01/08/24

Signals and Systems Assignment - I

Name : M. Vamshilal Naik Roll.no : 23071A1012 Class: EIE-B

1.) Evaluate the following integrals :

ⓐ $\int_{-\infty}^{\infty} e^{-at^2}\delta(t-5)\,dt$

Given $\int_{-\infty}^{\infty} e^{-at^2}\delta(t-5)\,dt$

We know that $\delta(t-5) = \begin{cases} 1 & \text{for } t=5 \\ 0 & \text{elsewhere} \end{cases}$

$\therefore \int_{-\infty}^{\infty} e^{-at^2}\delta(t-5)\,dt = \left[e^{-at^2}\right]_{t=5} = e^{-25a}$ //

(b) $\int_{-\infty}^{\infty} t^2\delta(t-6)\,dt$

We know that $\delta(t-6) = \begin{cases} 1 & \text{for } t=6 \\ 0 & \text{elsewhere} \end{cases}$

$\therefore \int_{-\infty}^{\infty} t^2\delta(t-6)\,dt = \left[t^2\right]_{t=6} = 36$ //

(c) $\int_{0}^{3} \delta(t)\sin 5\pi t\,dt$

WKT $\delta(t) = \begin{cases} 1 & \text{for } t=0 \\ 0 & \text{elsewhere} \end{cases}$

$\therefore \int_{0}^{3} \delta(t)\sin 5\pi t\,dt = \left[\sin 5\pi t\right]_{t=0} = 0$

(d) $\int_{-\infty}^{\infty} \delta(t+2)e^{-2t}\,dt$

WKT $\delta(t+2) = \begin{cases} 1 & \text{for } t=-2 \\ 0 & \text{elsewhere} \end{cases}$

$\therefore \int_{-\infty}^{\infty} \delta(t+2)e^{-2t}\,dt = \left[e^{-2t}\right]_{t=-2} = e^{4}$ //

(e) $\int_{-\infty}^{\infty} (t-2)^3 \delta(t-2)\,dt$

WKT $\delta(t-2) = \begin{cases} 1 & \text{for } t=2 \\ 0 & \text{elsewhere} \end{cases}$

$\therefore \int_{-\infty}^{\infty} (t-2)^3 \delta(t-2)\,dt = \left[(t-2)^3\right]_{t=2} = 0$ //

(f) $\int_{-\infty}^{\infty} \delta(t)e^{-j\omega t}\,dt$

WKT $\delta(t) = \begin{cases} 1 & \text{for } t=0 \\ 0 & \text{elsewhere} \end{cases}$

$\therefore \int_{-\infty}^{\infty} \delta(t)e^{-j\omega t}\,dt = \left[e^{-j\omega t}\right]_{t=0} = 1$ //

(g) $\int_{-\infty}^{\infty} [\delta(t)\cos 2t + \delta(t-2)\sin 2t]\,dt$

WKT $\delta(t) = \begin{cases} 0 & \text{elsewhere} \\ 1 & \end{cases}$ and $\delta(t-2) = \begin{cases} 1 & \text{for } t=2 \\ 0 & \text{elsewhere} \end{cases}$

$\therefore \int_{-\infty}^{\infty} [\delta(t)\cos 2t + \delta(t-2)\sin 2t]\,dt = [\cos 2t]_{t=0} + [\sin 2t]_{t=2} = 1 + \sin 4t$ //

2.) Find the following summations

(a) $\sum_{n=-\infty}^{\infty} e^{3n}\,\delta(n-3)$

WKT $\delta(n-3) = \begin{cases} 1 & \text{for } n=3 \\ 0 & \text{elsewhere} \end{cases}$

$\therefore \sum_{n=-\infty}^{\infty} e^{3n}\,\delta(n-3) = \left[e^{3n}\right]_{n=3} = e^{9}$

(b) $\sum_{n=-\infty}^{\infty} \delta(n-2)\cos 3n$

WKT $\delta(n-2) = \begin{cases} 1 & \text{for } n=2 \\ 0 & \text{elsewhere} \end{cases}$

$\therefore \sum_{n=-\infty}^{\infty} \delta(n-2)\cos 3n = \left[\cos 3n\right]_{n=2} = \cos 6$

(c) $\sum_{n=-\infty}^{\infty} n^2\,\delta(n+4)$

WKT $\delta(n+4) = \begin{cases} 1 & \text{for } n=-4 \\ 0 & \text{elsewhere} \end{cases}$

$\therefore \sum_{n=-\infty}^{\infty} \delta(n+4)\,n^2 = \left[n^2\right]_{n=-4} = 16$

(d) $\sum_{n=-\infty}^{\infty} \delta(n-2)\,e^{n^2}$

WKT $\delta(n-2) = \begin{cases} 1 & \text{for } n=2 \\ 0 & \text{elsewhere} \end{cases}$

$$\therefore \sum_{n=-\infty}^{\infty} \delta(n-2) e^{n^2} = \left[e^{n^2}\right]_{n=2} = e^{2^2} = e^4$$ //

(c) $\sum_{n=0}^{\infty} \delta(n+1) 4^n$

WKT $\delta(n+1) = \begin{cases} 1 & \text{for } n=-1 \\ 0 & \text{elsewhere for } n \neq -1 \end{cases}$

$$\therefore \sum_{n=0}^{\infty} \delta(n+1) 4^n = \left[4^n\right]_{n=a} = 0$$ //

3.) Prove the properties of impulse function.

Sol:- First property $\int_{-\infty}^{\infty} x(t)\delta(t)\,dt = x(0)$

proof :- Let $x(t)$ be continuous at $t=0$; the value of $x(t)$ at $t=0$ is $x(0)$. The impulse $\delta(t)$ exists only at $t=0$. For all other time $\delta(t)=0$. Therefore, the integration of $x(t)\delta(t)$ from $t=-\infty$ to ∞ has a value only at $t=0$.

$$\therefore \int_{-\infty}^{\infty} x(t)\delta(t)\,dt = \int_{-\infty}^{\infty} x(0)\delta(t)\,dt = x(0)\int_{-\infty}^{\infty} \delta(t)\,dt = x(0).$$

Second property $x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0)$

proof :- Let the signal $x(t)$ be continuous at $t=t_0$. The value of $x(t)$ at $t=t_0$ is $x(t_0)$. $\delta(t-t_0)$ is an impulse function that exists only at $t = t_0$ Therefore,

$$x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$$

Third property $\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt = x(t_0)$

proof :- Let $x(t)$ be continuous at $t = t_0$ and let its value at $t = t_0$ be $x(t_0)$. We know that $\delta(t-t_0) = 1$ only for $t = t_0$. For all other time it is equal to zero. Therefore, the integration of the product term $x(t)\,\delta(t-t_0)$ from $-\infty$ to ∞ has a value only at $t = t_0$.

$$\therefore \int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt = \int_{-\infty}^{\infty} x(t_0)\,\delta(t-t_0)\,dt = x(t_0)\int_{-\infty}^{\infty} \delta(t-t_0)\,dt$$

$$= x(t_0)$$

Fourth property $\int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau = x(t)$

proof :- Let $x(\tau)$ be continuous at $\tau = t$. Let the value of $x(\tau)$ at $\tau = t$ be $x(t)$. We know that $\delta(t-\tau) = 1$ only at $\tau = t$. For all other τ it is zero. So the integration of the product $x(\tau)\,\delta(t-\tau)$ has a value only at $\tau = t$.

$$\therefore \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau = x(\tau)\Big|_{\tau = t} = x(t)$$

This is the formula for convolution of $x(t)$ with $\delta(t)$. This property says that the convolution of any signal with an impulse results in the original signal itself.

Fifth property $\delta(at) = \frac{1}{|a|}\delta(t)$ [Scaling property]

proof :- Let $x(t)$ be some function. Consider the integral $\int_{-\infty}^{\infty} x(t)\,\delta(at)\,dt$ for $a>0$

Let $at = T$

$\therefore$ $t = \frac{T}{a}$ and $dt = \frac{dT}{a}$

If $a>0$ $\int_{-\infty}^{\infty} x(t)\,\delta(at)\,dt = \frac{1}{a}\int_{-\infty}^{\infty} x\left(\frac{T}{a}\right)\delta(T)\,dT$

$$= \left[\frac{1}{a}x\left(\frac{T}{a}\right)\right]_{T=0}$$

$$= \frac{1}{a}x(0)$$

similarly, for $a<0$ $\int_{-\infty}^{\infty} x(t)\,\delta(at)\,dt = \frac{1}{-a}x(0)$

$$\int_{-\infty}^{\infty} x(t)\,\delta(at)\,dt = \frac{1}{|a|}x(0)$$

Now, consider $\frac{1}{|a|}x(0)$

We know that $x(0) = \int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt$

$$\frac{1}{|a|}x(0) = \frac{1}{|a|}\int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt.$$

$$= \int_{-\infty}^{\infty} x(t) \frac{1}{|a|} \delta(t) dt$$

which indicates that $\delta(at) = \frac{1}{|a|} \delta(t)$

Sixth property $\delta(t) = \delta(-t)$

i.e. impulse function is an even function.

proof :- Consider the scaling property

$$\delta(at) = \frac{1}{|a|} \delta(t)$$

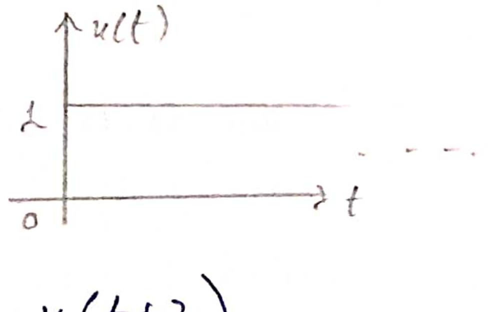
Let $a = -1$

$$\therefore \quad \delta(-t) = \frac{1}{|-1|} \delta(t) = \delta(t)$$

4.) Sketch the following signals

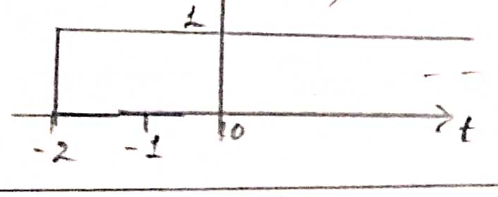
(a). $u(-t+2)$.

The basic signal is $u(t)$

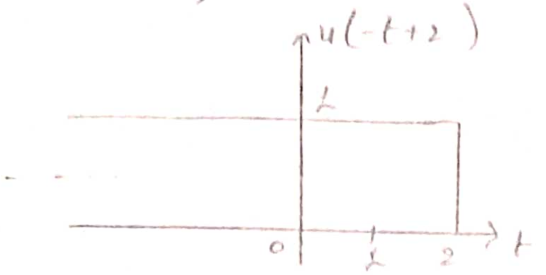


(i) $u(t+2)$

$t = -2$ $\quad$ ∴ time shifting.



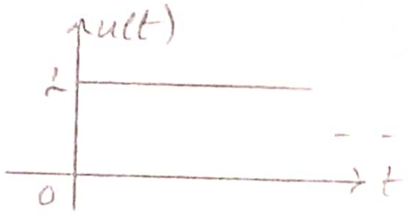
(ii) $u(-t+2)$



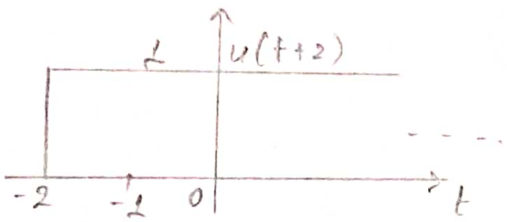
∴ time reversal

(b) $x(t) = -2u(t+2)$

The basic unit step signal.

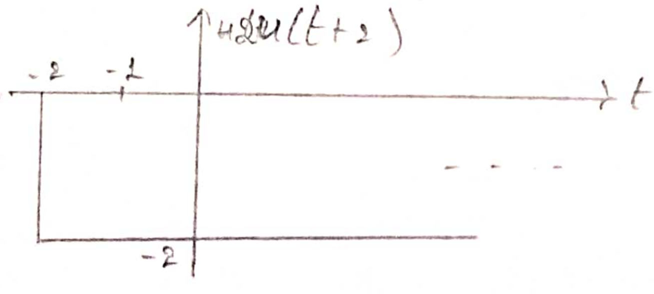


(i) $u(t+2) \Rightarrow t = -2$



∴ time shifting

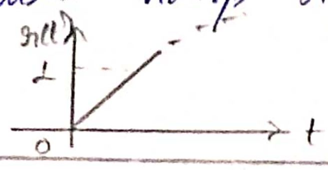
(ii) $-2u(t+2)$



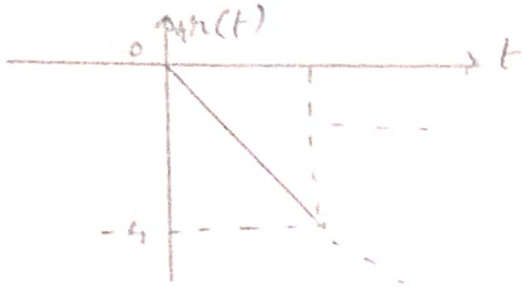
∴ Amplitude scaling

(c) $x(t) = -4r(t)$

The basic ramp signal.



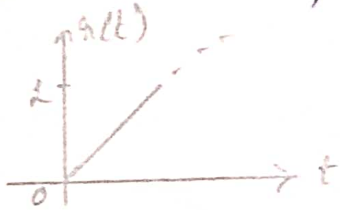
(ii) $-4r(t)$



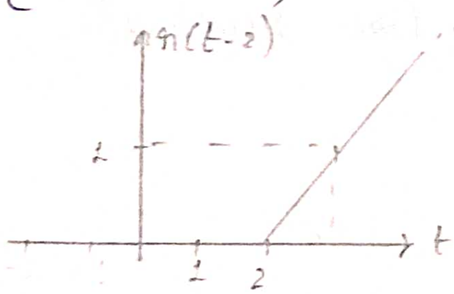
: Amplitude scaling

d.) $x(t) = 2r(t-2)$

The basic ramp signal.



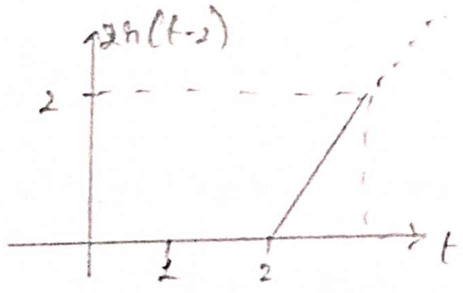
(i) $r(t-2)$



$t=2$

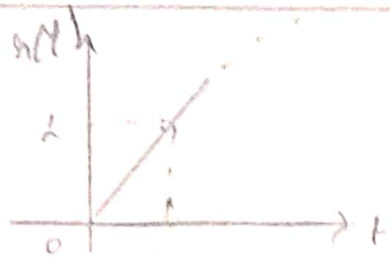
time shifting.

(ii) $2r(t-2)$

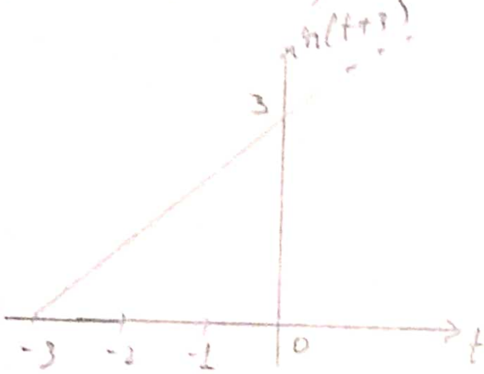


e.) $x(t) = r(-t+3)$

The basic ramp signal.



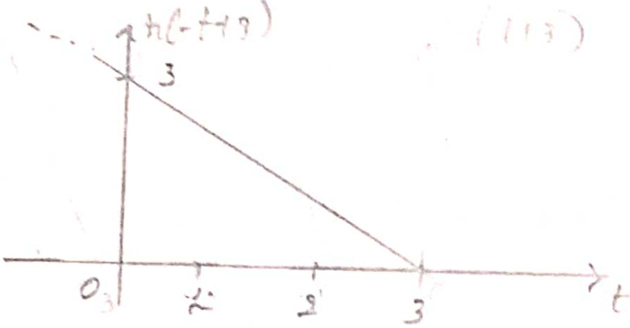
(i). $x(t+3)$



$t = -3$

$\therefore$ time shifting.

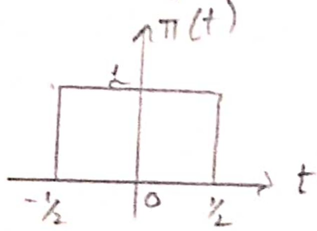
(ii) $x(-t+3)$



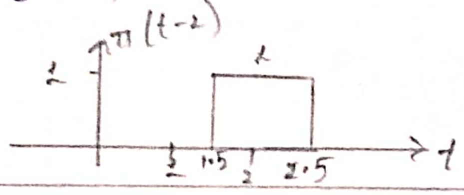
$\therefore$ time reversal.

f.) $x(t) = \Pi(t-2)$

The basic gate signal is.



(i) $\Pi(t-2)$

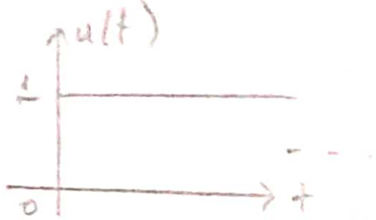


$\therefore$ time shifting.

