Politecnico di Milano

Facoltà di ingegneria
Dipartimento di Elettronica e Informazione

RF Systems

Notes on Exercises
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Professore:
Giuseppe Macchiarella

Studente:
Edoardo Contini

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Chapter 1

Antennas

1.1 Antennas Parameters

1.1.1 Directive Gain

The power radiated by an antenna depends on the direction. The *directive gain* toward a given direction is expressed as:

$$D(\theta,\phi) = \frac{Radiation\ \ Intensity}{Isotropic\ \ Intensity} = \frac{U(\theta,\phi)}{P_{rad}/4\pi} \tag{1.1}$$

1.1.2 Directivity function and D_M definition

There is a direction $(\theta_{MAX}, \phi_{MAX})$ where D is maximum. The directivity function is the function D normalized to D_M:

$$f(\theta,\phi) = \frac{D(\theta,\phi)}{D_M} \quad \Longrightarrow S_R(R,\theta,\phi) = \frac{P_{rad}}{4\pi R^2} D_M f(\theta,\phi) \tag{1.2}$$

D_M represents the ratio between the radiated power density in the direction where it is maximum divided by the power density obtained with an isotropic radiator:

$$D_M = \frac{4\pi}{\int_0^{2\pi}\int_0^{2\pi} f(\theta,\phi) sin(\phi) d\theta d\phi} \tag{1.3}$$

1.1.3 Radiated Power Density

It is usefull to evaluate the radiated power density as it can be used for an analysis of propagation of power towards the threedimensional space.

$$S_R = \frac{dP_{rad}}{ds} = \frac{1}{2} Re(E \times H^*) = \frac{1}{R^2} U(\theta,\phi) \tag{1.4}$$

Isotropic Radiation

In case of isotropic radiation the radiation intensity $U(\theta,\phi) = \frac{P_{rad}}{4\pi}$

Non Isotropic Radiation

In non isotropic cases we get that the radiotion intensity is:

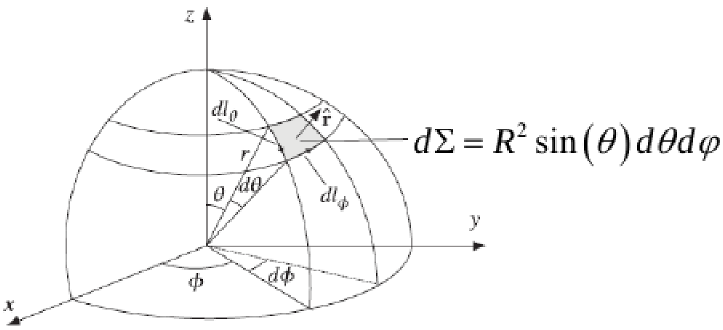
$$U(\theta, \phi) = \frac{P_{rad}}{4\pi} D(\theta, \phi) \tag{1.5}$$

The radiated power intensity results:

$$S_R(R, \theta, \phi) = \frac{P_{rad}}{4\pi R^2} D(\theta, \phi) \tag{1.6}$$

From the formula of the power density at distance R we can derive the net radiated power:

$$P_{rad} = \iint_\Sigma S_R(R, \theta, \phi)\, d\Sigma = \frac{P_{rad} D_{MAX}}{4\pi R^2} \iint_\Sigma f(\theta, \phi)\, d\Sigma \tag{1.7}$$



$$P_{rad} = \frac{P_{rad} D_{MAX}}{4\pi} \int_0^\pi \int_0^{2\pi} \sin(\theta) f(\theta, \phi) d\theta d\phi \tag{1.8}$$

from which:

$$\frac{1}{D_{MAX}} = \frac{\eta}{G} = \frac{1}{4\pi} \int_0^\pi \int_0^{2\pi} \sin(\theta) f(\theta, \phi) d\theta d\phi \tag{1.9}$$

and :

$$G = \frac{4\pi\eta}{\int_0^\pi \int_0^{2\pi} \sin(\theta) f(\theta, \phi) d\theta d\phi} \tag{1.10}$$

1.1.4 Fields Intensity

The radiated Power can be also related to the intensity of the electromagnetic fields:

$$S_R = \frac{1}{2} \frac{\sqrt{\epsilon_r}}{Z_0} |E|^2 = \frac{1}{2} \frac{Z_0}{\sqrt{\epsilon_r}} |H|^2 \tag{1.11}$$

The power density from a transmitting antenna (PT , G) at distance R along the direction of maximum radiation is given by:

$$S_R = \frac{P_T G}{4\pi R^2} \tag{1.12}$$

Then:

$$|E| = \frac{1}{R}\sqrt{\frac{Z_0 P_{ERP}}{2\pi\sqrt{\epsilon_r}}} \quad [V/m] \tag{1.13}$$

$$|H| = \frac{1}{R}\sqrt{\frac{\sqrt{\epsilon_r} P_{ERP}}{2\pi Z_0}} \quad [A/m] \tag{1.14}$$

1.1.5 Efficienty of Antennas

The impedance "seen" by the transmitter is determined by 2 contributes: Z_R [1] and R_p [2]. The power dissipated by Z_R constitutes the radiated power[3].

