Assignment 1

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1. Introduction

Value-at-risk(VaR) and Expected Shortfalls(ES) are probably the most prevailing risk measure in the financial industry. In this report, we construct a portfolio consisting of three indices and one bond in Table 1. There are some standard methods widely used in the financial industry measuring the market risk. Through implementing unconditional methods like Variance-Covariance method (both with normal and student-t assumption), Historical Simulation(HS) and conditional methods like CCC-GARCH and FHS-EWMA, we evaluate the VaR and ES at 97.5% and 99% confidence level over the one-day time horizon. Through backtesting, we would compare the performance of each method. Besides, we introduce four extreme but possible scenarios into the risk factor during stress testing to evaluate how the portfolio would respond to the dramatic changes.

Table 1: The portfolio

Type	Currency	Weight
DJ Life Insurance	USD	250,000
S&P 500	USD	250,000
AEX	Euro	250,000
NL 3Y Bond	Euro	250,000
Portfolio	Euro	1,000,000

2. Theory

2.1. Loss operator

The first step is to identify the loss operator of our portfolio. We assume that the value of a portfolio V_t is a function of time and a set of risk factors Z_t, denoted by $V_t = f(t, Z_t)$. We have three types of risk factors in our model: the logarithmic price of indices, yield and logarithmic exchange rate. The risk factor changes is defined by $X_t := Z_t - Z_{t-1}$. Using mapping $f(t, Z_t)$, the loss of portfolio is $L_{t+1} = -(f(t+1, \mathbf{Z}_t + X_{t+1}) - f(t, Z_t))$. The loss operator at time t is $l_{[t]}(\mathbf{x}) := -(f(t+1, \mathbf{z}_t + \mathbf{x}) - f(t, \mathbf{z}_t))$, so that $L_{t+1} = l_{[t]}(\mathbf{X}_{t+1})$.

Assuming the mapping f is differentiable, we can accurately approximate the portfolio loss by the linearized loss:

$$L_{t+1}^{\Delta} = -\left(f_t(t, \mathbf{Z}_t) + \sum_{i=1}^{d} f_{z_i}(t, \mathbf{Z}_t) X_{t+1,i} \right) \tag{1}$$

2.2. Value-at-Risk and Expected Shortfalls

The formal definition of VaR given some confidence level $\alpha \in (0,1)$ is:

$$\mathrm{VaR}_{\alpha} = \inf\{l \in \mathbb{R} : P(L > l) \leqslant 1 - \alpha\} = \inf\{l \in \mathbb{R} : F_L(l) \geqslant \alpha\} \tag{2}$$

which statistically means a quantile of the loss distribution in time horizon Δ. A flaw of VaR is that it only considers the frequency of large losses, but does not capture their size. Thus, Expected Shortfall(ES) is introduced. The ES is defined by the average value of the $1-\alpha$ largest losses:

$$ES_{\alpha}(L) = E(L \mid L > VaR_{\alpha}) \tag{3}$$

2.3. Variance-Covariance method

In this method, the risk factor change X_{t+1} is treated as following a multivariate normal distribution $X_{t+1} \sim N_d(\mu, \Sigma)$, where μ is the mean factor and Σ is the covariance(or variance-covariance) matrix [1].

2.3.1. Normal distribution

If we assume that the risk factor change X_t follows a normal distribution, then for a single asset portfolio, the VaR is calculated by:

$$\mathrm{VaR}_\alpha = \mu + \sigma\Phi^{-1}(\alpha), \tag{4}$$

Meanwhile, the ES is given by:

$$ES_\alpha = \mu + \sigma\frac{\phi\left(\Phi^{-1}(\alpha)\right)}{1-\alpha} \tag{5}$$

where μ is mean loss and $\Phi^{-1}(\alpha)$ is the α-quantile of Φ. Furthermore, if there are multiple assets in our portfolio, the linear combination of these univariate normal distributed variables also follows a univariate normal distribution. The portfolio loss is:

$$L_{t+1}^{\Delta} = l_t^{\Delta}(X_{t+1}) \sim N\left(-c_t - b_t'\mu_t, b_t'\Sigma_t b_t\right) \tag{6}$$

. Where $\Sigma_t$is the variance-covariance matrix of x_t at time t, and c_t and b_t is time-dependent constants. Then the VaR and ES of multi-assets portfolio can be computed by equation 4 and 5.

2.3.2. Student-t distribution

The normal distribution is not a very suitable assumption since the tails of financial asset returns are often heavier than normal. Thus we can assume the returns follow student-t distribution, the VaR and ES can be computed as follows:

$$VaR_\alpha = \mu + \sigma t_v^{-1}(\alpha) \tag{7}$$

$$ES_\alpha = \mu + \sigma\frac{t_v(t_v^{-1}(\alpha))}{1-\alpha}\left(\frac{v+(t_v^{-1}(\alpha))^2}{v-1}\right) \tag{8}$$

Where the σ used here is calculated using the original volatility divided by $\sqrt{v/(v-2)}$, and t_v represents the students-t pdf with v degrees of freedom.

2.4. *Historical simulation method*

This method simply depends on the historical data we observe in the market. If we have n historical portfolio returns, the VaR_α can be obtained by sorting losses from low to high and finding the losses at the confidence level α. Additionally, the ES_α can be calculated by the average of $n*(1-\alpha)$ highest losses. Hence, this approach is easy to understand and use when the historical data is available for all assets in the portfolio.