5.) Sketch the following signals

a.) $u(n+2)\,u(-n+3)$

The basic unit step signal

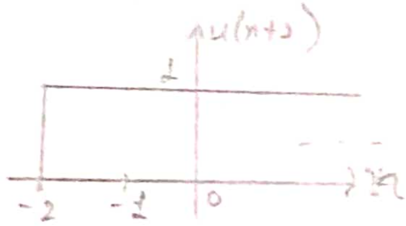


(i) $u(n+2)$

$\therefore t = n$

$n = -2$

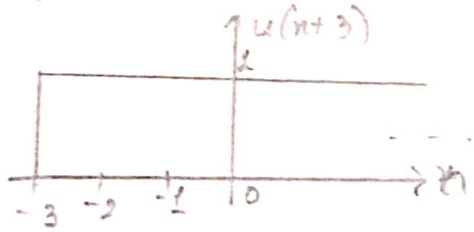
$\therefore$ time shifting



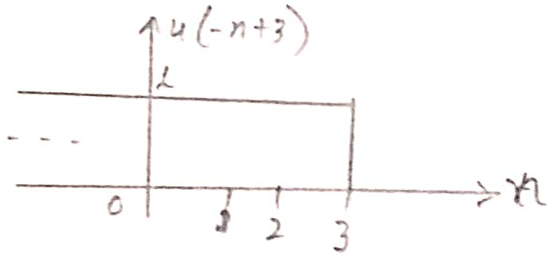
(ii) $u(-n+3)$

$n = -3$

$\therefore$ time shifting



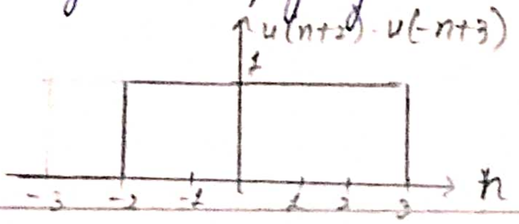
$\therefore$ time folding



(iii) $u(n+2)\cdot u(-n+3)$

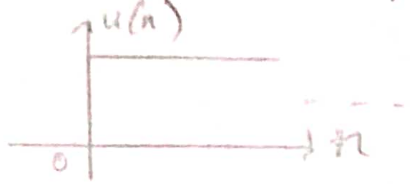
$\therefore$ Using Signal multiplation.

~~Adding~~ Multiplying their amplitudes.



b.) $x(n) = u(n+4) - u(n-2)$

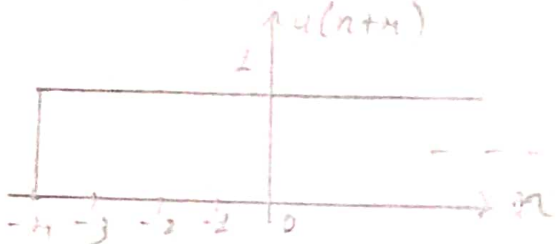
The basic unit step signal



(i) $u(n+4)$

$n = -4$

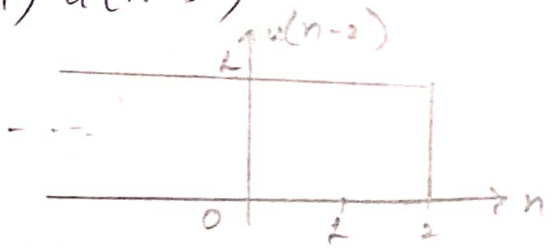
$\therefore$ Time shifting.



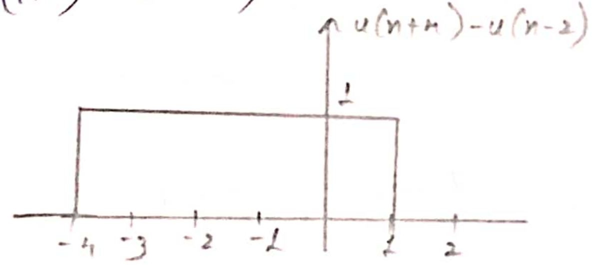
(ii) $u(n-2)$

$n = 2$

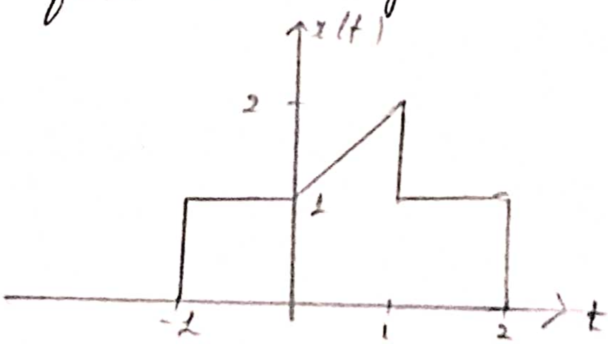
$\therefore$ Time shifting



(iii) $u(n+4) - u(n-2)$



6.) For the signal $x(t)$ shown in figure and find the signals.

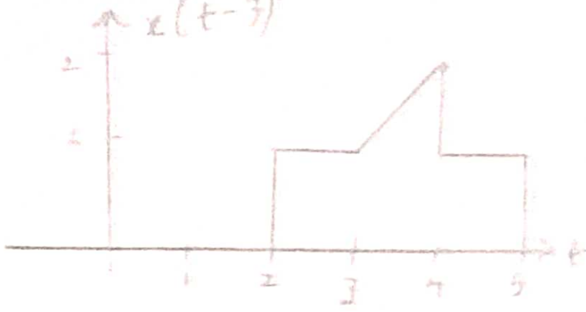


a.) $x(t-3)$ and $x(t+3)$

(i) $x(t-3)$

$\therefore t = 3$

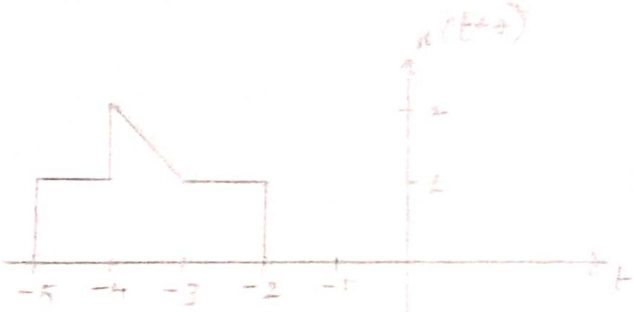
time shifting



(ii) $x(t+3)$

$t = -3$

$\therefore$ time shifting

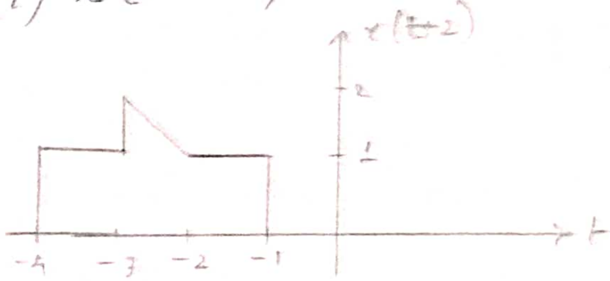


b.) $x(2t+2)$ and $x(\frac{1}{2}t - 2)$

(i) $x(2t+2)$

$t = -2$

$\therefore$ Time shifting



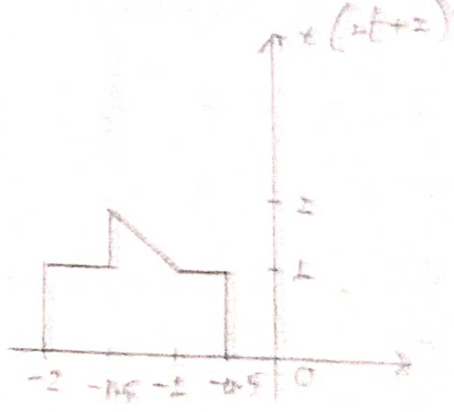
$\therefore$ Time scaling

$x(-1) = x(\frac{-1}{2}) = -0.5$

$x(-2) = x(-\frac{2}{2}) = -1$

$x(-3) = x(\frac{-3}{2}) = -1.5$

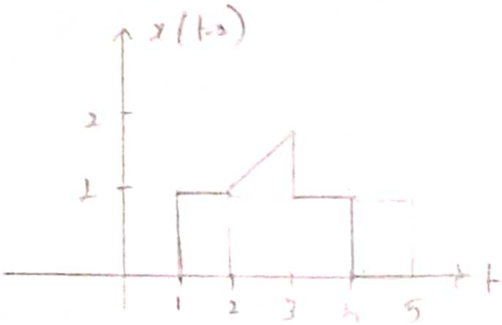
$x(-4) = x(-\frac{4}{2}) = -2$



(ii) $x\left(\frac{1}{2}t-2\right)$

$t=2$

$\therefore$ Time shifting



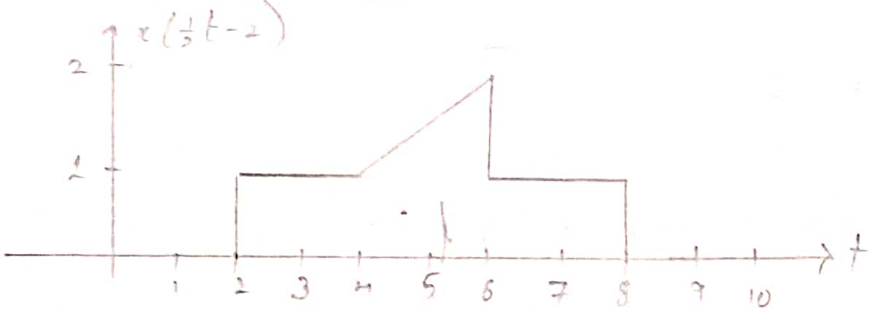
Time scaling

$x(2) = x(2\cdot 2) = x(4) = 4$

$x(3) = x(2\cdot 3) = 6$

$x(4) = x(2\cdot 4) = 8$

$x(5) = x(2\cdot 5) =$ [illegible]



c) $x\left(\frac{5}{3}t\right)$ and $x\left(\frac{3}{5}t\right)$

(i) $x\left(\frac{5}{3}t\right)$

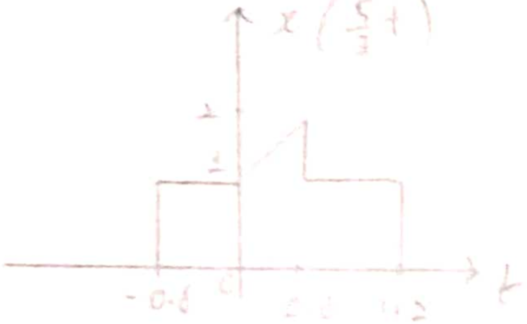
$\therefore$ Time scaling

$x(-1) = x\left(-\frac{3}{5}\right) = (-0.6)$

$x(0) = x(0) = 0$

$x(1) = x\left(\frac{3}{5}\right) = 0.6$

$x(2) = x\left(\frac{2\cdot 3}{5}\right) = 1.2$



(ii) $x\left(\frac{3}{5}t\right)$

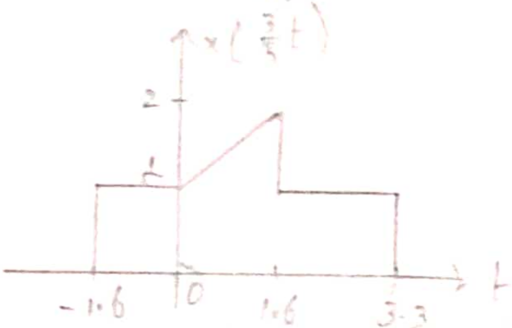
∴ Time scaling

$x(-1) = x\left(-\frac{5}{3}\right) = -1.6$

$x(0) = x\left(\frac{5}{3}(0)\right) = 0$

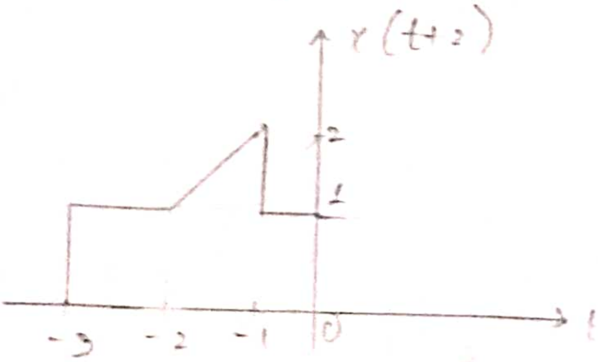
$x(1) = x\left(\frac{5}{3}\right) = 1.6$

$x(2) = x\left(2\cdot\frac{5}{3}\right) = 3.3$



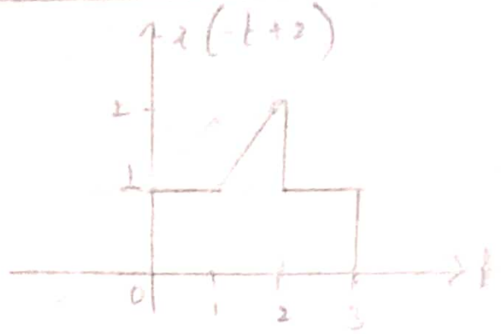
d.) $x(-t+2)$ and $x(-t-2)$

(i) $x(-t+2)$



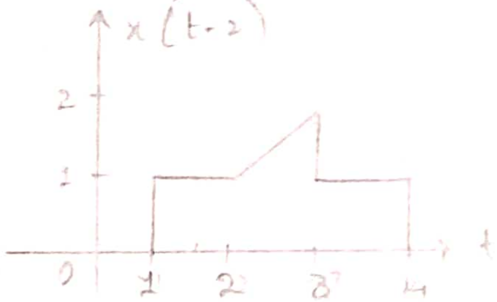
Time shifiting

$\therefore t = -2$



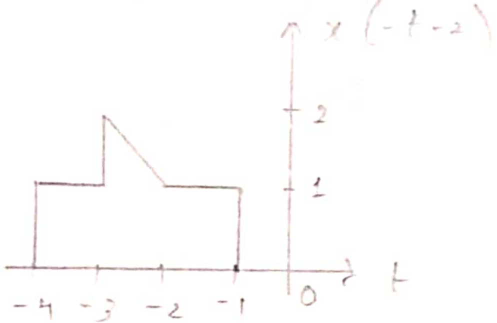
$\therefore$ Time reversal

(iii) $x(-t-2)$



$\therefore t = 2$

Time shifting

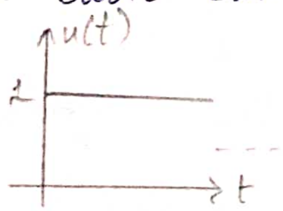


$\therefore$ Time reversal

7.) Sketch the following signals

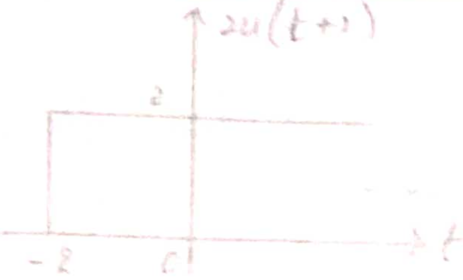
a.) $2u(t+2) - 2u(t-3)$

The basic unit step signal



(i) $2u(t+2)$

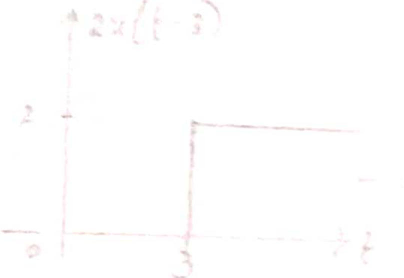
$\therefore t = -2$



$\therefore t = -2$

Time shifting

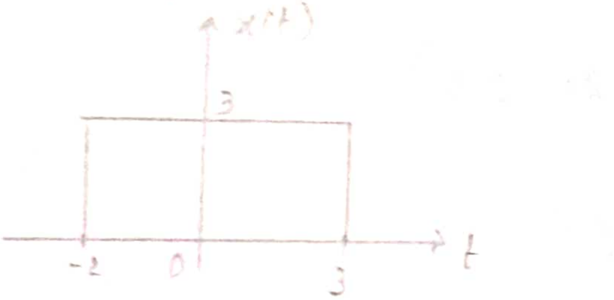
(ii) $2u(t-3)$



$\therefore t = 3$

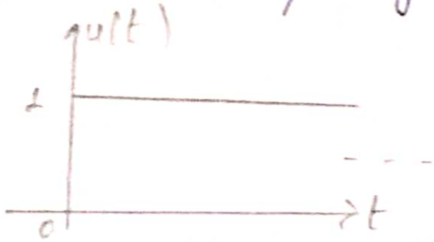
Time shifting

(iii) $2u(t+2) - 2u(t-3)$

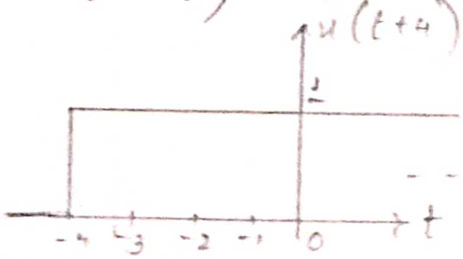


b) $u(t+4)u(-t+4)$

The basic unit step signal is



(i) $u(t+4)$

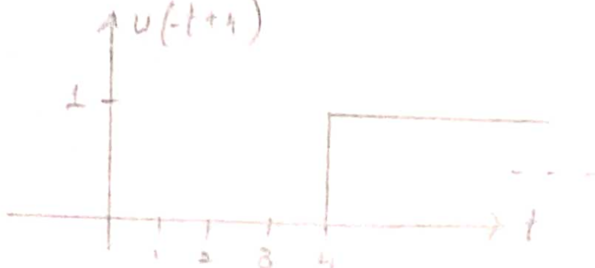


$\therefore t = -4$

Time shifting.

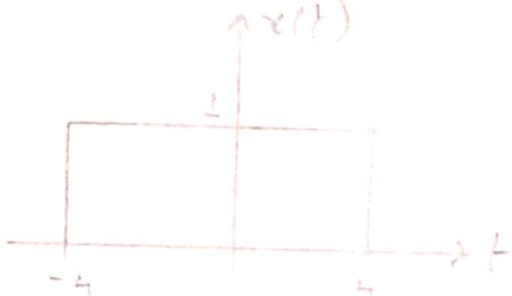
(ii) $u(-t+4)$

$\therefore$ Time folding.



