Aufgabe 1.

a)

$$A = \begin{pmatrix} 1 & -2 & 3 \\ -5 & 4 & 1 \\ 2 & -1 & 3 \end{pmatrix} \qquad b = \begin{pmatrix} 1 \\ 9 \\ 5 \end{pmatrix}$$

$$a_1 = \begin{pmatrix} 1 \\ -5 \\ 2 \end{pmatrix} \quad a_2 = \begin{pmatrix} -2 \\ 4 \\ -1 \end{pmatrix} \quad a_3 = \begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix}$$

$$q_1^T a_3 = \frac{1}{\sqrt{30}} (1 \; -5 \; 2) \begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix} = \frac{3-5+6}{\sqrt{30}} = \frac{4}{\sqrt{30}}$$

$$v_1 = \sqrt{1^2 + (-5)^2 + 2^2} = \sqrt{30}$$

$$q_1 = \frac{1}{\sqrt{30}} \cdot \begin{pmatrix} 1 \\ -5 \\ 2 \end{pmatrix} = \begin{pmatrix} 0.1826 \\ -0.9129 \\ 0.3651 \end{pmatrix}$$

$$\frac{4}{30} \begin{pmatrix} 1 \\ -5 \\ 2 \end{pmatrix} = \begin{pmatrix} 0.1333 \\ -0.6667 \\ 0.2667 \end{pmatrix} = \text{proj } q_1$$

$$\text{diff } v_3 = a_3 - \text{proj}_{q_1} - \text{proj}\, q_2 = \begin{pmatrix} 1.667 \\ 1.667 \\ 3.333 \end{pmatrix}$$

$$q_2^T a_3 = -1.3416$$

$$\text{Betrag } v_3 = \sqrt{16.66666} = 4.0825$$

$$\text{proj}_{q_2}(a_3) = -1.3416 \cdot \begin{pmatrix} -0.8944 \\ 0 \\ 0.4472 \end{pmatrix}$$

$$\text{Proj } q_3 = \begin{pmatrix} 0.4082 \\ 0.4082 \\ 0.8165 \end{pmatrix}$$

$$= \begin{pmatrix} 1.2 \\ 0 \\ -0.6 \end{pmatrix}$$

$$Q_{123} = \begin{pmatrix} 0.1826 & -0.8944 & 0.4082 \\ -0.9129 & 0 & 0.4082 \\ 0.3651 & 0.4472 & 0.8165 \end{pmatrix}$$

$$Q^T A = \begin{pmatrix} 5.4772 & -4.3818 & 4.000 \\ 0 & 1.3416 & -1.3416 \\ 0 & 0 & 4.0825 \end{pmatrix} = R$$

b.) $Ax=b$ $\quad QRx=b$ $\rightarrow Rx=Q^Tb$

$$\begin{pmatrix} 5.4772 & -4.3818 & 4.000 \\ 0 & 1.3416 & -1.3416 \\ 0 & 0 & 4.0825 \end{pmatrix}$$

$q_1^T = \frac{-34}{\sqrt{30}} \approx -6.2083$

$q_2 = -0.8945 + 2.236 = 1.3417$

$q_3 = 0.4082 + 3.6738 + 4.0825$

$= 8.1645$

$$Y = \begin{pmatrix} -6.2083 \\ 1.3417 \\ 8.1645 \end{pmatrix}$$

$$\begin{pmatrix} 5.4772 & -4.3818 & 4.000 \\ 0 & 1.3416 & -1.3416 \\ 0 & 0 & 4.0825 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -6.2083 \\ 1.3417 \\ 8.1645 \end{pmatrix}$$

$x_3 = 2$ $\quad 5.4772x_1 - 13.1454 + 8 = -6.2083$

$x_2 = 3$

$x_1 = -0.194$ $\quad x = \begin{pmatrix} -0.193 \\ 3 \\ 2 \end{pmatrix}$