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Contents

Chapter 1

Semiconductors

Silicon has a thetrahedral structure repeated periodically in the crystal. Due to this periodic configuration , electrons are subject to periodic potential and, as we know from Shrodinger equation, there only spedific bands of energy allowed.

Valence band

It's the last band filled at 0K.

Conduction band

It's the first band empty band at 0K.

1.0.1 Semiconductors band Structure

In semiconductors we have the Valence band completely filled and the conduction band completely empty. In this condition also if you apply a electric field you cannot have flow of electrons as there are no free states in the valences band in which electrons are able to move. Usually the energy gap between VB and CB is something near 1eV and if we increase the temperature some electron can be excited to the conduction band[1].

Now that we have some electrons in CB if we apply an electric field we are able to obtain some current. In particular we can have electrons moving in the CB and holes moving in the VB. Note that in dielectric we can't have current as the the energy gap is much more higher than in semiconductors, so electrons are not able to reach the conduction band due to the thermal energy.

1.0.2 Metals band structure

In metals the VB and the CB are overlapped , so electrons are free to move in any case as there in no energy gap to be jumped.

1.1 Silicon band structure

Silicon is a semiconductor material , in particular at room temperature it has $E_{gap} = 1,12eV$

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[1]Remember the magnitude of the thermal energy : $KT = 25meV$

1.1.1 Energy gap

The silicon's energy gap changes with temperature following the law:

$$E_g(T) = E_g(0) - \frac{\alpha T^2}{\beta + T} \tag{1.1}$$

in which:

- $\alpha = 4.73 * 10^{-4} \frac{eV}{K}$

- $\beta = 636K$

- $E_g(0) = 1.17eV$

It's intresting to notice the derivative of the energy gap:

$$\frac{dE_g}{dT}(T) = \frac{-2\alpha T(\beta + T)^2 + \alpha T^2}{(\beta + T)^2} \tag{1.2}$$

For example we have:

$$\frac{dE_g^{Si}}{dT}(300K) = -2.5 * 10^{-4} \frac{eV}{K} \tag{1.3}$$

So we have $25meV$ of variation over $100K$ excursion. This value looks negligible in front of the energy gap , but we have compare it with ....

1.1.2 Density of States

Let's compute how many states we have in the valence band.

$$E - E_c = \frac{\hbar^2 k_x^2}{2m_x} + \frac{\hbar^2 k_y^2}{2m_y} + \frac{\hbar^2 k_z^2}{2m_z} \tag{1.4}$$

We cannot plot E as it should be a 4-dimensional plot but we can have a look on constant energy surfaces.

$$E - E_c = \frac{\hbar^2 k_x^2}{2m_x} \tag{1.5}$$

And we can get the intersection with axes:

$$k_x = \sqrt{\frac{2m_x(E - E_c)}{\hbar^2}} \tag{1.6}$$

$$k_y = \sqrt{\frac{2m_y(E - E_c)}{\hbar^2}} \tag{1.7}$$

$$k_z = \sqrt{\frac{2m_z(E - E_c)}{\hbar^2}} \tag{1.8}$$

Note as the larger is the energy the larger are the intersections, and all points inside the ellipsoide have a lower energy in comparison on the surface.

We are able to compute the volume of the ellipsoid:

$$V = \frac{4}{3}\pi\sqrt{\frac{2m_x(E - E_c)}{\hbar^2}}\sqrt{\frac{2m_y(E - E_c)}{\hbar^2}}\sqrt{\frac{2m_z(E - E_c)}{\hbar^2}} = \frac{4}{3}\pi\frac{\sqrt{8m_x m_y m_z}}{\hbar^3}\sqrt{(E - E_c)} \tag{1.9}$$

1.1.3 Number of states in conduction band

$$N = \frac{V}{(\frac{2\pi}{L})^3} = \frac{Ellipsoide\ Volume}{Volume\ of\ one\ state} = \frac{4}{3}\frac{\pi}{(2\pi)^3}\frac{\sqrt{8m_xm_ym_z}}{\hbar^3}\sqrt{(E-E_c)^3} = \frac{4}{3}\frac{\pi}{h^3}\sqrt{8m_xm_ym_z}\sqrt{(E-E_c)^3}$$

(1.10)

from which:

$$dN = \frac{4}{3}\frac{\pi}{h^3}\sqrt{8m_xm_ym_z}\frac{3}{2}\sqrt{(E-E_c)}dE$$

(1.11)

