Politecnico di Milano

Facoltà di ingegneria
Dipartimento di Elettronica e Informazione

RF Systems

Notes on Exercises
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Professore:
Giuseppe Macchiarella

Author: *Edoardo Contini*
Editor: *Francesco Lamarca*

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Chapter 1

Antennas

1.1 Antennas Parameters

1.1.1 Directive Gain

The power radiated by an antenna depends on the direction. The *directive gain* toward a given direction is expressed as:

$$D(\theta, \phi) = \frac{Radiation \;\; Intensity}{Isotropic \;\; Intensity} = \frac{U(\theta, \phi)}{P_{rad}/4\pi} \tag{1.1}$$

1.1.2 Directivity function and D_M definition

There is a direction $(\theta_{MAX}, \phi_{MAX})$ where D is maximum. The directivity function is the function D normalized to D_M:

$$f(\theta, \phi) = \frac{D(\theta, \phi)}{D_M} \quad \Longrightarrow S_R(R, \theta, \phi) = \frac{P_{rad}}{4\pi R^2} D_M f(\theta, \phi) \tag{1.2}$$

D_M represents the ratio between the radiated power density in the direction where it is maximum divided by the power density obtained with an isotropic radiator:

$$D_M = \frac{4\pi}{\int_0^{2\pi} \int_0^{2\pi} f(\theta, \phi) sin(\phi) d\theta d\phi} \tag{1.3}$$

1.1.3 Radiated Power Density

It is usefull to evaluate the radiated power density as it can be used for an analysis of propagation of power towards the threedimensional space.

$$S_R = \frac{dP_{rad}}{ds} = \frac{1}{2} Re(E \times H^*) = \frac{1}{R^2} U(\theta, \phi) \tag{1.4}$$

Isotropic Radiation

In case of isotropic radiation the radiation intensity $U(\theta, \phi) = \frac{P_{rad}}{4\pi}$

Non Isotropic Radiation

In non isotropic cases we get that the radiotion intensity is:

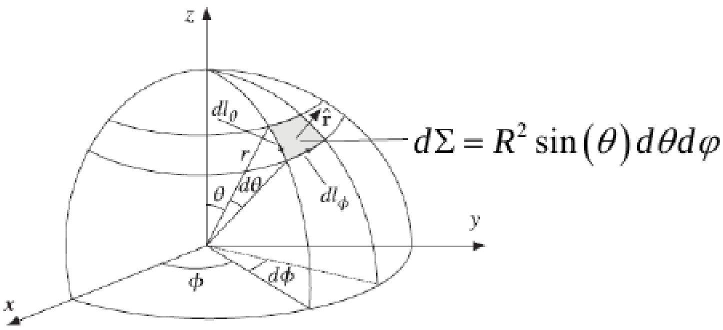
$$U(\theta, \phi) = \frac{P_{rad}}{4\pi} D(\theta, \phi) \tag{1.5}$$

The radiated power intensity results:

$$S_R(R, \theta, \phi) = \frac{P_{rad}}{4\pi R^2} D(\theta, \phi) \tag{1.6}$$

From the formula of the power density at distance R we can derive the net radiated power:

$$P_{rad} = \iint_\Sigma S_R(R, \theta, \phi)\, d\Sigma = \frac{P_{rad} D_{MAX}}{4\pi R^2} \iint_\Sigma f(\theta, \phi)\, d\Sigma \tag{1.7}$$



$$P_{rad} = \frac{P_{rad} D_{MAX}}{4\pi} \int_0^\pi \int_0^{2\pi} \sin(\theta) f(\theta, \phi) d\theta d\phi \tag{1.8}$$

from which:

$$\frac{1}{D_{MAX}} = \frac{\eta}{G} = \frac{1}{4\pi} \int_0^\pi \int_0^{2\pi} \sin(\theta) f(\theta, \phi) d\theta d\phi \tag{1.9}$$

and :

$$G = \frac{4\pi\eta}{\int_0^\pi \int_0^{2\pi} \sin(\theta) f(\theta, \phi) d\theta d\phi} \tag{1.10}$$

1.1.4 Fields Intensity

The radiated Power can be also related to the intensity of the electromagnetic fields:

$$S_R = \frac{1}{2} \frac{\sqrt{\epsilon_r}}{Z_0} |E|^2 = \frac{1}{2} \frac{Z_0}{\sqrt{\epsilon_r}} |H|^2 \tag{1.11}$$

The power density from a transmitting antenna (PT , G) at distance R along the direction of maximum radiation is given by:

$$S_R = \frac{P_T G}{4\pi R^2} \tag{1.12}$$

Then:

$$|E| = \frac{1}{R}\sqrt{\frac{Z_0 P_{ERP}}{2\pi\sqrt{\epsilon_r}}} \quad [V/m] \tag{1.13}$$

$$|H| = \frac{1}{R}\sqrt{\frac{\sqrt{\epsilon_r} P_{ERP}}{2\pi Z_0}} \quad [A/m] \tag{1.14}$$

1.1.5 Efficiency of Antennas

The impedance “seen” by the transmitter is determined by 2 contributes: Z_R [1] and R_p [2]. The power dissipated by Z_R constitutes the radiated power[3].