3. Method

The 10-years dataset we use is taken from 01/04/2011 to 01/04/2021, remaining 2493 daily data after synchronizing. For all the methods, VaRs and ESs are calculated at confidence levels 0.975 and 0.99 respectively and for the time horizon of 1 trading day except specific statement.

3.1. *Variance-Covariance method*

To investigate the sensitivity of this method to the length of the estimation period with normal distribution, we compare the resulting estimated VaR and ES based on four different past periods: first two years (500 days) of ten years; first five years (1250 days) of ten years; the whole ten years; the whole ten years without stress period(10/01/2020-29/04/2020). We also use student-t distribution with 3, 4, 5 and 6 degrees of freedom to compare the VaRs and ESs based on ten-years data and ten-years data without stress period respectively. Additionally, we assess the validity of the normal assumption and student-t assumption respectively using daily returns of each asset and daily portfolio loss.

The h-day historical VaR could be calculated by 1-day VaR through:

$$\mathrm{VaR}(h,\alpha) = \sqrt{h}\cdot\mathrm{VaR}(1,\alpha) \tag{9}$$

This property holds only if daily return is uncorrelated, but we would compare the empirical 5-day and 10-day VaRs with that obtained by this property. Besides, the associated ES is also compared with annually ES.

3.2. *Historical simulation method*

To compare the estimated VaRs and ESs using HS method, we apply a rolling window with two-year window size and the results are estimated yearly for the first five years and the whole ten years respectively.

3.3. *Constant conditional correlation model with GARCH(1,1)*

Both VS method and historical simulation method only consider constant volatility. For a more plausible VaR, we apply models in which volatility is not constant but time-dependent. GARCH(1,1) is a suitable time series model in this task, which could be described by:

$$\sigma_n^2 = \omega + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2 \tag{10}$$

The CCC model is usually a helpful starting point to proceed to a more complex model. Although the constancy of conditional correlation is unreal, it could give an adequate performance in our task [1]. Through GARCH, we could obtain the subsequent day volatility $\hat{\sigma}_{t,i}$ $\hat{\sigma}_{t,j}$ for each asset separately. Using the long-term constant correlation matrix of returns and the predicted volatilities, we could keep updating the covariance matrix at time t by:

$$\mathrm{cov}(t,i,j) = \rho_{i,j}\hat{\sigma}_{t,i}\hat{\sigma}_{t,j} \tag{11}$$

. Then we could this the time-varying variance-covariance matrix to calculate VaR and ES according to the VC method mentioned above. During calculation, we apply a rolling window with three-year window size and the results are estimated daily for seven years(the first three years are the first window of ten years).

3.4. *Filtered historical simulation with EWMA*

The FHS model is constructed in two steps: firstly we can generate the distribution of each risk factor by computing daily return divided by an estimate of volatility on the day of the return-$Z_t = \frac{X_t}{\sigma_t}$; secondly we can multiply them by an estimate of volatility on the day of the VaR mesure-$\hat{X}_{t+1} = Z_t * \sigma_{t+1}$ [2]. Then we can simply calculate the VaR and ES of the day by adding every risk factor of the portfolio with weights and applying the HS method mentioned above.

Regarding the estimate of volatility of every day in the first step, we use EWMA model, where the weights assigned to the square of returns-u_n^2 declined exponentially as we move back through time. Hence, the volatility can be obtained as follows:

$$\sigma_n^2 = \lambda * \sigma_{n-1}^2 + (1-\lambda) * u_{n-1}^2 \tag{12}$$

where RiskMetrics uses $\lambda = 0.94$ for daily volatility forecasting.

To compare the estimated VaRs and ESs using FHS-EWMA, we apply a rolling window with three-year window size and the results are estimated daily for seven years(the first three years is the first window of ten years).

3.5. *Backtesting VaR*

To investigate the performance of VaR systems, we carry out backtesting using the historical data(08/04/2014-01/04/2021, 1742 days in total). All methods are applied by a rolling window with three-year window size and the VaRs and ESs are estimated daily for seven years(the first three years is the first window of ten years).

The estimated loss is compared with the actual loss during a specific time horizon. We count the exact number when the actual loss exceeds the estimated VaR per year and the average violations between these two over all the years. If the estimation method is reliable, we would expect that the violation of VaR should behave like Bernoulli random variables with success probability close to $1-\alpha$. We also compare the expected shortfalls with average shortfalls yearly for each method.

3.6. *Stress testing*

One of the limitations of VaR is that it can not include all the possible scenarios. VaR can not capture unforeseen, dramatic changes such as the financial crisis in 2008 and the extreme situation in the Pandemic. To fill the gap, we would implement stress testing, in which we manually add extreme but plausible conditions to see how our portfolio would react. We perform these four scenarios: currency up for 10%; currency down for 10%; interest rate up for 2%; equity index values up for 20%. And the periods we choose to apply these scenarios are the relatively stressed period in the ten years: 02/08/2011-15/08/2011; 17/08/2015-28/08/2015; 05/03/2020-25/03/2020.

4. Results

4.1. Variance-Covariance method

4.1.1. Normal distribution

As shown in Table 2, VaRs(99%) are all larger than VaRs(97.5%) and ESs(99%) are all larger than ESs(97.5%) for two, five and ten years. Interestingly, the VaRs and ESs of five years are the lowest, followed by two years and ten years scenarios. As for scenarios without the stress period, the VaRs and ESs are significantly lower than those of ten years including the stress period.

