Computational Finance Assignment 2

Group 9: Chun Cheng-13137727, Meifang Li-13043390, Anzhen Zhang-13012843

March 2021

1 Introduction

When there is no arbitrage opportunity in the market, we can calculate the price of an option as the discounted value of the expected payoff of this option. There are different styles of options and the calculation of their payoff can be different. In this report, we will mainly focus on European put option, digital call option and discrete Asian call option. For all these options, we only consider those that can only be exercised at maturity. If exercising the option is not profitable at maturity, it will not be exercised and the corresponding payoff is 0. When it's profitable to exercise the option at maturity, the option will be exercised. For European put option, the payoff can be calculated as the strike price minus the price of the underlying asset at maturity. For discrete Asian call option, suppose we observe the price of the underlying asset N times before the maturity, the payoff can be calculated as the average of the N prices minus the strike price. We use geometric average when calculating the payoff for the geometric average Asian option and arithmetic average when calculating the payoff for the arithmetic average Asian option. For the digital call option, the payoff is 1 when the price of the underlying asset at maturity is higher than the strike price.

For European options, the expected payoff can be calculated analytically using the Black-Scholes model. However, for some options like arithmetic average Asian option, the expected payoff can not be calculated analytically. Monte Carlo simulations can solve this problem by simulating the possible future prices of the underlying asset and then calculate the expected payoff of the option. We can estimate the sensitivities in the Monte Carlo method using the bump and revalue method, pathwise method and likelihood ratio method, etc. In the Monte Carlo method, the more times we repeat the simulations, the more accurate results we get. This can be inefficient when we do not have much time but still need an accurate result with a small standard deviation. There are many variance reduction techniques to solve this problem, antithetic variable technique, control variate technique and importance sampling technique, etc. In this report, we will first use Monte Carlo simulation to calculate the price of an European put option and compare the results we get from the simulations with the analytical value from the Black-Scholes model. We also study how this method behaves when we change the strike price and the volatility of the European put option. Then we estimate the hedging parameter δ of an European put option and a digital call option in Monte Carlo using the methods mentioned above and compared the results we get. Finally, we use Monte Carlo simulations to calculate the price of a discrete arithmetic average Asian option and apply the control variate technique to accelerate the valuation of this option.

This report is organized as follows: Section 2 will introduce the methods that we use in this report, including the Monte Carlo method, bump and revalue method, pathwise method, likelihood ratio method and control variate techniques, etc. Section 3 presents the results that our experiments obtained and gives further discussions about these results.

2 Methods

2.1 Monte Carlo method

Monte-Carlo method is a technique that uses random numbers from a probability distribution to simulate possible outcomes in a process that involves uncertainty. It can generate multiple samples of the outcomes. Based on the samples, we can calculate the expected value of the outcome using the statistical method. In the financial market, Monte-Carlo simulation can help us calculate the option price. As we have mentioned before, the option price is related to the expected payoff of this option and the payoff of the option is related to the price of the underlying asset in the future, which is hard to predict due to uncertainty. We can use Monte Carlo simulations to simulate possible paths of the price of the underlying asset and solve the uncertainty problem in the option pricing process.

When we use the Monte Carlo method to calculate the option price, we first simulate M price paths of the underlying asset, then calculate the mean payoff of this option and discount the mean payoff at the risk-free rate to get a sample of the option price. We repeat this process multiple times and get multiple samples of the option price, then based on the law of large numbers, the mean of samples will converge to the expected value of the option price. When we use this method to evaluate the price of a European put option that is based on a stock, the steps are as follows:

1. Simulate one path of the stock price. Since we assume that the stock price follows a geometric Brownian motion

$$dS = rSdt + \sigma SdZ \tag{1}$$

where r is the risk-free rate, σ is the volatility, S is stock price and Z is the standard Brownian motion. We can assume $G = ln(S)$ and apply *Itô* lemma, we can get

$$dln(S_t) = (r - 0.5\sigma^2)dt + \sigma dZ \tag{2}$$

If we assume r and σ to be constant, we can integrate this equation and obtain the following equation

$$S_t = S_0 e^{(r-0.5\sigma^2)t+\sigma\sqrt{t}Z} \tag{3}$$

where S_t is the stock price at time t and S_0 is the stock price at $t = 0$.

2. Calculate the payoff $max\{K - S_T, 0\}$

3. Repeat Step 1-2 for M times and calculate the option price as the mean of the payoffs discounted at risk-free rate.

4. Repeat Step 1-3 multiple times and get multiple samples of the option price, then calculate the mean of the option price as the expected value of the option price.

It's worthwhile to mention that if we increase M or the number of times we repeat the process of obtaining one sample of the option price, the option prices we get will converge to the same expected value. If we ignore Step 4, we can still calculate the standard deviation of the option price as the standard error of the discounted payoff because the discounted payoffs we get are independent and identically distributed random variables.

$$\sigma(option\ price) = \frac{\sigma(discounted\ payoff)}{\sqrt{M}} \tag{4}$$

Since the true value of the option price $E(X)$ always lies within the confidence interval and the confidence interval is calculated as $\hat{X} \pm 1.96 * se(X)$ where $\hat{X}$ denotes the mean of the option price and $se(X)$ denotes the standard error of the option price, we can see that the confidence interval becomes smaller as $se(X)$ becomes smaller, the absolute difference $|E(X) - \hat{X}|$ also becomes smaller, thus our estimate $\hat{X}$ becomes more accurate when the standard error of our estimate becomes smaller.