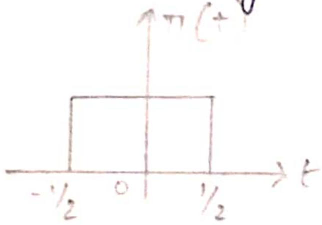
(iii) $x(t) = u(t+4)u(-t+4)$

$\therefore$ signal multiplication

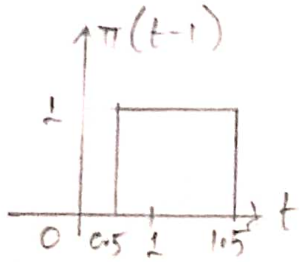


c.) $\Pi\left(\frac{t-2}{2}\right) + \Pi(2t-3.5)$

The basic gate signal



(i) $\Pi\left(\frac{t-2}{2}\right) = \Pi\left(\frac{1}{2}t-1\right)$



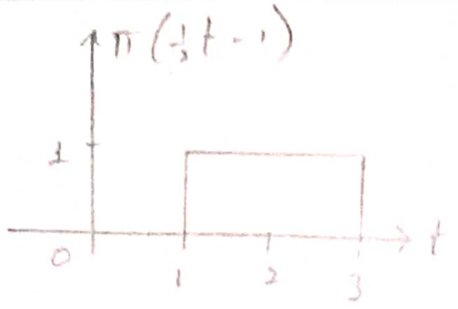
$\therefore t = 1$

Time shifting.

$x(0) = x(\frac{1}{2}(0.5)) = \cancel{0}\ 1$

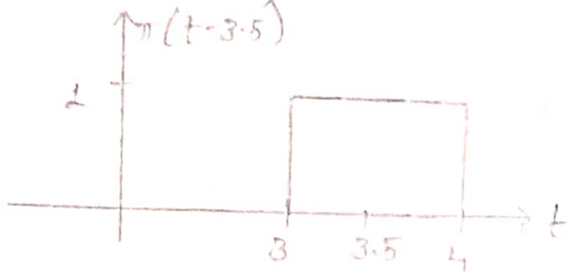
$x(1) = x(2\cdot(1)) = 2$

$x(1.5) = x(2(1.5)) = 3$



∴ Amplitude scaling
Time

(ii) $\Pi(2t-3.5)$

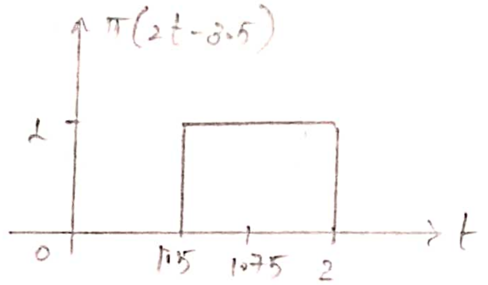


∴ Time shifting.

$x(3) = x(\frac{3}{2}) = 1.5$

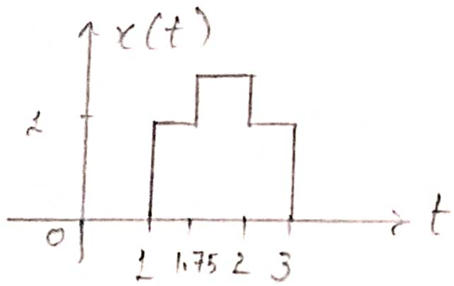
$x(3.5) = x(0.5 \times 3.5) = 1.75$

$x(4) = x(\frac{4}{2}) = 2$



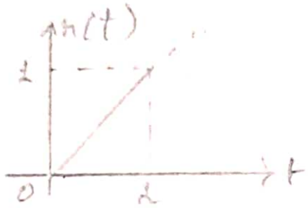
∴ Time scaling.

(iii) $\Pi(\frac{1}{2}t-1) + \Pi(2t-3.5) = x(t)$

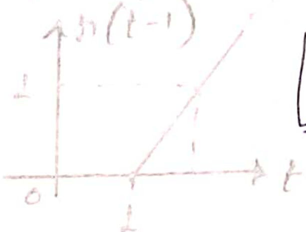


d.) $r(t) - r(t-1) - r(t-3) + r(t-4)$

(i) $r(t)$

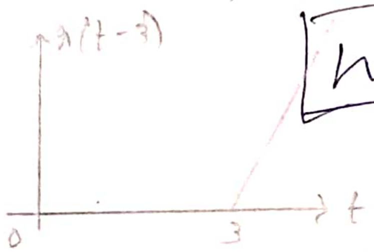


(ii) $r(t-1)$



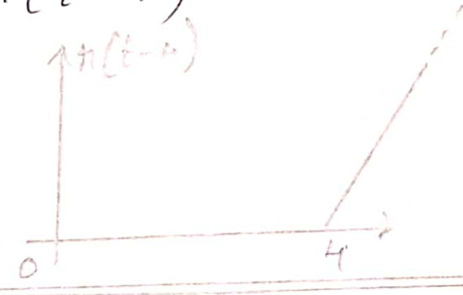
Wrong

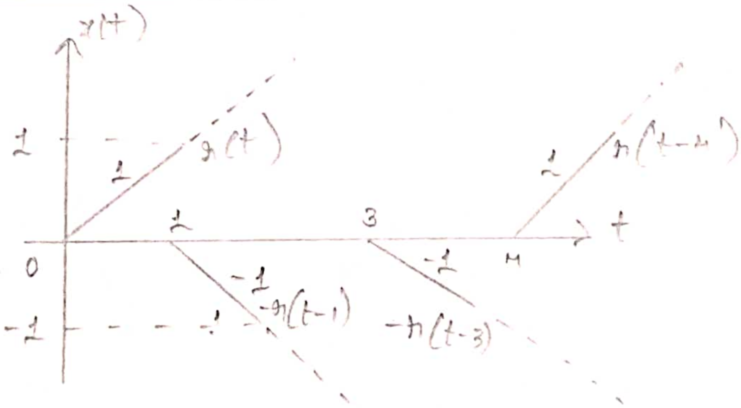
(iii) $r(t-3)$

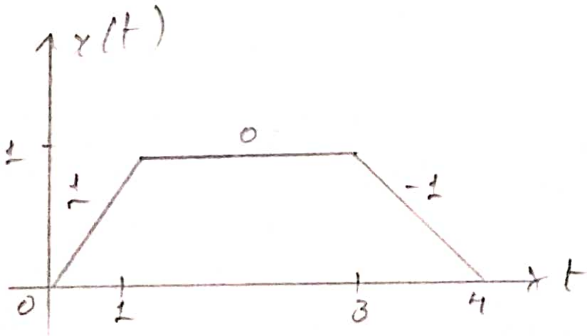


Wrong

(iv) $r(t-4)$

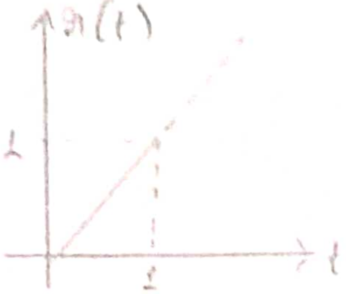


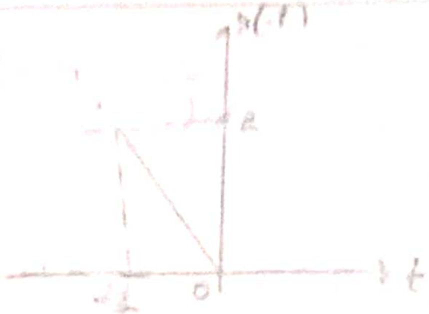




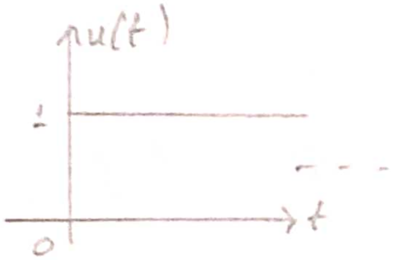
e.) $r(-t) \cdot u(t+2)$

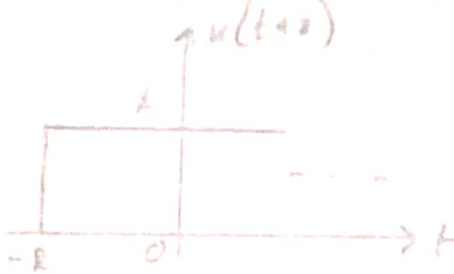
(i) The basic ramp signal is .



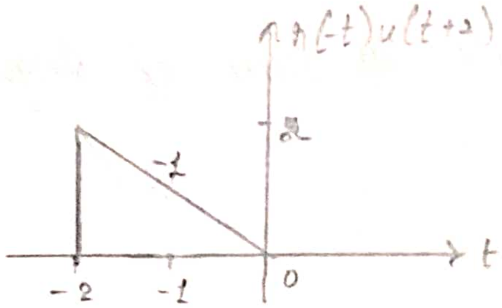


(ii) $u(t+2)$



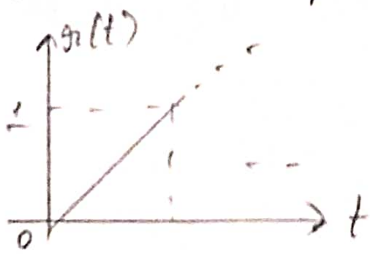


(iii) $x(-t) \cdot u(t+2)$

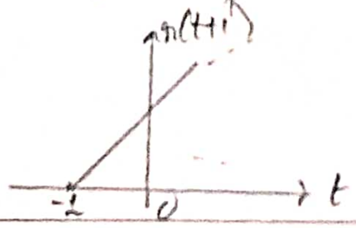


f) $x(-0.25t+1)$

The basic ramp signal is



(i) $r(t+1)$



$t = -1$

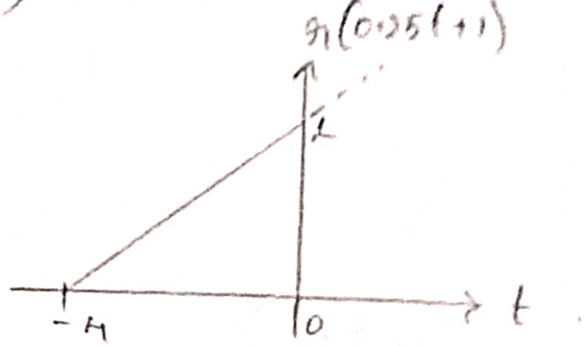
$\therefore$ Time shifting.

(ii) $x(0.25t+1) = x\left(\frac{1}{4}t+1\right)$

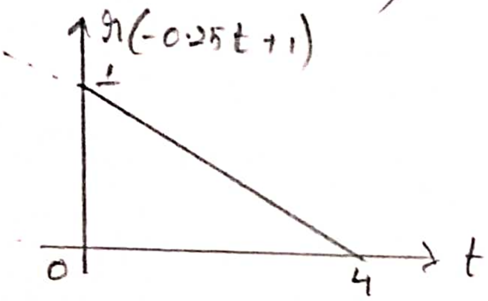
$\therefore$ Time scaling.

$x(-1) = x(4(-1)) = -4$

$x(0) = x(4(0)) = 0$



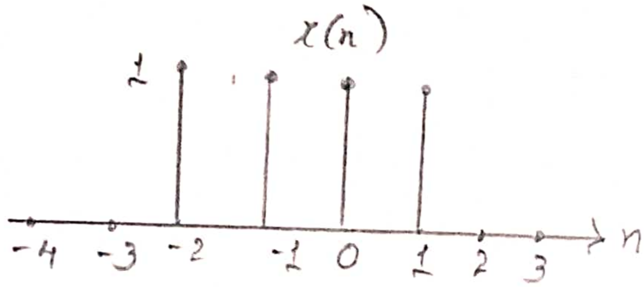
(iii) $x(-0.25t+1)$



$\therefore$ Time folding.

8.) Express the following signals as sum of singular functions :

(i)

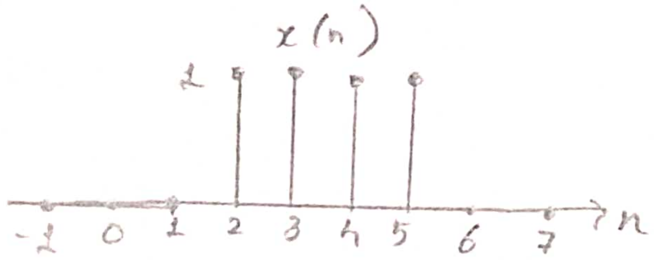


$$x(n) = \delta(n+2) + \delta(n+1) + \delta(n) + \delta(n-1)$$

$$x(n) = \begin{cases} 0 & \text{for } n \leq -3 \\ 1 & \text{for } -2 \leq n \leq 1 \\ 0 & \text{for } n \geq 2 \end{cases}$$

$$\therefore x(n) = u(n+2) - u(n-2)$$

(ii)

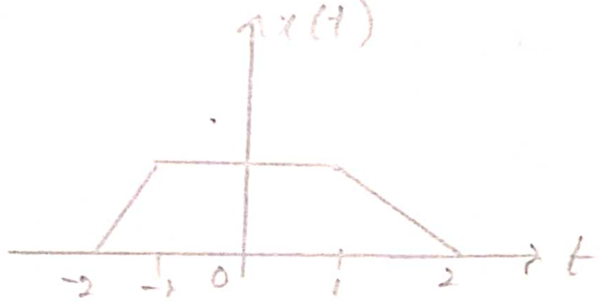


$$x(n) = \delta(n-2) + \delta(n-3) + \delta(n-4) + \delta(n-5)$$

$$x(n) = \begin{cases} 0 & \text{for } n \leq 1 \\ 1 & \text{for } 2 \leq n \leq 4 \\ 0 & \text{for } n \geq 6 \end{cases}$$

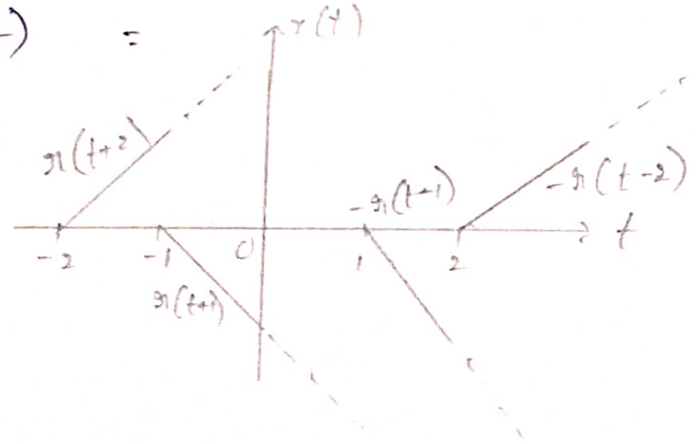
$$\therefore x(n) = u(n-2) - u(n-6)$$

(iii)



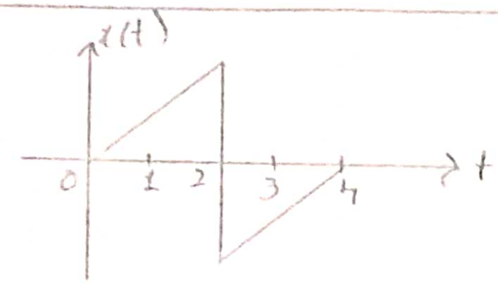
Sol :-

$x(t) =$

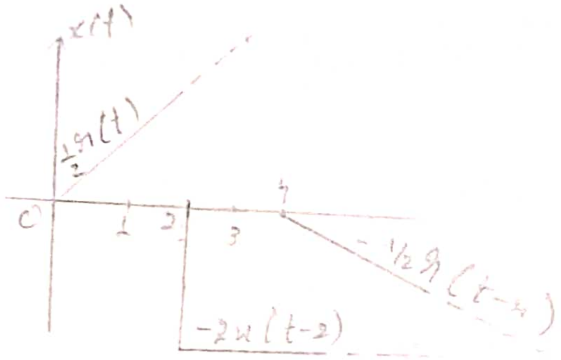


$$x(t) = r(t+2) - r(t+1) - r(t-1) + r(t-2)$$

(iv.)

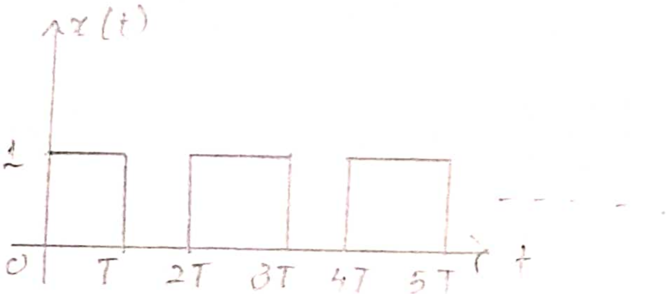


Sol:-

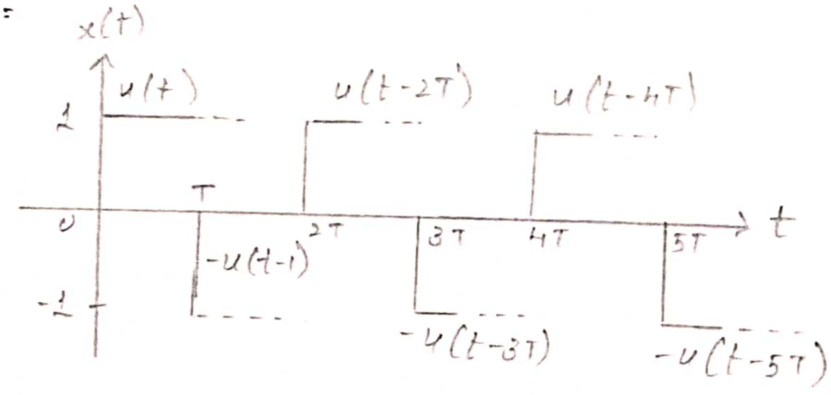


$$x(t) = \frac{1}{2}r(t) - 2u(t-2) - \frac{1}{2}r(t-4)$$

(V.)

