

1. $f(x) = \frac{x}{\sqrt{1-3x}}$ Pela derivada do quociente

$$h'(x) = \frac{f'(x) \cdot g(x) + f(x) \cdot g'(x)}{(g(x))^2}$$

$$h'(x) = \frac{1 \cdot \sqrt{1-3x} - x \left(\frac{-1}{2\sqrt{1-3x}} \right) \cdot (-3)}{(g(x))^2}$$

$$* \frac{d}{dx} [g(x)]^n = n[g(x)]^{n-1} \cdot g'(x)$$

$$(1-3x)^{\frac{1}{2}} \rightarrow \frac{1}{2} \cdot (1-3x)^{-\frac{1}{2}} \cdot (0-3) = \frac{(1-3x)^{-\frac{1}{2}} \cdot (-3)}{2}$$

$$* \frac{1}{2\sqrt{1-3x}} \cdot (-3)$$

$$h'(x) = \sqrt{1-3x} - x \left(\frac{1}{2\sqrt{1-3x}} \cdot (-3) \right)$$

$$h'(x) = \sqrt{1-3x} - \left(\frac{-3x}{2\sqrt{1-3x}} \right) = \frac{2(\sqrt{1-3x})^2 + 3x}{2\sqrt{1-3x}}$$

$$h'(x) = \frac{2-6x+3x}{2\sqrt{1-3x}} = \frac{2-3x}{2\sqrt{1-3x}} \cdot \frac{1}{(1-3x)}$$

$$h'(x) = \frac{2-3x}{2(\sqrt{1-3x}) \cdot (1-3x)}$$

$$2. g(t) = \frac{f(t^4)}{2[f(t^4)]^2 + 1} \quad / \quad f(1) = f'(1) = 1 \quad / \quad t = 1$$

$$u = t^4 \Rightarrow u' = 4t^3 \Rightarrow h(t) = 2[f(u)]^2 + 1$$

$$\frac{d}{dx} [2[f(u)]^2 + 1] = 2 \cdot (f(u))^2 + 0$$

$$\text{usando } \frac{df}{dx} = \frac{df}{du} \cdot \frac{du}{dx} \Rightarrow$$

$$4f(u) \cdot f'(u) \cdot 4t^3 = 4f(t^4) \cdot f'(t^4) \cdot 4t^3$$

$$2(f(u))^2 = 4(f(u) \cdot f'(u) \cdot (2(2(f(u))^2 + 1) - (f(u) \cdot 4f(u) \cdot f'(u) \cdot 4t^3)))$$

(2(f(t^4))^2 + 1) = (quociente)

$$g'(t) = \frac{f'(t^4) \cdot 4t^3 \cdot (2(f(t^4))^2 + 1) - (f(t^4) \cdot 4f(t^4) \cdot f'(t^4) \cdot 4t^3)}{(2(f(t^4))^2 + 1)^2}$$

$$g'(t) = \frac{4(2+1) - (16)}{9} = \frac{72 - 16}{9} = \frac{-4}{9}$$

$$3. f(x) = x^3 + ax^2 + bx + c \quad / \quad g(x) = 5 - 3\sin(x)$$

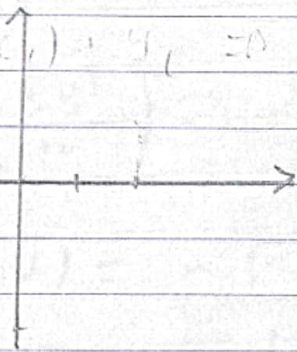
$$g(x) = 5 - 3\sin(x) \quad : \quad g(0) = 5 - 3 \cdot \sin(0) = 5$$

$$g(0 + \Delta x) = 5 - 3 \cdot \sin(\Delta x)$$

$$m(1) = \lim_{\Delta x \rightarrow 0} \frac{5 - 3 \cdot \sin(\Delta x) - 5}{\Delta x} = \frac{-3 \sin(\Delta x)}{\Delta x}$$

$$m(1) = -3 \lim_{\Delta x \rightarrow 0} \frac{\sin(\Delta x)}{\Delta x} = \frac{0}{1}$$

$$y = m(1) \cdot (x - 0) + y_1 = 0 \cdot (x - 0) + 5$$



$$4a, \quad y \sin(x^2) = x \sin(y^2) - \operatorname{cosec}\left(\frac{8y^4}{\pi}\right) + 1$$

$$\frac{d}{dx} y \sin(x^2) = \frac{d}{dx} x \sin(y^2) - \operatorname{cosec}\left(\frac{8y^4}{\pi}\right)$$

$$\# \frac{d}{dx} y \sin(x^2) = \frac{dy}{dx} \cdot \sin(x^2) + y \cdot (2x \cos(x^2))$$

$$\# \frac{d}{dx} x \sin(y^2) = 1 \cdot \sin(y^2) + x \cdot \left(\cos(y^2) \cdot 2y \frac{dy}{dx} \right)$$

$$\frac{d}{dx} \sin(y^2) = \cos(y^2) \cdot 2y \frac{dy}{dx}$$

$$\# \frac{d}{dx} \operatorname{cosec}\left(\frac{8y^4}{\pi}\right) = -\frac{8y^4}{\pi} \left(\frac{1}{u} \right) \left(\frac{1}{u^2} \right)$$

$$\operatorname{cosec}(u) = -\operatorname{cosec}(u) \cdot \cot(u)$$

$$u' = \frac{8 \cdot y^4 \cdot \pi^{-1}}{\pi}$$

$$\frac{d}{dx} y^4 = \frac{d}{dx} (f(x)^4) = 4(f(x)^3) \cdot (f(x))' = 4 \cdot y^3 \frac{dy}{dx}$$