Table 2: The VaRs and ESs based on different past periods by VC method(normal distribution)

	VaR(97.5%)	VaR(99%)	ES(97.5%)	ES(99%)
Two years	17570.70	20871.08	20974.27	23923.58
Five years	16748.25	19922.69	20021.94	22858.72
Ten years	18033.45	21454.63	21561.60	24618.87
Ten years(without stress period)	15445.75	18395.52	18487.74	21123.74

We can see in Figure 1, 2, 3, 4, 5 that the tails of all the risk factors are heavier than normal distribution. Thus, the normal assumption fails.

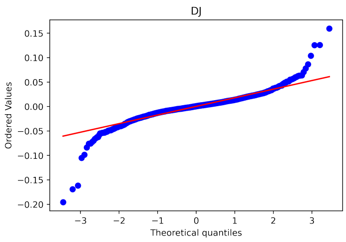

Fig. 1: QQ plot for DJ(normal)

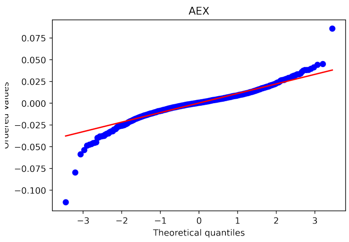

Fig. 2: QQ plot for AEX(normal)

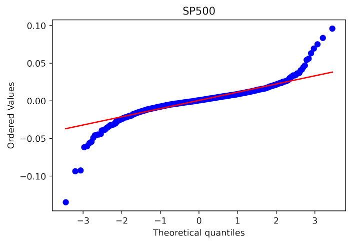

Fig. 3: QQ plot for SP500(normal)

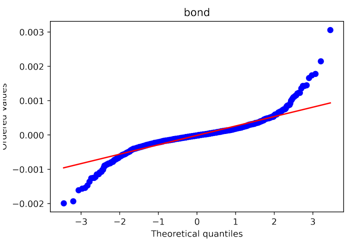

Fig. 4: QQ plot for Bond(normal)

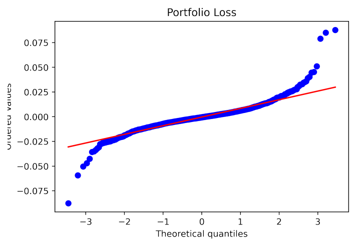

Fig. 5: QQ plot for Loss(normal)

4.1.2. Student-t distribution

As shown in Table 3 and Table 4, VaRs(99%) are all larger than VaRs(97.5%) and ESs(99%) are all larger than ESs(97.5%) for each degree of freedom. Additionally, the higher degree of freedom, the higher VaR(97.5%) and the lower ESs for both with and without stress period scenarios. As for scenarios without the stress peiod, the VaRs and ESs of each degree of freedom are significantly lower than those of ten years including the stress period.

We can see in Figure 6, 7, 8, 9, 10 that all the risk factors fit almost perfectly with student-t distribution with df=3. And we can also tell from Figure 10, 11, 12, 13 that the student-t with df=3 is the best assumption comparing to df=4,5,6 since the deviations from straight line are much bigger with df=4,5,6. Thus, the student-t distribution with degree of freedom equal to 3 is the best assumption for our portfolio so we choose this one for the following comparison and discussion.

Table 3: The VaRs and ESs based on ten years by VC method(student t distribution)

	VaR(97.5%)	VaR(99%)	ES(97.5%)	ES(99%)
df=3	16888.85	24211.39	26900.92	37486.39
df=4	18064.07	24472.06	26100.36	34202.12
df=5	18324.79	24070.27	25203.30	31936.11
df=6	18387.64	23692.23	24557.45	30476.70

Table 4: The VaRs and ESs based on ten years(without stress period) by VC method(student t distribution)

	VaR(97.5%)	VaR(99%)	ES(97.5%)	ES(99%)
df=3	14458.87	20772.41	23091.34	32218.20
df=4	15472.15	20997.16	22401.09	29386.48
df=5	15696.94	20650.74	21627.64	27432.71
df=6	15751.14	20324.79	21070.78	26174.40

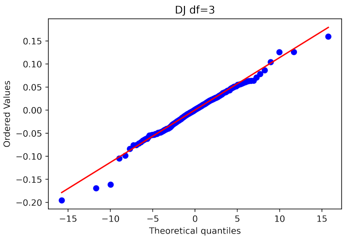

Fig. 6: QQ plot for DJ(t, df=3)

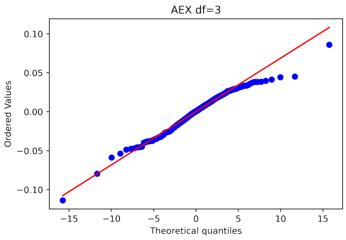

Fig. 7: QQ plot for AEX(t, df=3)

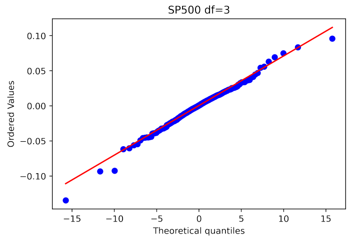

Fig. 8: QQ plot for SP500(t, df=3)

