Aufg. 1.

a) $\text{cond } A = \|A\|_\infty \cdot \|A^{-1}\|_\infty$

$\|A\|_\infty = 1 + 0 + 2 = 3 = A_\infty$

$0 + 1 + 0 = 1$

$10^{-4} + 0 + 10^{-4} = 2 \cdot 10^{-4}$.

$3 \cdot 20'001 = \underline{\underline{60'003}}$

$$A^{-1} = \begin{matrix} -1 & 0 & 20'000 \\ 0 & 1 & 0 \\ 1 & 0 & -10'000 \end{matrix} = 20'001 = \|A^{-1}\|_\infty$$

b) $\frac{\|x - \tilde{x}\|}{\|x\|_\infty} \leq \text{cond } A \cdot \frac{\|\tilde{b} - b\|}{\|b\|_\infty}$ bei $\tilde{b} = \begin{pmatrix} 1 \\ 1 \\ \varepsilon \end{pmatrix}$

geg: $\|\tilde{b} - b\|_\infty = |\varepsilon|$

$\tilde{b} = (1, 1, \varepsilon)^T$

$\|b\|_\infty = 1$

$60'003 \cdot \frac{|\varepsilon|}{1} \leq 0.01$

$|\varepsilon| = \frac{0.01}{60'003} = \underline{\underline{1.666 \cdot 10^{-7}}}$

c) $\frac{\|\tilde{x} - x\|_\infty}{\|x\|_\infty}$ $\quad Ax = b \quad x = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ (Python)

x_1 $20'000 \cdot \frac{0.01}{60003} = \frac{-1 + 0.0033}{-0.9967}$

x_3 $10'000 \cdot \frac{0.01}{60003} = \frac{1 - 0.00166}{0.9983}$

$\tilde{x} = \begin{pmatrix} -0.9967 \\ 1 \\ 0.9983 \end{pmatrix}$

$\|\tilde{x} - x\|_\infty = 20'000 \quad 20'000 \cdot \frac{0.01}{60003} = 0.00333331$

$\approx 33.33\%$

d) bei Störung v. Maximal. 10^{-7}: $\quad \alpha = \frac{\|\Delta A\|_\infty}{\|A\|_\infty} = |\varepsilon|$

$\|\Delta A\| \leq 3 \cdot 10^{-7}$

$\alpha \leq \frac{3 \cdot 10^{-7}}{3} = 10^{-7}$

$\text{cond}(A) \cdot \alpha < 1$

$\text{cond}(A) \cdot \alpha \leq 60'003 \cdot 10^{-7} = 0.006003 < 1$ ✓

mit cond $= 60'003$ &

$|\varepsilon| \leq \frac{0.01(1 - \text{cond}\,\alpha)}{\text{cond}} - \alpha \qquad \alpha = 10^{-7}$

$\frac{0.0099}{60003} - 10^{-7} = 6.5658 \cdot 10^{-8}$