3} + \frac{1}{3}\left(1 + 2b^2 + 2b\sqrt{b^2+1}\right)^{-1/3}\right)^{1/2}$$

with $b = \frac{3\sqrt{3}}{2}\beta$

It is useful to define the function $\xi(a) = Root[x \to x^3 - x^2 - a^2]$ which is defined as the real root when $a \neq 0$. This satisfies $\xi(0) = 1$ and $\lim_{a\to\infty} \xi(a)a^{-2/3} = 1$:

$$\phi'(r) = \frac{m}{r^2}\xi\left(\frac{\Lambda r^4}{m^2}\right)^{1/2}$$

We see it deviates from GR when $\beta \gg 0$, which is just the region of the outer edges of a galaxy. We call this the '**MoG Radius**' (**Mo**dified **G**ravity Radius) in Planck units:

$$r_{MoG} = \frac{\sqrt{m}}{\sqrt[4]{\Lambda}}$$

or $r_{MoG} = \frac{\sqrt{mG}}{c\sqrt[4]{\Lambda}}$ in SI units. This radius is not particular to the Lagrangian we chose but is the same for *any* modified gravity theory in which can be formerly expanded out as a power series of contracted curvature tensors with coefficients of the order of Λ^{-n}. The MoG Radius is one of the three 'special' radii in physics for any object, the other two being it's de-Broglie radius and the Schwarzschild radius. We shall see later that important physics occurs in the regions where these radii coincide.

In Planck units, where mass of the sun is $m_\odot \approx 10^{38}$ and the distance to Pluto is $r = 10^{48}$, we get $\beta \approx 10^{-6}$ meaning modified gravity has negligible effect in our solar system right out to beyond Pluto.

Beyond this radius the acceleration is approximately proportional to:

$$\phi'(r) \to m\frac{\sqrt[3]{b}}{r^2} \propto \frac{r^{4/3}}{r^2} = \frac{1}{r^{2/3}}$$

thus it decreases from an inverse square law to an inverse two-thirds power law.

The equation departs from usual Newtonian gravity at the same region where MOND-like theories predict.[3] However, we have arrived at this result a different way.

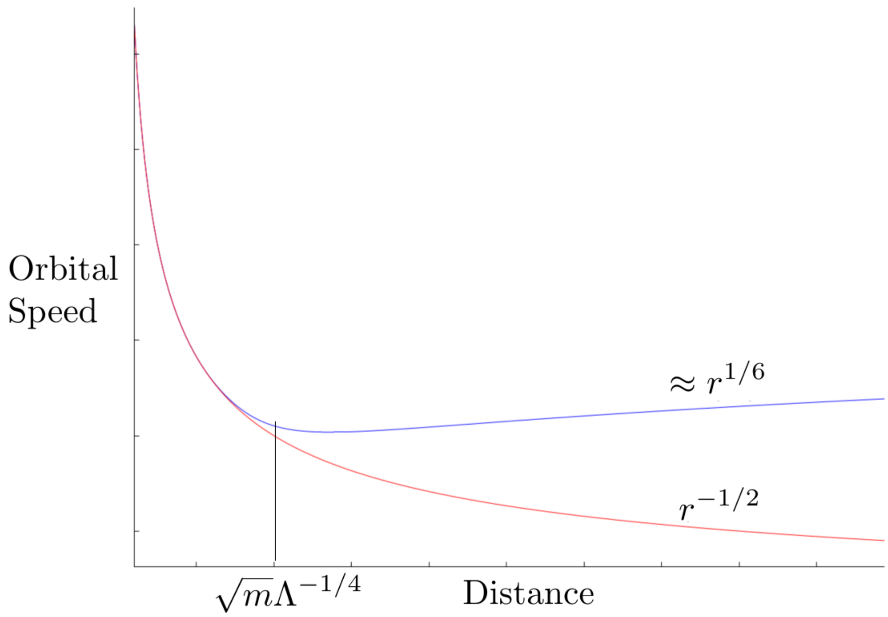
3.1.1 Orbital Speed

The speed of a body orbiting a mass is calculated by $v = \sqrt{r\phi'(r)}$. Past the MoG Radius the speed needed to stay in orbit very gradually increases as

$v \propto (\Lambda m r)^{1/6}$. Interestingly we can calculate that for any mass that can be contained in the universe, even at the distance of the radius of the observable universe this orbital speed is well below the speed of light. This is in contrast to the standard MOND model which predicts orbital speeds will be constant past the MOND radius. This is a feature the particular Lagrangian we chose.