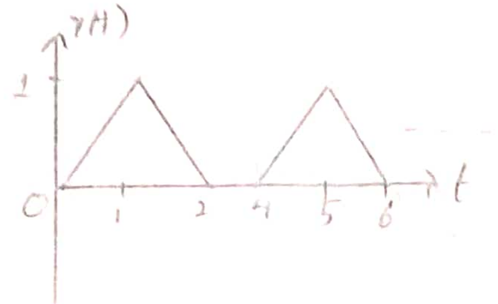


Sol:-

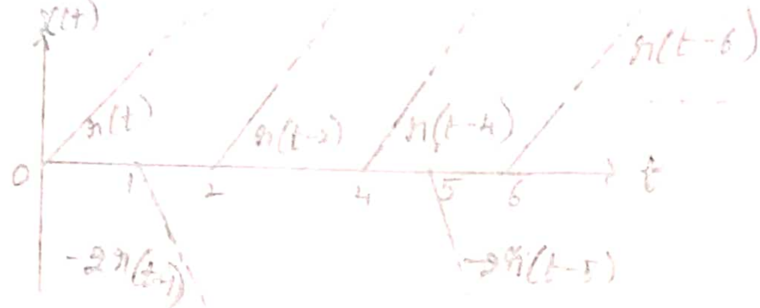


$$x(t) = u(t) - u(t-1) + u(t-2T) - u(t-3T) + u(t-4T) - u(t-5T)$$

f.)



Sol :-



$$x(t) = r(t) - 2r(t-1) + r(t-2) + r(t-4) - 2r(t-5) + r(t-6) + \dots$$

9.) Show that the complex exponential signal $x(t) = e^{j\omega_0 t}$ is periodic with period $2\pi/\omega_0$

Sol:- Given $x(t) = e^{j\omega_0 t}$

$x(t)$ will be periodic if $x(t+T) = x(t)$

if $$e^{j\omega_0 (t+T)} = e^{j\omega_0 t}$$

$$e^{j\omega_0 t} e^{j\omega_0 T} = e^{j\omega_0 t}$$

$$e^{j\omega_0 T} = 1$$

This is valid if $\omega_0 T = n(2\pi)$

where n is an integer.

$$\therefore T = \frac{2\pi n}{\omega_0}$$

Thus, the fundamental period, i.e. the smallest positive T of $x(t)$

is given by $T = \frac{2\pi}{\omega_0}$

10.) Show that the sinusoidal signal $x(t) = \sin(\omega_0 t + \theta)$ is periodic with period $2\pi / \omega_0$.

Sol:- Given $x(t) = \sin(\omega_0 t + \theta)$

$x(t)$ will be periodic if $x(t) = x(t+T)$

But $x(t+T) = \sin[\omega_0 (t+T) + \theta] = \sin[\omega_0 t + \theta + \omega_0 T]$

For $\sin(\omega_0 t + \theta)$ to be equal to $\sin(\omega_0 t + \theta + \omega_0 T)$, $\omega_0 T$ must be equal to $2n\pi$, where n is a positive integer.

Therefore, $T = \frac{2\pi n}{\omega_0}$

This shows that the sinusoidal signal $x(t) = \sin(\omega_0 t + \theta)$ is periodic with fundamental period $T = \frac{2\pi}{\omega_0}$

11.) Let $x_1(t)$ and $x_2(t)$ be periodic signals with fundamental periods T_1 and T_2 respectively. Under what conditions is the sum $x(t) = x_1(t) + x_2(t)$ periodic and what is the fundamental period of $x(t)$ if it is periodic?

(or)

Show that a composite signal is periodic if the ratio of their fundamental periods is a rational number.

Sol:- Given signals $x_1(t)$ and $x_2(t)$ are periodic ~~if the~~ ~~ratio of~~ with their fundamental periods ~~is a~~ T_1 and T_2 respectively. So

$$x_1(t) = x_1(t+T_1) = x_1(t+mT_1), \; m \text{ is a +ve integer}$$

$$x_2(t) = x_2(t+T_2) = x_2(t+kT_2), \; k \text{ is a +ve integer}$$

Given signal is $x(t) = x_1(t) + x_2(t)$

$$\therefore x(t) = x_1(t+mT_1) + x_2(t+kT_2)$$

For $x(t)$ to be periodic with period T, we require

$$x(t) = x(t+T) = x_1(t+T) + x_2(t+T)$$

$$= x_1(t+mT_1) + x_2(t+kT_2)$$

$$mT_1 = kT_2 = T$$

$$\frac{T_1}{T_2} = \frac{k}{m} = \text{rational number}$$

Then the fundamental period T of $x(t)$ is the LCM of T_1 and T_2. Therefore,

$$T = mT_1 = kT_2$$

So we can say that, the sum of two periodic signals is periodic only if the ratio of their respective periods is a rational number. Then the fundametal period is the LCM of the respective fundamental periods.

12.) Examine whether the following signals are periodic or not? If periodic determine the fundamental period.

a.) $\sin 12\pi t$.

$x(t) = \sin 12\pi t$

$\omega_0 = 12\pi$ $\therefore$ It is periodic

$T = \frac{2\pi}{\omega_0} = \frac{2\pi}{12\pi} = \frac{1}{6}$ sec

b.) $e^{j4\pi t}$

$x(t) = e^{j4\pi t}$ $\therefore$ It is periodic

$\omega_0 = 4\pi$

$T = \frac{2\pi}{\omega_0} = \frac{2\pi}{4\pi} = \frac{1}{2}$ sec

c.) $\sin(\pi t) \cdot u(t)$

$x(t) = \sin(\pi t) \cdot u(t)$

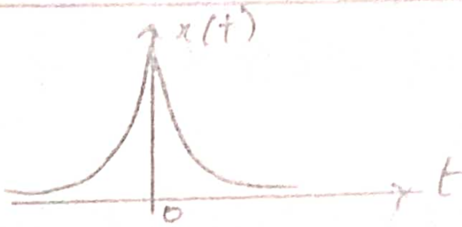
$\sin \pi t$ is periodic with period $T = \frac{2\pi}{\omega_0} = \frac{2\pi}{\pi}$

$\therefore T = 2$ sec

and $u(t)$ exists only b/w $t=0$ to $t=\infty$, hence it is not periodic.

$\therefore \sin(\pi t) \cdot u(t)$ is not periodic d.) $e^{-|t|}$

$$x(t) = e^{-|t|}$$



It is not periodic because it does not repeat at all. So it is aperiodic.

e.) $\cos 2t + \sin\sqrt{3}t$

$$x(t) = \cos 2t + \sin\sqrt{3}t$$

$$x(t) = x_1(t) + x_2(t)$$

$$x_1(t) = \cos 2t$$

$$\therefore \omega_0 = 2 \Rightarrow T_1 = \frac{2\pi}{\omega_0} = \frac{2\pi}{2} = \pi.$$

$$x_2(t) = \sin\sqrt{3}\,t$$

$$\therefore \omega_0 = \sqrt{3} \Rightarrow T_2 = \frac{2\pi}{\sqrt{3}}$$

The ratio of two periods $T = \frac{T_1}{T_2} = \frac{\pi}{\frac{2\pi}{\sqrt{3}}} = \frac{\sqrt{3}}{2}$

Since T_1/T_2 is not a ratio of two integers (i.e. not a rational number), the given signal $x(t)$ is non periodic.

f.) $3\sin 200\pi t + 4\cos 100\pi t$

$$x(t) = \underbrace{3\sin 200\pi t}_{x_1(t)} + \underbrace{4\cos 100\pi t}_{x_2(t)}$$

$$x_1(t) = 3\sin 200\pi t$$

$\therefore \omega_0 = 200\pi \quad \Rightarrow T_1 = \frac{2\pi}{\omega_0} = \frac{2\pi}{200\pi} = \frac{1}{100}$ sec

$x_2(t) = 4\cos 100t$

$\omega_0 = 100 \quad \Rightarrow \quad T_2 = \frac{2\pi}{\omega_0} = \frac{2\pi}{100} = \frac{\pi}{50}$ sec

The ratio of two periods $= \frac{T_1}{T_2} = \frac{\frac{1}{100}}{\frac{\pi}{50}} = \frac{1}{2\pi}$ sec

$\therefore$ It is not periodic, because it is not a rational number.

g.) $\sin 10\pi t + \cos 20\pi t$

$x(t) = \underbrace{\sin 10\pi t}_{x_1(t)} + \underbrace{\cos 20\pi t}_{x_2(t)}$

$x_1(t) = \sin 10\pi t$

$\therefore \omega_0 = 10\pi \quad \Rightarrow T_1 = \frac{2\pi}{\omega_0} = \frac{2\pi}{10\pi} = \frac{1}{5}$ sec

$x_2(t) = \cos 20\pi t$

$\therefore \omega_0 = 20\pi \quad \Rightarrow T_2 = \frac{2\pi}{\omega_0} = \frac{2\pi}{20\pi} = \frac{1}{10}$ sec

The ratio of two periods $= \frac{T_1}{T_2} = \frac{\frac{1}{5}}{\frac{1}{10}} = 2$ sec

$\therefore T_1 = 2T_2$

$\therefore$ It is periodic

h.) $\sin(10t+1) - 2\cos(5t-2)$

$$x(t) = \sin(10t+1) - 2\cos(5t-2)$$

$$x_1(t) = \sin(10t+1)$$

$$\omega_0 = 10 \Rightarrow T_1 = \frac{2\pi}{\omega_0} = \frac{2\pi}{10} = \frac{\pi}{5} \text{ sec}$$

$$x_2(t) = 2\cos(5t-2)$$

$$\omega_0 = 5 \Rightarrow T_2 = \frac{2\pi}{\omega_0} = \frac{2\pi}{5} \text{ sec}$$

$$\therefore \frac{T_1}{T_2} = \frac{\frac{\pi}{\cancel{5}}}{\frac{2\pi}{\cancel{5}}} = 2 \text{ sec}.$$

$$\Rightarrow 2T_1 = T_2 \qquad \therefore \text{It is periodic}$$

The Fundamental periodic $T = 2T_1 = T_2 = \frac{2\pi}{5}$ sec

i.) je^{j6t}

$$x(t) = je^{j6t}$$

$$\therefore \omega_0 = 6 \Rightarrow T = \frac{2\pi}{\omega_0} = \frac{2\pi}{6} = \frac{\pi}{3} \text{ sec}$$

$\therefore$ It is periodic

j.) $3u(t) + 2\sin 2t$

$$x(t) = 3u(t) + 2\sin 2t$$

$$x_1(t) = 3u(t)$$

$x_1(t) = 0$ $\therefore$ it is aperiodic

$x_2(t) = 2\sin 2t$

$\omega_0 = 2 \Rightarrow T = \frac{2\pi}{\omega_0} = \frac{2\pi}{2} = \pi$ sec.

$\therefore$ It is periodic

$\Rightarrow$ The sum of aperiodic and periodic signals, is aperiodic.

k.) $6e^{j[4t+(\pi/3)]} + 8e^{j[3\pi t+(\pi/4)]}$

$x(t) = 6e^{j[4t+(\pi/3)]} + 8e^{j[3\pi t+(\pi/4)]}$

$x_1(t) = 6e^{j[4t+(\pi/3)]}$

$\omega_0 = 4 \Rightarrow T_1 = \frac{2\pi}{\omega_0} = \frac{2\pi}{4} = \frac{\pi}{4}$ sec

$x_2(t) = 8e^{j[3\pi t+(\frac{\pi}{4})]}$

$\omega_0 = 3\pi \Rightarrow T_2 = \frac{2\pi}{\omega_0} = \frac{2\pi}{3\pi} = \frac{2}{3}$ sec

$\therefore \frac{T_1}{T_2} = \frac{\frac{\pi}{4}}{\frac{2}{3}} = \frac{3\pi}{4}$

$\therefore$ The ratio is not rational, the signal $x(t)$ is not periodic.

L.) $u(t) - 2u(t-5)$

$x(t) = u(t) - 2u(t-5)$

$\therefore$ The given two signals are aperiodic. $\therefore$ It is aperiodic m.) $2 + \cos 2\pi t$

$x(t) = 2 + \cos 2\pi t$

Here 2 is a dc signal extending from $-\infty$ to ∞.
The time period of $\cos 2\pi t$ is $T = \frac{2\pi}{2\pi} = 1$ sec.
Since it is a rational number, $\cos 2\pi t$ is periodic.
The signal $x(t)$ is nothing but $\cos 2\pi t$ shifted upwards by 2. So $x(t)$ is also periodic with a fundamental period $T = 1$ sec.

13.) Show that the complex exponential sequence $x(n) = e^{j\omega_0 n}$ is periodic only if $\omega_0 / 2\pi$ is a rational number.

Sol:- Given $x(n) = e^{j\omega_0 n}$

$x(n)$ will be periodic if $x(n+N) = x(n)$

$$e^{j[\omega_0(n+N)]} = e^{j\omega_0 n}$$

$$e^{j\omega_0 N} e^{j\omega_0 n} = e^{j\omega_0 n}$$

This is possible only if $e^{j\omega_0 N} = 1$

This is true only if $\omega_0 N = 2\pi k$

∵ Where k is a positive integer

$$\therefore \frac{\omega_0}{2\pi} = \frac{k}{N} = \text{rational number}$$

This shows that the complex exponential sequence $x(n) = e^{j\omega_0 n}$ is periodic if $\omega_0/2\pi$ is a rational number.

14.) Let $x(t)$ be the complex exponential signal, $x(t) = e^{j\omega_0 t}$ with radian frequency ω_0 and fundamental period $T = 2\pi/\omega_0$. Consider the discrete-time sequence $x(n)$ obtained by the uniform sampling of $x(t)$ with sampling interval T_s, i.e.

$$x(n) = x(nT_s) = e^{jn\omega_0 T_s}$$

Show that $x(n)$ is periodic if the ratio of the sampling interval T_s to the fundamental period T of $x(t)$, i.e. T_s/T is a rational number.

Sol:- Given $x(t) = e^{j\omega_0 t}$

$$x(n) = x(nT_s) = e^{jn\omega_0 T_s}$$

T_s is the sampling interval.

Now, fundamental period $T = \frac{2\pi}{\omega_0}$

$$\therefore \omega_0 = \frac{2\pi}{T}$$

If $x(n)$ is periodic with the fundamental period N, then $x(n+N) = x(n)$

$$e^{j(n+N)\omega_0 T_s} = e^{jn\omega_0 T_s}$$

$$e^{jn\omega_0 T_s} e^{jN\omega_0 T_s} = e^{jn\omega_0 T_s}$$

This is true only if $e^{jN\omega_0 T_s} = 1$

$$N\omega_0 T_s = 2\pi m$$

where m is a positive integer.

$$N \cdot \frac{2\pi}{T} T_s = 2\pi m$$

$$\frac{T_s}{T} = \frac{m}{N} = \text{Rational number}$$

$\therefore x(n)$ is periodic if ratio of the sampling interval to the fundamental period of $x(t)$, T_s/T is a rational number.

15.) Obtain the condition for discrete-time sinusoidal signal to be periodic.

Sol: In case of continuous-time signals, all sinusoidal signals are periodic. But in discrete-time case, not all sinusoidal sequences are periodic.

Consider a discrete-time signal given by

$$x(n) = A\sin(\omega_0 n + \theta)$$

where A is amplitude, ω_0 is frequency and θ is phase shift.

A discrete-time signal is periodic if and only if

$$x(n) = x(n+N) \text{ for all } n$$

Now, $x(n+N) = A\sin[\omega_0(n+N)+\theta] = A\sin(\omega_0 n + \theta + \omega_0 N)$

Therefore, $x(n)$ and $x(n+N)$ are equal if $\omega_0 N = 2\pi m$
That is, there must be an integer m such that

$$\omega_0 = \frac{2\pi m}{N} = 2\pi\left[\frac{m}{N}\right]$$

(or) $$N = 2\pi\left[\frac{m}{\omega_0}\right]$$

From the above equation we find that, for the discrete time signal to be periodic, the fundamental frequency ω_0 must be a rational multiple of 2π. Otherwise the discrete-time signal is aperiodic.