Now we can compute the number of states in the conduction band as:

$$dN = \frac{4}{3}\frac{\pi}{h^3}\sqrt{8m_xm_ym_z}\frac{3}{2}\sqrt{(E-E_c)}dE$$

(1.12)

$$g_c(E) = \frac{4\pi}{h^3}\sqrt{2m_xm_ym_z}\sqrt{E-E_c}*2*6$$

(1.13)

Where we multiply by 2 due the possibility of the presence of both spins in the same state and for 6 due to the degeneracy. In conculsion we have:

$$g_c(E) = \frac{48\pi}{h^3}\sqrt{2m_t^2m_l}\sqrt{E-E_c} = \frac{48\pi}{h^3}\sqrt{2m_{dos}^3}\sqrt{E-E_c}$$

(1.14)

In which we define $m_{dos} = \sqrt[3]{m_t^2m_l}$ remembering that $m_x = m_z = m_t = 0.19m_0$ and $m_y = m_l = 0.92m_0$.[2]

1.1.4 Electron density

Only under thermodynamic equilibrium[3] we can use the Fermi-Dirac distribution to determine how many states of the conduction band are occupied by electrons:

$$f(E) = \frac{1}{1+e^{\frac{E-E_f}{KT}}}$$

(1.15)

then if we have $E - E_f >> KT$ we can use the Maxwell-Boltzman approximation:

$$f_{MB}(E) = e^{\frac{E-E_f}{KT}}$$

(1.16)

Note that in the M.B. approximation we have $f_{MB}(E_f) = 1$, but as we know this is not true as by definition we have $f(E_f) = \frac{1}{2}$. Let's compute now the number of electrons in the conduction band multiplying the number of states by the probability to have them filled:

$$n = \int_{E_c}^{\infty} g_c(E)f(E)dE = \frac{48\pi}{h^3}\sqrt{2m_{dos}^3}\sqrt{E-E_c}\frac{1}{1+e^{\frac{E-E_f}{KT}}}$$

(1.17)

This integral cannot be solved analiticaly but we are able to simplify the expression using some variable changes:

- $\frac{E-E_c}{KT} = x$

- $\frac{E_f-E}{KT} = \mu$

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[2]this is valid for silicon
[3]Condition in which there are no net processes

- $dE = KTdx$

- $x - \mu = \frac{E - E_f}{KT}$

we get:

$$n = \frac{48\pi}{h^3}\sqrt{2m_t^2 m_l}\int_0^\infty \frac{\sqrt{KT}\sqrt{x}}{1 + e^{x-\mu}}KTdx = \frac{48\pi}{h^3}\sqrt{2m_t^2 m_l}(KT)^{\frac{3}{2}}\frac{\sqrt{\pi}}{2}\frac{2}{\sqrt{\pi}}\int_0^\infty \frac{\sqrt{x}}{1 + e^{x-\mu}}dx = N_c F_{\frac{1}{2}}(\mu)$$

(1.18)

In which we define:

- $N_c = \frac{48\pi}{h^3}\sqrt{2m_t^2 m_l}(KT)^{\frac{3}{2}}\frac{\sqrt{\pi}}{2}$

- $F_{\frac{1}{2}}(\mu) = \frac{2}{\sqrt{\pi}}\int_0^\infty \frac{\sqrt{x}}{1 + e^{x-\mu}}dx$

If $x - \mu >> 1$ we can make an approximation and write : $F_{\frac{1}{2}}(\mu) = \frac{2}{\sqrt{x}}\int_0^\infty \sqrt{x}e^{-x+\mu}dx = e^\mu$.[4]
So we have:

$$n = N_c e^\mu = N_c e^{-\frac{E - E_f}{KT}}$$

(1.19)

The number of holes can be found with the same approach:

$$p = \int_{-\infty}^{E_v} g_c(E)(1 - f(E))dE = N_v F_{\frac{1}{2}}(\mu)$$

(1.20)

from which we get:

$$p = N_v e^{-\frac{E_f - E_v}{KT}}$$

(1.21)

1.1.5 Intrinsic Fermi level

In intrinsic silicon we have $p = n$

$$N_c e^\mu = N_c e^{-\frac{E - E_f}{KT}} = N_v F_{\frac{1}{2}}(\mu)$$

from which we get the intrinsic Fermi level:

$$E_f = \frac{E_c + E_v}{2} + KTln(\frac{N_c}{N_v})$$

(1.22)

1.1.6 Intrinsic carriers concentration

Now we can compute the number of instrinsic carriers for silicon:

$$n = p = n_i = N_c e^{-\frac{E_c - E_f}{KT}} = N_c e^{-\frac{E_g}{2KT}}e^{ln(\frac{N_c}{N_v})^{-\frac{1}{2}}} = \sqrt{N_c N_v}e^{-\frac{E_g}{2KT}}$$

(1.23)

$$n_i(300K) = 1.45 * 10^{10}cm^{-3}$$

Let's notice as there is an exponential dependence from the energy gap which is stricly related to the temperature; if the temperature increase the carriers density increase a lot.