2.2 Bump-and-revalue method

As mentioned before, the price of an option $V(S)$ at time $t = 0$ can be calculated by computing the discounted value of the expected payoffs, then we can use the Euler formula to obtain the defined delta-parameter $\delta_t = \frac{\partial V_t}{\partial S_t}$ at time $t = 0$:

$$\delta_0 = \frac{V(S+\epsilon) - V(S)}{\epsilon} \tag{5}$$

where $V(S)$ represent the option price calculated with stock price S through a Monte Carlo simulation, ϵ is the bump size, $V(S+\epsilon)$ is the option price computed with stock price $S+\epsilon$ through the same number of trials(paths) with $V(S)$. When calculating the option price $V(S)$ and $V(S+\epsilon)$, we notice that the bump size ϵ could affect the results. Furthermore, the bumped and unbumped estimate of option price could use different seeds or the same seeds for calculating S_T in Monte Carlo.

In terms of accuracy, the bump-and-revalue method not only introduces the bias for computing the delta by forward-difference estimator and ignoring the higher-order derivatives but also relies on the bump size ϵ. Although we can reach a more accurate value by taking a reasonable bump size ϵ and a rather large number of trials, the bias in this approach is still inevitable.

The analytical value of European put option can be obtained by Black-Scholes model:

$$\Delta_t = -N(-d_1) \tag{6}$$

$$d_1 = \frac{ln(\frac{S_t}{K}) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \tag{7}$$

where $N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}u^2} du$ denote the standard normal cumulative distribution.

2.3 Pathwise method

The pathwise method is about the differentiation of each simulated outcome with respect to the parameter of interest but this approach is not suitable for calculating the delta of a digital option because the digital options(or binary options)have a discontinuous payoff. The payoff used in the pathwise method should be smooth and differentiable almost everywhere so we use the standard logistic function to smooth the payoff function of the digital option:

$$f(x) = \frac{1}{1+e^{-x}} \tag{8}$$

where x can be obtained by $S_T - K$. This smoothing process is shown in Figure 1. The hedge parameter-δ of

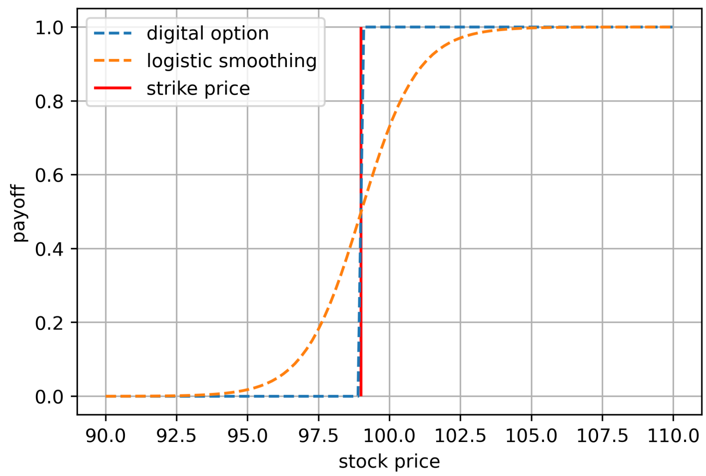

Figure 1: Logistic function and digital option payoff

European call option can be calculated as follows:

$$\Delta = \frac{dC}{dS_0} = E[e^{-rT}\frac{d\max(S_T - K, 0)}{dS_0}] \tag{9}$$

To obtain the delta for a digital call option, the payoff function $\max(S_T - K)$ should be substituted by Equation 8 and get

$$\frac{df(S_T-K)}{dS_0} = \frac{df(S_T-K)}{dS_T}\frac{dS_T}{dS_0} = \frac{e^{(S_T-K)}}{(1+e^{(S_T-K)})^2}\frac{dS_T}{dS_0} \tag{10}$$

For a geometric Brownian Motion, $\frac{dS_T}{dS_0} = \frac{S_T}{S_0}$. Hence, the delta parameter of the binary call option using the application of smoothing method can be computed as follows:

$$\hat{\Delta} = e^{-rT}\frac{e^{(S_T-K)}}{(1+e^{(S_T-K)})^2}\frac{S_T}{S_0} \tag{11}$$

$$\Delta = E[\hat{\Delta}] \tag{12}$$

2.4 Likelihood ratio method

The likelihood ratio method differentiates the probability density rather than the payoff function, thus it could be directly used in the digital call option as follows:

$$C(S_0) = E[f(S_T)] = \int f(S_T)g(S_T, S_0)dS_T \tag{13}$$

Where $f(S_T) = e^{-rT}1_{\{S_T \geq K\}}$, $g(S_T, S_0)$ is the log-normal density of S_T:

$$g(S_T, S_0) = \frac{1}{S_T\sigma\sqrt{2\pi T}}\exp\Big[-\frac{1}{2}\Big(\frac{\ln(S_T/S_0) - (r - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}\Big)^2\Big] \tag{14}$$

and

$$\frac{\partial \log(g(S_T, S_0))}{\partial S_0} = \frac{Z}{S_0 \sigma \sqrt{T}} \tag{15}$$

where Z represents the same random numbers as we calculated in Equation 3. Thus, for the digital call option, the delta-parameter can be obtained by:

$$\Delta = \frac{dC(S_0)}{dS_0} = E[f(S_T)\frac{\partial \log(g(S_T, S_0))}{\partial S_0}] = E[e^{-rT} 1_{\{S_T \geq K\}} \frac{Z}{S_0 \sigma \sqrt{T}}] \tag{16}$$

