Computational Neuroscience
Assignment 1

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Problem 1

Given the following data: $A = 0.025mm^2$, $C_m = 10\frac{nF}{mm^2}$, $R_m = 1M\Omega$ and $E = -70mV$.

- The total membrane capacitance of a neuron C_m is given by:

$$C_m = A * c_m = 0.025mm^2 * 10\frac{nF}{mm^2} = 0.25nF \tag{1}$$

- The total membrane resistance R_m is given by:

$$R_m = \frac{r_m}{A} = \frac{1M\Omega}{0.025mm^2} = 40M\Omega \tag{2}$$

- The membrane time constant τ_m is given by:

$$\tau_m = R_m * C_m = 40M\Omega * 0.25nF = 10ms \tag{3}$$

- The current required for holding the cell at a potential of $-65mV$ is, based on Ohm law:

$$I = \frac{V - E}{R_m} = \frac{-65mV + 70mV}{40M\Omega} = 0.125nA \tag{4}$$

- The time needed for a neuron to reach a potential of $-67mV$, starting from time $t = 0$, and injecting the above computed current can be computed using the solution of the differential equation of the integrate and fire model as:

$$V(t) = V + (V_0 - V) * e^{-\frac{t}{\tau}} \tag{5}$$

  Which solving for t results in:

$$t = -\tau * ln(\frac{V(t) - V}{V_0 - V}) = -10ms * ln(\frac{-67mV + 65mV}{-70mV + 65mV}) = 9.16ms \tag{6}$$

Problem 2

An integrate and fire model (IF) was generated in Python by integrating with Euler method the following differential equation:

$$\tau_m * \frac{dV}{dt} = E - V + (R_m * Ie) \tag{7}$$

For the accompanying code see the repository: `https://github.com/elequiroli/Computational_Neuroscience_Course/tree/main/HMW1`. By injecting into the model currents at increasing intensities, we can see the increase in voltage and the spiking activity (Figure 1). An increased injected current lead to an increase in the firing rate of the neuron.

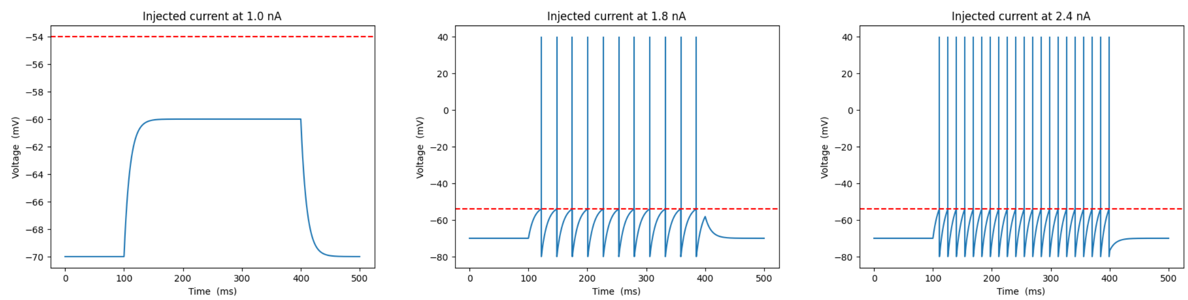

Figure 1: Voltage changes in the IF model at increased current injections

It is important to note that a refractory period was not implemented (set to zero) in the displayed figure 1 and 2. The firing rate of the model was computed at increased current injection and computed as $fr = N_{spikes}/T$ and compared with the theoretical calculations by employing the following formula:

$$r_{isi} = (\tau_m * ln(\frac{R_m Ie + E - V_{reset}}{R_m Ie + E - V_{thr}}))^{-1} \tag{8}$$

In the Figure 2 you can see the computed firing rate and the firing rate obtained from the calculation. The plot show absence of spiking in case of injected current less than 1.75 nA. Then the firing rate reaches 40 Hz and grow with increased current injections.

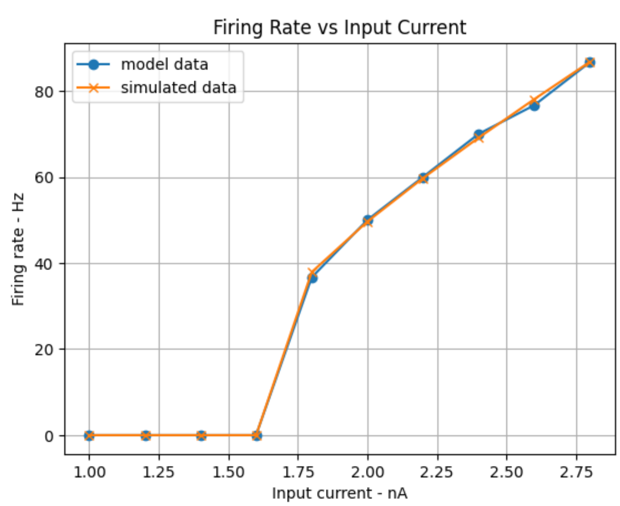

Figure 2: Firing rate obtained from the IF model (blue) and the simulation using the equation 8 (orange)

Problem 3

A Hodgkin and Huxley (HH) model of a neuron was implemented in Python by integrating the HH differential equations, using the Euler method. The complementary code can be found in: `https://github.com/elequiroli/Computational_Neuroscience_Course/tree/main/HMW1`.