$$u' = 0 \cdot y^4 \cdot \pi^{-1} - 8 \cdot 4 y^3 \frac{dy}{dx} \cdot \pi^{-1} - (8 \cdot y^4 \cdot (\pi)^{-2})$$

$$u' = -8 \cdot 4 y^3 \frac{dy}{dx} \cdot \pi^{-1} + 8 \cdot y^4 \cdot \pi^{-2}$$

$$\frac{d}{dx} \sin(x^2) = 2x \cos(x^2)$$

$$\frac{dy}{dx} \sin(x^2) + y (2x \cos(x^2)) = \sin(y^2) + x(\cos(y^2)).$$

$$2y \frac{dy}{dx} - \left(-\operatorname{cosec}\left(\frac{8y^4}{\pi}\right) \cotg\left(\frac{8y^4}{\pi}\right) \cdot -84y^3 \frac{dy}{dx} \pi^{-1} + 8y^4 \cdot \pi^{-2} \right)$$

$$\frac{dy}{dx} \sin(x^2) = \sin(y^2) + x(\cos(y^2)) \cdot 2y \frac{dy}{dx} + \operatorname{cosec}\left(\frac{8y^4}{\pi}\right) \cotg\left(\frac{8y^4}{\pi}\right)$$

$$+ 64y^3 \frac{dy}{dx} \cdot \pi^{-1} + 8y^4 \cdot \pi^{-2}$$

$$5 \quad f(x) = \ln \left(\operatorname{arctg} \left(\sqrt{\frac{1-x}{x+1}} \right) \right)$$

$$\operatorname{arctg} \left(\sqrt{\frac{1-x}{x+1}} \right) = g \Rightarrow \frac{d}{dg} (\ln(g)) \cdot \frac{d}{dx} \left(\operatorname{arctg} \left(\sqrt{\frac{1-x}{x+1}} \right) \right)$$

$$\frac{1}{g} \cdot \frac{d}{dx} \left(\operatorname{arctg} \left(\sqrt{\frac{1-x}{x+1}} \right) \right) \Rightarrow h = \sqrt{\frac{1-x}{x+1}} \Rightarrow$$

$$\frac{d}{dh} (\operatorname{arctg}(h)) \cdot \frac{d}{dx} \left(\sqrt{\frac{1-x}{x+1}} \right) \Rightarrow$$

$$\frac{1}{1+h^2} \cdot \frac{1}{2\sqrt{\frac{1-x}{x+1}}} \cdot \frac{-1(x+1) - (1-x)}{(x+1)^2}$$

$$\frac{1}{1+\left(\sqrt{\frac{1-x}{x+1}}\right)^2} \cdot \frac{1}{2\sqrt{\frac{1-x}{x+1}}} \cdot \frac{-1(x+1) - (1-x)}{(x+1)^2}$$

$$\frac{1}{\operatorname{arctg} \left(\sqrt{\frac{1-x}{x+1}} \right)} \cdot \frac{1}{1+\left(\sqrt{\frac{1-x}{x+1}}\right)^2} \cdot \frac{1}{2\sqrt{\frac{1-x}{x+1}}} \cdot \frac{-x-1+1+x}{(x+1)^2}$$

$$\frac{1}{\operatorname{arctg} \left(\sqrt{\frac{1-x}{x+1}} \right)} \cdot \frac{1}{2+\frac{1-x}{x+1}} \cdot \frac{1}{\sqrt{\frac{1-x}{x+1}}} \cdot \frac{1}{(x+1)^2}$$

$$* \frac{x+1+1-x}{x+1} = \frac{2}{x+1}$$

$$5 - \frac{1}{\operatorname{arctg}\left(\sqrt{\frac{1-x}{x+1}}\right)} \cdot \frac{x+1}{2} \cdot \frac{\sqrt{x+1}}{\sqrt{1-x}} \cdot \frac{1}{(x+1)^2}$$

$$- \frac{\sqrt{x+1}}{2 \operatorname{arctg}\left(\sqrt{\frac{1-x}{x+1}}\right) \cdot \sqrt{1-x} \cdot (x+1)}$$

$$6. f(x) = |x^3| \Rightarrow \frac{d}{dx} \sqrt{(x^3)^2} = \sqrt{x^6} \quad (x^{\frac{6}{2}} = 3)$$

$$f'(x) = \frac{1}{2\sqrt{x^6}} \cdot 6x^5 = \frac{1}{|x^3|} \cdot 3x^5 = \frac{3x^5}{|x^3|}$$

$$f''(x) = \left(\frac{3x^5}{\sqrt{(x^3)^2}} \right) = \frac{3x^5}{\sqrt{x^6}} \Rightarrow \text{Quociente}$$

$$f''(x) = \frac{3 \cdot 5x^4 \cdot \sqrt{x^6} - 3x^5 \cdot \frac{1}{2\sqrt{x^6}} \cdot 6x^5}{(\sqrt{x^6})^2}$$

$$f''(x) = \frac{15x^4 \cdot |x^3| - 3x^5 \cdot \frac{3x^5}{|x^3|}}{x^6} \Rightarrow$$

$$f''(x) = \frac{15x^4 \cdot |x^3| - \frac{9x^{10}}{|x^3|}}{x^6} \quad \text{e } |a|^2 = a^2$$

$$f''(x) = \frac{15x^4 \cdot (x^3)^2 - 9x^{10}}{|x^3| \cdot x^6} = \frac{15x^4 \cdot x^6 - 9x^{10}}{|x^3| \cdot x^6}$$

$$f''(x) = \frac{15x^{10} - 9x^{10}}{|x^3| \cdot x^6} = \frac{6x^{10}}{|x^3| \cdot x^6} =$$

$$f''(x) = \frac{6x^4}{|x^3|} \quad \text{e}$$