2.5 Control variate technique

Suppose we want to use simulations to estimate $E(X)$ which is the expected value of a random variable and X is the output of a simulation. If we can find another variable Y that is also an output of a simulation and the expected value of this variable $E(Y)$ is known, we can use a new variable $Z = X - \beta(Y - E(Y))$ to estimate the same value. Since $E(Y - E(Y)) = 0$, we can get $E(Z) = E(X - \beta(Y - E(Y))) = E(X) - \beta E(Y - E(Y)) = E(X)$. The variance of Z is

$$Var(Z) = Var(X - \beta(Y - E(Y))) = Var(X) + \beta^2 * Var(Y) - 2\beta * Cov(X, Y) \tag{17}$$

If X and Y are not independent, we can find choose a β such that $\beta^2 * Var(Y) - 2\beta * Cov(X, Y) < 0$ and Var(Z) reaches minimum. Suppose $f(\beta) = \beta^2 * Var(Y) - 2\beta * Cov(X, Y)$, the first order derivative $f^{'}(\beta) = 2\beta * Var(Y) - 2 * Cov(X, Y)$, we can see that $f(\beta)$ reach minimum when $f^{'}(\beta_{min}) = 0, \beta_{min} = \frac{Cov(X,Y)}{Var(Y)}$. If we use β_{min} in equation 17, we can get

$$Var(Z) = Var(X) - \frac{(Cov(X, Y))^2}{Var(Y)} \tag{18}$$

Thus, we can get our estimate $E(X)$ while obtaining a smaller variance of the variable if we use variable Z instead of X in the simulation. To apply this method, we first need to find the random variable Y that is related to X and its expected value is known, then we need to find the optimal value β_{min}.

3 Results and Discussion

3.1 Basic Option Valuation

In this part, we mainly focus on a European put option on a stock. The default parameters of this option are strike price $K =$ €99, interest rate $r = 0.06$, initial stock price $S_0 =$ €100, volatility $\sigma = 0.2$, maturity $T = 1$.

3.1.1 Comparison of Monte-Carlo method with Black-Scholes model

First we construct an MC model to investigate the European put option. Given 20000 trials (paths) and parameters mentioned above, the estimated put option price we get is €4.7914 (by a single simulation run) and standard deviation of the estimated option price (calculated by equation 4) is 0.0557. The Black-Scholes option price for the same parameters is €4.7783. Given 30 simulations and parameters setting, we get another estimated MC put option value €4.7749, which is closer to the analytical option value. Figure 2 shows that the MC put option price with the standard deviation, calculated as the standard error of the discounted payoff in one single simulation, converges to the BS analytical value as the number of trials increases from 100 to 20000.

Figure 3 shows that the standard deviation of the option price decreases rapidly in the beginning and becomes flat with more trials. By Law of Large Number, the accuracy of our estimate will be higher with more trials, which also corresponds with our observation. Besides, we see that the standard deviation of the option price for 30 simulations is very similar to the standard deviation of the estimated option price computed by equation 4. Hence, we only study one of them in our following discussions.

3.1.2 Tests for different strike price and volatility

In this question, we set the number of trials at 20000 and repeat the simulation 30 times to study how European put option price and the corresponding standard deviation change with different strike prices.

As shown in Figure 4, we can see that the put option price increases as the strike price grows from 80 to 110. Furthermore, Figure 5 shows that with a higher strike price we generally get a higher estimated standard

