Prova - Cálculo - Modulo I

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1a $f(-x) = (-x)^2 + 4|-x|$

$f(-x) = x^2 + 4|x|$

módulo de x = módulo de $-x$

Verdadeiro

1b $f(x) = \sec\left(x - \frac{\pi}{2}\right) \Rightarrow \frac{1}{\cos\left(x - \frac{\pi}{2}\right)}$

t	x	y
0	$\pi/2$	1
$\pi/2$	π	0
π	$3\pi/2$	-1
$3\pi/2$	2π	0
2π	$\frac{5\pi}{2}$	1

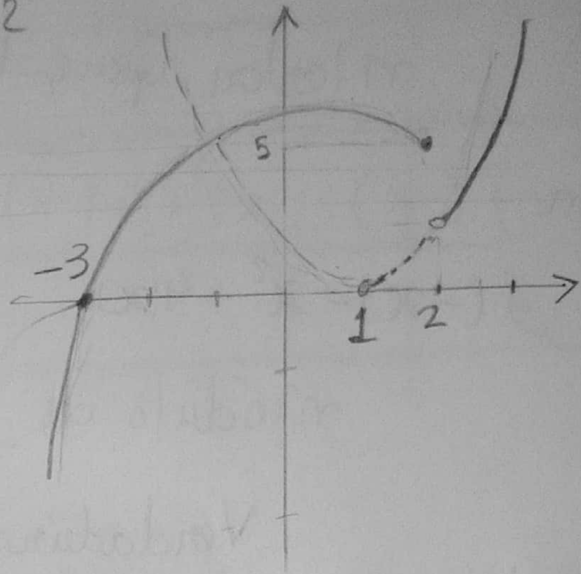
$\cos\left(x - \frac{\pi}{2}\right) \Rightarrow x - \frac{\pi}{2} = t$

$x = t + \frac{\pi}{2}$

$x = \frac{2t + \pi}{2}$

Verdadeiro, pois a função não admite valores nulos no denominador, ou seja $\cos\left(x - \frac{\pi}{2}\right) \neq 0$

1c $f(x) = \begin{cases} 9 - x^2, & x \leq 2 \\ x^2 - 2x + 1, & x > 2 \end{cases}$



$f(x) = 9 - x^2, \quad x \leq 2$

$-x^2 = -9 \cdot (-1)$

$x^2 = 9$

$x = \pm 3 \Rightarrow -3$

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$\Delta = b^2 - 4ac$

$\Delta = 4 - 4$

$\Delta = 0$

$\frac{-b \pm \sqrt{\Delta}}{2 \cdot a} = \frac{2}{2} = 1$

Falso, pois na bijetora cada elemento de x só poderia se ligar a um único y

2. $(m-2)x^2 + 4\cdot(m-2) + m$

$m - 2 < 0$

$m < 2$

$\Delta = b^2 - 4ac$

$\Delta = (4(m-2))^2 - 4\cdot(m-2)\cdot(m)$

$\Delta = (4m-8)^2 - 4m\cdot(m-2)$

$\Delta = (4m-8)\cdot(4m-8) - 4m^2 + 8m$

$\Delta = 16m^2 - 32m - 32m + 64 - 4m^2 + 8m$

$\Delta = 16m^2 - 4m^2 - 32m - 32m + 8m + 64$

$\Delta = 12m^2 - 56m + 64$

$\frac{-\Delta}{4a} = y_v$

$y_v = \frac{-(12m^2 - 56m + 64)}{4(m-2)}$

$14 = \frac{-12m^2 + 56m - 64}{4m - 8}$

$\underline{m = -2}$

Pois para que se possa ter valor máximo, o termo "a" da função necessita ser $a < 0$