Assuming the reader aware of maximum power transfer theorem, neglecting the losses resistances because as stated before negligible in respect of the radiation impedence, we will assume impedence matching.

To understand how much of the electrical power is actually translated in radiated power the antenna efficiency can be defined as:

$$\eta = \frac{P_{rad}}{P_T} = \frac{Re\{Z_R\}}{Re\{Z_R\} + R_p} \tag{1.15}$$

1.1.6 Effective Radiated Power

The effective radiated power if simply defined as:

$$P_{ERP} = P_T G \tag{1.16}$$

1.1.7 Gain of Antennas

Defined as:

$$G = \eta D_M = \frac{2\eta}{1 - cos(\frac{\phi}{2})} \tag{1.17}$$

the gain is one of the fundamental paraments that describes an antenna.

1.1.8 Received Power

Given the power density (S_R)[4] and the direction of arrival (θ) of an incident radiation the received power can be calculated as:

$$P_r = A_e S_R f(\theta) \tag{1.18}$$

where:

[1] radiation impedance
[2] loss resistance
[3] Typically $R_p \ll |Z_R|$
[4] magnitude of the Poynting vector

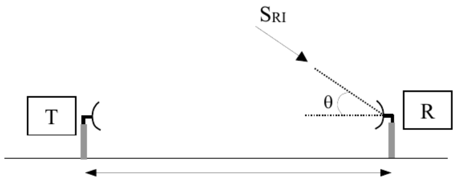


Figure 1.1: Simplified scheme of a Radio Link with an additiona power density from an non optimal direction

- A_e is the **effective Area** of the anttenna
- $f(\theta)$ is the directivity function

1.1.9 Effective Area

$$\frac{G}{A_e} = \frac{4\pi}{\lambda^2} \tag{1.19}$$

1.1.10 Equivalent length

The equivalent length can be related to the effective area using the following formula:

1.2 Dish Antennas

1.2.1 Gain

$$G = A_e \frac{4\pi}{\lambda^2} \tag{1.20}$$

1.2.2 Effective Area

$$A_e = e_a \frac{1}{4} \pi d^2 \tag{1.21}$$

where:

- e_a is the aperture efficiency (0.55 - 0.65)
- d = Dish diameter

1.2.3 Beamwidth

$$cos(\phi_B) = 1 - \frac{2}{D_{MAX}} = 1 - \frac{2\eta}{\pi^2 e_a^2} \left(\frac{\lambda}{d}\right)^2 \tag{1.22}$$

1.2.4 Free-space Attenuation

While traveling in the atmosphere, fields are subject to non negligible attenuations, which obviously increases with disance and can be avaluated as:

$$A_{dB} = 20log_{10}\left(\frac{4\pi L}{\lambda}\right) \tag{1.23}$$

Doing this calculation we should be carefull to use the wavelenght of the received power.

1.2.5 Radar

Let's analyse a Radar setup in which we have a target with $\sigma = 1m^2$ of reflective surface and a Dish antenna as RX-TX device. We may add an additional term to the **Friis equation**:

$$P_r = P_t - 20log_{10}\left(\frac{4\pi L}{\lambda}\right) + G_t + G_r + 10log_{10}\left(\frac{4\pi\sigma}{\lambda^2}\right) \tag{1.24}$$

1.3 Radio Link

1.3.1 Link Budget

Figure 1.2: Simplified scheme of a Radio Link

Supposing to have a Radio Link as in Figure 3.3 between 2 Antennas at a certain relative distance R the link budget may be evaluated by means of the **Friis equation** which at the end leds to a simple analitical expression for the receiving power[5]:

$$P_{r|_{dB}} = P_{t|_{dB}} - 10\log frac\lambda 4\pi R + G_{t|_{dB}} + G_{r|_{dB}} \tag{1.25}$$

in which:

- $P_{r|_{dB}}$ is the effective power received by the antenna.
- $P_{t|_{dB}}$ is the overall transmitted power of the TX side.
- $G_{r|_{dB}}$ is the gain of the receiving antenna.
- $G_{t|_{dB}}$ is the gain of the transmitting antenna.

while the term $20\log\frac{\lambda}{4\pi R}$ stands for the free space losses. Note from a practical point of view there as at least a couple of DoF in the design flow to obtain a certain receiving power.