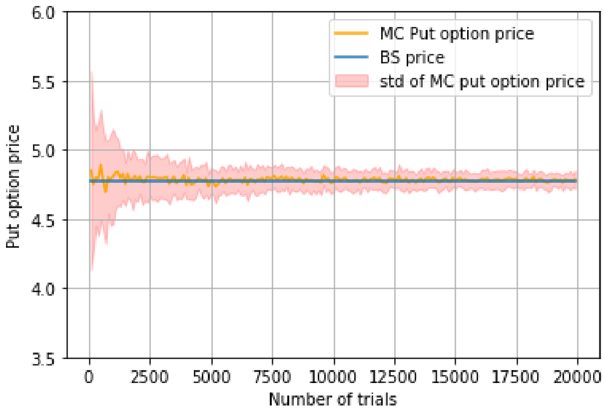

Figure 2: Comparison of Monte Carlo put option price with BS put option price

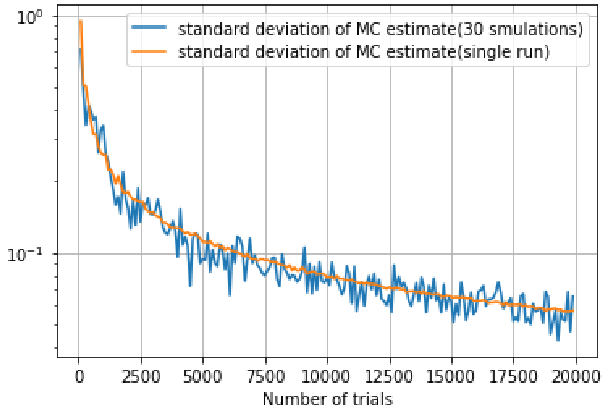

Figure 3: standard error of MC estimate

Figure 4: MC put option price with different strike price

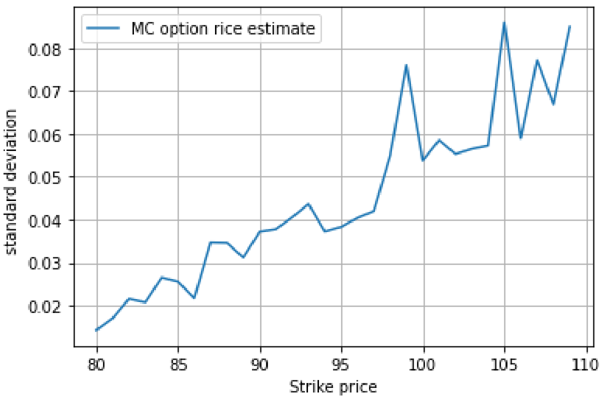

Figure 5: MC put option price with different volatility

deviation. The reason behind this is that the intrinsic value of the put option grows as the strike price grows, leading to the larger standard deviation of our estimated option price.

Then we study the relationship between the volatility and the put option price. We can tell in Figure 6 that the put option price increases almost linearly as the volatility increases from 0.1 to 1.1. This is mainly because increasing volatility means increasing risk, investors want to get a higher return for higher risk, thus the option price increases. Moreover, in Figure 7, with a higher volatility, we generally get a higher standard deviation. This is because the probability that the put option is in the money at maturity increases as the volatility increases. This increased probability also leads to an increased standard deviation of the option price.

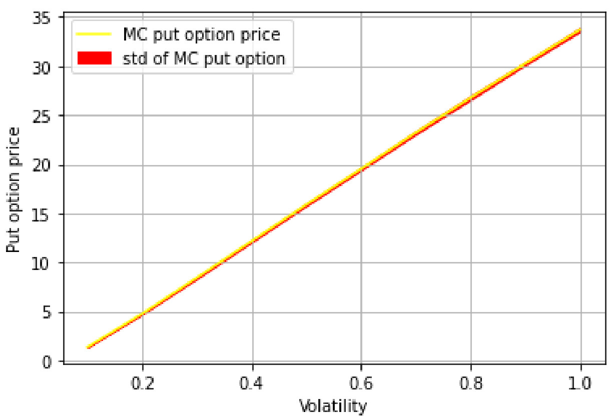

Figure 6: MC put option price with different volatility

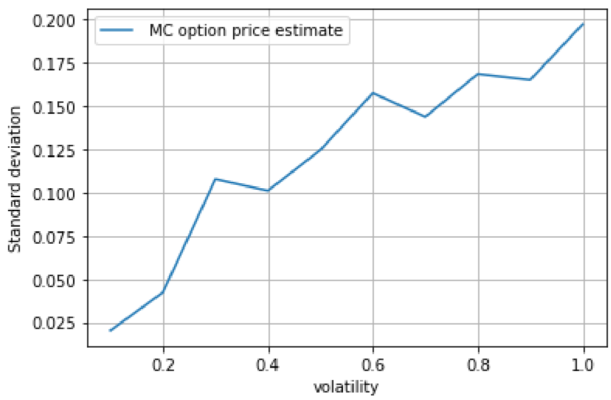

Figure 7: standard error of MC estimate with different volatility

3.2 Estimation of Sensitivities in MC

In this part, we calculate the hedge parameter of European put option and digital call option using the bump-and-revalue method with different seeds and same seeds in each case. We also use sophisticated methods like the pathwise and likelihood ratio methods to further investigate the digital call option. The default parameters of both options are strike price $K = €99$, interest rate $r = 0.06$, initial stock price $S_0 = €100$, volatility $\sigma = 0.2$, maturity $T = 1$, simulation=30.

3.2.1 The hedge parameter of European put option

The analytical hedge parameter of European put option calculated by Equation 6 is -0.3262. We first investigate the bump size ϵ we should use in both different and same seed methods. We can see in Table 1, the relative error increases as ϵ decreases when the number of paths is fixed and the relative error decreases as number of paths increases for all ϵ in different seed scenario. As for the same seed scenario, when number of paths is larger than 10^4 the relative error increases as the number of paths grows in Table 2. Thus, we fix the bump size at $\epsilon = 0.5$ to further study the difference between these two seed methods.

Table 1: The absolute relative error of European put option with different seed, simulation=30

Paths	$\epsilon = 0.01$	$\epsilon = 0.02$	$\epsilon = 0.5$
10^4	458.17%	79.76%	13.29%
10^5	341.26%	79.25%	0.36%
10^6	122.32%	11.45%	1.28%
10^7	4.19%	9.69%	0.62%

Table 2: The absolute relative error of European put option with same seed, simulation=30

Paths	$\epsilon = 0.01$	$\epsilon = 0.02$	$\epsilon = 0.5$
10^4	0.185%	0.154%	1.191%
10^5	0.028%	0.055%	1.373%
10^6	0.037%	0.065%	1.389%
10^7	0.027%	0.055%	1.373%

As shown in Figure 8, these two seed methods both converge to the analytical value as the number of trials grows from 100 to 20000. And Figure 9 can indicate that the standard deviation decreases as the number of trials increases in both methods but the standard deviation of the same seed method is always smaller than that of the different seed method through all number of trials. This is because the variance of delta is $Var(\delta) = \frac{1}{\delta^2}[Var(V(S+\epsilon)) + Var(V(S)) - 2Cov(V(S+\epsilon), V(S))]$, when we use different seed, $V(S)$ and $V(S+\epsilon)$ is independent, we can assume the covariance between them are 0, as for the same seed method, $V(S)$ and $V(S+\epsilon)$ are dependent and the covariance between these two is positive. Hence, using the same seed method will reduce the standard deviation of the hedge parameter-δ.