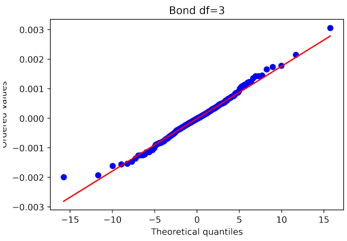

Fig. 9: QQ plot for Bond(t, df=3)

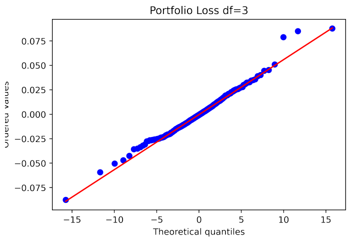

Fig. 10: QQ plot for Loss(t, df=3)

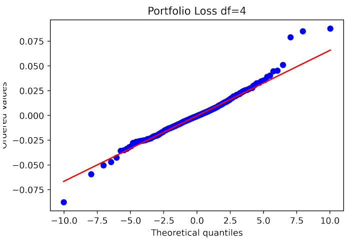

Fig. 11: QQ plot for Loss(t, df=4)

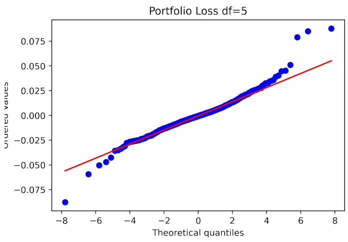

Fig. 12: QQ plot for Loss(t, df=5)

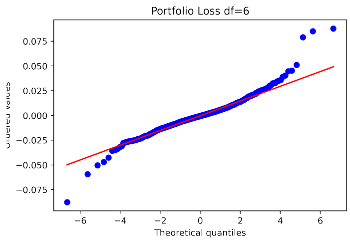

Fig. 13: QQ plot for Loss(t, df=6)

4.1.3. Empirical VaR

Figure 14 and 15 reveals the overall difference between the empirical 5-day and 10-day VaRs compared with those obtained by 1-day VaR using the equation: $\sqrt{h} \cdot \mathrm{VaR}(1, \alpha)$. Expect for the last year, it is clear that the estimated 5-day and 10-day VaRs are mostly greater than the empirical values. Roughly after the first quarter in 2020, the empirical VaR exceed the evaluated value. A similar trend again occurs in Figure 15 for 10-day VaR. It is noticeable that there is a cross point at the end of 2018, which means that the empirical and estimated values are close.

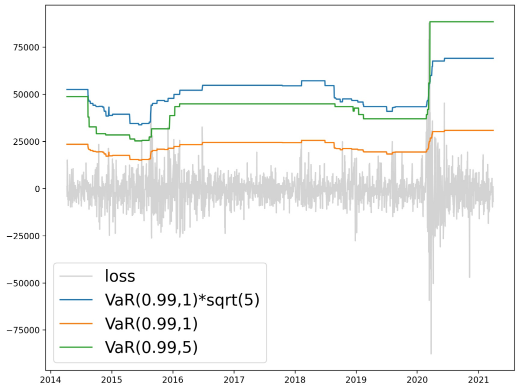

Fig. 14: The empirical 5-day VaR compared with 1-day VaR

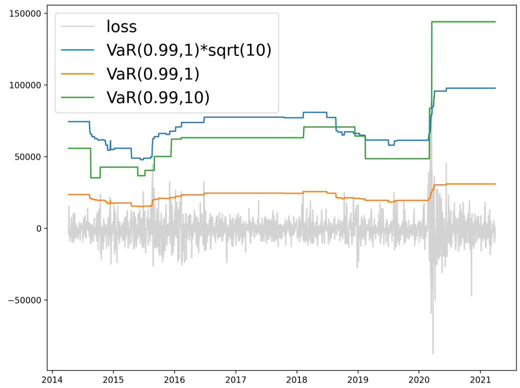

Fig. 15: The empirical 10-day VaR compared with 1-day VaR

4.2. Historical simulation method

As shown in Figure 16 and Figure 17, VaRs(99%) are all larger than VaRs(97.5%) and ESs(99%) are all larger than ESs(97.5%) for both 5 years and 10 years scenarios. The results of 5 years only show three steps because the first two years is the first window and are the same as the first-three-years results of 10 years.

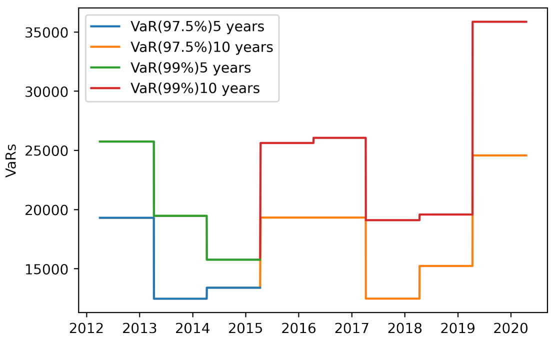

Fig. 16: The VaRs for HS

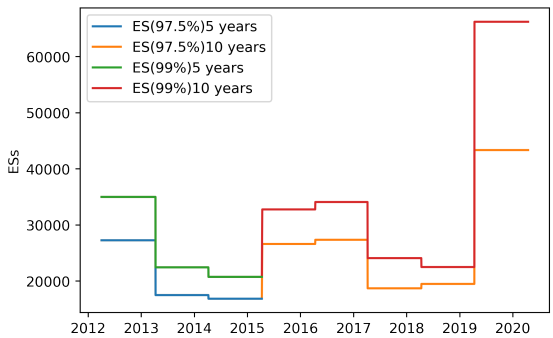

Fig. 17: The ESs for HS

4.3. CCC-GARCH(1,1) and FHS with EWMA

The results of CCC-GARCH(1,1) and FHS with EWMA are showed in Figure 18 and 19. The general trends of these two models are very much the same, but the VaRs of FHS are much volatile than those in GARCH from 2015 to 2017. As for VaR and ES of different confidence levels, the ES is constantly larger than VaR; meanwhile, the values under 99% confidence level are always greater than those under 97.5%. This phenomenon applies to both models.