$14\cdot(4m-8) = -12m^2 + 56m - 64$

$56m - 112 = -12m^2 + 56m - 64$

$0 = -12m^2 + 56m - 64 - 56m + 112$

$-12m^2 + \cancel{56m} - \cancel{56m} - 64 + 112 = 0$

$-12m^2 + 48 = 0$

$-12m^2 = -48$

$m^2 = \frac{-48}{-12} = 4$

$m = \sqrt{4}$

$m = \pm 2$

3. $f(x) = \begin{cases} -2x^2 + x - 2, & \text{se } x < 0 \\ 2x + 1, & \text{se } x \geq 0 \end{cases}$

$g(x) = \begin{cases} -x^2 + 2, & \text{se } x > -3 \\ 4x - 1, & \text{se } x \leq -3 \end{cases}$

a) $g(f(x)) = -(-2x^2 + x - 2)^2 + 2, \text{ se } -2x^2 + x - 2 > -3$

$g(f(x)) = -((-2x^2 + x - 2) \cdot (-2x^2 + x - 2)) + 2$

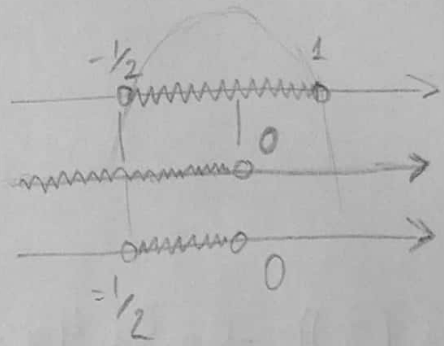
$g(f(x)) = -(+4x^4 - 2x^3 + 4x^2 - 2x^3 + x^2 - 2x + 4x^2 - 2x + 4) + 2$

$g(f(x)) = -(4x^4 - 2x^3 - 2x^3 + 4x^2 + x^2 + 4x^2 - 2x - 2x + 4) + 2$

$g(f(x)) = -(4x^4 - 4x^3 + 9x^2 - 4x + 4) + 2$

$g(f(x)) = -4x^4 + 4x^3 - 9x^2 + 4x - 4 + 2$

$\underline{g(f(x)) = -4x^4 + 4x^3 - 9x^2 + 4x - 2}$



$-2x^2 + x - 2 > -3$

$-2x^2 + x - 2 + 3 > 0$

$-2x^2 + x + 1 = 0$

$\Delta = b^2 - 4ac \qquad \dfrac{-b \pm \sqrt{\Delta}}{2a}$

$\Delta = 1 + 8$

$\Delta = 9$

$\dfrac{-1 \pm 3}{-4} \quad \dfrac{2}{-4} = -\dfrac{1}{2} \quad \dfrac{-4}{-4} = 1$

