17 v) $\cot g\, x = \dfrac{\cos x}{\operatorname{sen} x}$

$$\lim_{x\to 0} \cot g(2x)\cot g\left(\frac{\pi}{2}-x\right) = \lim_{x\to 0}\left(\frac{\cos(2x)}{\operatorname{sen}(2x)}\right)\cdot\left(\frac{\cos\frac{\pi}{2}-x}{\operatorname{sen}\frac{\pi}{2}-x}\right) \Rightarrow$$

$$* \lim_{x\to 0}\left(\frac{\cos\frac{\pi}{2}-x}{\operatorname{sen}\frac{\pi}{2}-x}\right) = \lim_{x\to 0} \frac{\overset{0}{\cancel{\cos\frac{\pi}{2}}}\cos x + \overset{1}{\cancel{\operatorname{sen}\frac{\pi}{2}}}\cdot\operatorname{sen} x}{\underset{1}{\cancel{\operatorname{sen}\frac{\pi}{2}}}\cos x - \operatorname{sen} x\cdot\underset{0}{\cancel{\cos\frac{\pi}{2}}}} =$$

$$* \left(\frac{\operatorname{sen} x}{\cos x}\right) \Rightarrow \lim_{x\to 0}\left(\frac{\cos 2x}{\operatorname{sen} 2x}\right)\cdot\left(\frac{\operatorname{sen} x}{\cos x}\right) \Rightarrow$$

$$\lim_{x\to 0}\left(\frac{\cos 2x}{2x\,\frac{\operatorname{sen} 2x}{2x}}\right)\cdot\left(\frac{\operatorname{sen} x}{\cos x}\right) \Rightarrow \lim_{x\to 0}\left(\frac{\cos 2x}{\frac{\operatorname{sen} 2x}{2x}}\right)\cdot\left(\frac{\operatorname{sen} x}{2x\cos x}\right)$$

$$\lim_{x\to 0}\left(\frac{\cos 2x}{\frac{\operatorname{sen} 2x}{2x}}\right)\left(\frac{\operatorname{sen} x}{x}\right)\cdot\left(\frac{1}{2\cdot\cos x}\right) \Rightarrow$$

$$\frac{\cos 0}{1}\cdot 1\cdot\frac{1}{2} = \frac{1}{2}$$

18 a) $\lim_{x\to+\infty} \left(\dfrac{x^2+1}{x^2-3}\right)^{x^2}$

$* \lim_{x\to\pm\infty} \left(1+\dfrac{1}{x}\right)^{x} = e$

$t = \underline{x^2-3} \Rightarrow \underline{x^2 = t+3} = \begin{cases} x\to+\infty \\ t\to+\infty \end{cases}$

$\lim_{x\to+\infty} \left(\dfrac{x^2+1}{t}\right)^{x^2}$

$\lim_{t\to+\infty} \left(\dfrac{t+3+1}{t}\right)^{t+3} \Rightarrow \lim_{t\to+\infty} \left(\dfrac{t+4}{t}\right)^{t+3} \Rightarrow$

$\lim_{t\to+\infty} \left(\dfrac{\cancel{t}}{\cancel{t}}+\dfrac{4}{t}\right)^{t+3} \Rightarrow \lim_{x\to+\infty} \left(1+\dfrac{4}{t}\right)^{t+3} \Rightarrow$

$\lim_{t\to+\infty} \left(1+\dfrac{4}{t}\right)^{t\cdot\frac{t+3}{t}} \Rightarrow \lim_{t\to+\infty} \left[\left(1+\dfrac{4}{t}\right)^{t}\right]^{\lim_{t\to+\infty}\left(\frac{t+3}{t}\right)}$

$* \lim_{t\to+\infty} \left[\left(1+m\right)^{\frac{4}{m}}\right] \quad m = \dfrac{+4}{t^+} \quad \begin{cases} t\to+\infty \\ m\to 0^+ \end{cases} \Rightarrow {}^{*}\lim_{m\to 0^+} \left(\left(1+m\right)^{\frac{1}{m}}\right)^{4} = e^4$

$t = \dfrac{4}{m}$

$\left(e^4\right)^{\lim_{t\to+\infty}\left(\frac{t+3}{t}\right)} \Rightarrow * \lim_{t\to+\infty} \left(\dfrac{t+3}{t}\right) \Rightarrow \lim_{t\to+\infty} \dfrac{\cancel{t}\cdot\left(1+\frac{3}{t}\right)}{\cancel{t}} \Rightarrow$

$\lim_{t\to+\infty} \left(1+\dfrac{3}{t}\right) \Rightarrow \left(1+0\right) = \underline{\underline{1}}$

$\left(e^4\right)^1 = \underline{\underline{e^4}}$

19(a) $\lim_{x\to 0} \frac{\log(1+3x)}{x} \Rightarrow \lim_{x\to 0} \frac{1}{x} \log(1+3x) \Rightarrow$

$\lim_{x\to 0} \log(1+3x)^{\frac{1}{x}} \Rightarrow 3x = t \begin{cases} x \to 0 \\ t \to 0 \end{cases}$

$\log \lim_{t\to 0} (1+t)^{\frac{1}{t/3}} \Rightarrow * \frac{1}{t/3} = \frac{1}{1} \cdot \frac{3}{t} = \frac{3}{t}$

$\log \lim_{t\to 0} [(1+t)^{\frac{1}{t}}]^3 \Rightarrow \log(e)^3 = \underline{\log e^3}$

20d, $f(x) = \begin{cases} \frac{1-\cos(x)}{sen(x)}, se\ x \neq 0 \\ 1, \quad se\ x = 0 \end{cases}$ no ponto $x = 0$

$f(0) = 1$ ✓

$\lim_{x\to 0} \frac{1-\cos(x)}{sen(x)} = \frac{0}{0} \Rightarrow \lim_{x\to 0} tg\left(\frac{x}{2}\right) \Rightarrow tg\left(\lim_{x\to 0} \left(\frac{x}{2}\right)\right)$

$tg\left(\frac{0}{2}\right) \Rightarrow tg(0) \Rightarrow 0 \qquad f(0) \neq \lim_{x\to 0} f(x)$

A função não é contínua no ponto 0

Pois o $\lim_{x\to 0} f(x)$, é diferente de $f(0)$

20 g) $f(x) = \dfrac{2}{3x^2 + x^3 - x - 3}$ no ponto $x = -3$

$$f(-3) = \frac{2}{3(-3)^2 + (-3)^3 - (-3) - 3} = \frac{2}{\cancel{27} - \cancel{27} + \cancel{3} - \cancel{3}} = \frac{2}{0}$$

$f(x) = \cancel{\exists}$ O valor -3 não é definido para o domínio dessa função.

dessa forma ela não é contínua

$-3 \notin D(f)$. Não é contínua.