We may also report the linear form of the **Friis equation**:

$$P_r = P_t\left(\frac{\lambda}{4\pi L}^2\right)G_T G_R \tag{1.26}$$

1.3.2 Attenuators' Noise

In any RF receiver we are going to have at least some losses between the antenna and the LNA. Sometimes also in other sections of the receiver path attenuation couldn't be avoided. We may evaluate the additional noise introduced my an attenuator A_{f1} for example the one in Figure 1.3 as:

[5]which is the logaritmic form of the Friis equation

$$T_{f1} = T_0(10^{\frac{A_{f1}}{10}} - 1) \tag{1.27}$$

1.3.3 Equivalent Temperature

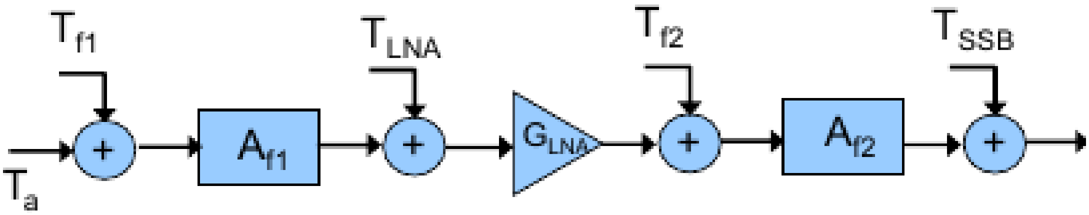


Figure 1.3: General structure of a noisy receiving chain

Supposing to have a receiving chain as in Figure[6] 1.3 we may want to rappresent the system with noiseless components and an input referred noise considering all contribution. This could be very usefull beacuse using this model the Signal-to-Noise Ratio evalutation turns out to be extremly simple, while we can just compare the receiving power with the overall noise directly at the input port.

To get an $T_{equivalent}$ we have to refer at the input any additional noise by dividing it by the Gain from to input to the point where the noise is injected.

At the end for the example in figure we get:

$$T_{sys} = T_{eq} = T_a + T_{f1} + T_{LNA}A_{f1} + T_{f2}\frac{A_{f1}}{G_{LNA}} + T_{SSB}\frac{A_{f1}A_{f2}}{G_{LNA}} \tag{1.28}$$

where:

- T_{f1} is the noise introduced by the first attenuator
- T_{LNA} is the noise introduced by the LNA
- T_{f2} is the noise introduced by the second attenuator
- T_{SSB} is the noise introduced by the mizer

With the same concept we may find the *Output Referred Noise* T_out by simply applyng the relative transfer function at each noise to the ouput:

$$T_out = T_a\frac{G_{LNA}}{A_{f1}A_{f2}} + T_{f1}\frac{G_{LNA}}{A_{f1}A_{f2}} + T_{LNA}\frac{G_{LNA}}{A_{f2}} + T_{f2}\frac{1}{A_{f2}} + T_{SSB} \tag{1.29}$$

and to get T_{eq} we may divide T_{out} by the noiseless gain $\frac{G_{LNA}}{A_{f1}A_{f2}}$

$$T_{eq} = \frac{T_out}{\frac{G_{LNA}}{A_{f1}A_{f2}}} \tag{1.30}$$

after some algebra it's clear that the two different approaches are absolutely equivalent.

[6]note that here A_x stands for an attenuation $= \frac{1}{Gain}$ and G stands for a Gain

1.3.4 Noise Figure

The Noise Figure is very usefull as it gives an immediate evaluation of the overall degration that a system gives to the SNR.

Noise Figure is usually calculate diving the overall noise of the system by the noise given by the source; this operation is usually done at the output but can be done in every point of the system.

1.3.5 Noise figue and equivalent temperature relationship

$$T_{eq} = T_0(10^{\frac{NF_{eq}}{10}} - 1) \tag{1.31}$$

where T_0 it's usually the room temperature considered $290°C$.

1.3.6 Datarate Limitation

Given the modulation and demodulation scheme[7], the bandwidth (B) and the minimum SNR required the maximum possible datarate (R) is limited by noise.

