# Atividade Algebra Linear -

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1 - $T(1,0) = 0(x,0) + 0(0,y)$
$T(0,1) = 1(x,0) + 2(0,y)$

Dessa forma temos:

$$[T] = \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix}$$

A verificação de que $\lambda = 2$ $T(v) = \lambda v$, se da:

$$\begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 2 \begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow y = 2x \quad (x, 2x) = v$$
$$\Rightarrow 2y = 2y \quad x(1,2)$$

Portanto é verdadeiro.

2. $T: R^2 \to R^2$ tal que $T(x,y) = (2y, x)$

$T(1,0) = (0,1) = 0(1,0) + 1(0,1)$
$T(1,0) = (2,0) = 2(1,0) + 0(0,1)$

$[T] = \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}$ Sabendo que $\det([T] - \lambda I) = 0$
Visto que $\lambda I = \lambda$ pelas propriedades

$\begin{vmatrix} 0-\lambda & 2 \\ 1 & 0-\lambda \end{vmatrix} = \lambda^2 - 2 = 0$ dado que $\lambda = \pm 2^{1/2}$

$Tv = \lambda v$ (A matriz T (Transformação) multiplica v)

$\Rightarrow$ Para $\lambda_1 = -2^{1/2}$ $\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = -2^{1/2} \begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow \begin{matrix} 2y = -2^{1/2} x \\ x = -2^{1/2} y \end{matrix}$

Assim, a combinação geradora do espaço V

$(-2^{1/2}, y) = y(-\sqrt{2}, 1)$

É o autovetor

$v_1 = (-\sqrt{2}, 1)$

Dessa forma o V é gerado por $v_1$, isto pois

$V_{-\sqrt{2}} = [(-\sqrt{2}, 1)]$ se $\lambda = -\sqrt{2}$

$\Rightarrow$ Para $\lambda_2 = 2^{1/2}$

$$\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 2^{1/2} \begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow \begin{matrix} 2y = 2^{1/2} x \\ x = 2^{1/2} y \end{matrix}$$

Assim, a combinação geradora do espaço V

$(\sqrt{2}y, y) = y(\sqrt{2}, 1)$

É o autovetor

$v_2 = (\sqrt{2}, 1)$

$V_{\sqrt{2}} = [(\sqrt{2}, 1)]$ se $\lambda = \sqrt{2}$

3. $T(1,0) = (1,2) = 1(1,0) + 2(0,1)$
$T(0,1) = (1,1) = 1(1,0) + 1(0,1)$

$$[T] = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$$ Autovalores. $\det([T] - \lambda I) = 0$

$$\begin{bmatrix} 1-\lambda & 1 \\ 2 & 1-\lambda \end{bmatrix} = 0 \Rightarrow (1-\lambda)^2 - 2 = 0 \Rightarrow \lambda^2 - 2\lambda - 1 = 0$$

$$\frac{-b \pm \sqrt{\Delta}}{2a} \Rightarrow \frac{2 \pm 2\sqrt{2}}{2} < \begin{matrix} 1+\sqrt{2} \\ 1-\sqrt{2} \end{matrix} \Rightarrow Tv = \lambda v$$

$\Rightarrow$ Para $\lambda_1 = 1+\sqrt{2}$

$$\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 1+\sqrt{2} \begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow \begin{matrix} x+y = (1+\sqrt{2})x \\ 2x+y = (1+\sqrt{2})y \end{matrix} \Rightarrow$$

$x + y = x + \sqrt{2}x$
$2x + y = y + \sqrt{2}y$

Dessa forma:

$(x, \sqrt{2}x) = x(1, \sqrt{2}) \Rightarrow v_1 = (1, \sqrt{2})$

$V_{1+\sqrt{2}} = [(1, \sqrt{2})]$

$\Rightarrow$ Para $\lambda_2 = 1 - \sqrt{2}$

$x + y = (1-\sqrt{2})x \Rightarrow y = -\sqrt{2}x$
$2x + y = (1-\sqrt{2})y \Rightarrow 2x = -\sqrt{2}y$