16.) Determine whether the following discrete-time signals are periodic or not? If periodic, determine the fundamental period.

sol: a.) $\sin(0.02\pi n)$

Given $x(n) = \sin(0.02\pi n)$

Comparing with $x(n) = \sin(2\pi f n)$

we have $0.02\pi = 2\pi f$ or $f = \frac{0.02\pi}{2\pi} = 0.01 = \frac{1}{100} = \frac{k}{N}$

The given signal $x(n)$ is periodic with fundamental period $N = 100$

b.) $\sin(5\pi n)$

$x(n) = \sin(5\pi n)$

$$2\pi f = 5\pi \Rightarrow f = \frac{5\pi}{2\pi} = \frac{5}{2} = \frac{k}{N}$$

$\therefore$ It is periodic with period $N = 2$.

c.) $\cos 4n$

$$x(t) = \cos 4n$$

$$2\pi f = 4 \Rightarrow f = \frac{4}{2\pi} = \frac{2}{\pi}$$

Since $f = \frac{2}{\pi}$ is not a rational number, $x(n)$ is not periodic.

d.) $\sin \frac{2\pi n}{3} + \cos \frac{2\pi n}{5}$

$$x(t) = \sin \frac{2\pi n}{3} + \cos \frac{2\pi n}{5}$$

$$2\pi f_1 = \frac{2\pi}{3} \Rightarrow \frac{1}{3} = f_1 = \frac{k_1}{N_1} \qquad \therefore N_1 = 3$$

$$2\pi f_2 = \frac{2\pi}{5} \Rightarrow f_2 = \frac{1}{5} = \frac{k_2}{N_2} \qquad \therefore N_2 = 5$$

$$\therefore \frac{N_1}{N_2} = \frac{3}{5}$$ $\therefore$ It is periodic.

e.) $\cos\left(\frac{n}{6}\right)\cos\left(\frac{n\pi}{6}\right)$

$$x(n) = \cos\left(\frac{n}{6}\right)\cos\left(\frac{n\pi}{6}\right)$$

$$2\pi f_1 = \frac{1}{6} \Rightarrow f_1 = \frac{1}{12\pi} = \frac{k_1}{N_1}$$

$2\pi f_2 = \frac{\pi}{6} \Rightarrow f_2 = \frac{1}{12}$

$\therefore$ Thus $\cos(n/6)$ is non-periodic and $\cos\left(\frac{n\pi}{6}\right)$ is periodic. $x(n)$ is non-periodic.

f.) $\cos\left(\frac{\pi}{2} + 0.3n\right)$

$x(n) = \cos\left(\frac{\pi}{2} + 0.3n\right)$

$2\pi f = 0.3 \Rightarrow f = \frac{0.3}{2\pi} = \frac{3}{20\pi}$

$\therefore$ It is not periodic

g.) $e^{j(\pi/2)n}$

$x(n) = e^{j(\pi/2)n}$

$2\pi f = \frac{\pi}{2} \Rightarrow f = \frac{\pi}{4\pi} = \frac{1}{4} = \frac{k}{N}$

$\therefore$ It is periodic with $N = 4$

h.) $1 + e^{j2\pi n/3} - e^{j4\pi n/7}$

$x(n) = 1 + e^{j2\pi n/3} - e^{j4\pi n/7}$

Let $x(n) = 1 + e^{j2\pi n/3} - e^{j4\pi n/7} = x_1(n) + x_2(n) + x_3(n)$

$x_1(n) = 1$, $x_2(n) = e^{j2\pi n/3}$ and $x_3(n) = e^{j4\pi n/7}$

$x_1(n) = 1$ is a dc signal with an arbitary period $N_1 = 1$

$$x_2(n) = e^{j2\pi n/3} = e^{j2\pi f_2 n}$$

$$\therefore \frac{2\pi n}{3} = 2\pi f_2 n \quad \text{or} \quad f_2 = \frac{1}{3} = \frac{k_2}{N_2} \quad \text{where } N_2 = 3$$

$\therefore$ It is periodic

$$x_3(n) = e^{j4\pi n/7} = e^{j2\pi f_3 n}$$

$$2\pi f_3 = \frac{4\pi}{7} \Rightarrow f_3 = \frac{2}{7} = \frac{k_3}{N_3} \text{ where } N_3 = \frac{7}{2}$$

Now, $\frac{N_1}{N_2} = \frac{1}{3}$ = rational number.

$$\frac{N_1}{N_3} = \frac{1}{7/2} = \frac{2}{7} = \text{rational number.}$$

The LCM of $N_1, N_2, N_3 = \frac{7}{2} \times 3 = \frac{21}{2}$

The given signal $x(n)$ is periodic with fundamental period $N = 10.5$.

17.) Determine the power and rms value of signal $x(t) = A\sin(\omega_0 t + \theta)$.

Sol:

$$x(t) = A\sin(\omega_0 t + \theta)$$

$$\text{Average power } P = \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{-T}^{T} |A\sin(\omega_0 t + \theta)|^2 dt$$

$$= \lim_{T\to\infty} \frac{A^2}{2T} \int_{-T}^{T} \left[\frac{1-\cos(2\omega_0 t+2\theta)}{2}\right] dt$$

$$= \lim_{T\to\infty} \frac{A^2}{4T} \int_{-T}^{T} dt - \lim_{T\to\infty} \frac{A^2}{4T} \int_{-T}^{T} \cos(2\omega_0 t+2\theta)\, dt$$

$$= \lim_{T\to\infty} \frac{A^2}{4T} \int_{-T}^{T} dt - 0$$

∴ integration of cosine function over one full cycle is always zero.

$$= \lim_{T\to\infty} \frac{A^2}{4T} [T+T]$$

$$= \frac{A^2}{2}$$

rms value $= \sqrt{\frac{A^2}{2}} = \frac{A}{\sqrt{2}}$

So we can conclude that the power of a sinusoidal signal with amplitude A is equal to $A^2/2$ and rms value is equal to $A/\sqrt{2}$

Normalized energy $E = \int_{-\infty}^{\infty} |A\sin(\omega_0 t+\theta)|^2 dt$

$$= \int_{-\infty}^{\infty} \frac{A^2[1-\cos 2(\omega_0 t+\theta)]dt}{2}$$

$$= \frac{A^2}{2}\int_{-\infty}^{\infty} dt - \frac{A^2}{2}\int_{-\infty}^{\infty}[\cos(2\omega_0 t+2\theta)]\,dt = \frac{A^2}{2}[t]_{-\infty}^{\infty} - 0$$

$$= \infty$$

∴ Energy $E = \infty$.

18.) Prove the following :

a.) The power of the energy signal is zero over infinite time.

b.) The energy of the power signal is infinite over infinite time.

Sol:- a.) Power of the energy signal.

Let $x(t)$ be an energy signal, i.e. $E = \int_{-\infty}^{\infty} |x(t)|^2 dt$ is finite.

power of the signal $P = \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T} \left[\underset{T\to\infty}{Lt} \int_{-T}^{T} |x(t)|^2 dt \right] = \underset{T\to\infty}{Lt} \frac{1}{2T} \left[\int_{-\infty}^{\infty} |x(t)|^2 dt \right]$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T} [E] \quad \because \text{Since } E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$= 0 \times E = 0 \quad \because \underset{T\to\infty}{Lt} \frac{1}{2T} = 0 .$$

Thus, the power of the energy signal is zero over infinite time.

b.) Energy of the power signal

Let $x(t)$ be a power signal, i.e. $P = \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$ is finite

Energy of the signal $E = \int_{-\infty}^{\infty} |x(t)|^2 dt$

i.e. $E = \underset{T\to\infty}{Lt} \int_{-T}^{T} |x(t)|^2 dt$

$= \underset{T\to\infty}{Lt} \left[2T \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt \right] = \underset{T\to\infty}{Lt}\ 2T \left[\underset{T\to\infty}{Lt} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt \right]$

$= \underset{T\to\infty}{Lt}\ 2TP \qquad \therefore P = \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$

$= \infty$

Thus, the energy of the power signal is infinite over infinite time.

19.) Determine the power and rms value of the following signals:

a.) $x(t) = 7\cos\left(20t + \frac{\pi}{2}\right)$

It is of the form $A\cos(\omega_0 t + \theta)$

Then power of the signal $P = \frac{7^2}{2} = 24.5\,W$

The rms value of the signal is $\sqrt{24.5}$.

b.) $x(t) = 12\cos\left(20t + \frac{\pi}{3}\right) + 16\sin\left(30t + \frac{\pi}{2}\right)$

$P = \frac{12^2}{2} + \frac{16^2}{2} = 200\,W$

rms $= \sqrt{200}$

c.) $x(t) = 8\cos 4t \cos 6t$

$x(t) = 8\left[\frac{\cos 10t + \cos 2t}{2}\right]$

$= 4\cos 10t + 4\cos 2t$

$P = \frac{4^2}{2} + \frac{4^2}{2} = 16\ W$

$rms = \sqrt{16} = 4.$

d.) $x(t) = e^{j2t}\cos 10t$

$x(t) = (\cos 2t + j\sin 2t)\cos 10t$

$= \cos 2t \cos 10t + j\sin 2t \cos 10t$

$= \frac{1}{2}[\cos 12t + \cos 8t + j(\sin 12t - \sin 8t)]$

$P = \frac{(\frac{1}{2})^2}{2} + \frac{(\frac{1}{2})^2}{2} + \frac{(\frac{1}{2})^2}{2} + \frac{(\frac{1}{2})^2}{2} = \frac{1}{2}\ W$

$rms = \sqrt{\frac{1}{2}}$

e.) $x(t) = Ae^{j5t}$

$x(t) = A(\cos 5t + j\sin 5t)$

$P = \frac{A^2}{2} + \frac{A^2}{2} = A^2$

$rms = \sqrt{A^2} = A$

20.) Determine whether the following signals are energy signals or power signals and calculate their energy or power :

a.) $x(t) = \sin^2 \omega_0 t$

This is a squared sine wave. Hence it is periodic signal. So it can be a power signal and calculate the power directly

$$P = \underset{T\to\infty}{Lt} \frac{1}{2T}\int_{-T}^{T} |x(t)|^2 dt$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T}\int_{-T}^{T} \sin^4 \omega_0 t \, dt$$

$$P = \underset{T\to\infty}{Lt} \frac{1}{2T}\int_{-T}^{T} \frac{1}{8}\left[3 - 4\cos 2\omega_0 t + \cos 4\omega_0 t\right] dt$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T}\int_{-T}^{T} \frac{3}{8} dt - \underset{T\to\infty}{Lt} \frac{1}{2T}\int_{-T}^{T} 4\cos 2\omega_0 t \, dt + \underset{T\to\infty}{Lt} \frac{1}{2T}\int_{-T}^{T} \cos 4\omega_0 t \, dt$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T}\frac{3}{8}(t)_{-T}^{T} - 0 + 0$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T}\frac{3}{8}\left[T + T\right] = \frac{3}{8} W$$

The power of the signal is finite and non zero.

Now, $E = \int_{-\infty}^{\infty} |x(t)|^2 dt$

$$= \underset{T\to\infty}{Lt} \int_{-T}^{T} (\sin^2 \omega_0 t)^2 dt$$

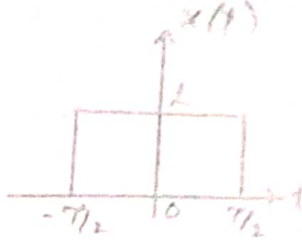
$$= \underset{T\to\infty}{Lt} \int_{-T}^{T} \frac{1}{8}\left[3 - 4\cos 2\omega_0 t + \cos 4\omega_0 t\right] dt$$

$$= \underset{T\to\infty}{Lt} \frac{3}{8}\left[t\right]_{-T}^{T} = \underset{T\to\infty}{Lt} \frac{3}{8} 2T = \infty$$

Hence it is a power signal with $P = \frac{3}{8}$ watts.

b.) $x(t) = rect\left(\frac{t}{\tau}\right)$

$$rect\left(\frac{t}{\tau}\right) = \begin{cases} 1 & \text{for } -\frac{\tau}{2} < t < \frac{\tau}{2} \\ 0 & \text{other wise.} \end{cases}$$



∴ It is non-periodic function.

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$= \int_{-\tau/2}^{\tau/2} (1)^2 dt = \frac{\tau}{2} + \frac{\tau}{2} = \tau$$

$$P = \underset{T \to \infty}{Lt} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

$$= \underset{T \to \infty}{Lt} \frac{1}{2T} \int_{-\tau/2}^{\tau/2} (1)^2 dt = \underset{T \to \infty}{Lt} \frac{1}{2T} [\tau] = 0$$

∴ It is energy signal with $E = \tau$ joules.

c.) $x(t) = rect(t/\tau) \sin\omega_0 t$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$= \int_{-\tau/2}^{\tau/2} [\sin\omega_0 t]^2 dt = \int_{-\tau/2}^{\tau/2} \left[\frac{1 - \cos 2\omega_0 t}{2}\right] dt$$

$$= \frac{1}{2} \int_{-\tau/2}^{\tau/2} dt - \frac{1}{2} \int_{-\tau/2}^{\tau/2} \cos 2\omega_0 t\, dt = \frac{\tau}{2} - 0$$

$$= \frac{\tau}{2} \text{ joules.}$$

d.) $x(t) = Ae^{-at}u(t)$, $a > 0$

$$x(t) = Ae^{-at}u(t) = \begin{cases} Ae^{-at} , & t \geq 0 \\ 0 , & t < 0 \end{cases}$$

$\therefore |x(t)|^2 = A^2 e^{-2at}$ for $t \geq 0$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$= \int_0^{\infty} |A e^{-at}|^2 dt = A^2 \int_0^{\infty} e^{-2at} dt$$

$$= A^2 \left[\frac{e^{-2at}}{-2a} \right]_0^{\infty}$$

$$E = \frac{A^2}{2a} \text{ J.}$$

e.) $x(t) = u(t)$

$$P = \underset{T \to \infty}{Lt} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

$$= \underset{T \to \infty}{Lt} \frac{1}{2T} \int_0^{T} (1)^2 dt = \frac{1}{2}$$

It is power signal $P = \frac{1}{2}$ W.

f.) $x(t) = t \cdot u(t)$

$$x(t) = tu(t) = \begin{cases} t , & t \geq 0 \\ 0 , & t < 0 \end{cases}$$

$\therefore |x(t)|^2 = t^2$ for $t \geq 0$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$= \underset{T\to\infty}{Lt} \int_0^T (t)^2 dt = \underset{T\to\infty}{Lt} \left[\frac{t^3}{3}\right]_0^T = \underset{T\to\infty}{Lt} \frac{T^3}{3} = \infty$$

$$P = \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{-T}^{T} t^2 dt = \underset{T\to\infty}{Lt} \frac{1}{2T} \left[\frac{t^3}{3}\right]_0^T = \infty$$

Hence the given signal is neither energy signal or power signal.

g.) $x(t) = e^{j[3t + (\pi/2)]}$

The given signal is an infinite duration periodic signal with is a combination of sine and cosine signals. So it can be a power signal.

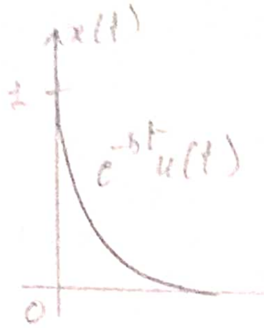
$$P = \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{-T}^{T} \left|e^{j[3t + (\frac{\pi}{2})]}\right|^2 dt$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T} [2T] = 1$$

21.) Sketch the following signals and say whether they are energy signals or power signals.

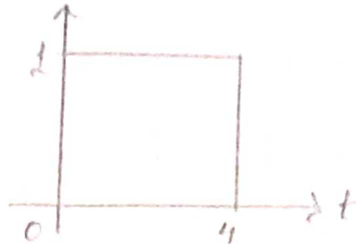
a.) $e^{-5t} u(t)$.



$E=\int_0^{\infty}|e^{-5t}|^2dt$

$=\frac{1}{10}$ J

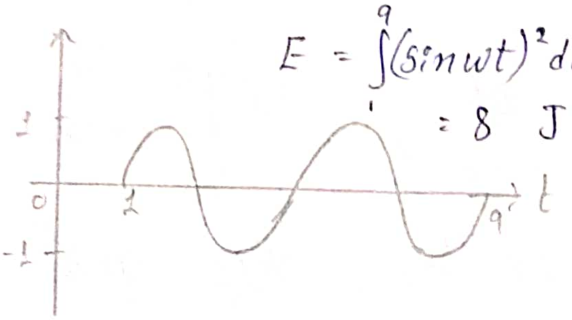
b.) $u(t)-u(t-4)$



$E=\int_0^{4}(1)^2dt$

$=4$ J

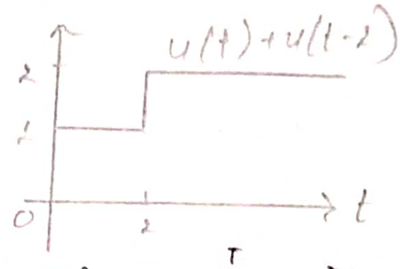
c.) $\sin wt\, u(t-1)u(9-t)$



$E=\int_1^{9}(\sin wt)^2dt$

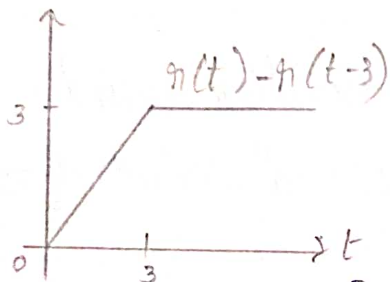
$=8$ J

d.) $u(t)+u(t-2)$



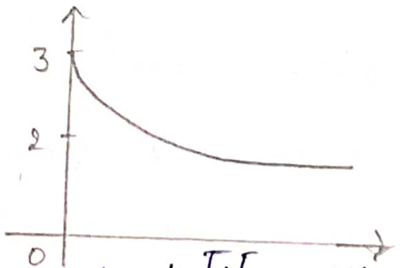
$P=\lim_{T\to\infty}\frac{1}{2T}\left[\int_0^{2}(1)^2dt+\int_2^{T}(2)^2dt\right]=2$ W

e.) $r(t)-r(t-3)$



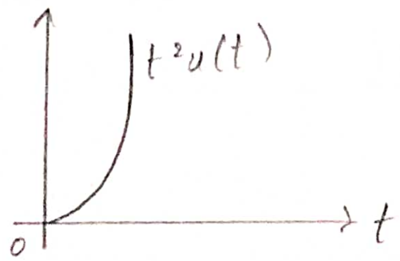
$P=\lim_{T\to\infty}\frac{1}{2T}\left[\int_0^{3}(t)^2dt+\int_3^{T}(3)^2dt\right]=4.5$ W

f.) $(2+e^{-6t})u(t)$



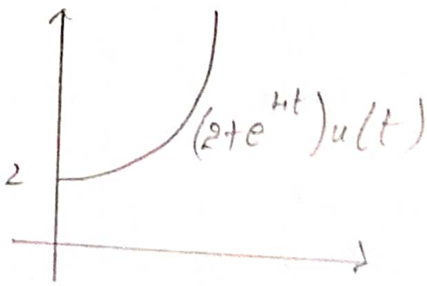
$P=\lim_{T\to\infty}\frac{1}{2T}\left[\int_0^{T}(2+e^{-6t})^2dt\right]=2$ W

g.) $t^2u(t)$



$P=\lim_{T\to\infty}\frac{1}{2T}\left[\int_0^{T}(t^2)^2dt\right]=\infty$

h.) $(2+e^{4t})u(t)$



∴ It is neither power signal or energy signal

22.) Find which of the following signals are energy signals, power signals, neither energy nor power signals :

a.) $\left(\frac{1}{2}\right)^n u(n)$

$$x(n) = \left(\frac{1}{2}\right)^n u(n)$$

$$E = \underset{N\to\infty}{Lt} \sum_{n=-N}^{N} \left[\left(\frac{1}{2}\right)^n\right]^2 u(n)$$

$$= \underset{N\to\infty}{Lt} \sum_{n=0}^{N} \left(\frac{1}{4}\right)^4$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n = \frac{1}{1-\left(\frac{1}{4}\right)} = \frac{4}{3} \cdot J$$

$$P = \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=0}^{N} \left(\frac{1}{4}\right)^n$$

$$= \underset{N\to\infty}{Lt} \frac{1}{2N+1} \left[\frac{1-(1/4)^{N+1}}{1-(1/4)}\right] = 0$$