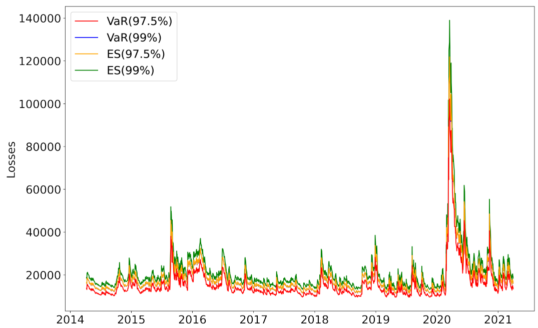

Fig. 18: The VaRs and ESs for CCC-GARCH

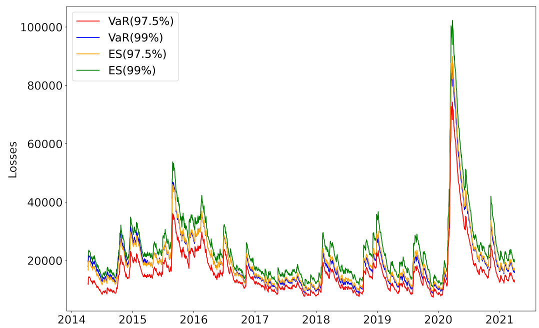

Fig. 19: The VaRs and ESs for FHS-EWMA

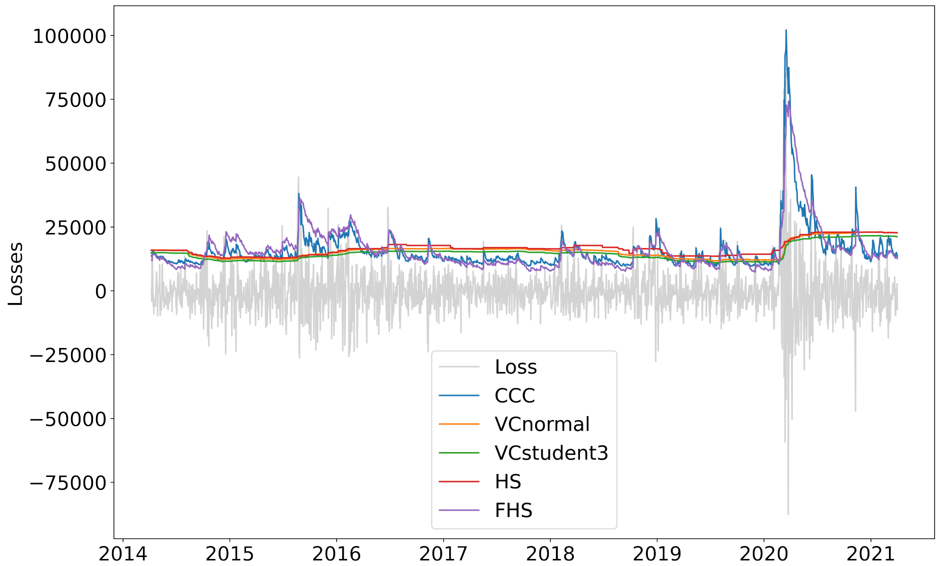

Fig. 20: The VaR(97.5%)results for five methods

4.4. Backtesting VaR

We can see in Table 5 and Table 6 that at the year between 2015.4-2016.4 and 2019.4-2020.4, the numbers of violation for all the methods are significantly larger than the other years. For example, HS has 10 and 15 violations respectively during these two periods, while there are only 1 or 2 in the other periods. As for the statistical testing result, all the p-values, both under 99% and 97.5% confidence level, are significantly small. Thus, the violations do not follow a Bernoulli random variable distribution.

As shown in Figure 21, 22 and Figure 23, 24, the VaR violations occurred in clusters in 2015-2016 and the first quarter in 2020 for both the VC normal and VC student methods. And Figure 25 and Figure 26 indicate that the HS method also shows clusters during the same periods. However, Figure 27, 28 and Figure 25, 26 show that the VaR violations in CCC-GARCH and FHS-EWMA methods spread more evenly throughout all the years. There is not a clear pattern indicating when the violations would occur.