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[4]Remembering the average value of a normal Gaussian curve: $\int_0^\infty \sqrt{x}e^{-x}dx = \frac{\sqrt{\pi}}{2}$

$$n = n_i e^{\frac{E_f - E_i}{KT}} \qquad (1.24)$$

$$p = n_i e^{\frac{E_i - E_v}{KT}} \qquad (1.25)$$

These are general expressions, valid also for non intrinsic silicon.

Chapter 2

Doped semiconductors

In general semiconductors of the IV group are doped with elements from the III and V group, with a substitutional doping process.

In a doped material we have:

$$n = p + N_d^+ = N_c e^{-\frac{E_c - E_f}{KT}} = N_v e^{-\frac{E_f - E_v}{KT}} + \frac{N_d}{1 + 2e^{\frac{E_v - E_f}{KT}}} \qquad (2.1)$$

In which holes can be neglected onbtaining, and considering the material at room temperature we can assume to be in the complete ionization condition, in which $n = N_d$.[1]

2.1 Action mass law

From previous considerations we obtain the action mass law:

$$p = \frac{N_i^2}{N_d} \qquad (2.2)$$

But note now that increasing a lot the donors concentration we have to consider the incomplete ionization condition in which the Maxwell-Bolzman approximation is not still valid.[2]

2.1.1 Relationship between Fermi level and Temperature

Having in mind the 1.22 we notice that the higher is the temperature , the higher is the probability to have occupied states in the conduction band. But E_f cannot keep decreasing; infact if we increase the temperature intrinsic carriers concentration increase and cannot be neglected, so E_f asintotically approches E_i. On the other side if we decrease the temperature E_f approches E_d and incomplete ionization must be considered.[3]

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[1]In which N_d is the donors density

[2]High doped silicon is generally called degenerated silicon

[3]All these considerations can be repeated for acceptors dopants

2.2 Carriers transport

2.2.1 Drift

If we aplly an electric field it will accelerate as electrons as hole [4] , and we will aspect a linear relationship, but carriers are moving in a crystal and they are subjected to scattering events.

We can find for both carriers an average value of the velocity given by:

$$V_{d_n} = \mu_n F \tag{2.3}$$

$$V_{d_p} = \mu_p F \tag{2.4}$$

Where μ_n and μ_p are the mobilities for respectively electrons and holes.

$$J_{drift_n} = qn\mu_n F \qquad J_{drift_p} = qp\mu_p F \tag{2.5}$$

2.2.2 Mobility

Mobility changes has a relationship with:

- Doping concentration

- Temperature

- Dimensionality

Mobility and doping concentration

If we increase the doping concentration we increase the number of impurities and the probability of scattering events with them. There are 2 different types particles with which carriers can scatter:

- Phonons

- Impurities

Mobility and temperature

If the temperature increase also the probability of scattering increase and so the mobility is smaller.

Mobility and dimensionality

Near surface the surface the number of impurities is much more higher and so increasing the scattering probability the mobility deacrease.

The relationship given for the drift velocity is still linear and it seems that we are able to increase the drift velocity freely; this is not true as when we reach high levels of electric field the scattering contribution given by optical phonons[5] is not still negligible.

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[4]In opposite directions

[5]High energetic particles, generated by lattice vibrations

2.2.3 Diffusion

Carriers transport is also given by the diffusion phenomena which is simply described by:

$$J_{diff_n} = qD_n\frac{dn}{dx} \qquad J_{diff_p} = qD_p\frac{dp}{dx} \qquad (2.6)$$

In which D_n and D_p are the diffusion coefficients given by the Einstein's relation:

$$D_n = \frac{KT\mu_n}{q} \qquad D_p = \frac{KT\mu_p}{q} \qquad (2.7)$$

2.2.4 Total density of current

$$J_n = qn\mu_n F + D_n\frac{dn}{dx} \qquad J_p = qp\mu_p F + D_p\frac{dp}{dx} \qquad (2.8)$$