$\underline{-\tfrac{1}{2} < x < 0}$

$g(f(x)) = -(2x+1)^2 + 2$, se $2x+1 > -3$

$g(f(x)) = -((2x+1)\cdot(2x+1)) + 2$

$g(f(x)) = -(+4x^2 + 2x + 2x + 1) + 2$

$g(f(x)) = -(4x^2 + 4x + 1) + 2$

$g(f(x)) = -4x^2 - 4x - 1 + 2$

$\underline{g(f(x)) = -4x^2 - 4x + 1}$

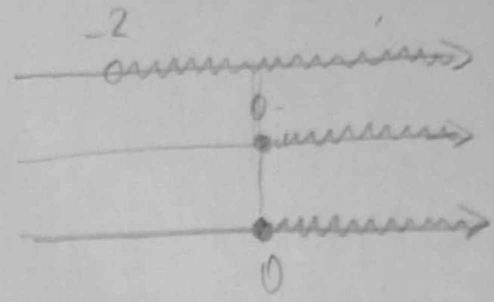
$2x + 1 > -3$

$2x > -3 - 1$

$2x > -4$

$x > -\frac{4}{2} = \underline{\underline{-2}}$

Se $x \geq 0$



$g(f(x)) = 4\cdot(-2x^2 + x - 2) - 1$, se $-2x^2 + x - 2 \leq -3$

$g(f(x)) = -8x^2 + 4x - 8 - 1$

$\underline{g(f(x)) = -8x^2 + 4x - 9}$

Se $x \leq -\frac{1}{2}$

$-2x^2 + x - 2 + 3 \leq 0$

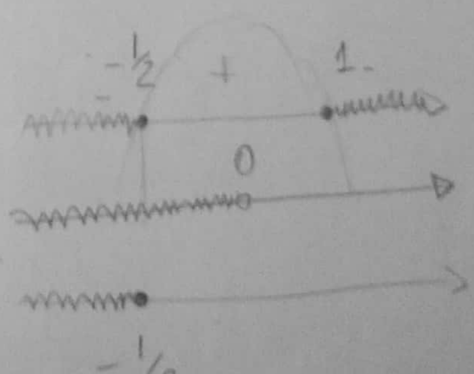
$-2x^2 + x + 1 \leq 0$

$\Delta = b^2 - 4ac \quad \frac{-b \pm \sqrt{\Delta}}{2 \cdot a}$

$\Delta = 1 + 8$

$\underline{\underline{\Delta = 9}}$

$\frac{-1 \pm 3}{-4} \begin{cases} -\frac{1}{2} \\ 1 \end{cases}$



$g(f(x)) = 4\cdot(2x+1) - 1$, se $2x + 1 \leq -3$

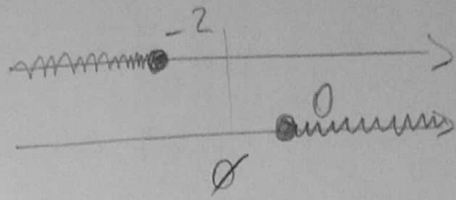
$g(f(x)) = 8x + 4 - 1$

$g(f(x)) = \underline{\underline{8x + 3}}$

$2x \leq -3 - 1$

$2x \leq -4$

$x \leq -\frac{4}{2} = -2$



3a $g(f(x)) = \begin{cases} -4x^4 + 4x^3 - 9x^2 + 4x - 2, & \text{se } -\frac{1}{2} < x < 0 \\ -4x^2 - 4x + 1, & \text{se } x \geq 0 \\ -8x^2 + 4x - 9, & \text{se } x \leq -\frac{1}{2} \end{cases}$

b) $-8x^2 + 4x - 9$

$g(f(-1)) = -8\cdot(-1)^2 + 4(-1) - 9$

$g(f(-1)) = -8 - 4 - 9$

$g(f(-1)) = -12 - 9 = \underline{\underline{-21}}$

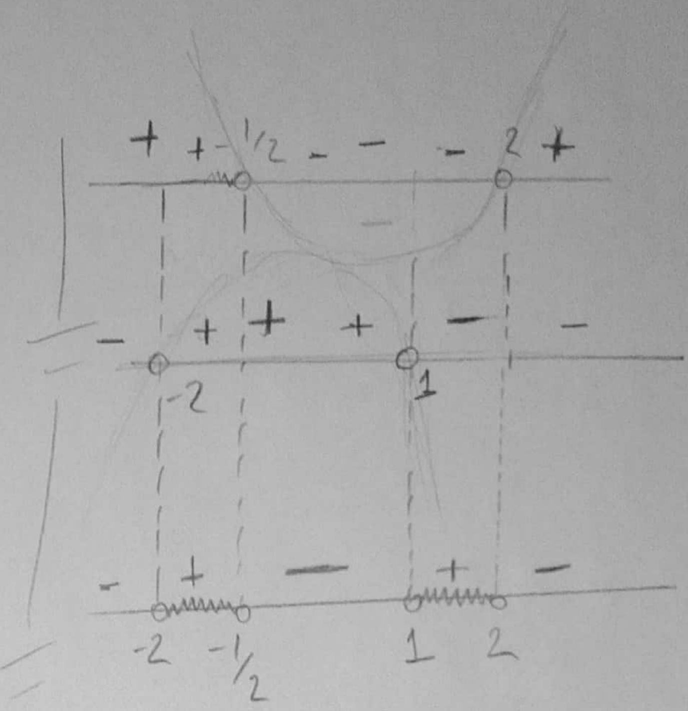
$g(f(1)) = -4\cdot(1)^2 - 4\cdot(1) + 1$