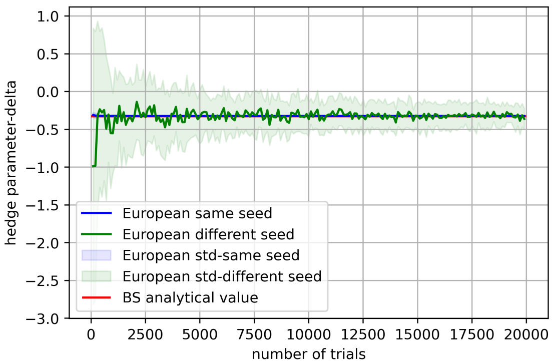

Figure 8: Hedge parameter delta with the increase of the number of trials

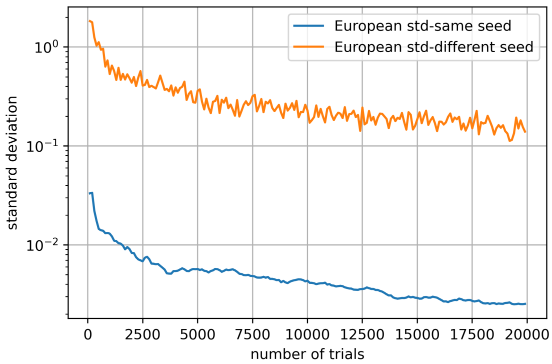

Figure 9: Standard deviation with different seed and same seed

3.2.2 The hedge parameter of Digital option

In regard to the digital option, we also fix bump size at $\epsilon = 0.5$, then use bump-and-revalue method with different and same seed methods to calculate the hedge parameter. As can be seen in Figure 10, these two methods both show convergence behavior to $\delta = 0.018$ approximately as the number of trials grows from 0 to 20000. Furthermore, we implement the pathwise and likelihood ratio methods and the results in Figure 11 can indicate that both methods also converge to $\delta = 0.018$ approximately as the number of trials increases.

As for the standard deviation of δ, we can tell in Figure 12 that though the standard deviations of the four different methods all drop significantly as the number of trials grows, the different seed method still shows the largest standard deviation, followed by the same seed method, the pathwise method and the smallest one-the likelihood ratio method. When we introduce the logistic smoothing method to estimate the payoff of digital option, we also take the bias between digital payoff and logistic payoff into account, so the pathwise method shows a bit larger standard deviation than the likelihood ratio method. However, the pathwise and likelihood methods both improve the results with smaller standard deviation compared to the bump-and-revalue method. This is mainly due to the simulation times we need to get a single δ. We only have to simulate once to obtain one δ in the pathwise and likelihood ratio method whereas we have to take twice Monte Carlo simulations to calculate one δ in bump-and-revalue method, taking more randomization into account and leading to a larger standard deviation.