Chapter 3

Valence band of Silicon

The valence band of silicon is made by the overlapping of three subbands:

- Heavy holes band

- Light oholes bans

- Spin-off band

These 3 subbands are all isotropic which means that they have the same effective mass in all directions:

$$m_x = m_y = m_z \tag{3.1}$$

In detail we have:

- $m_{hh} = 0.49m_0$ and we are able to write the dispersion relationship for the heavy holes band as: $E_v - E = \frac{\hbar^2 k^2}{2m_{hh}}$

- $m_{lh} = 0.16m_0$ and we are able to write the dispersion relationship for the light holes band as: $E_v - E = \frac{\hbar^2 k^2}{2m_{lh}}$

- $m_{so} = 0.29m_0$ and we are able to write the dispersion relationship for the spin-off band as: $E_v - E = \frac{\hbar^2 k^2}{2m_{so}}$

in which $k^2 = k_x^2 + k_y^2 + k_z^2$ As we have isotropic bands we are able to plot E over k obtaining 3 parabolic curves, while in the k_x,k_y and k_z space we are going to have 3 spheres as larger as the effective mass is larger.

Considering the dispersion relations we can find the intersection with axes of the spheres:

$$K = \pm\sqrt{\frac{2m_*(E_v - E)}{\hbar^2}} \tag{3.2}$$

and starting from them we can find the volume of sphere which will be usefull to calculate the density of states. For example we can evaluate the volume of the sphere related to the heavy holes band.

$$V_{hh} = \frac{4}{3}\pi \left[\sqrt{\frac{2m_{hh}(E_V - E)}{\hbar^2}} \right]^3 \tag{3.3}$$

Let's now evaluate the number of states in the same way we have done for the conduction band of generic semiconductors in chapter 1.1.3:

$$N = \frac{V}{(\frac{2\pi}{L})^3 L^3} = \frac{4}{3}\pi \frac{1}{h^3}\sqrt{8m_{hh}^3}\sqrt{(E_V - E)^3} \tag{3.4}$$

$$dN = \frac{4}{3}\pi \frac{1}{h^3}\sqrt{8m_{hh}^3}\frac{3}{2}\sqrt{(E_V - E)}dE \tag{3.5}$$

Now we are able to get the densities of states as:

$$g_v^{hh} = \frac{8\pi}{h^3}\sqrt{2m_{hh}^3}\sqrt{E_v - E} \tag{3.6}$$

$$g_v^{lh} = \frac{8\pi}{h^3}\sqrt{2m_{lh}^3}\sqrt{E_v - E} \tag{3.7}$$

$$g_v^{so} = \frac{8\pi}{h^3}\sqrt{2m_{so}^3}\sqrt{E_s o - E} \tag{3.8}$$

from which we can find the number of carriers evaluating the number of occupied states.

$$p^{hh} = \int_{-\infty}^{E_v} g_v^{hh}(1 - f(E)) = \int_{-\infty}^{E_v} \frac{8\pi}{h^3}\sqrt{2m_{hh}^3}\sqrt{E_v - E}\frac{1}{1 + e^{-\frac{E - E_f}{KT}}}dE \tag{3.9}$$

Equation which can be simplified as done for the conduction band usign some variable changes:

$$\left[x = \frac{E_v - E}{KT} \qquad dE = -KTdx \qquad \mu = \frac{E_v - E_f}{KT} \qquad x - \mu = \frac{E_f - E}{KT}\right]$$

$$p^{hh} = \frac{8\pi}{h^3}\sqrt{2m_{hh}^3}(KT)^{\frac{3}{2}}\frac{\sqrt{\pi}}{2}\frac{2}{\sqrt{\pi}}\int_0^{+\infty}\frac{\sqrt{x}}{1 + e^{x-\mu}}dx = N_v F_{\frac{1}{2}}(\mu) \tag{3.10}$$

we can also introduce:

- $N_v^{hh} = \frac{8\pi}{h^3}\sqrt{2m_{hh}^3}(KT)^{\frac{3}{2}}\frac{\sqrt{\pi}}{2}$

- $N_v^{lh} = \frac{8\pi}{h^3}\sqrt{2m_{lh}^3}(KT)^{\frac{3}{2}}\frac{\sqrt{\pi}}{2}$

- $N_v^{so} = \frac{8\pi}{h^3}\sqrt{2m_{so}^3}(KT)^{\frac{3}{2}}\frac{\sqrt{\pi}}{2}$