$g(f(1)) = -4 - 4 + 1$

$g(f(1)) = -8 + 1 = \underline{\underline{-7}}$

4a. $f(x) = \log_{(x^2-1)}\left(\frac{2x^2-3x-2}{-x^2-x+2}\right)$

$$\frac{2x^2-3x-2}{-x^2-x+2} > 0$$



$2x^2-3x-2=0$

$\Delta = b^2-4ac$ | $\frac{-b\pm\sqrt{\Delta}}{2a}$

$\Delta = 9+16$ | $\frac{3\pm5}{4}$ → $\frac{8}{4}=2$; $-\frac{2}{4} = -\frac{1}{2}$

$\Delta = 25$

$-x^2-x+2$

$\Delta = b^2-4ac$ | $\frac{-b\pm\sqrt{\Delta}}{2a}$

$\Delta = 1+8$ | $\frac{+1\pm3}{-2}$ → -2 ; 1

$\Delta = 9$

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$x^2-1>0$

$x^2-1=0$

$x^2=1$

$x=\sqrt{1}$

$x=\pm1$

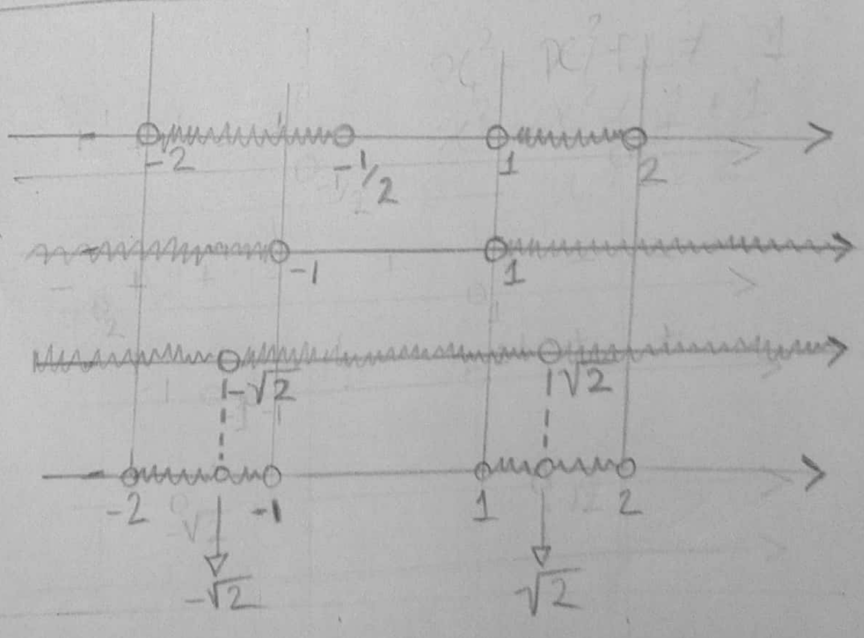
//

$x^2-1\neq1$

$x^2\neq1+1$

$x^2\neq2$

$x\neq\sqrt{2}$



$D(f) = \{x\in\mathbb{R} \mid -2<x<-\sqrt{2}$ ou $-\sqrt{2}<x<-1$ ou $1<x<\sqrt{2}$ ou $\sqrt{2}<x<2\}$