Thus we make the prediction that the orbital speeds of stars in the outer regions follow a $r^{1/6}$ speed law (when subtracting any effects of dark matter). This is much closer to the observed orbital speeds than the original Einstein-Hilbert action predicts.

For a body in empty space the speed needed to stay in orbit differs from the inverse square law according to the graph:



3.1.2 Consequence for Dark Matter search

Taking the example of the Milky Way galaxy. Using the approximate values of $m \approx 10^{40} kg$, (where we removed mass attributed to Dark Matter) the MoG radius is calculated to be around $20,000$ light years. The radius of the Milky Way is thought to be around $100,000$ light years. Thus as we have observed with most galaxies the deviation from Newton's laws occurs from about 20%-80% of the radius and this fits those observations.

We can do the same calculation with a typical galaxy cluster such as the Perseus cluster with a mass of about $10^{44} kg$ (excluding Dark Matter) and find the MoG radius of about $300,000$ light years which is only small fraction of the $11,600,000$ light years so modified gravity will have a huge effect in this regime also when we consider the velocities of galaxies in the outer regions.

From this analysis we predict that if deviations from General Relativity exist then they will precisely show up in the regime of the outer edges of galaxies and galaxy clusters.

This makes the search for dark matter extremely difficult because without the reliability of the inverse square law to galactic distances, any conclusions based on this law cannot be trusted.

3.2 Atomic Scale Calculations

Applying the MoG Radius formula to atoms, we find that the electrons are outside the MoG Radius of the nucleus (from Hydrogen to beyond Uranium) by a factor of thousands. So that to calculate the (negligible) gravitational force of the proton on the electrons we would need the modified potential. The same goes for a pair of atoms in a vacuum.

An interesting exercise is to find the radius of a sphere of water which matches its own MoG radius. Plugging in the values for the density of water, we get a value of the order of $1mm$. Thus the gravitational force between two droplets of rain however close will need the modified gravitational formula.

Of course in both these situations the gravitational force is so small as to be undetectable by any known means hence why we wouldn't have noticed it. But it shows that the prediction is that the inverse square law of gravity breaks down both on the galactic scale as well as the atomic scale.

3.3 Three Important Radii

An object of mass m has three important distances associated with it: The de-Broglie radius ($1/m$), the Schwarzschild radius ($2m$) and now, we can add to that, the MoG Radius ($\sqrt{m}\Lambda^{-1/4}$). It is informative to find the places where these coincide as these are usually places where interesting physics occurs. A simple calculation tells us:

- de-Broglie and Schwarzschild coincide at the Planck mass ($m \approx 1$)
- de-Broglie and MoG Radius coincides at the electron-proton mass scale of around 50MeV ($m \approx \Lambda^{1/6}$)
- Schwarzschild and MoG coincide at ($m \approx \Lambda^{-1/2}$) the mass of the observable Universe.

Thus we find the overlaps occur in the three main mass scales in physics; the quantum gravity scale, the elementary particle scale and the cosmological scale. Here is a summary of the places where the 3 radii collide:

Mass	*Schwarzschild Radius*	*MoG Radius*	*de-Broglie Radius*
$\Lambda^{-1/2}$	Radius of Universe	Radius of Universe	*(Insignificant)*
1	Planck Length	$\approx 0.05mm$	Planck Length
$\Lambda^{1/6}$	*(Insignificant)*	Particle Radius*	Particle Radius*

*The classical radius for an elementary particle of about 50Mev.

It is intriguing that although we have been dealing completely in cosmological and gravitational considerations, the elementary particles energy scale has appeared. The Higgs potential is responsible for the scale of the elementary particle masses. Thus our analysis would not be complete without investigating this sector.

4 Higgs Sector

In this section we will give an argument for why the particle masses are so small compared to the Planck mass. Gravity and the Higgs field have a connection in that gravitational fields act on the total energy of a particle while the Higgs couples via the rest mass of a particle which is simply the energy of a particle at rest. So we should not be surprised if some kind of unification can be achieved.