3.0.1 Holes concetration in valence band

we can evaluate the number of carriers considering the contribution of the three subbands:

$$p_{v_tot} = p^{hh} + p^{lh} + p^{so} \tag{3.11}$$

But we know that $E_v - E_{so} = 44meV$ and as $44meV >> KT$ we are able to neglect the contribution of the spin-off band.

$$p = (N_v^{hh} + N_v^{lh})F_{\frac{1}{2}}(\mu) \tag{3.12}$$

Where we have $N_v^{hh} + N_v^{lh} = 1.83 * 10^{19}cm^{-3}$

3.0.2 Carriers density in Metals

Let's consider the case of $E_f > E_c$. In this case tha Maxwell-Bolzmann approximation cannot be used, but we can consider $f(E)$ as step function.

$$n = \int_{E_c}^{E_f} g_v(E)dE = \frac{48\pi}{h^3}\sqrt{2m_t^2 m_h}\sqrt{E - E_c}dE = \left[\frac{48\pi}{h^3}\sqrt{2m_t^2 m_h}\frac{2}{3}E - E_c\right]_{E_c}^{E_f} =$$

$$= \frac{48\pi}{h^3}\sqrt{2m_t^2 m_l}(KT)^{\frac{3}{2}}\frac{\sqrt{x}}{2}\frac{2}{\sqrt{x}}\frac{2}{3}\left(\frac{E_f - E_c}{KT}\right)^{\frac{3}{2}} = N_c\frac{2}{3}\frac{2}{\sqrt{x}}\left(\frac{E_f - E_c}{KT}\right)^{\frac{3}{2}} = N_c\frac{2}{3}\frac{2}{\sqrt{\pi}}\mu^{\frac{3}{2}} \quad (3.13)$$

We can compare this result with the Maxwell-Bolzman approximation ($n = N_c e^{-\frac{E_c - E_f}{KT}} = N_c e^{\mu}$) and note that it's valid only for $\mu < -3$ and that the approximation error increase with E_f.

Chapter 4

Incomplete ionization

Let's now consider the incomplete ionization.

Example: $T = 77K$ $N_d = 10^{16} cm^{-3}$ $E_c - E_d = 54 meV$

In this case we are at low temperature and we can neglect the holes concentration as the fermi level increase , so we have $n \approx N_d^+$

$$n = N_c e^{-\frac{E_c - E_f}{KT}} = N_d \frac{1}{1 + 2e^{-\frac{E_d - E_f}{KT}}} = N_d^+ \tag{4.1}$$

$$N_d = N_c e^{-\frac{E_c - E_f}{KT}} 1 + 2e^{-\frac{E_d - E_f}{KT}} \tag{4.2}$$

with $x = e^{-\frac{E_c - E_f}{KT}}$ we have this equation:

$$2N_c e^{\frac{E_c - E_f}{KT}} x^2 + N_c x - N_d = 0 \tag{4.3}$$

We can easly calculate x:

$$x = \frac{-N_c \pm \sqrt{N_c^2 + 8N_d N_c e^{\frac{E_c - E_d}{KT}}}}{4N_c e^{\frac{E_c - E_d}{KT}}} = e^{-\frac{E_c - E_d}{KT}} \tag{4.4}$$

$$E_c - E_f = -KT ln \left(-N_c + \frac{\sqrt{N_c^2 + 8N_d N_c e^{\frac{E_c - E_d}{KT}}}}{4N_c e^{\frac{E_c - E_d}{KT}}} \right) \tag{4.5}$$

Considering our example:

- $N_c(77K) = 3.68 * 10^1 8 cm^{-3}$

- $KT(77K) = 6.64 meV$

we get: $E_c - E_f = 49.6 meV = 7.48 KT$ despite $E_c - E_f$ looks large , at this temperature KT is quiet low and so as $E_C - E_f >> KT$ the Maxwell-Bolzaman approximation is stiil valid.