4b $f(x) = \dfrac{\sqrt{x-3}}{\sqrt{5-x} - 1}$

$x - 3 \geq 0$

$x \geq 3$

$\sqrt{5-x} - 1 \neq 0$

$\sqrt{5-x} \neq 1$

$5 - x \neq 1^2$

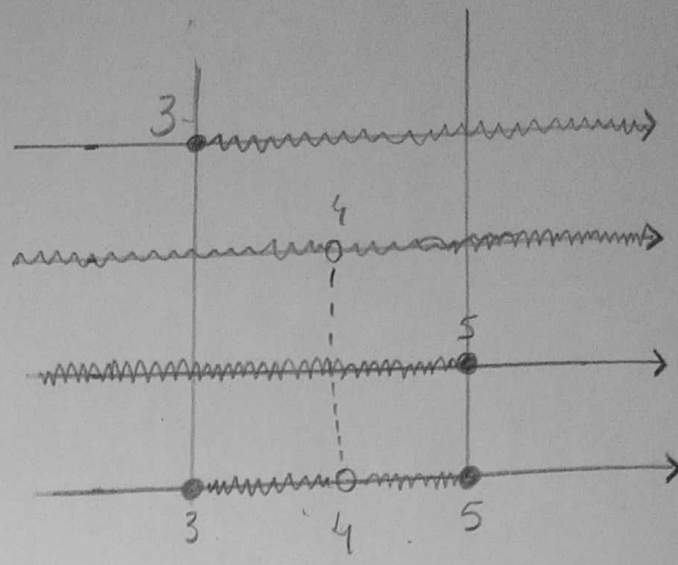
$-x \neq 1^2 - 5$

$-x \neq -4$

$x \neq 4$

$5 - x \geq 0$

$5 \geq x$



$D(f) = \{x \in \mathbb{R} \mid 3 \leq x < 4 \text{ ou } 4 < x \leq 5\}$

5a, $a = \text{arc sen}\left(\frac{4}{5}\right)$

$* \text{cossec}^2 x = 1 + \text{cotg}^2 x$

$$\frac{1}{\text{sen}^2 a} = 1 + \frac{1}{\text{tg}^2 a}$$

$$\frac{1}{\left(\frac{4}{5}\right)^2} = 1 + \frac{1}{\text{tg}^2 a}$$

$$\frac{1}{\frac{16}{25}} = 1 + \frac{1}{\text{tg}^2 a}$$

$$\frac{25}{16} = 1 + \frac{1}{\text{tg}^2 a}$$

$$\frac{25}{16} - 1 = \frac{1}{\text{tg}^2 a}$$

$$\frac{9}{16} = \frac{1}{\text{tg}^2 a}$$

$$9\,\text{tg}^2 a = 16$$

$$\text{tg}^2 a = \frac{16}{9}$$

$$\text{tg}\, a = \sqrt{\frac{16}{9}}$$

$$\text{tg}\, a = \frac{4}{3}$$

$b = \text{arc cotg}\left(-\frac{2}{3}\right)$

$$\text{tg}\, b = \frac{1}{-\frac{2}{3}}$$

$\sec^2 x = 1 + \text{tg}^2 x$

$$\frac{1}{\cos^2 b} = 1 + \left(-\frac{3}{2}\right)^2$$

$$\frac{1}{\cos^2 b} = 1 + \frac{9}{4} \Rightarrow \frac{13}{4}$$

$$\frac{1}{\cos^2 b} = \frac{13}{4}$$

$$13\cos^2 b = 4 \Rightarrow \cos^2 b = \frac{4}{13}$$

$$\cos b = \sqrt{\frac{4}{13}} \Rightarrow \cos b = \frac{2}{\sqrt{13}}$$

$$\cos b = \frac{2}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = \frac{2 \cdot \sqrt{13}}{13}$$

$$\frac{4}{3} - \frac{2\sqrt{13}}{13} = \frac{52 - 6\sqrt{13}}{39}$$

5b $3^{2x+3} - 3^{2x} = 5^{2x-1} + 5^{2x}$

$$3^{2x+3}(1 - 3^{-3}) = 5^{2x-1}(1 + 5^{+1})$$

$$3^{2x} \cdot 3^3\left(1 - \frac{1}{3^3}\right) = 5^{2x} \cdot 5^{-1}(6)$$

$$9^x \cdot \cancel{27}\left(\frac{26}{\cancel{27}}\right) = 5^{2x} \cdot \frac{6}{5}$$

$$9^x \cdot 26 = 25^x \cdot \frac{6}{5}$$

$$\frac{9^x}{25^x} = \frac{6/5}{26}$$

$$\frac{9^x}{25^x} = \frac{6}{130}$$