- By injecting in the model a current of $I_e/A = 0.2\mu A/mm^2$ we register an increase of the voltage which leads to generations of action potentials, as seen in Figure 3. In the same figure, it is possible to see that during the action potential, the sodium activation variable m rapidly increases, opening sodium channels. Meanwhile, the sodium inactivation variable h slowly decreases, closing sodium channels and helping to end the spike. The potassium activation variable n rises more slowly, opening potassium channels that hyperpolarize the membrane after the peak.

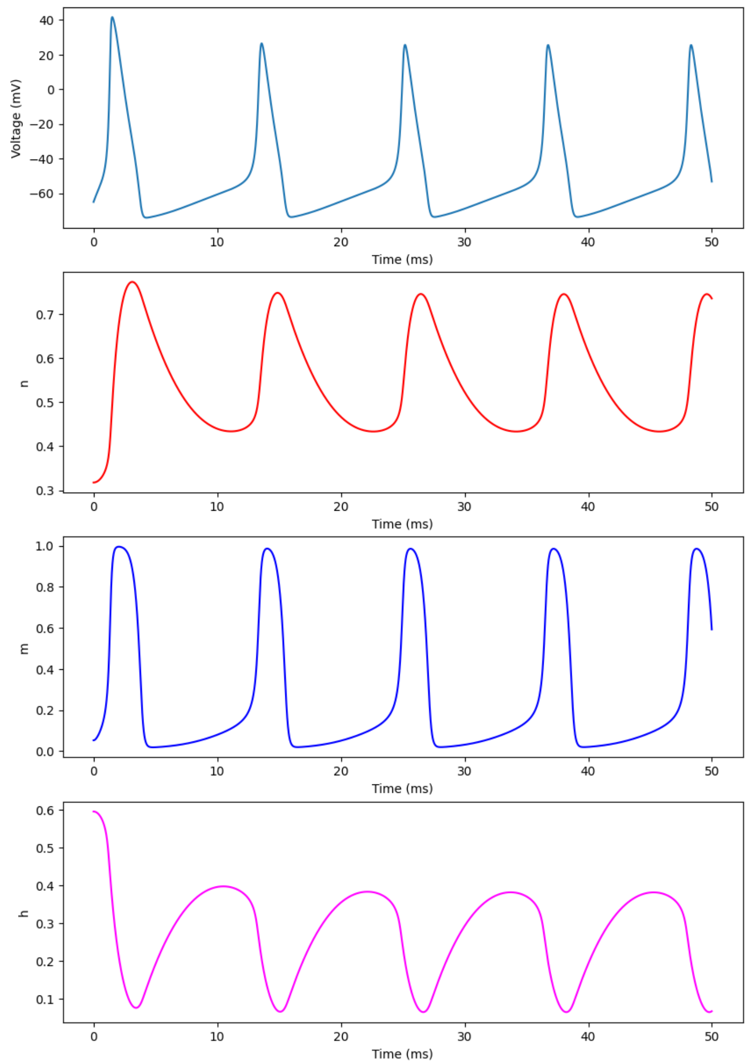

Figure 3: Voltage responses,n,m and h in time

- By injecting increasing currents in the model, we obtain the firing rate curve in figure 4. If you compare it with the IF model firing rate in figure 2, we can say that in the IF model, firing only occurs if the input current is large enough to make the membrane potential reach a fixed threshold. However, once above the threshold, the firing rate increases approximately linearly with current. In contrast, the

Hodgkin-Huxley model exhibits a sharper onset because action potentials depend on the activation of voltage-gated sodium channels, requiring a critical level of depolarization. Moreover, at higher current injection, the curve does not have a linear behavior.

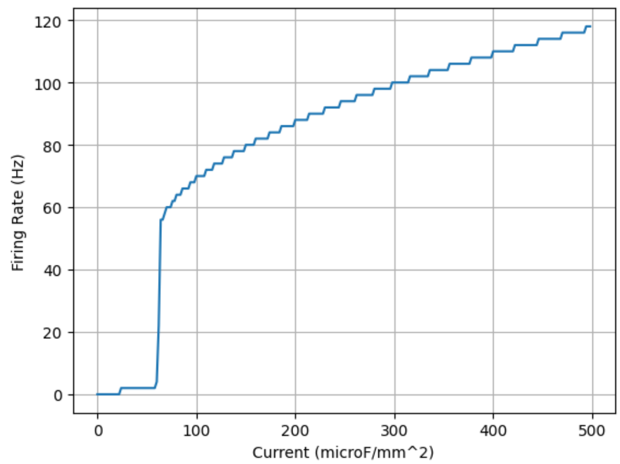

Figure 4: Firing rate curve for the HH model

- By injecting in the model a current of $I_e/A = -0.05\mu A/mm^2$ for 5 ms, the membrane potential hyperpolarize. After the pulse ends and the current returns to zero, the membrane potential increases towards the resting potential. In this case, in Figure 5, we can see a rebound depolarization happening, which generates an action potential (Figure 5).

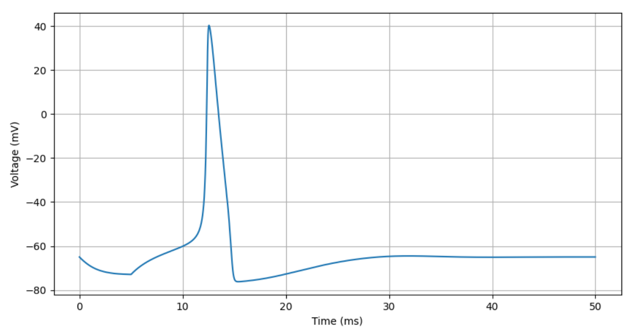

Figure 5: Example of hyperpolarization with rebound depolarization in the HH model