Dessa forma:

$$(x, -\sqrt{2}x) \quad x(1, -\sqrt{2}) \Rightarrow v_1 = (1, -\sqrt{2})$$

$$V_{1-\sqrt{2}} = [(1, -\sqrt{2})]$$

4) $T(1,0,0) = (1,1,2) = 1(1,0,0) + 1(0,1,0) + 2(0,0,1)$
$T(0,1,0) = (1,-1,1) = 1(1,0,0) - 1(0,1,0) + 1(0,0,1)$
$T(0,0,1) = (0,2,-1) = 0(1,0,0) + 2(0,1,0) - 1(0,0,1)$

$$[T] = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 2 \\ 2 & 1 & 1 \end{bmatrix}$$

$$\det([T] - \lambda I) = 0 \quad \begin{bmatrix} 1-\lambda & 1 & 0 \\ 1 & -1-\lambda & 2 \\ 2 & 1 & -1-\lambda \end{bmatrix} = 0$$

$$(-1-\lambda)(-1)^{1+1} \begin{bmatrix} -1-\lambda & 2 \\ 1 & -1-\lambda \end{bmatrix} + 1(-1)^{1+2} \begin{bmatrix} 1 & 2 \\ 2 & -1-\lambda \end{bmatrix} = 0$$

$(1-\lambda)[(-\lambda-1)^2 - 2] - [(-\lambda-1) - 4] = 0 \Rightarrow$
$(1-\lambda)[\lambda^2 + 2\lambda + 1 - 2] + (\lambda + 5) = 0 \Rightarrow$
$(1-\lambda)[\lambda^2 + 2\lambda - 1] + (\lambda + 5) = 0 \Rightarrow$
$\lambda^2 + 2\lambda - 1 - \lambda^3 - 2\lambda^2 + \lambda + \lambda + 5 = 0 \Rightarrow$
$-\lambda^3 - \lambda^2 + 4\lambda + 4 = 0 \cdot (-1)$
$\lambda^3 + \lambda^2 - 4\lambda - 4 = 0$

Os divisores de $-4$ são $1, -1, 2, -2, 4$ e $-4$.

$\lambda_1 = -2 \quad \lambda_2 = -1 \quad \lambda_3 = 2$

$\Rightarrow$ Para $\lambda_1 = -2$

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -2 \begin{bmatrix} x \\ y \\ z \end{bmatrix} \Rightarrow \begin{cases} x + y = -2x \\ x - y + 2z = -2y \\ 2x + y - z = -2z \end{cases} \Rightarrow$$

$x = y/3 \rightarrow 2z = -\frac{y}{3} - y \Rightarrow z = -\frac{2}{3}y$

Dessa forma:

$(y/3, y, -2/3 y) = (y, 3y, -2y) = y(1, 3, -2)$

$v_1 = (1, 3, -2)$ e $V_2 = [(1, 3, -2)]$

$\Rightarrow$ Para $\lambda_2 = -1$

$x + y = -x \quad \Rightarrow y = -2x$

$x - y + 2z = -y \quad x - (-2x) + 2z \Rightarrow z = x/2$

$2x + y - z = -z$

Dessa forma:

$(x, -2x, x/2) = (2x, -4x, x) = x(2, -4, 1)$

$v_1 = (2, -4, 1) \quad V_1 = [(2, -4, 1)]$

$\Rightarrow$ Para $\lambda_3 = 2$

$x + y = 2x \quad \Rightarrow y = x$

$x - y + 2z = 2y \quad x - x + 2z = 2x \Rightarrow z = x$

$2x + y - z = 2z$

Dessa forma:

$(x, x, x) = x(1,1,1)$

$v_1 = (1,1,1)$ $V_2 = [(1,1,1)]$

5. $T: P_2 \to P_2$

$T(ax^2+bx+c) = ax^2 + cx + b$

$T(1) = 1x = 0(1) + 1(x) + 0(x^2)$ $\beta\{1, x, x^2\}$

$T(x) = 1 = 1(1) + 0(x) + 0(x^2)$

$T(x^2) = x^2 = 0(1) + 0(x) + 1(x^2)$

$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\det\left(\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}\right) = 0 \Rightarrow \det\left(\begin{bmatrix} 0-\lambda & 1 & 0 \\ 1 & 0-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{bmatrix}\right) = 0$