Table 5: The expected and actual number of VaR(97.5%) violations per year(from previous April to next April) and p-value of binomial test

year	2014	2015	2016	2017	2018	2019	2020	average	p-value
Trading days	250	250	250	250	250	250	242	250	-
expected number	6	6	6	6	6	6	6	6	-
VC_{normal}	8	22	3	6	10	30	5	12	$3.35*10^{-8}$
$VC_{stu(df3)}$	10	24	3	6	11	31	6	13	$1.53*10^{-10}$
HS	4	19	3	5	5	24	5	9.29	$3.39*10^{-27}$
CCC-GARCH	5	15	3	7	8	18	6	8.86	0.007
FHS-EWMA	6	7	4	10	9	22	7	9.29	0.002

Table 6: The expected and actual number of VaR(99%) violations per year(from previous April to next April) and p-value of binomial test

year	2014	2015	2016	2017	2018	2019	2020	average	p-value
Trading days	250	250	250	250	250	250	242	250	-
expected number	3	3	3	3	3	3	3	3	-
VC_{normal}	3	12	2	2	5	23	2	7	$3.26*10^{-10}$
$VC_{stu(df3)}$	2	11	1	1	4	19	1	5.57	$5.54*10^{-6}$
HS	2	10	1	1	1	15	1	4.43	0.0024
CCC-GARCH	3	8	3	5	3	13	3	5.43	$1.62*10^{-5}$
FHS-EWMA	3	5	2	7	6	17	2	6	$3.8*10^{-7}$