Assuming the reader aware of maximum power transfer theorem, neglecting the losses resistances because as stated before negligible in respect of the radiation impedence, we will assume impedence matching.

To understand how much of the electrical power is actually translated in radiated power the antenna efficiency can be defined as:

$$\eta = \frac{P_{rad}}{P_T} = \frac{Re\{Z_R\}}{Re\{Z_R\} + R_p} \tag{1.15}$$

1.1.6 Effective Radiated Power

The effective radiated power if simply defined as:

$$P_{ERP} = P_T G \tag{1.16}$$

1.1.7 Gain of Antennas

Defined as:

$$G = \eta D_M \tag{1.17}$$

the gain is one of the fundamental paraments that describes an antenna.

1.1.8 Received Power

Given the power density (S_R)[4] and the direction of arrival (θ) of an incident radiation the received power can be calculated as:

$$P_r = A_e S_R f(\theta) \tag{1.18}$$

where:

[1] radiation impedance
[2] loss resistance
[3] Typically $R_p \ll |Z_R|$
[4] magnitude of the Poynting vector

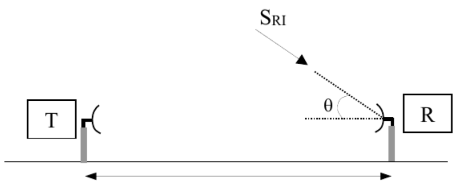


Figure 1.1: Simplified scheme of a Radio Link with an additiona power density from an non optimal direction

- A_e is the **effective Area** of the anttenna
- $f(\theta)$ is the directivity function

1.1.9 Effective Area

$$\frac{G}{A_e} = \frac{4\pi}{\lambda^2} \tag{1.19}$$

1.1.10 Equivalent length

The equivalent length can be related to the effective area using the following formula:

1.2 Dish Antennas

1.2.1 Gain

$$G = A_e \frac{4\pi}{\lambda^2} \tag{1.20}$$

1.2.2 Effective Area

$$A_e = e_a \frac{1}{4} \pi d^2 \tag{1.21}$$

where:

- e_a is the aperture efficiency (0.55 - 0.65)
- d = Dish diameter

1.2.3 Beamwidth

$$cos(\phi_B) = 1 - \frac{2}{D_{MAX}} = 1 - \frac{2\eta}{\pi^2 e_a^2} \left(\frac{\lambda}{d}\right)^2 \tag{1.22}$$

1.2.4 Free-space Attenuation

$$A_{dB} = 20 log_{10} \left(\frac{4\pi L}{\lambda}\right) \tag{1.23}$$

1.3 Radio Link

1.3.1 Link Budget

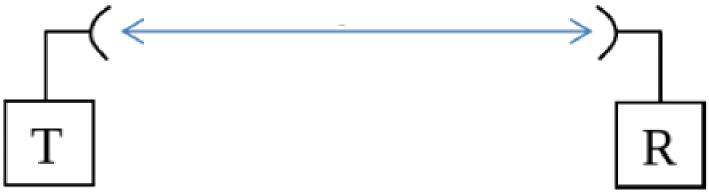


Figure 1.2: Simplified scheme of a Radio Link

Supposing to have a Radio Link as in Figure 1.2 between 2 Antennas at a certain relative distance R the link budget may be evaluated by means of the **Friis equation** which at the end leds to a simple analitical expression for the receiving power[5]:

$$P_{r|_{dB}} = P_{t|_{dB}} - 10 \log frac\lambda 4\pi R + G_{t|_{dB}} + G_{r|_{dB}} \tag{1.24}$$

in which:

- $P_{r|_{dB}}$ is the effective power received by the antenna.
- $P_{t|_{dB}}$ is the overall transmitted power of the TX side.
- $G_{r|_{dB}}$ is the gain of the receiving antenna.
- $G_{t|_{dB}}$ is the gain of the transmitting antenna.

while the term $20 \log \frac{\lambda}{4\pi R}$ stands for the free space losses. Note from a practical point of view there as at least a couple of DoF in the design flow to obtain a certain receiving power.

We may also report the linear form of the **Friis equation**:

$$P_r = P_t \left(\frac{\lambda}{4\pi L}^2 \right) G_T G_R \tag{1.25}$$

1.3.2 Attenuators' Noise

In any RF receiver we are going to have at least some losses between the antenna and the LNA. Sometimes also in other sections of the receiver path attenuation couldn't be avoided. We may evaluate the additional noise introduced my an attenuator A_{f1} for example the one in Figure 1.3 as:

$$T_{f1} = T_0 (10^{\frac{A_{f1}}{10}} - 1) \tag{1.26}$$

1.3.3 Equivalent Temperature

Supposing to have a receiving chain as in Figure[6] 1.3 we may want to rappresent the system with noiseless components and an input referred noise considering all contribution. This could

[5] which is the logaritmic form of the Friis equation

[6] note that here A_x stands for an attenuation $= \frac{1}{Gain}$ and G stands for a Gain