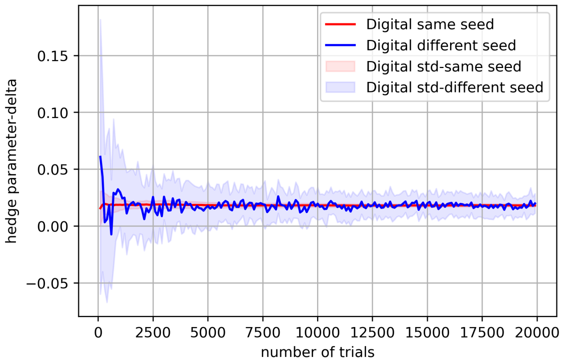

Figure 10: Hedge parameter delta with different and same seed

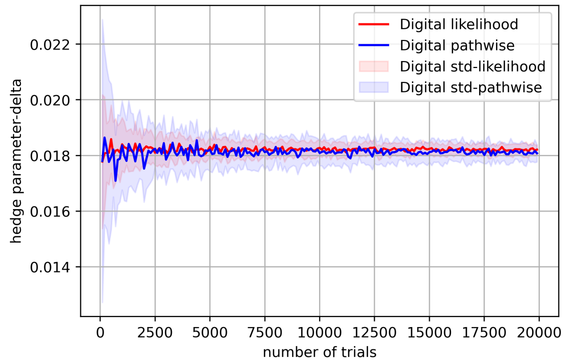

Figure 11: Hedge parameter delta with pathwise and likelihood method

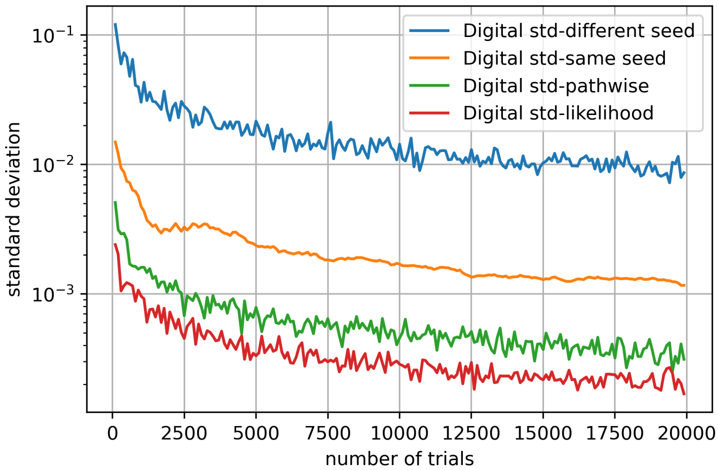

Figure 12: Standard deviation for four different methods

3.3 Variance reduction by control variate

In this part, we use Monte Carlo method to calculate the price of a discrete arithmetic average Asian call option with the strike price K=99, volatility $\sigma = 0.2$, the risk-free rate $r = 0.06$, maturity $T = 1$, the initial stock price $S0 = 100$ and the number of observation time $N = 50$.

3.3.1 Use Monte Carlo method to estimate the price of a geometric average Asian call option

Although the price of the discrete arithmetic average Asian call option can not be calculated analytically, the price of the corresponding discrete geometric average Asian call option can because we assume that the stock price $S(t)$ follows geometric Brownian motion. We can get that the geometric average of the stock prices S_N:

$$S_N = (\prod_{i=0}^{N} S(\frac{iT}{N}))^{\frac{1}{1+N}} = S(0) \times exp((r_N - 0.5\sigma_N^2)T + \sigma_N \sqrt{T} Z) \tag{19}$$

Where N denotes the number of times we observe the stock price, $r_N = 0.5(r - 0.5\sigma^2 + \sigma_N^2)$, $\sigma_N = \sigma\sqrt{\frac{2N+1}{6N+6}}$ and Z is a standard normal random variable. The payoff of the discrete geometric average Asian call option is $S_N - K$ and we can calculate the expected payoff analytically. The price of this option is

$$C_g(S(0), T) = exp(r_N T - rT)C(S(0), T) \tag{20}$$

Where C(S(0),T) denotes the price of a European call option with risk-free rate r_N and volatility σ_N. We use the black-schloes formula to calculate $C(S(0), T)$ and then get the value of $C_g(S(0), T)$.

We use equation 20 to calculate the analytical price of the discrete geometric average Asian call option and then use the Monte Carlo method to estimate this price. M denotes the number of paths we simulate in the

Monte Carlo simulation when calculating the option price. We increase M from 100 to 100,000 at a step size of 200 and run the simulations to calculate the option price. Figure 13 shows both the analytical price and the estimated price from the Monte Carlo simulation with the standard deviation (std) that is calculated as the standard error of the discounted payoff. Since the decrease of std is invisible in figure 13, we also plot the std against M in Figure 14. We can see that the option price generally converges to the analytical value and the corresponding std decreases as the number of trials increases.

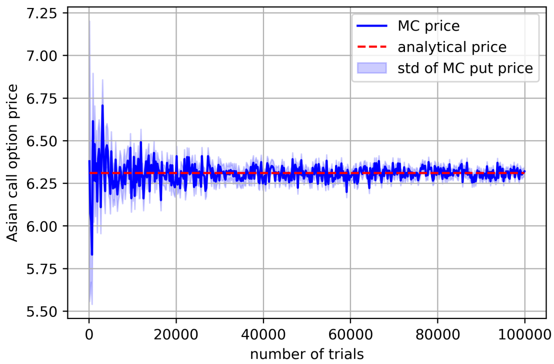

Figure 13: Monte Carlo estimate of the geometric average Asian option price

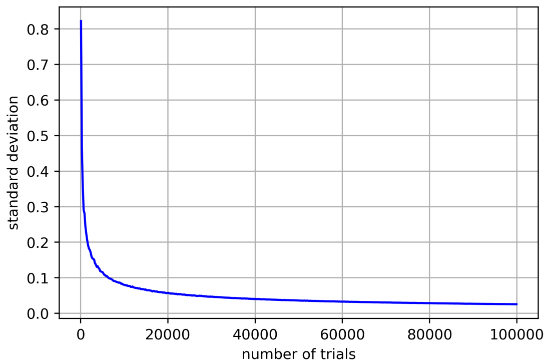

Figure 14: std of the geometric average Asian option price in Monte Carlo simulations

3.3.2 using geometric average Asian option price as control variate

If we use the prices from the same paths when calculating the prices of the arithmetic and geometric average Asian option, we can assume that these two prices are correlated. We want to estimate the expected value of the arithmetic average Asian call option price denoted as $E(X)$ where X is the price of this option that we get from the Monte Carlo simulation. Since we can also use the Monte Carlo method to estimate the price of the geometric average Asian option Y and we know the corresponding analytical value $E(Y)$, we can use $Z = X - \beta(Y - E(Y))$ as the new estimator. This estimator is unbiased because we have proved in section 2.4 that $E(Z) = E(X)$ and the corresponding variance of Z is much lower than that of X.