$\begin{bmatrix} -\lambda & 1 & 0 \\ 1 & -\lambda & 0 \\ 0 & 0 & 1-\lambda \end{bmatrix} \begin{matrix} -\lambda & 1 \\ 1 & -\lambda \\ 0 & 0 \end{matrix}$ $-\lambda \cdot -\lambda \cdot (1-\lambda) - 1 + \lambda = \lambda^2 \cdot (1-\lambda) - 1 + \lambda$

$\lambda^2 - \lambda^3 - 1 + \lambda = -\lambda^3 + \lambda^2 + \lambda - 1$

$$\begin{array}{l|l} -\lambda^3 + \lambda^2 + \lambda - 1 & -\lambda + 1 \\ \hline \lambda^3 - \lambda^2 & \lambda^2 - 1 \\ \quad \lambda - 1 & \\ \quad \underline{-\lambda + 1} & \\ \qquad 0 & \end{array}$$

$(-\lambda + 1)(\lambda^2 - 1) = 0 = 1$ $\lambda = \pm 1$

$(-1+1) \cdot (1^2 - 1) = 0 \Rightarrow 0 = 0$

$(1+1) \cdot ((-1)^2 - 1) = 0$

$\Rightarrow$ Para $\lambda = 1$

Faremos pela ordem da base.

$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c \\ b \\ a \end{bmatrix} = \begin{bmatrix} c \\ b \\ a \end{bmatrix} \Rightarrow$ $b = c$, $c = b$, $a = a$

Dessa forma

$v_1 = ax^2 + bx + b$

Para $\lambda = -1$

$b = -c \Rightarrow c = -b$
$c = -b$
$a = -a \Rightarrow a = 0$

$v_1 = bx - b$

6 - $T\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = 1\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + 0\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + 0\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + 0\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

$T\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = 0\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + 0\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + 1\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + 0\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

$T\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = 0\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + 1\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + 0\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + 0\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

$T\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = 0\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + 0\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + 0\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + 1\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

$$[T] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\det([T] - \lambda I) = 0$

$$\begin{bmatrix} 1-\lambda & 0 & 0 & 0 \\ 0 & -\lambda & 1 & 0 \\ 0 & 1 & -\lambda & 0 \\ 0 & 0 & 0 & 1-\lambda \end{bmatrix} \quad (1-\lambda)\begin{bmatrix} 1-\lambda & 0 & 0 \\ 0 & -\lambda & 1 \\ 0 & 1 & \lambda \end{bmatrix}$$

$$(1-\lambda)(1-\lambda)(\lambda^2-1)=0$$

$$(1-\lambda)\begin{bmatrix} -\lambda & 1 \\ 1 & -\lambda \end{bmatrix} \qquad \lambda_1 = 1 \quad \lambda_2 = -1$$

$$(1-\lambda)(\lambda^2-1)=0$$

$\Rightarrow$ Para $\lambda_1 = 1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow \begin{matrix} x = x \\ z = y \\ y = z \\ w = w \end{matrix}$$

$(x, z, z, w) = x(1,0,0,0) + z(0,1,1,0) + w(0,0,0,1)$

$v_1 = (1,0,0,0)(0,1,1,0)(0,0,0,1)$

$V_1 = [(1,0,0,0)(0,1,1,0)(0,0,0,1)]$

$\Rightarrow$ Para $\lambda_2 = -1$

$x = -x \qquad 2x = 0$

$z = -y \qquad y = -z$

$y = -z$

$w = -w \qquad 2w = 0$

Dessa forma:

$(0, -z, z, 0) = z(0,-1,1,0)$

$v_1 = (0, -1, 1, 0)$
$V_1 = [(0, -1, 1, 0)]$

Assim o Autovetor é $(0, -1, 1, 0)$