4.1 Higgs with U(3) Family Symmetry

From the observation of 3 families of elementary particles, the existence of CKM mixing matrices and certain empirical mass relations between the charged leptons, it has been speculated that there may exist a $U(3)$ family symmetry. Thus we need another assumption to add to our list:

- The Higgs sector has a U(3) family symmetry

The effect of this symmetry is that we should consider a Higgs field as a $3 \times \overline{3}$ Hermitian matrix, Φ_{IJ}. We want the Lagrangian to be invariant under $U(3)$ rotations of this matrix $\Phi \to M^{-1}\Phi M$ where $M \in U(3)$. This means, for example, all traces of Φ^n are invariant.

The mass term of the 3 families of charged leptons, for example, becomes $\Phi^{IJ}\overline{\psi}_I\psi_J$. Φ can be decomposed into $U^\dagger M U$, where M is a diagonal matrix of real eigenvalues which give the masses of the 3 leptons and U will contribute to the family mixing matrices when we consider other triplets.

Let us postulate that we can get the Higgs action by a symbolic substitution $R \to R + \partial^\mu\Phi\partial_\mu\Phi + \Phi^n$ then stripping away the kinetic terms we arrive at a potential of the form:

$$V(\Phi) = \sqrt{\Lambda^2 + c_0\Lambda\Phi^n + \Phi^{2n}}$$

where Φ^n symbolically stands for some combination of traces of the Φ matrices which we write as $[\Phi]$. The argument doesn't depend on this exact form of the potential, only that it's constant term is Λ and that the terms grow in powers of Λ^{-1} to match the gravitational sector.

To work out the eigenvalues of a 3×3 matrix it is enough to know the three values of $[\Phi]$, $[\Phi^2]$ and $[\Phi^3]$ where [..] stands for the trace. Therefore we can define a potential which is minimised when these are set to certain values. Let us define the useful function: $a_n = (m_1^n + m_2^n + m_3^n)^{1/n}$. (In this language the empirical Koide correspondence for charged leptons is written as $a_1 \approx \frac{2}{3}a_{1/2}$) Thus (one of many) potential functions could be the 12th degree:

$$V(\Phi)^2 = ([\Phi]^6 - a_1^6)^2 + ([\Phi^2]^3 - a_2^6)^2 + ([\Phi^3]^2 - a_3^6)^2$$

which is of the same form and as above provided we set $a_1^{12} + a_2^{12} + a_3^{12} \approx \Lambda^2$. And this gives masses in the range $m \approx \Lambda^{1/6}$ in Planck units which is of the order of about $50MeV$. This happens to be close to the geometric mean of electron and proton masses or the 3 charged lepton masses.

Another candidate is the 6th degree Higgs potential:

$$V(\Phi)^2 = ([\Phi]^3 - a_1^3)^2 + ([\Phi^2][\Phi] - a_2^2 a_1)^2 + ([\Phi^3] - a_3^3)^2$$

This one unfortunately predicts masses in the range $\Lambda^{1/3}$ which, although a lot smaller than the Planck mass, is far too big for the elementary particle range. This one can still be made to work if we consider $\Phi = \phi^\dagger \phi$ with the mass term $\overline{\psi}\phi\psi$.

Different arguments could suggest different potentials but in general the mass scale will be of the form $\Lambda^{1/n}$, $n \geq 3$. However, the point of the argument is to show the naturalness of the smallness of the particle masses compared to the 'natural' Planck scale. We find $n = 6$ happens to be a good fit.

Thus in conclusion, we can say that the reason why the particle masses are much smaller than the Planck mass and in the order of some inverse power of the Cosmological constant is related to the number of families. The more families, the more powers of Φ are needed in the Higgs potential to uniquely determine the masses and mixing angles, and consequently, the more the masses approach the Planck mass.

4.2 Quarks and Neutrinos

The argument has so far been restricted to the charged leptons, as these are the particles we have most data on. Quark mass is measured indirectly and neutrino masses also inferred indirectly from solar experiments. The quark masses are not vastly different from the lepton masses (e.g. the top quark is only 2 orders of magnitude above the tau-lepton, which is not very much in comparison to the 120 orders of magnitude for Λ in Planck units.) The neutrino masses on the other hand may be 4 or more orders of magnitude below the electron mass which is more of a mystery.