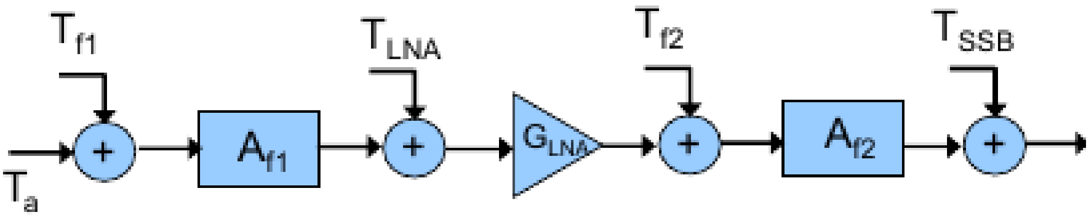


Figure 1.3: General structure of a noisy receiving chain

be very usefull beacuse using this model the Signal-to-Noise Ratio evalutation turns out to be extremly simple, while we can just compare the receiving power with the overall noise directly at the input port.

To get an $T_{equivalent}$ we have to refer at the input any additional noise by dividing it by the Gain from to input to the point where the noise is injected.

At the end for the example in figure we get:

$$T_{sys} = T_{eq} = T_a + T_{f1} + T_{LNA}A_{f1} + T_{f2}\frac{A_{f1}}{G_{LNA}} + T_{SSB}\frac{A_{f1}A_{f2}}{G_{LNA}} \tag{1.27}$$

where:

- T_{f1} is the noise introduced by the first attenuator
- T_{LNA} is the noise introduced by the LNA
- T_{f2} is the noise introduced by the second attenuator
- T_{SSB} is the noise introduced by the mizer

With the same concept we may find the *Output Referred Noise* T_out by simply applyng the relative transfer function at each noise to the ouput:

$$T_out = T_a\frac{G_{LNA}}{A_{f1}A_{f2}} + T_{f1}\frac{G_{LNA}}{A_{f1}A_{f2}} + T_{LNA}\frac{G_{LNA}}{A_{f2}} + T_{f2}\frac{1}{A_{f2}} + T_{SSB} \tag{1.28}$$

and to get T_{eq} we may divide T_{out} by the noiseless gain $\frac{G_{LNA}}{A_{f1}A_{f2}}$

$$T_{eq} = \frac{T_out}{\frac{G_{LNA}}{A_{f1}A_{f2}}} \tag{1.29}$$

after some algebra it's clear that the two different approaches are absolutely equivalent.

1.3.4 Noise Figure

The Noise Figure is very usefull as it gives an immediate evaluation of the overall degration that a system gives to the SNR.

Noise Figure is usually calculate diving the overall noise of the system by the noise given by the source; this operation is usually done at the output but can be done in every point of the system.

1.3.5 Noise figue and equivalent temperature relationship

$$T_{eq} = T_0(10^{\frac{NF_{eq}}{10}} - 1) \tag{1.30}$$

where T_0 it's usually the room temperature considered $290°C$.

1.3.6 Datarate Limitation

Given the modulation and demodulation scheme[7], the bandwidth (B) and the minimum SNR required the maximum possible datarate (R) is limited by noise.

Known the equivalent noise temperature ($T_{eq} = T_{sys}$) e can write:

$$SNR = \frac{P_r}{KT_{sys}B} = \left(\frac{E_b}{N_0}\right)\left(\frac{R}{B}\right) \tag{1.31}$$

[7]Given the mod/demod scheme the energy transmitter per bit is fixed

Chapter 2

Receivers

2.1 Image

The problem of the image rises during the domodulation process when the RF signal is downconverted to an Intermediate Frequncy (IF) because of the mixer by means of the LO downconverts to IF both the RF signal and the Image which any signal at the same opposite distance[1] from the LO with respect to RF signal.

Example

Let's report an example for clarification[2]:

- $f_{RF} = 12Ghz$
- $f_{IF} = 140Mhz$

we know that the local oscillator frequency (fLO) is above the signal band.
Find the frequncy of the image is quite simple:

$$f_{IF} = |f_{RF} - f_{LO}| \tag{2.1}$$

from this we are able to find f_{LO} and then:

$$f_{IM} = f_{LO} - f_{IF} = f_{RF} - 2f_{IF} \tag{2.2}$$

2.2 Cascaded Noise Figure

The evalutation of the Noise figure of cascaded stage could be performed with this formula:

$$NF_{eq} = NF_1 + \frac{NF_2 - 1}{G_1} + \frac{NF_3 - 1}{G_1 G_2} \tag{2.3}$$

[1] Note that this deistance is by definition IF
[2] 1^{st} February 2017 exercise 1 question a)

2.3 Cascaded Stage IIP3

$$\left(\frac{1}{IIP3}\right)^2 = \left(\frac{1}{IIP3_1}\right)^2 + \left(\frac{G_1}{IIP3_2}\right)^2 + \left(\frac{G_1 G_2}{IIP3_3}\right)^2 + ... \left(\frac{G_1 G_2 ... G_{n-1}}{IIP3_n}\right)^2 \tag{2.4}$$