$\therefore$ It is energy signal.

b.) $e^{j[(\pi/3)n + (\pi/2)]}$

$$x(t) = e^{j[(\pi/3)n + (\pi/2)]}$$

$$E = \underset{N\to\infty}{Lt} \sum_{n=-N}^{N} \left|e^{j(\pi/3)n + (\pi/2)}\right|^2$$

$$E = \underset{N\to\infty}{Lt} \sum_{n=-N}^{N} 1 = \underset{N\to\infty}{Lt} [2N+1] = \infty$$

$$P = \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} \left| e^{j[(\pi/3)n + (\pi/2)]} \right|^2$$

$$= \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} 1 = \underset{N\to\infty}{Lt} \frac{1}{2N+1} [2N+1]$$

$$= 1 \text{ W}$$

$\therefore$ It is power signal.

c.) $\sin\left(\frac{\pi}{3}n\right)$

$$x(n) = \sin\left(\frac{\pi}{3}n\right)$$

$$E = \underset{N\to\infty}{Lt} \sum_{n=-N}^{N} \sin^2\left(\frac{\pi}{3}n\right)$$

$$= \underset{N\to\infty}{Lt} \sum_{n=-N}^{N} \frac{1-\cos[(2\pi/3)n]}{2} = \frac{1}{2} \underset{N\to\infty}{Lt} \sum_{n=-N}^{N} \left(1-\cos\frac{2\pi}{3}n\right)$$

$$= \infty$$

$$P = \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} \sin^2\left(\frac{\pi}{3}n\right)$$

$$= \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} \frac{1-\cos[(2\pi/3)n]}{2}$$

$$= \frac{1}{2} \underset{N\to\infty}{Lt} \frac{1}{2N+1} [2N+1] = \frac{1}{2} \text{ W}$$

$\therefore$ It is a power signal.

d.) $u(n) - u(n-6)$

$$x(n) = u(n) - u(n-6)$$

$$E = \underset{N\to\infty}{Lt} \sum_{n=-N}^{N} [u(n) - u(n-6)]^2$$

$$= \underset{N\to\infty}{Lt} \sum_{n=0}^{5} 1 = 6 \text{ J}.$$

$$P = \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} [u(n) - u(n-6)]^2$$

$$= \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=0}^{5} 1 = 0$$

$\therefore$ It is an energy signal.

e.) $nu(n)$

$$x(n) = nu(n)$$

$$E = \underset{N\to\infty}{Lt} \sum_{n=-N}^{N} [n]^2 u(n)$$

$$= \underset{N\to\infty}{Lt} \sum_{n=0}^{N} [n^2] = \infty$$

$$P = \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} [n]^2 u(n)$$

$$= \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=0}^{N} n^2 = \infty$$

$\therefore$ It is neither energy signal nor power signal.

f.) $r(n) - r(n-4)$

$$x(n) = r(n) - r(n-4)$$

$$E = \underset{N\to\infty}{Lt} \sum_{n=-N}^{N} [r(n) - r(n-4)]^2$$

$$E = \underset{N\to\infty}{Lt} \sum_{n=0}^{4}\left[n^2 + \sum_{n=5}^{N} (4)^2\right] = \infty$$

$$P = \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} [x(n) - x(n-4)]^2$$

$$= \underset{N\to\infty}{Lt} \frac{1}{2N+1} \left[\sum_{n=0}^{4} n^2 + \sum_{n=5}^{N} (4)^2\right] = 8\ W$$

$\therefore$ It is power signal.

23.) Find whether the signal

$$x(t) = \begin{cases} t-2 & -2 \le t \le 0 \\ 2-t & 0 \le t \le 2 \\ 0 & \text{otherwise} \end{cases}$$

is energy signal or power signal. Also find the energy and power of the signal.

sol:-
$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$= \left[\int_{-2}^{0} (t-2)^2 dt + \int_{0}^{2} (2-t)^2 dt\right]$$

$$= \int_{-2}^{0} (t^2 - 4t + 4) dt + \int_{0}^{2} (4 + t^2 - 4t) dt$$

$$= \left[\frac{t^3}{3} - \frac{4t^2}{2} + 4t\right]_{-2}^{0} + \left[4t + \frac{t^3}{3} - \frac{4t^2}{2}\right]_{0}^{2}$$

$$= \frac{64}{3}\ J$$

$$P = \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T} \left[\int_{-2}^{0} (t-2)^2 \, dt + \int_{0}^{2} (2-t)^2 \, dt \right]$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T} \left[\frac{64}{3} \right] = 0$$

$\therefore$ It is an energy signal

24.) Find the whether the signal

$$x(n) = \begin{cases} n^2 & 0 \le n \le 3 \\ 10-n & 4 \le n \le 6 \\ n & 7 \le n \le 9 \\ 0 & \text{otherwise.} \end{cases}$$

is a power signal or energy signal. Also find the energy and power of the signal.

sol:-

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=0}^{3} (n^2)^2 + \sum_{n=4}^{6} (10-n)^2 + \sum_{n=7}^{9} (n)^2$$

$$= \sum_{n=0}^{3} n^4 + \sum_{n=4}^{6} (100 + n^2 - 20n) + \sum_{n=7}^{9} n^2$$

$$= (0+1+16+81) + (36+25+16) + (49+64+81)$$

$$= 369 < \infty \text{ joules}$$

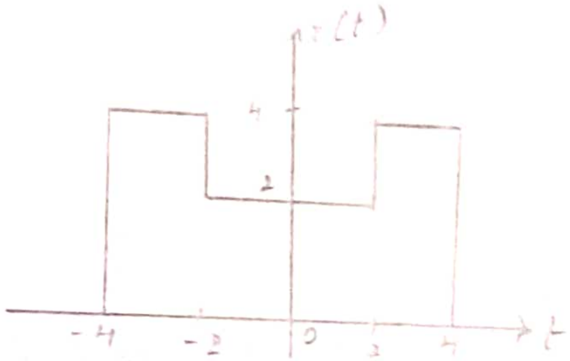
$$P = \underset{N \to \infty}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \underset{N \to \infty}{Lt} \frac{1}{2N+1} \left[\sum_{n=0}^{3} (n^2)^2 + \sum_{n=4}^{6} (10-n)^2 + \sum_{n=7}^{9} (n)^2 \right]$$

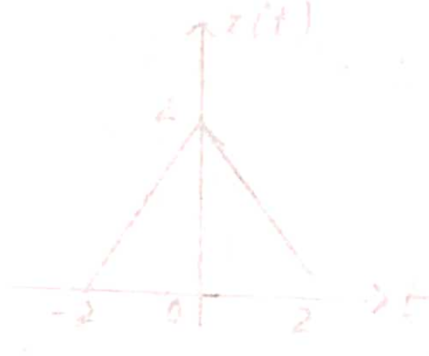
$$= \underset{N \to \infty}{Lt} \frac{1}{2N+1} [369] = 0$$

$\therefore$ It is an energy signal.

25.) Find the energy of the signals shown in figure.



(a)



(b)

Sol:-

(a) $$x(t) = \begin{cases} 4 & -4 \le t \le -2 \\ 2 & -2 \le t \le 2 \\ 4 & 2 \le t \le 4 \\ 0 & \text{otherwise} \end{cases}$$

$$E = \int_{-4}^{-2} (4)^2 \, dt + \int_{-2}^{2} (2)^2 dt + \int_{2}^{4} (4)^2 dt$$

$$= 16 (t)_{-4}^{-2} + 4 (t)_{-2}^{2} + 16 (t)_{2}^{4}$$

$$= 16(-2+4) + 4(2+2) + 16(4-2)$$

$$= 80 \text{ J}$$

b.) $x(t) = \begin{cases} t+2 & -2 \le t \le 0 \\ 2-t & 0 \le t \le 2 \\ 0 & \text{otherwise} \end{cases}$

$$E = \int_{-2}^{0} [t+2]^2\, dt + \int_{0}^{2} [2-t]^2\, dt$$

$$= \int_{-2}^{0} [t^2+4t+4]\, dt + \int_{0}^{2} [4+t^2-4t]\, dt$$

$$= \frac{16}{3} \text{ J}$$

26.) Find which of the following signals are causal or non-causal.

a.) $x(t) = e^{2t} u(t-1)$

The signal $x(t)$ is causal because $x(t) = 0$ for $t < 0$

b.) $x(t) = e^{-3t} u(-t+2)$

The given signal $x(t)$ is non causal because $x(t) \neq 0$ for $t < 0$

c.) $x(t) = 3 \operatorname{sinc} 2t$.

A sinc signal $x(t)$ is non causal because it exists for $t < 0$ also

d.) $x(t) = u(t+2) - u(t-2)$

The given signal exists from $t = -2$ to $t = +2$. Since $x(t) \neq 0$ for $t < 0$ it is non causal

e.) $x(t) = \cos 2t$.

The given signal exists from $-\infty$ to ∞, since $x(t) \neq 0$ for $t < 0$, the signal is non causal.

f.) $x(t) = \sin 2t \cdot u(t)$

The given signal is causal because $x(t) = 0$ for $t < 0$

g.) $x(n) = u(n+4) - u(n-2)$

The given signal exists from $n = -4$ to $n = 1$. Since $x(n) \neq 0$ for $n < 0$, it is non causal

h.) $x(n) = \left(\frac{1}{4}\right)^n u(n+2) - \left(\frac{1}{2}\right)^n u(n-4)$

The given signal exists for $n < 0$ also. So it is non-causal

i.) $x(t) = 2u(-t)$

The given signal exists only for $t < 0$. So it is anticausal. It can be called non-causal also.

j.) $x(n) = u(-n)$

The given signal exists only for $n < 0$. So it is anti-causal. It can be called non-causal also.

27) Find the even and odd components of the following signals :

a) $x(t) = e^{j2t}$

$\therefore x(-t) = e^{-j2t}$

$x_e(t) = \frac{1}{2}[x(t) + x(-t)] = \frac{1}{2}[e^{j2t} + e^{-j2t}] = \cos 2t$

$x_o(t) = \frac{1}{2}[x(t) - x(-t)] = \frac{1}{2}[e^{j2t} - e^{-j2t}] = j\sin 2t$

b) $x(t) = \cos\left(\omega_o t + \frac{\pi}{3}\right)$

$\therefore x(-t) = \cos\left(-\omega_o t + \frac{\pi}{3}\right) = \cos\left(\omega_o t - \frac{\pi}{3}\right)$

$x_e(t) = \frac{1}{2}[x(t) + x(-t)] = \frac{1}{2}\left[\cos\left(\omega_o t + \frac{\pi}{3}\right) + \cos\left(\omega_o t - \frac{\pi}{3}\right)\right]$

$= \cos\omega_o t \cos\frac{\pi}{3} = \frac{1}{2}\cos\omega_o t$

$x_o(t) = \frac{1}{2}[x(t) - x(-t)] = \frac{1}{2}\left[\cos\left(\omega_o t + \frac{\pi}{3}\right) - \cos\left(\omega_o t - \frac{\pi}{3}\right)\right]$

$= -\sin\omega_o t \sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}\sin\omega_o t$

c) $x(t) = (1 + t^2 + t^3)\cos^2 10t$

$\therefore x(-t) = (1 + t^2 - t^3)\cos^2 10t$

$x_e(t) = \frac{1}{2}[x(t) + x(-t)] = \frac{1}{2}[(1+t^2+t^3)\cos^2 10t + (1+t^2-t^3)\cos^2 10t]$

$= \frac{1}{2}[2(1+t^2)\cos^2 10t] = (1+t^2)\cos^2 10t$

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)] = \frac{1}{2}[(1+t^2+t^3)\cos^2 10t - (1+t^2-t^3)\cos^2 10t]$$

$$= \frac{1}{2}[2t^3\cos^2 10t] = t^3\cos^2 10t$$

d) $x(t) = \sin 2t + \sin 2t\cos 2t + \cos 2t$

$$\therefore x(-t) = \sin(-2t) + \sin(-2t)\cos(-2t) + \cos(-2t)$$

$$= -\sin 2t - \sin 2t\cos 2t + \cos 2t$$

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

$$= \frac{1}{2}[(\sin 2t + \sin 2t\cos 2t + \cos 2t) + (-\sin 2t - \sin 2t\cos 2t + \cos 2t)]$$

$$= \frac{1}{2}[2\cos 2t] = \cos 2t$$

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

$$= \frac{1}{2}[\sin 2t + \sin 2t\cos 2t + \cos 2t) + (-\sin 2t - \sin 2t\cos 2t + \cos 2t)]$$

$$= \frac{1}{2}2[\sin 2t + \sin 2t\cos 2t] = \sin 2t + \sin 2t\cos 2t$$

e) $x(t) = 1 + 2t + 3t^2 + 4t^3$

$$\therefore x(-t) = 1 - 2t + 3t^2 - 4t^3$$

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

$$= \frac{1}{2}[(1 + 2t + 3t^2 + 4t^3) + (1 - 2t + 3t^2 - 4t^3)]$$

$$= \frac{1}{2}[2(1 + 3t^2)] = 1 + 3t^2$$

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

$$= \frac{1}{2}[(1+2t+3t^2+4t^3) - (1-2t+3t^2-4t^3)]$$

$$= \frac{1}{2}2[2t+4t^3] = 2t+4t^3$$

28.) Find whether the following signals are even or odd.

a.) $x(t) = e^{-3t}$

$$x(-t) = e^{3t}$$

$$-x(t) = -e^{-3t}$$

Since $x(-t) \neq x(t)$ and $x(-t) \neq -x(t)$, the given signal is neither even signal nor odd signal.

b.) $x(t) = u(t+2)$

$$x(-t) = u(-t+2)$$

$$-x(t) = -u(t+2)$$

Then $$u(t+2) = \begin{cases} 1 & \text{for } t \geq -2 \\ 0 & \text{for } t < -2 \end{cases}$$

$$u(-t+2) = \begin{cases} 1 & \text{for } t \leq 2 \\ 0 & \text{for } t > 2 \end{cases}$$

$$-u(t+2) = \begin{cases} -1 & \text{for } t > -2 \\ 0 & \text{for } t < -2 \end{cases}$$

Since $x(-t) \neq x(t)$ and $x(-t) \neq -x(t)$, the given signal is neither even signal nor odd signal c.) $x(t) = 3e^{j4\pi t}$

$x(-t) = 3e^{-j4\pi t}$

$-x(t) = -3e^{j4\pi t}$

since $x(-t) \neq x(t)$ and $x(-t) \neq -x(t)$, the given signal is neither even signal nor odd signal

d.) $x(t) = u(t+4) - u(t-2)$

$x(-t) = u(-t+4) - u(-t-2)$

$-x(t) = -u(t+4) + u(t-2)$

since $x(-t) \neq x(t)$ and $x(-t) \neq -x(t)$, the given signal is neither even signal nor odd signal.