In a similar manner to the above we can postulate $3 \times \overline{3}$ complex Higgs field for each of the charge groups: charged leptons, neutrinos, up-quark family, down-quark family: E^{ab}, V^{ab}, U^{ab}, D^{ab}. A potential involving each of the Higgs separately will give masses to the particles. To fix the mixing angles, the potential must also contain cross-terms such as $[UD]$, $[U^2D]$ and $[UDE]$. These fix values for the absolute values of the CKM matrix, the PMNS matrix, and a hypothetical XY matrix which is the mixing matrix for interactions involving the hypothetical X and Y bosons converting leptons to quarks. There are enough combinations just considering cubic terms to fix all these values as expectation values of a Higgs potential.

The general argument is that we should expect particle masses to be in a scale $\Lambda^{1/n}$ for some $3 \leq n \leq 6$ still holds.

We note that using this scheme means there is now an $U(3)$ family symmetry broken by the Higgs potential. But it currently doesn't give any additional information about the particle mass ratios since we have deliberately constructed the Higgs potential to give the known values.

The explanation for there being 4 different mass matrices and their exact values can only be found through some more general broken symmetry on the charges as $SU(5)$ or $O(10)$ for example. Combining both the family and gauge symmetries in one Higgs potential is a very hard problem and probably is futile using the bottom up approach.

Thus, the 'mystery' of the different particle mass ratios and mixing angles is now transferred to the mystery of the form of the Higgs potential. One can still use anthropic arguments as to why the Higgs potential has the form it does, but this seems unlikely that anthropic arguments should only apply to the scalar fields in the theory.

4.3 Charge Families and Mixing Matrices

The previous potential was for a single charge type, e.g. charged leptons (-1). We can include the other charge types: up-quarks $(+\frac{2}{3})$, down-quarks $(-\frac{1}{3})$ and neutrinos (0) in a similar potential without increasing the power of the Higgs. Thus we can form the following potential, for example with $\Phi_a = \{V, D, U, E\}$. With the index $a = 3|Q| \in \{0, 1, 2, 3\}$:

$$V(\Phi)^2 = \sum_{abc} {}^{([\Phi_a\Phi_b\Phi_c]-\alpha_{abc})^2+([\Phi_a\Phi_b][\Phi_c]-\alpha_{ab}\alpha_c)^2}_{+([\Phi_a][\Phi_b][\Phi_c]-\alpha_a\alpha_b\alpha_c)^2}$$

Again, when expanded out this is in the general form $\approx \Phi^6 - \Lambda\Phi^3 + \Lambda^2$

All the masses of the fermions and mixing angles are thus encoded in the values of $\{\alpha_{abc}, \alpha_{ab}, \alpha_a\}$. (34 values in total allowing for symmetries, 12 of which we know the value of experimentally). In the approximation of zero neutrino masses $\Phi_0 \approx 0$ this leaves 19 values (of which we know 14 of them).

$$\alpha_a = \sum_i m_{a,i} \quad , \quad \alpha_{ab} = \sum_{ij} |C^{ij}_{ab}|^2 m_{a,i} m_{b,j}$$

where C_{ab} is the mixing matrix from charge-group a to group b. When $a = b$ it is the identity. It satisfies $C_{ab}C_{bc} = C_{ac}$. C_{03} is the PMNS matrix and C_{12} is the CKM matrix. A third independent mixing matrix we could specify is C_{20} can call that the XY mixing matrix since it would only be relevant in interactions with hypothetical X and Y bosons which can change leptons to quarks.

Since all the constants are functions of the charge, it is likely they come from vacuum expectation values (v.e.v's) of another Higgs field. Conversely a Higgs which has both flavour and family quantum numbers could serve both roles.

This analysis confirms that this form of the Higgs potential is compatible with the Standard Model.

4.4 Yang-Mills Fields

For the Yang-Mills fields which include electromagnetism, we know we can create them from any gravitational theory by incorporating additional curled up dimensions. Thus we can immediately include the electromagnetic tensor as $R \to R + F_{\mu\nu}F^{\mu\nu}$ and something more complicated for the $\mathcal{R}^2$ term. When gravity is negligible we can approximate this with a Lagrangian of the form $L = \sqrt{-g}\sqrt{\Lambda^2 + (F_{\mu\nu}F^{\mu\nu} + A_\mu j^\mu)^2}$ for example. (The precise form is not important.) This gives us a slight deviation from the inverse square law for electromagnetism. Unfortunately this will be undetectable for all known experiments since the effect will only deviate on scales comparable to the radius of the Universe. It is already hopeless for the electric field since matter is on average electrically neutral so the only hope for experiment would have been measuring the effect of long range magnetic fields for example from the sun.