Then we need to decide the optimal value of β such that $Var(Z)$ reaches its minimum. Since we do not know the exact correlation of the two prices X and Y in the simulations, we need to use Monte Carlo method to estimate their correlation and then compute the optimal value of β. We use the option parameters mentioned above, set the number of trials to be 1000 and increase the sample size of X and Y from 100 to $10,000$ at a step size of 500, the corresponding optimal beta we get from the simulations are shown in Figure 15. When we increase the sample size of X and Y, we can assume that the $Cov(X,Y)$ and $Var(Y)$ will be more accurate. We can see that the optimal value of β generally fluctuate between 1.035 and 1.036 as we increase the sample size and it's more close to 1.035 in most of the simulations, so we set the value of β to be 1.035 in the following experiments.

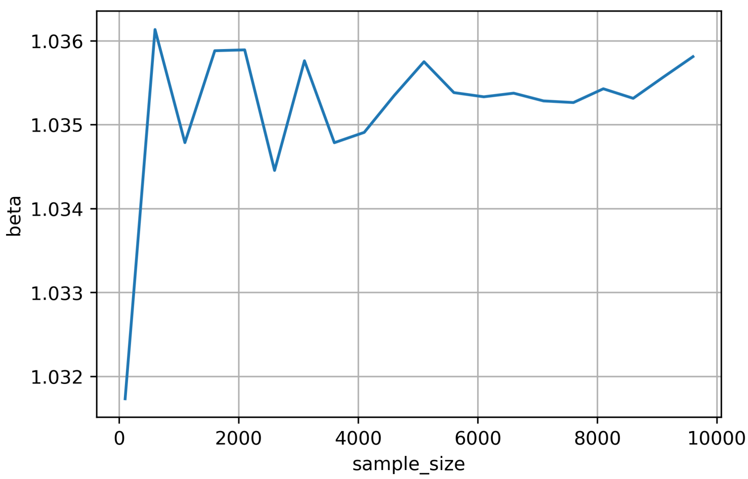

Figure 15: Optimal value of β when the number of paths is 1000

3.3.3 Apply the control variate technique

We study the performance of this technique for different number of paths, number of time points, strike price and volatility in the default parameter settings. The normal Monte Carlo method (MC) is denoted as "before" and the Monte Carlo method with the control variate technique(MCWCV) is denoted as "after" in the figures below.

We first increase the number of trials (paths) that we simulate in the Monte Carlo method. The mean and standard error (*se*) of the option price we estimate in the Monte Carlo simulations are shown in Figure 16. We can see that the mean of the option price we get in these two methods generally converges to the same value and the corresponding standard error of the option price becomes smaller when we increase the number of paths. Besides, the mean of the option price we get in MCWCV is more stable than the mean in MC. Figure 17 presents the difference between *se* in these two methods. We can see that MCWCV produces much lower *se*.

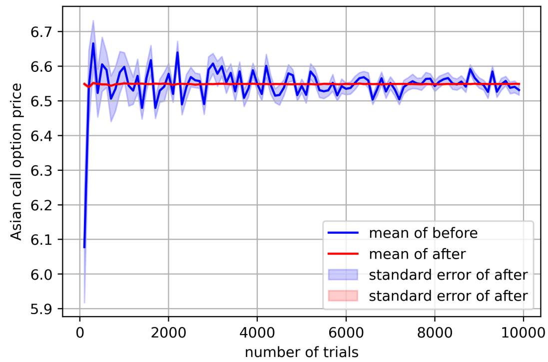

Figure 16: mean and *se* of the option price in MC and MCWCV for different number of paths

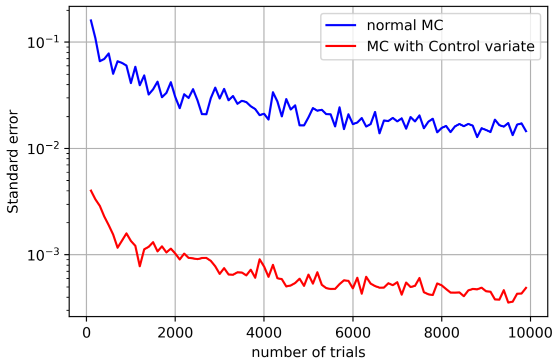

Figure 17: *se* of the option price in MC and MCWCV for different number of paths

Due to limited computational power, we set the number of paths to be 1,000 in the following experiments. We then studied the performance of the control variate technique for different number of time points N. We change N from 10 to 95 at a step size of 5, the results are shown in Figure 18 and Figure 19. We can see that the mean of the option price in MCWCV is much stabler and the corresponding *se* is much lower. Besides, the *se* in these two methods show a similar trend and fluctuate around a specific value when the number of time points increases. We can also see that the discrete arithmetic average Asian call option price generally increases as N increases, this is because arithmetic average of the stock prices generally increases as N increases, and this leads to a higher expected payoff of the option.