4.0.1 Extrinsic zone

Let's consider now $T >> RT$, we cannot neglect the number of holes but we can make the complete ionization assumption, so we have:

$$n = p + N_d^+ \approx \frac{n_i^2}{N_d} + N_d \tag{4.6}$$

from which we can write a simple equation to find n:

$$n^2 - N_d n - n_i^2 = 0 \tag{4.7}$$

Electron density at high temperatures:

$$n = \frac{N_d + \sqrt{N_d^2 + 4n_i^2}}{2}$$

4.0.2 Freeze-out zone

If we consider instead $T << RT$ we neglect the holes concentration but we have to consider the incomplete ionization:

$$n \approx N_d^+ = N_d \frac{1}{1 + 2e^{-\frac{E_d - E_f + E_c - E_c}{KT}}} = N_d \frac{1}{1 + 2e^{-\frac{E_c - E_f}{KT}} e^{\frac{E_c - E_d}{KT}} \frac{N_c}{N_c}} = N_d \frac{1}{1 + 2\frac{n}{N_c} e^{-\frac{E_c - E_d}{KT}}} \tag{4.8}$$

we get a simple equation to find n:

$$\frac{2}{N_c} e^{\frac{E_c - E_d}{KT}} n^2 + n + -N_d = 0 \tag{4.9}$$

$$n = \frac{1 + \sqrt{1 + \frac{8N_d}{N_c} e^{\frac{E_c - E_d}{KT}}}}{\frac{4}{N_c} e^{\frac{E_c - E_d}{KT}}} \approx \frac{\sqrt{\frac{8N_d}{N_c} e^{\frac{E_c - E_d}{KT}}}}{\frac{4}{N_c} e^{\frac{E_c - E_d}{KT}}} \tag{4.10}$$

$$n = \sqrt{\frac{N_c N_d}{2}} e^{-\frac{E_c - E_d}{KT}} \tag{4.11}$$

Considering this result we can find an expression for the Fermi level at low temperature:

$$N_c e^{-\frac{E_c - E_f}{KT}} = \sqrt{\frac{N_c N_d}{2}} e^{-\frac{E_c - E_d}{KT}} \tag{4.12}$$

$$E_f = \frac{E_c + E_d}{2} + KT ln \sqrt{\frac{N_d}{2N_c}} \tag{4.13}$$

From which we can notice that at very low temperatures the Fermi level is placed in the middle point between E_c and E_d.

Chapter 5

Electrostatic analisys of semiconductor materials

Let's note how adding an exernal potential to the crystal the periodic potential configurations and the bands are shifted. When the external potential is positive the bands are bended downwards.

Under the effective mass approximation we can consider electrons in the crystal as free particles subjected only to external forces. So external potential is the only thing we need to invetigate the behaviour of our carriers.

$$\vec{F} = -\vec{\nabla}\phi \quad \implies \quad \vec{F} = -\frac{d\phi}{dx} \tag{5.1}$$

$$\vec{\nabla}\vec{F} = \frac{\rho}{\epsilon_{si}} \quad \implies \quad \frac{dF}{dx} = \frac{\rho}{\epsilon_{si}} \tag{5.2}$$

From which we get:

$$\vec{\nabla}(-\vec{\nabla}\phi) = -\vec{\nabla}^2\phi = \frac{\rho}{\epsilon_{si}} \tag{5.3}$$

And we are able to write the **Poisson equation for semiconductor materials**

$$\frac{d^2\phi}{dx^2} = -\frac{\rho^2}{\epsilon_{si}} = -\frac{q}{\epsilon_{si}}(p - n + Nd^+ - Na^-) \tag{5.4}$$

Assuming n doped silicon we can neglect holes and Na is nil, so we can write:

$$\frac{d^2\phi}{dx^2} = -\frac{q}{\epsilon_{si}}(Nd^+ - n) = -\frac{q}{\epsilon_{si}}(Nd + \Delta Nd(x) - n_i e^{\frac{q(\phi + \Delta\phi(x) - \phi_f)}{KT}}) \tag{5.5}$$

$$\frac{d^2\phi}{dx^2} = -\frac{q}{\epsilon_{si}}\Delta Nd(x) + \frac{q^2 Nd}{\epsilon_{si} KT}\Delta\phi(x) \tag{5.6}$$

This is a second order non omogeneous differential equation that has as result an exponential behaviour:

$$\Delta\phi(x) \propto e^{-\frac{x}{L_d}} \tag{5.7}$$

where Ld defined as the debye length is $L_d = \sqrt{\frac{\epsilon_{si} KT}{q^2 Nd}}$

Chapter 6

Current transport

$$J_n = qn\mu_n F + qD_n\frac{dn}{dx} = -qn\mu_n\frac{d\phi}{dx} + qD_n\frac{d}{dx}(n_i e^{\frac{q(\phi-\phi_f)}{KT}}) \tag{6.1}$$

Bibliography