5 Experimental Tests

We have seen that experimental tests of the deviation of the inverse square law of electromagnetism are all but hopeless due to the distances involved. The experimental tests for the gravitational part would involve measuring the speeds of stars in the outer edge of the galaxy. (But this is a post-diction as we already observed this phenomena and it is difficult to untangle this from the dark matter hypothesis). We should note that arguments that rely on vanilla General Relativity to prove the existence of dark matter should be viewed carefully as this is precisely the region at which we expect General Relativity to break down. Thus we expect in this region that there would be effects both from modification of GR *and* possible dark matter which makes untangling this very difficult.

The most promising experimental test would be to observe bodies orbiting the sun at a radius of about 7000AU, either by sending a probe or detecting a distant orbiting body.

In SI units of metres and kg, we have the surprising coincidence that a noticeable deviation from GR occurs at a radius at the order of $\sqrt{M}$ where M is the mass in kg and the conversion factor is $m/\sqrt{kg}$. This lucky coincidence in numbers means that the theory might be able to to be tested on human scales. The coincidence occurs because $1kg/m^2$ in Planck units is very close in order to $\sqrt{\Lambda}$ in Planck units. Unlike the other ‘coincidences’ this is pure chance as metres and kilograms have no relation to each other. Thus, for example, we note the deviation from the inverse square law of gravity for a pair of $10kg$ masses is expected to occur below the metre scale.

5.1 Binary Stars

To observe the effects of modified gravity beyond the MoG radius outside the clutter of galaxies, the best candidate may be widely separated binary stars if there is an independent way to deduce the mass of the stars (such as by observing orbiting planets). One can use the MoG radius formula as a quick check to find good stellar candidates. There has been some recent research into this [4].

5.2 Agreement with Experiment

From the basic assumptions we have found that the cosmological constant and outer galaxy gravity deviations naturally occur. These values have been measured very precisely and agree with the model.

The arguments concerning the Higgs sector suggest the particle masses are decreasing over time so that they are approximately proportional to $T^{-1/3}$. There has been little evidence for this but this is very hard to measure owing to the fact that electron's and quark's rest-masses make up very little of the mass of an atom and also the long time scales involved.

5.3 Possible Gravitational-Higgs Unification

The similarities with the metric $g^{\mu\nu}$ and the complex Higgs matrix Φ^{ab} (such as their power series action and their coupling to mass/energy of various kinds) suggest some stronger unification. In particular it suggests viewing the generation indices a, b as indices into $3 + \overline{3}$ additional bulk dimensions making 10 bulk dimensions in total. Thus relating the number of generations (3) to half the number of extra bulk dimensions (6). The general prediction that we have outlined is that the form of the power series for the gravitational sector should match, in general form, to that of the Higgs potential. So as particle experiments tell us more about the Higgs sector this should also tell us something about the gravitational sector.

6 Conclusion

From basic assumptions we have calculated the value of the Cosmological Constant, and shown that, provided the series converges to a function consistent with General Relativity, then the divergence from GR will occur precisely at the place where galaxy rotation curves deviate from GR (assuming no dark matter).

Hence, it shouldn't be considered a mystery why the cosmological constant has the value it does. Nor should it be surprising that the Einstein gravitational equations break down at precisely the point where $\Lambda r^4 > m^2$.

Lastly we considered the Higgs sector to be of the same basic form and from that we concluded that the mass scale of the elementary particles should be some inverse power of the Cosmological Constant in Planck units. Specifically for 3 families we arrived at a mass scale of $\Lambda^{1/6}$ which gives a good fit.

6.1 Historical Note

Paul Dirac (1902-1984) postulated his Large Number hypothesis in 1937. The galactic rotation anomaly was discovered in the 1960's. At around that time estimates of the age of the Universe settled on a value of about 13.7 billion years. The accelerated expansion of the Universe was discovered in 1998.

6.2 Epilogue

In conclusion, we must stop viewing the Cosmological Constant as an annoyance [7] whose value is either too big (because it is non-zero) or too small (because is it not the Planck mass) but just right - and as a fundamental clue that could guide us to a more complete action for the universe.

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