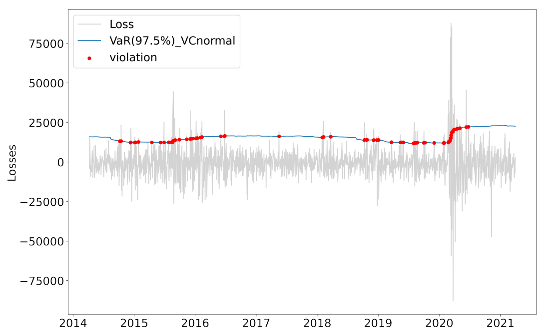

Fig. 21: The VaR(97.5%) Violation for VC normal

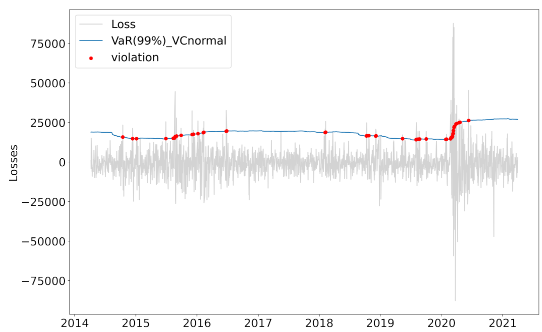

Fig. 22: The VaR(99%) Violation for VC normal

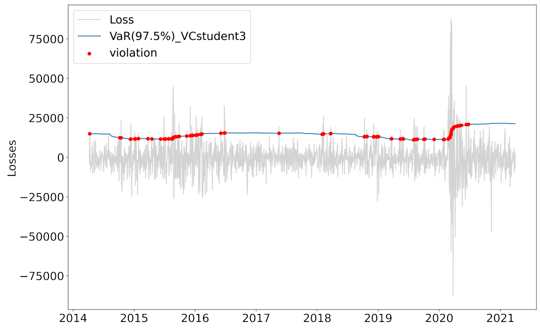

Fig. 23: The VaR(97.5%) Violation for VC student df3

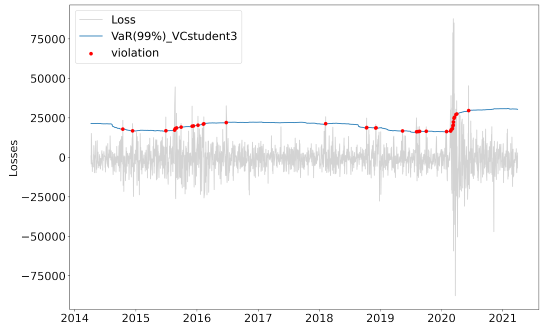

Fig. 24: The VaR(99%) Violation for VC student df3

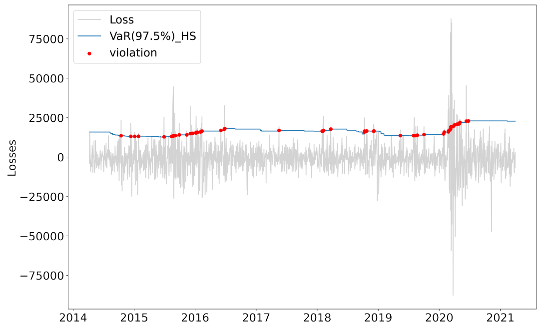

Fig. 25: The VaR(97.5%) Violation for HS

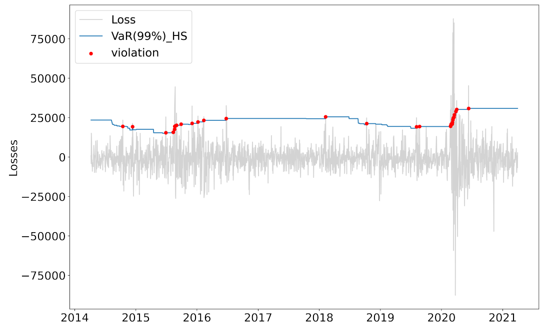

Fig. 26: The VaR(99%) Violation for HS

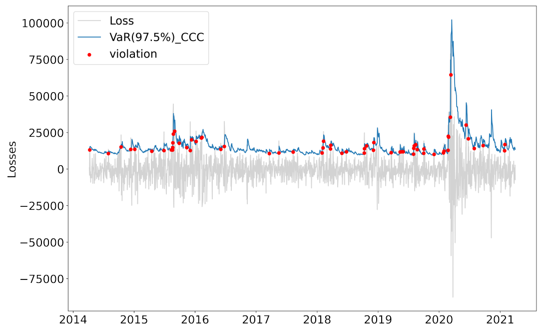

Fig. 27: The VaR(97.5%) Violation for CCC

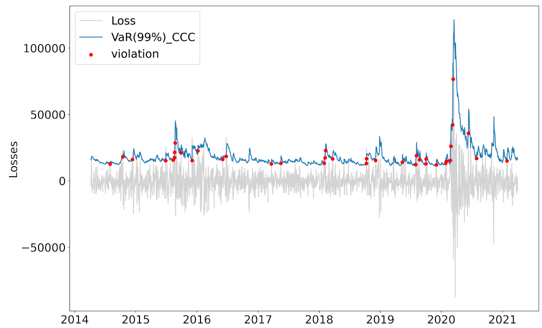

Fig. 28: The VaR(99%) Violation for CCC

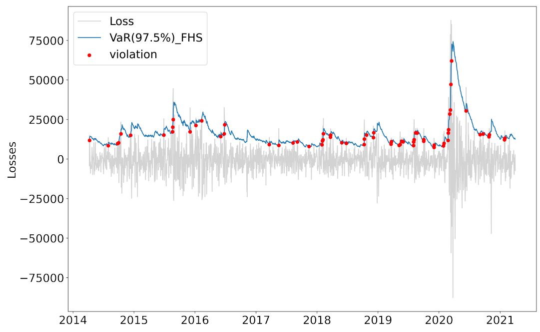

Fig. 29: The VaR(97.5%) Violation for FHS

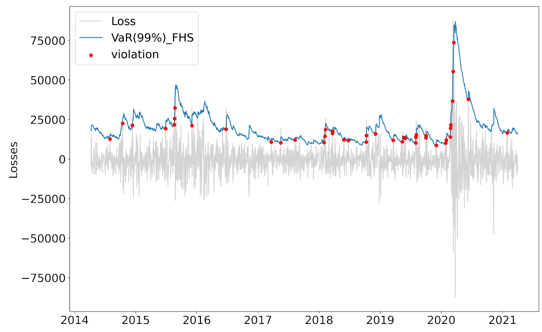

Fig. 30: The VaR(99%) Violation for FHS

As shown in Table 7 and Table 8, ESs(99%) are all larger than ESs(97.5%) for all the methods and for both expected and average scenarios. There are no clear trends for ESs of each method through these years but the relationship between expected and average can be revealed by $p - values > 0.05$, showing that there are no significant differences between expected and average for all the methods.

Table 7: The expected shortfalls with average (per year) shortfalls with CI=97.5% and p-value of t-test

year	2014	2015	2016	2017	2018	2019	2020	p-vale
Trading days	250	250	250	250	250	250	242	-
VC_{normal} Expected	15976.38	21466.27	24058.88	19729.72	17570.83	28344.72	28134.03	-
VC_{normal} Average	16743.99	16836.27	19676.81	19357.98	17207.48	15515.48	26863.97	0.21
$VC_{stu(df3)}$ Expected	15508.80	20738.63	24058.88	19729.72	17218.28	28016.23	26931.37	-
$VC_{stu(df3)}$ Average	20952.11	21069.15	24568.18	24160.82	21462.85	19357.04	33446.74	0.58
HS Expected	18905.75	22831.14	24058.88	20321.50	20598.51	31935.64	28134.03	-
HS Average	20394.65	20331.44	24836.68	24810.92	22348.64	20829.21	38336.33	0.96
CCC-GARCH Expected	17854.29	24079.00	21324.44	18652.32	17650.12	26368.85	22777.83	-
CCC-GARCH Average	15602.23	21414.76	16765.79	15194.33	15857.65	22765.56	24781.50	0.21
FHS-EWMA Expected	15779.13	30043.26	21461.00	16177.04	17180.31	27789.65	20962.79	-
FHS-EWMA Average	18571.33	27264.61	17124.28	13731.81	16381.53	21844.48	27254.09	0.96

Table 8: The expected shortfalls with average (per year) shortfalls with CI=99% and p-value of t-test

year	2014	2015	2016	2017	2018	2019	2020	p-value
Trading days	250	250	250	250	250	250	242	-
VC_{normal} Expected	20179.72	26427.00	27254.44	23342.00	20598.51	32484.12	35857.20	-
VC_{normal} Average	19153.54	19260.00	22477.59	22108.07	19644.09	17715.14	30633.24	0.21
$VC_{stu(df3)}$ Expected	22404.09	27393.00	32638.23	25710.81	21367.10	35399.00	45332.18	-
$VC_{stu(df3)}$ Average	29294.91	29461.08	34265.56	33682.71	29899.34	26973.14	46497.40	0.58
HS Expected	22404.09	27546.63	32638.23	25710.81	24860.26	39946.32	45332.18	-
HS Average	25464.59	25619.96	30719.69	30962.99	27186.56	25320.94	55151.98	0.96
CCC-GARCH Expected	19462.55	27741.19	21324.44	19854.80	21963.98	30065.86	25960.41	-
CCC-GARCH Average	17836.23	24464.55	19161.42	17371.18	18122.41	25996.74	28293.10	0.21
FHS-EWMA Expected	19462.55	31728.09	23152.78	18652.32	17350.06	30646.06	31606.76	-
FHS-EWMA Average	21773.90	31637.63	19638.81	16426.56	20043.72	25884.43	31857.95	0.96