29.) Find the even and odd components of the following signals.

a.) $x(n) = \left\{-3, 1, \underset{\uparrow}{2}, -4, 2\right\}$

$x(-n) = \left\{2, -4, \underset{\uparrow}{2}, 1, -3\right\}$

$\therefore x_e(n) = \frac{1}{2}\left[x(n) + x(-n)\right]$

$= \frac{1}{2}\left[-3+2, 1-4, 2+2, -4+1, 2-3\right]$

$= \left\{-0.5, -1.5, \underset{\uparrow}{2}, -1.5, -0.5\right\}$

$$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$$

$$= \frac{1}{2}[-3-2, 1+4, 2-2, -4-1, 2+3]$$

$$= \{-2.5, 2.5, \underset{\uparrow}{0}, -2.5, 2.5\}$$

b.) $x(n) = \{-2, 5, \underset{\uparrow}{1}, -3\}$

$$x(-n) = -3, \underset{\uparrow}{1}, 5, -2$$

$$\therefore x_e(n) = \frac{1}{2}[x(n) + x(-n)]$$

$$= \frac{1}{2}[-2+0, 5-3, 1+1, -3+5, 0-2]$$

$$= -1, 1, \underset{\uparrow}{1}, 1, -1$$

$$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$$

$$= \frac{1}{2}[-2-0, 5+3, 1-1, -3-5, 0+2]$$

$$= -1, 4, \underset{\uparrow}{0}, -4, 1$$

c.) $x(n) = \{\underset{\uparrow}{5}, 4, 3, 2, 1\}$

$$n = 0, 1, 2, 3, 4$$

$$\therefore x(n) = \underset{\uparrow}{5}, 4, 3, 2, 1$$

$$x(-n) = 1, 2, 3, 4, \underset{\uparrow}{5}$$

$$x_e(n) = \frac{1}{2}[1, 2, 3, 4, 5+5, 4, 3, 2, 1]$$

$$= \{0.5, 1, 1.5, 2, \underset{\uparrow}{5}, 2, 1.5, 1, 0.5\}$$

$$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$$

$$= \frac{1}{2}[-1, -2, -3, -4, 0, 4, 3, 2, 1]$$

$$= \{-0.5, -1, -1.5, -2, \underset{\uparrow}{0}, 2, 1.5, 1, 0.5\}$$

d.) $x(n) = \{5, 4, 3, 2, \underset{\uparrow}{1}\}$

$$n = -4, -3, -2, -1, 0$$

$$\therefore x(n) = 5, 4, 3, 2, \underset{\uparrow}{1}$$

$$x(-n) = \underset{\uparrow}{1}, 2, 3, 4, 5$$

$$\therefore x_e(n) = \frac{1}{2}[x(n) + x(-n)]$$

$$= \frac{1}{2}[5, 4, 3, 2, 1+1, 2, 3, 4, 5]$$

$$= [2.5, 2, 1.5, 1, 1, 1, 1.5, 2, 2.5]$$

$$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$$

$$= \frac{1}{2}[5, 4, 3, 2, 0, -2, -3, -4, -5]$$

$$= \frac{1}{2}[2.5, 2, 1.5, 1, 0, -1, -1.5, -2, -2.5]$$

Problems

1. Evaluate the following

a) $\int_{-\infty}^{\infty} e^{-t^2} \delta(t-3)\,dt$

WKT $\delta(t-3) = \begin{cases} 1 & \text{for } t=3 \\ 0 & \text{elsewhere} \end{cases}$

$$\therefore \int_{-\infty}^{\infty} e^{-t^2} \delta(t-3)\,dt = \left[e^{-t^2}\right]_{t=3} = e^{-9}$$

b) $\int_{0}^{\infty} t^3 \delta(t-2)\,dt$

WKT $\delta(t-2) = \begin{cases} 1 & \text{for } t=2 \\ 0 & \text{else where} \end{cases}$

$$\therefore \int_{0}^{\infty} t^3 \delta(t-2)\,dt = \left[t^3\right]_{t=2} = 8$$

c) $\int_{-\infty}^{\infty} \delta(t+3) e^{-2t}\,dt$

WKT $\delta(t+3) = \begin{cases} 1 & \text{for } t=-3 \\ 0 & \text{elsewhere} \end{cases}$

$$\therefore \int_{-\infty}^{\infty} \delta(t+3) e^{-2t} dt = \left[e^{-2t}\right]_{t=-3} = e^{6} \,//$$

d.) $\displaystyle\int_{-\infty}^{\infty} (t-1)^2 \, \delta(t-1) dt$

WKT $\quad \delta(t-1) = \begin{cases} 1 & \text{for } t=1 \\ 0 & \text{else where} \end{cases}$

$$\therefore \int_{-\infty}^{\infty} (t-1)^2 \delta(t-1) \, dt = \left[(t-1)^2\right]_{t=1} = 0 \,//$$

e.) $\displaystyle\sum_{n=-\infty}^{\infty} e^{2n} \, \delta(n-2)$

WKT $\quad \delta(n-2) = \begin{cases} 1 & \text{for } n=2 \\ 0 & \text{elsewhere} \end{cases}$

$$\therefore \sum_{n=-\infty}^{\infty} e^{2n} \delta(n-2) = \left[e^{2n}\right]_{n=2} = e^{4} \,//$$

f.) $\displaystyle\sum_{n=-\infty}^{\infty} n^2 \, \delta(n-3)$

WKT $\quad \delta(n-3) = \begin{cases} 1 & \text{for } n=3 \\ 0 & \text{elsewhere} \end{cases}$

$$\therefore \sum_{n=-\infty}^{\infty} n^2 \delta(n-3) = \left[n^2\right]_{n=3} = \cancel{47} \; 9 \,//$$

g.) $\displaystyle\sum_{n=-\infty}^{\infty} \delta(n) \, 5^{n}$

WKT $\quad \delta(n) = \begin{cases} 1 & \text{for } n=0 \\ 0 & \text{elsewhere} \end{cases}$

$$\therefore \sum_{n=-\infty}^{\infty} \delta(n) 5^n = [5^n]_{n=0} = 1$$

4.) $\sum_{n=-\infty}^{\infty} \delta(n) \sin 2n$

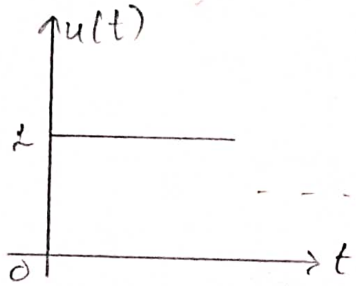
WKT $\delta(n) = \begin{cases} 1 & , n=0 \\ 0 & \text{elsewhere} \end{cases}$

$$\therefore \sum_{n=-\infty}^{\infty} \delta(n) \sin 2n = [\sin 2n]_{n=0} = 0$$

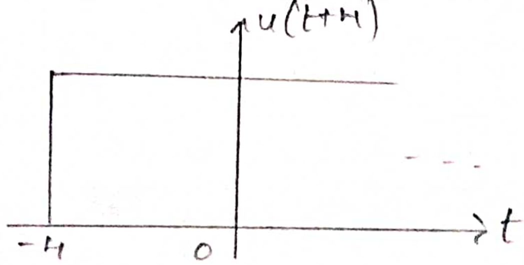
2.) Sketch the following signals :

a.) $u(-t+4)$

The basic unit step signal is



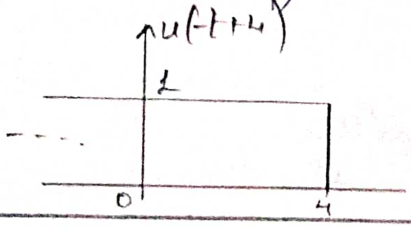
(i) $u(t+4)$



$t = -4$

$\therefore$ Time shifting

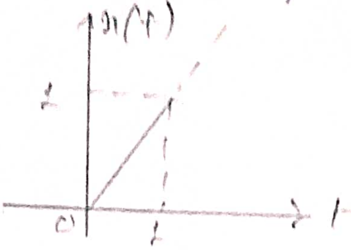
(ii) $u(-t+4)$



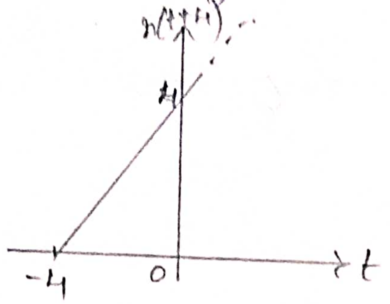
$\therefore$ Time folding

b.) $r(-t+4)$

The basic ramp signal is



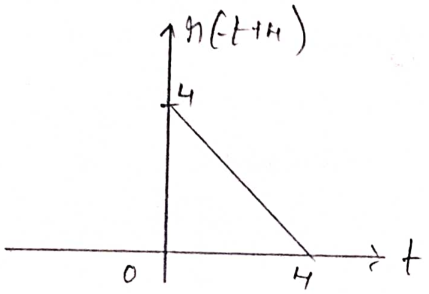
(i) $r(t+4)$



$t = -4$

$\therefore$ Time shifting

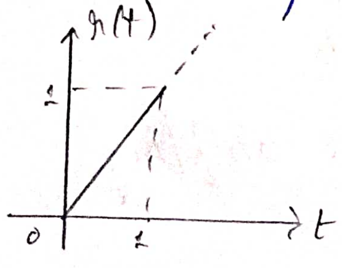
(ii) $r(-t+4)$



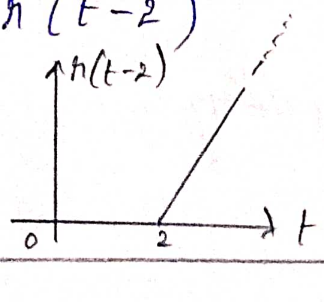
$\therefore$ time folding

c.) $-2r(t-2)$

The basic ramp signal is



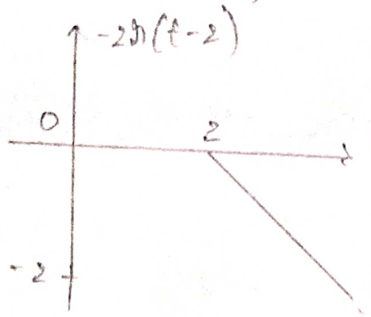
(i) $r(t-2)$



$\therefore t = 2$

Time shifting

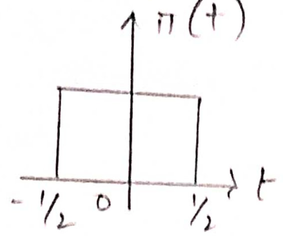
(ii) $-2r(t-2)$



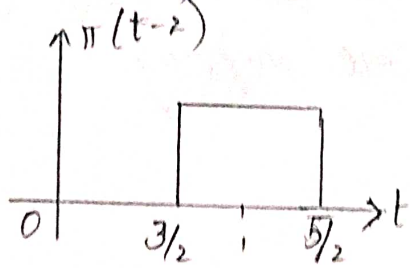
$\therefore$ Amplitude scaling.

d.) $\pi(t-2)$

The basic gate signal is.



(i) $\pi(t-2)$

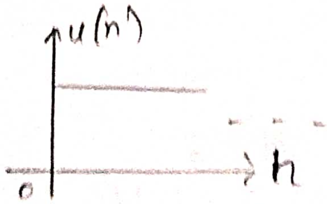


$\therefore t = 2$

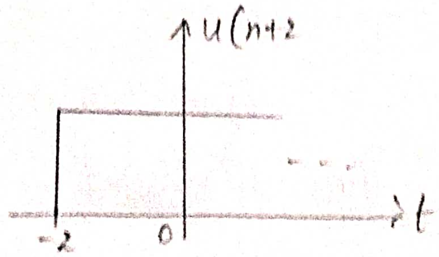
Time shifting

e.) $u(n+2) - u(n)$

The basic unit step signal is



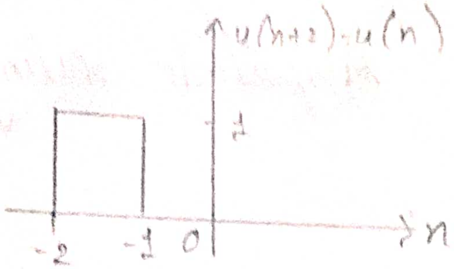
(i) $u(n+2)$



$\therefore n = -2$

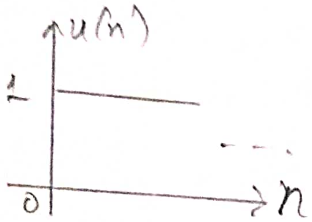
Time shifting

(ii) $u(n+2) - u(n)$

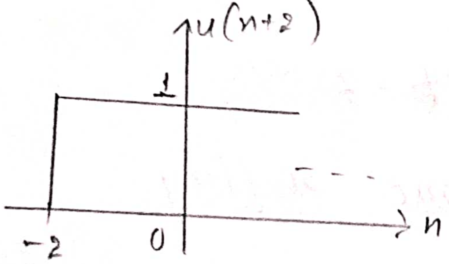


f.) $u(-n+2) - u(-n-2)$

The basic unit step signal is

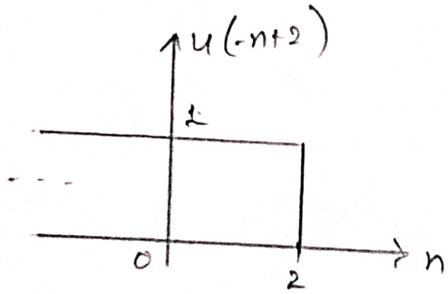


(i) $u(-n+2)$



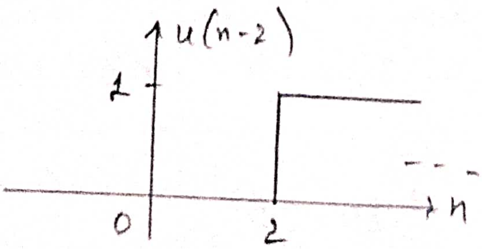
$\therefore n = -2$

Time shifting.



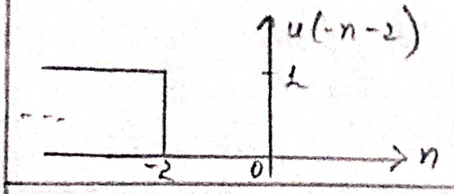
$\therefore$ Time reversal

(ii) $u(-n-2)$



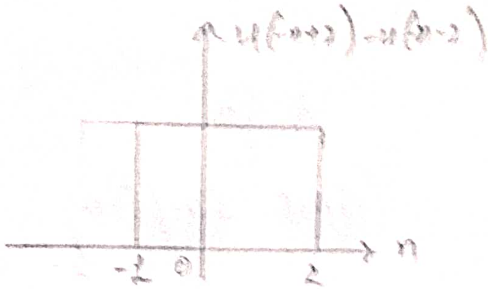
$\therefore n = 2$

Time shifting

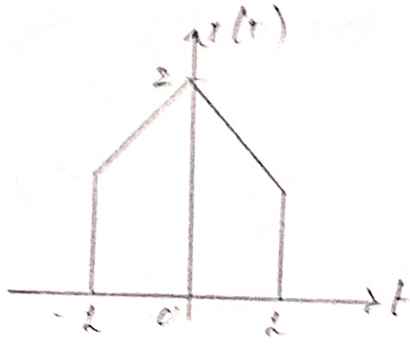


$\therefore$ Time reversal

(iii) $u(-n+2) - u(-n-2)$

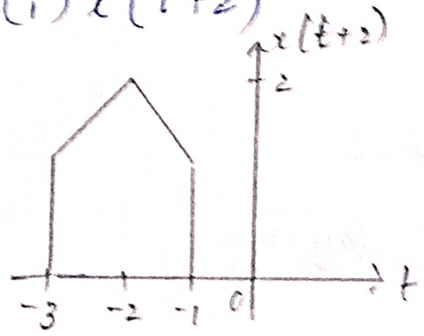


3.) For the signal $x(t)$ shown in figure, find the signals.



a.) $x(2t+2)$

(i) $x(t+2)$



$\therefore t = -2$

Time shifting.

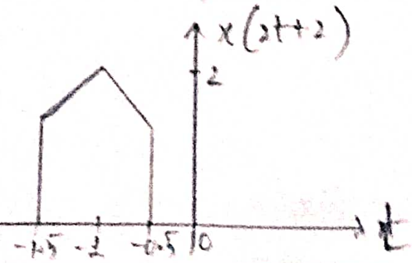
(ii) $x(2t+2)$

$x(-3) = x\left(\frac{-3}{2}\right) = -1.5$

$x(-2) = x\left(\frac{-2}{2}\right) = -1$

$x(-1) = x\left(\frac{-1}{2}\right) = -0.5$

$\therefore$ Time scaling.

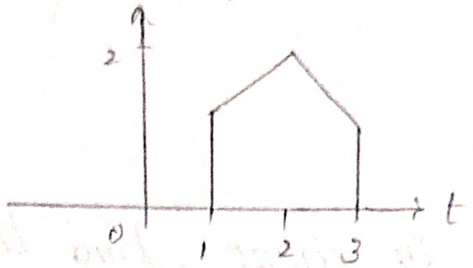


b.) $x(\frac{1}{2}t-2)$

(i) $x(t-2)$

$\therefore t=2$

Time shifting



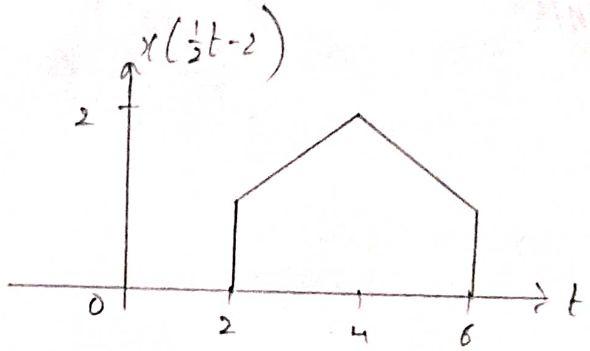
(ii) $x(\frac{1}{2}t-2)$

$\therefore$ Time scaling

$x(1) = x(2\cdot1) = 2$

$x(2) = x(2\cdot2) = 4$

$x(3) = x(2\cdot3) = 6$

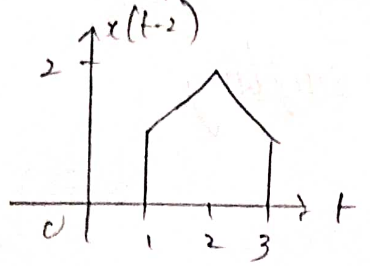


c.) $x(-t-2)$

(i) $x(t-2)$

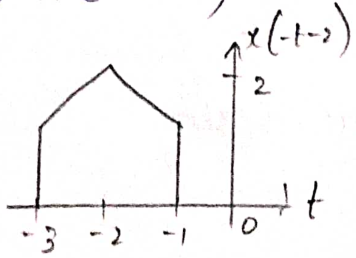
$\therefore t=2$

Time shifting



(ii) $x(-t-2)$

$\therefore$ Time folding



d.) $3x(5t)$

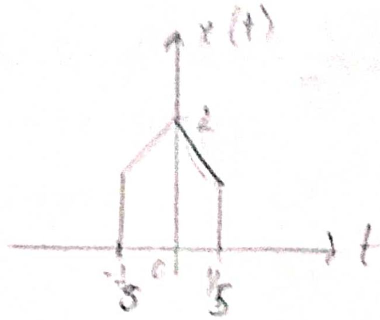
(i) $x(5t)$

$\therefore$ Time scaling.