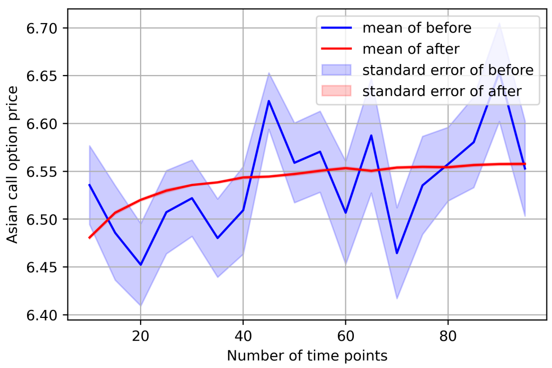

Figure 18: mean and *se* of the option price in MC and MCWCV for different number of time points

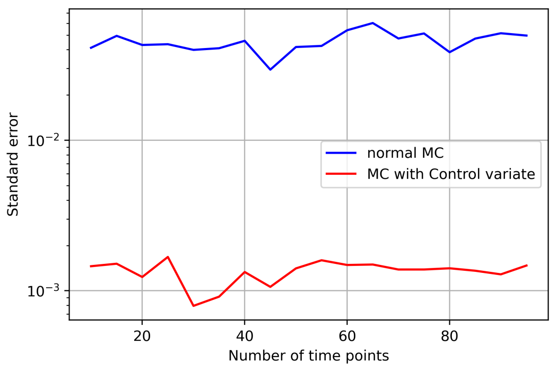

Figure 19: *se* of the option price in MC and MCWCV for different number of time points

We then change the strike price from 90 to 109 at a step size of 1. The mean and *se* of the option price we estimate are shown in Figure 20. We can see that the mean of the option price we get in MC and MCWCV are generally the same, the difference between them is too small compared to the scale of the y axis. The *se* in these two methods are invisible in Figure 20 so we present them in Figure 21. We can see that the *se* in these two methods decreases as the strike price increases and MCWCV still produces a much smaller *se* compared to MC. When we increase the strike price, the probability that the call option is out of the money at maturity increases, thus the expected payoff of this option decreases and the corresponding variance also decreases, this corresponds with the results shown in Figure 20 and Figure 21. Since the variance of the geometric average

Asian option price also decreases as the strike price increase, we think that might influence the optimal value of β, so we study whether the optimal value of β change when we change the strike price, the result is shown in Figure 22. We can see that the optimal value of $\beta = \frac{Cov(X,Y)}{Var(Y)}$ increases as the strike price increases. We think this is mainly due to the fact that $Var(Y)$ decreases as the strike price increase.

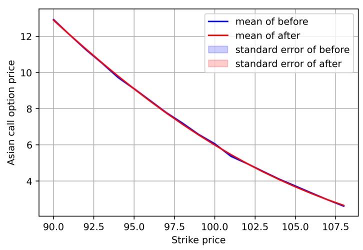

Figure 20: mean and *se* of the option price in MC and MCWCV for different strike price

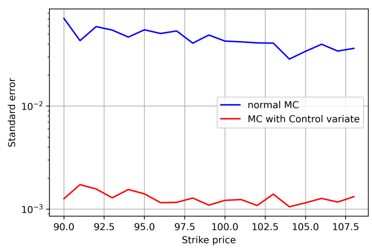

Figure 21: *se* of the option price in MC and MCWCV for different strike price

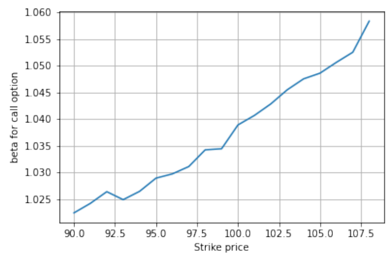

Figure 22: optimal value of β for different strike price

Finally, We change the volatility of the stock from 0.05 to 0.35 at a step size of 0.05. The mean and *se* of the corresponding option price are shown in Figure 23 and Figure 24. We can see that the mean of the option price we get in MC and MCWCV are generally the same, their differences are too small to be visible in Figure 23 and the option price increases as the volatility increases. This is because the probability that the call option is in the money at maturity increases as the volatility increases. This increased probability also leads to increased *se* of the option price in both methods shown in Figure 24. The variance of the geometric average Asian call option price increases as the volatility increases, we think this might influence the optimal value of β, so we studied this and present the results in Figure 25. Despite the fact that $Var(Y)$ increases as volatility increases, the optimal value of $\beta = \frac{Cov(X,Y)}{Var(Y)}$ also increases as the volatility increases. We believe this is because the correlation between the price of the geometric and arithmetic average Asian call option increases as the volatility increases.

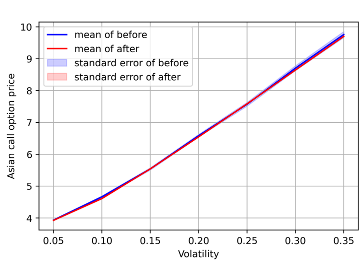

Figure 23: mean and se of the option price in MC and MCWCV for different volatilities

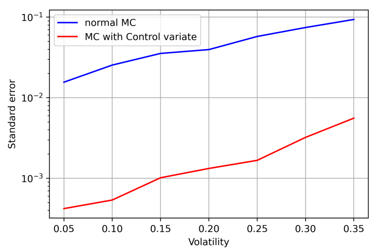

Figure 24: se of the option price in MC and MCWCV for different volatilities

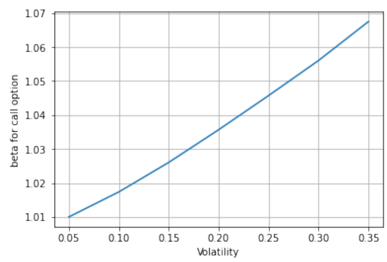

Figure 25: optimal value of β for different strike price