4.5. *Stress testing*

Table 9 presents the impacts of four different extreme scenarios on our portfolio, in which the portfolio loss is calculated at 01/04/2021. The realistic portfolio return is 0.8625 on the last day. The first scenario is the exchange rate going up/down 10%. When +10% increase is added to the currency in the abovementioned three periods, our portfolio return increases by 17.1%; on the other hand, a -10% decrease in currency would lead to an additional 17.6% more portfolio return. As for interest rate, a 2% increase would change the portfolio return to 0.9968. In the last scenario, a 20% up-movement in indices brings about a 24.4% boost in return.

Table 9: The stress testing results

Scenario	Portfolio Return	Portfolio Return Impact(%)
Realisation	0.8625	-
Currency +10%	1.0101	17.1
Currency -10%	1.0145	17.6
Interest rate +2%	0.9968	15.6
Equity index values +20%	1.0720	24.3

5. Discussion

According to the results, VC methods with both normal distribution and student-t distribution are sensitive to the estimation periods, especially when there are stressed periods. Without stressed periods, the higher values of portfolio loss would be excluded, resulting in lower VaRs and ESs. The normal assumption of portfolio loss is not feasible but the student-t distribution with 3 degrees of freedom is more suitable. This is because we recognize that our portfolio loss has volatility clustering and a heavy tail.

Figure 14 and 15 reveals that the empirical 5 and 10 days VaRs are lower than the VaRs obtained from one-day VaR with square root of time. This result is coherent with our expectation because the property holds only if the daily returns of the assets are uncorrelated. But, in this case, the assets in our portfolio are correlated. There is a discrepancy after the first quarter in 2020, in which the empirical value is larger than the estimated value. The outbreak of Covid-19 is a possible reason for this because the estimated value can not react to the dramatic change sufficiently. However, the empirical value is more accurate since it is calculated by real data.

When comparing all the methods together in Figure 20, it is apparent that CCC-GARCH and FHS describe the distribution of portfolio loss better. Although these methods have an identical moving step of 1 when applied rolling window, unconditional methods(VC and HS) still can not react sufficiently fast to the rapid change in the loss. We reckon that this is a drawback of the unconditional methods as they assume constant volatility. However, in CCC-GARCH and FHS, the volatility is time-varying so that it can predict time series properly.

As for the backtesting, we can not conclude that the VaR violations follow a Bernoulli distribution with success probability $1-\alpha, \alpha = 0.975, 0.99$ since all the p-values of statistical testing are not significant. Although this result is not ideal enough, we still notice that the performance of CCC-GARCH is the best among all the methods as the average number of violations is the smallest concerning the VaR(97.5%). To our surprise, when it comes to VaR(99%), HS performs best since the average number of violations is closest to the expected number 3. Additionally, we can say that there is no significant difference between expected ESs and average ESs for all the methods since all the p-values of statistical testing is larger than 0.05. After comparing the statistical result for VaR and ES, we prefer ES in this task because our methods have more accurate predictions for ES other than VaR.

According to Figure 21, 23, 25, 27 and 29, we could clearly notice that there are clear clustering in the unconditional methods(VC normal, VC student, HS) around 2015-2016 and the beginning of 2020, whereas in conditional methods(CCC-GARCH and FHS-EWMA) the actual VaR violates the expected VaR more evenly. The violation clusters in all the VC methods and HS method reveal that the estimated VaRs of these methods react slowly to the change of high volatility while CCC-Garch and FHS-EWMA could react in time. Hence, we believe that the VaRs obtained by CCC-Garch and FHS-EWMA are more reasonable than those by the other methods.

We apply four extreme but plausible scenarios during stress testing. The up-movement of the exchange rate, interest rate and indices could consequently lead to larger portfolio returns to different degrees. Intriguing, we find that even -10% change in the exchange rate could have 17.6% more return. A possible explanation could be that the exchange rate between Euro and USD is very close to one; thus, the down-movement in the exchange rate would not necessarily cause significant loss.

6. Conclusion

This report applies risk measurement based on the loss distribution to our portfolio, consisting of three indices and one bond. We calculate VaR and ES at 97.5% and 99% confidence level over a predetermined one-day time horizon through two types of methods. One of the types is the unconditional methods like VC method(normal and student-t distribution) and HS method. The QQ plots indicate that our portfolio loss is heavy-tailed, so that normal assumption is not feasible. Instead, a student-t distribution of degree 3 is a better option in this task. The 5-day and 10-day VaR calculated by 1-day VaR multiplying the square root of time is mostly larger than the empirical VaRs. This rule fails because the daily return of each asset is correlated in our portfolio, which violates its assumption. Another type is the conditional method like CCC-GARCH and FHS-EWMA. Empirical results in Figure 20 show that these two conditional methods could react to the volatility of the loss more concisely. We find that the unconditional methods have apparent violation clustering through back testing while the conditional methods do not. The statistical tests in Table 5 and 7 indicate that VaR fails the Bernoulli assumption, but expected ES statistically equals average ES. Therefore, ES might be a better risk management than VaR specifically in our portfolio.

References

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