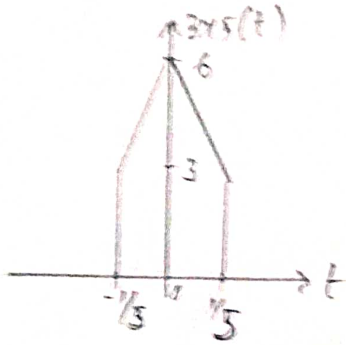
$x(-1) = x\left(\frac{-1}{5}\right) = \frac{-1}{5}$

$x(0) = x(0) = 0$

$x(1) = x\left(\frac{1}{5}\right) = \frac{1}{5}$



(ii) $3x5(t)$



∴ Amplitude scaling

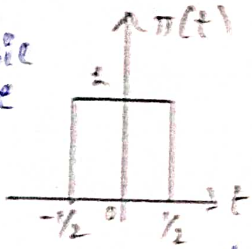
$2 \times 3 = 6$.

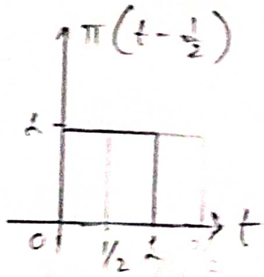
4.) Sketch the following signals

a.) $\Pi\left(\frac{t-2}{4}\right) + \Pi(t-2)$

(i) $\Pi\left(\frac{t-2}{4}\right) = \Pi\left(\frac{1}{4}t - \frac{1}{2}\right)$

basic gate





∴ $t = \frac{1}{2}$

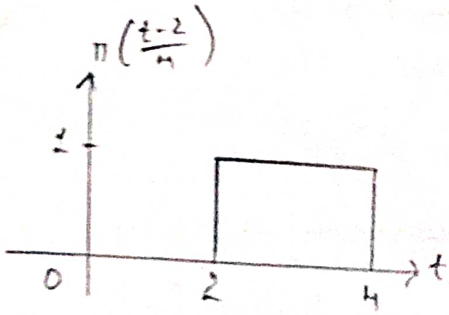
Time shifting.

∴ Time scaling

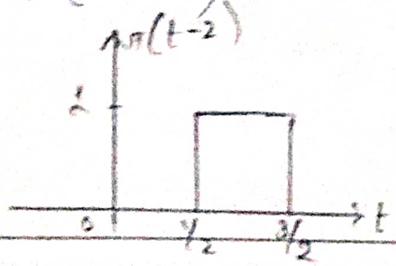
$x(0) = x(4(0)) = 0$

$x(1/2) = x\left(4 \times \frac{1}{2}\right) = 2$

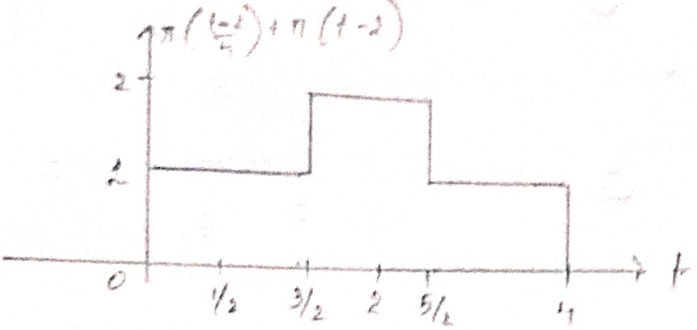
$x(1) = x(4 \times 1) = 1$



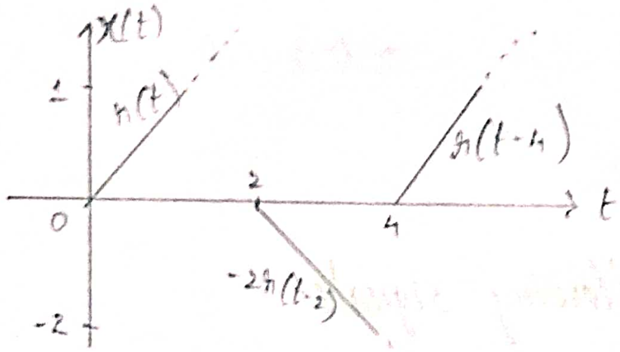
(ii) $\Pi(t-2)$

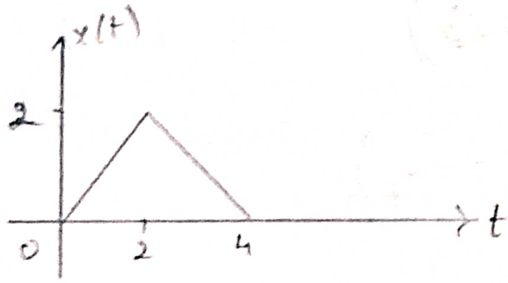


(iii) $\pi\left(\frac{t-2}{4}\right) + \pi(t-2)$



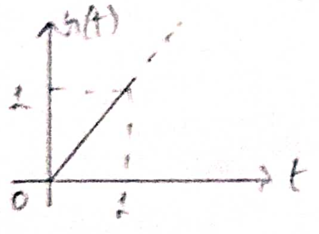
b.) $r(t) - 2r(t-2) + r(t-4)$



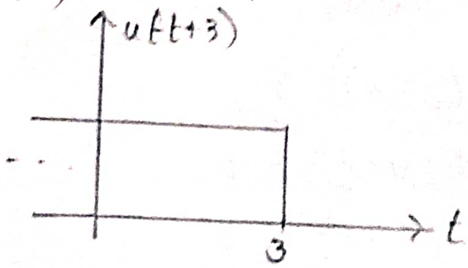


c.) $r(t) \cdot u(-t+3)$

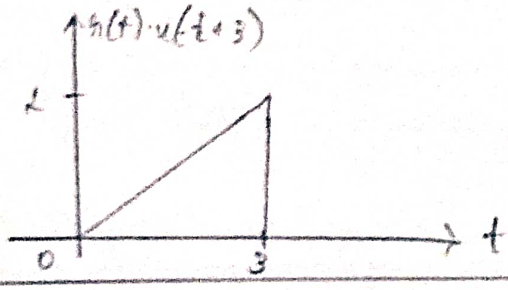
(i) $r(t)$



(ii) $u(-t+3)$

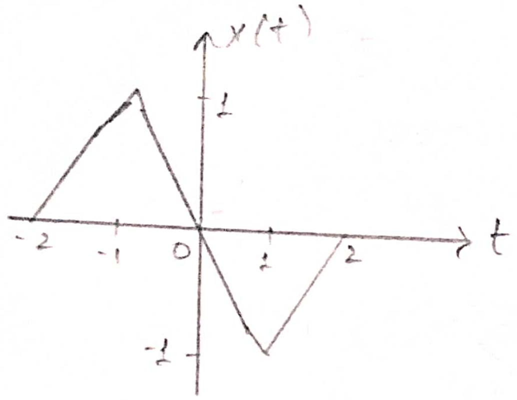


(iii) $r(t) \cdot u(-t+3)$

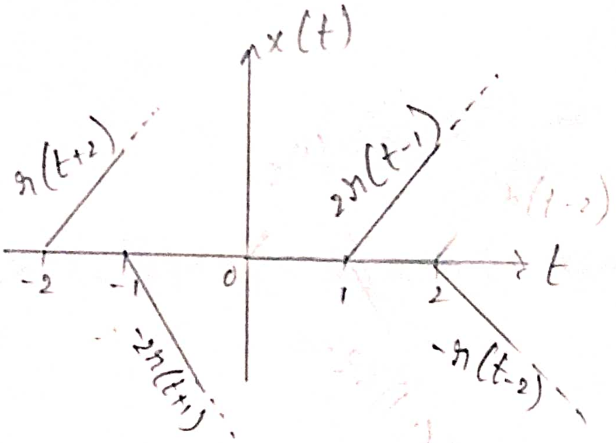


5. Express the signals shown in figures as sum of singular functions.

a.)

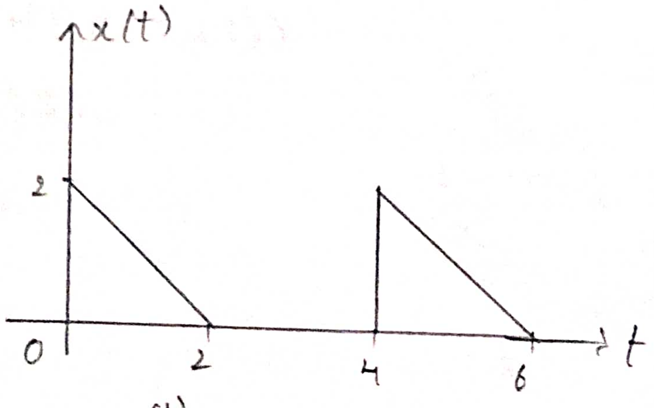


Sol :-

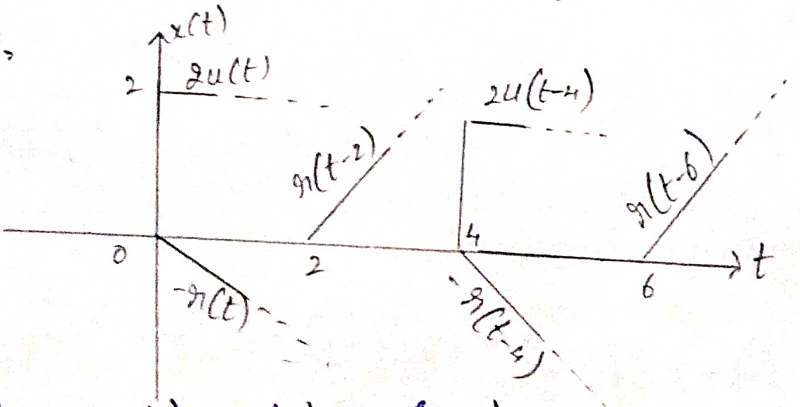


$$x(t) = r(t+2) - 2r(t+1) + 2r(t-1) - r(t-2)$$

b.)

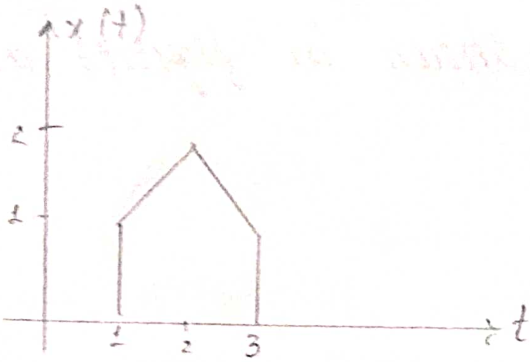


Sol :-

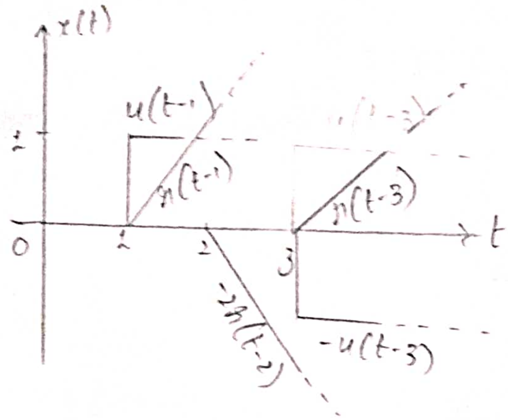


$$x(t) = 2u(t) - r(t) + r(t-2) + 2u(t-4) - r(t-4) + r(t-6)$$

c.)

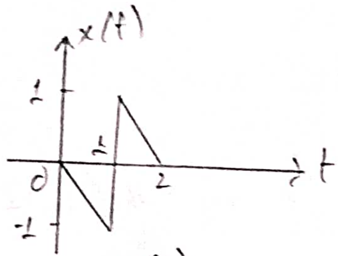


Sol :-

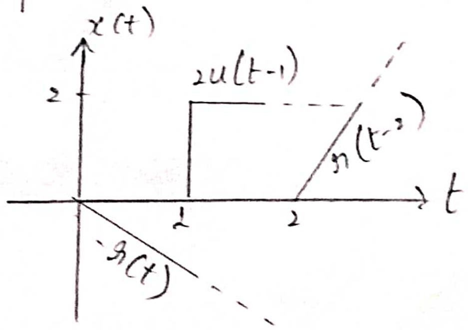


$$x(t) = u(t-1) + r(t-1) - 2r(t-2) + r(t-3) - u(t-3)$$

d.)

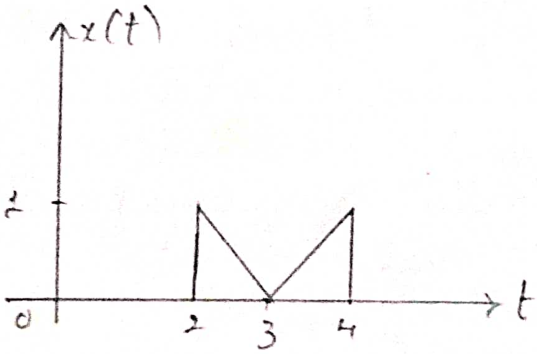


Sol :=



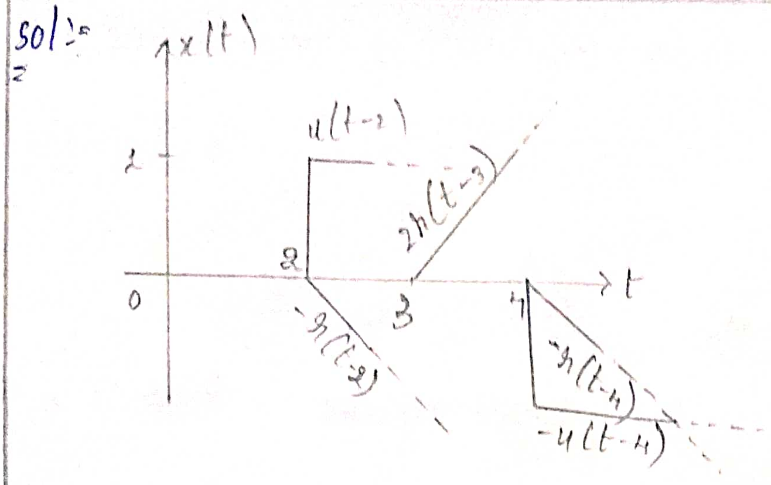
$$x(t) = -r(t) + 2u(t-1) + r(t-2)$$

e.)

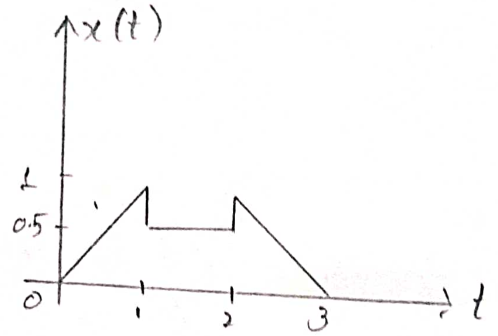


$$x(t) = u(t-2) - r(t-2) + 2r(t-3) - r(t-4) - u(t-4)$$

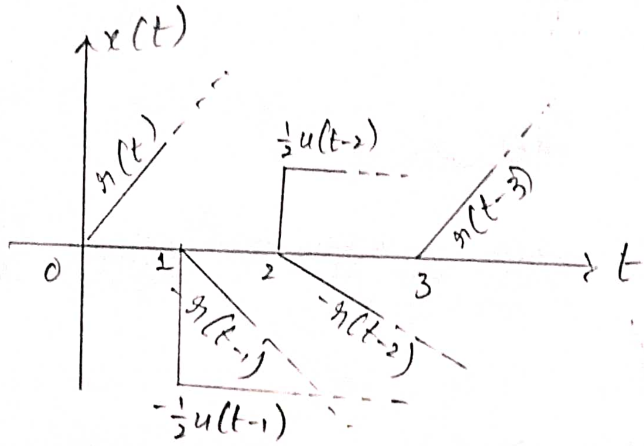
sol:-



f.)



sol:-



$$x(t) = r(t) - r(t-1) - \frac{1}{2}u(t-1) + \frac{1}{2}u(t-2) - r(t-2) + r(t-3)$$

6.) Examine whether the following signals are periodic or not. If periodic, determine the fundamental period.

a.) $\cos 6\pi t$

$x(t) = \cos 6\pi t$

$\therefore \omega_0 = 6\pi \Rightarrow T = \frac{2\pi}{\omega_0} = \frac{2\pi}{6\pi} = \frac{1}{3}$ sec

$\therefore$ It is periodic with fundamental period $T = \frac{1}{3}$ sec b.) $e^{j8\pi t}$

$x(t) = e^{j8\pi t}$

$\omega_0 = 8\pi \Rightarrow T = \frac{2\pi}{\omega_0} = \frac{2\pi}{8\pi} = \frac{1}{4}$ sec.

$\therefore$ It is periodic with fundamental period $T = \frac{1}{4}$ sec

c.) $2 + \sin 4\pi t$

$x(t) = 2 + \sin 4\pi t$

$x_1(t) = 2$ $\therefore$ ~~It is aperiodic~~ It is shifted to 2.

$x_2(t) = \sin 4\pi t$

$\omega_0 = 4\pi \Rightarrow T = \frac{2\pi}{4\pi} = \frac{1}{2}$ sec

$\therefore$ It is periodic with fundamental period $T = \frac{1}{2}$ sec

d.) $2u(t) + 3\cos 2\pi t$

$x(t) = 2u(t) + 3\cos 2\pi t$

$x_1(t) = 2u(t)$

$\therefore$ It is non-periodic

$x_2(t) = 3\cos 2\pi t$

$\omega_0 = 2\pi \Rightarrow T = \frac{2\pi}{\omega_0} = \frac{2\pi}{2\pi} = 1$ sec

$\therefore$ It is not periodic