Known the equivalent noise temperature ($T_{eq} = T_{sys}$) e can write:

$$SNR = \frac{P_r}{KT_{sys}B} = \left(\frac{E_b}{N_0}\right)\left(\frac{R}{B}\right) \tag{1.32}$$

1.3.7 N-QAM Bandwidth specification

Let's remember that in a N-QAM modulation and demodulation scheme the required bandwidth to obtain a certain datarate decreses as N increses. Supposing to have Nyquist shaped pulses which a certain roll-off factor α we are able to evaluate the required bandwidth as:

$$B = \frac{R(1+\alpha)}{log_2(N_{QAM})} \tag{1.33}$$

[7] Given the mod/demod scheme the energy transmitter per bit is fixed

Chapter 2

Receivers

2.1 Image

The problem of the image rises during the domodulation process when the RF signal is downconverted to an Intermediate Frequncy (IF) because of the mixer by means of the LO downconverts to IF both the RF signal and the Image which any signal at the same opposite distance[1] from the LO with respect to RF signal.

Example

Let's report an example for clarification[2]:

- $f_{RF} = 12Ghz$
- $f_{IF} = 140Mhz$

we know that the local oscillator frequency (fLO) is above the signal band.
Find the frequncy of the image is quite simple:

$$f_{IF} = |f_{RF} - f_{LO}| \tag{2.1}$$

from this we are able to find f_{LO} and then:

$$f_{IM} = f_{LO} - f_{IF} = f_{RF} - 2f_{IF} \tag{2.2}$$

2.2 Cascaded Noise Figure

The evalutation of the Noise figure of cascaded stage could be performed with this formula:

$$NF_{eq} = NF_1 + \frac{NF_2 - 1}{G_1} + \frac{NF_3 - 1}{G_1 G_2} \tag{2.3}$$

[1] Note that this deistance is by definition IF
[2] 1^{st} February 2017 exercise 1 question a)

Chapter 3

Non linearities

3.1 Multitone signals

Usually we deal with multitone signal as:

$$V = Acos(\omega_1 t) + Acos(\omega_2 t) + Acos(\omega_3 t) + ... \tag{3.1}$$

Supposing P_{t_n} as the power of one single tone which is proportional to A^2 we can define:

3.1.1 Average Power

$$P_a = NkA^2 = NP_t \tag{3.2}$$

3.1.2 Peak Power

The peak power in obtained when all the components reach their maximum peak leading to:

$$P_p = k(NA)^2 = N^2 P_t \tag{3.3}$$

3.1.3 Peak Factor

$$F = \frac{P_p}{P_a} \tag{3.4}$$

Note that if every tone has the same amplitude then $F = N$

3.2 Effects of distortion. Two-tone test and IIP3.

When two signals (ω_1, ω_2) at different frequencies are applied to a nonlinear system, the output exhibits components that are not harmonics of these frequencies, but are instead located at characteristic frequencies given by:

$$\omega = 2\omega_1 \pm \omega_2 \tag{3.5}$$

$$\omega = 2\omega_2 \pm \omega_1 \tag{3.6}$$

These components may corrupt the signal if the latter's frequency matches (or falls within the band of) these components, which are called intermodulation (IM) products. The amplitude of these components is given by:

$$A_{IM} = \frac{3\alpha_3 A_1^2 A_2}{4} \tag{3.7}$$

where A_1, A_2 are the amplitudes of the two interferers and α_3 is the third-harmonics coefficient of the modulated output y(t). Note that if these amplitudes, and particularly the one of the IM product matching the signal, becomes comparable with the latter, corruption becomes ineliminable even with the use of extremely sharp filters, since it falls in the bandwidth of interest. In order to measure intermodulation, a figure of merit is introduced named IIP3 (Input Third Intercept Point), generally measured by means of a "two-tone" test: two pure sinusoids[1] of equal amplitude are applied to the input, and the amplitude of the IM products at the output is measured together with the amplitude of the fundamental component:

$$A_{y,I} = \alpha_1 A \tag{3.8}$$

$$A_{y,IMP} = \frac{3}{4}\alpha_3 A^3 \tag{3.9}$$

It can be easily seen that, by plotting the output amplitudes on a log-log scale, the fundamental will show a linear behavior with a +20dB/dec slope, whereas the IM products show the same linear behavior, but with a +60dB/dec slope; the point at which the two lines cross is the IIP3, which can thus be derived by equating the amplitudes of the two measurements:

$$A_{y,I}(IIP3) = A_{y,IMP}(IIP3) \tag{3.10}$$

$$|\alpha_1 A_{IIP3}| = |\frac{3}{4}\alpha_3 A_{IIP3}^3| \tag{3.11}$$

$$A_{IIP3} = \sqrt{\frac{4}{3}|\frac{\alpha_1}{\alpha_3}|} \tag{3.12}$$

Note that this is actually an estimation: due to gain compression for higher amplitudes of the input tones, the crossing point between the fundamental's amplitude and the IM3 amplitude at the output shall lie at lower values of A_{in}, due to the reduced amplitude of the fundamental component; moreover, higher-order nonlinearities may manifest as A_{in} approaches A_{IIP3}, further complicating the picture. An alternative way to measure IIP3, therefore, consists in using an extremely low input level, so that the gain is not compressed and higher order nonlinearities are negligible: by then increasing A_{in} and plotting the resulting amplitudes, we can extrapolate the aforementioned linear plot and determine IIP3 more accurately. However, extrapolation can be a long and tedious process, so a shortcut providing a reasonable initial estimate is employed. Considering to have an input equal to A_{IIP3}, then the extrapolated output IM products are as ample as the fundamental tone. By reducing the input to a given level $A_{in,1}$, then the change of the input is given by:

$$\Delta P = 20\log(A_{IIP3}) - 20\log(A_{in,1}) \tag{3.13}$$

Since the output amplitude of the fundamental has a slope of +20dB/dec, whereas the amplitude of the IM3 component has a slope of +60dB/dec, the difference between the two plots has a slope of -40dB/dec. This means that by changing the input amplitude by a quantity ΔP, the difference between the two slopes changes by 2ΔP. We can therefore write:

$$\Delta P = 20\log(A_{IIP3}) - 20\log(A_{in,1}) = \frac{1}{2}(20\log(A_f) - 20\log(A_{IM})) \tag{3.14}$$

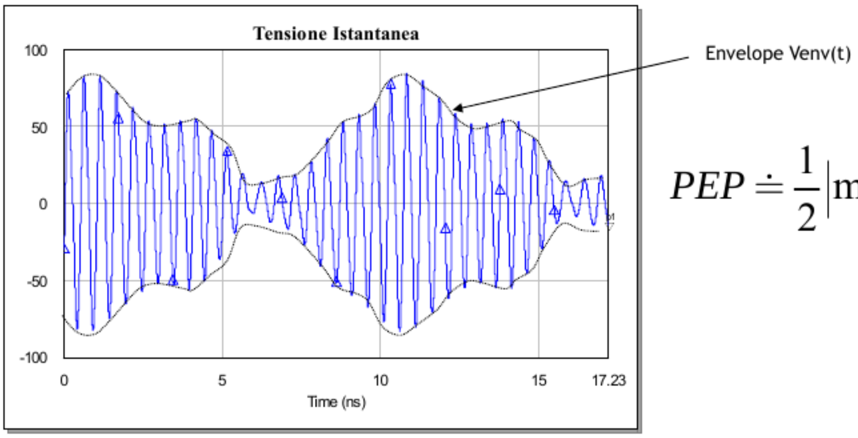
[1]A pure sinusoid at a given frequency is generally called "tone".

where A$_f$, A$_{IM}$ are the amplitudes of the fundamental component and of the IM products respectively, when the input signal has amplitude A$_{in,1}$. Therefore it is easy to recover:

$$20\log(A_{IIP3}) = \frac{\Delta P}{2} + 20\log(A_{in,1}) \tag{3.15}$$

3.3 Peak Envelope Power (PEP)

PEP it's the instantaneous power delivered at the peak of the modulated signal:



$$PEP \doteq \frac{1}{2}\left|\max\left(V_{\text{env}}(t)\right)\right|^2$$

3.4 1dB Compression Point

Given a Polinomial model of an non linear memoryless system:

$$v_o = a_0 + a_1 v_i + a_2 v_i^2 + a_3 v_i^3 + ... \tag{3.16}$$

and cosidering a 1-tone excitation $v_i = Acos(\omega_t)$ we get at the output:

$$v_o = b_0 + b_1 cos(\omega_t) + b_2 cos(2\omega_t) + b_3 cos(3\omega_t) + ... \tag{3.17}$$

with:

- $b_0 = \frac{1}{2} a_2 A^2$
- $b_1 = a_1 A + \frac{3}{4} a_3 A^3$
- $b_2 = \frac{1}{2} a_2 A^2$
- $b_3 = \frac{1}{4} a_3 A^3$

where b_1 rappresents the output of the signal at ω and we can notice that a non linear term is present $\frac{3}{4} a_3 A^3$. In case of amplifiers a_3 is usually negative this means that as we increase the input signal amplitude as the non linear negative increase in respect to the linear term thus giving gain compression.

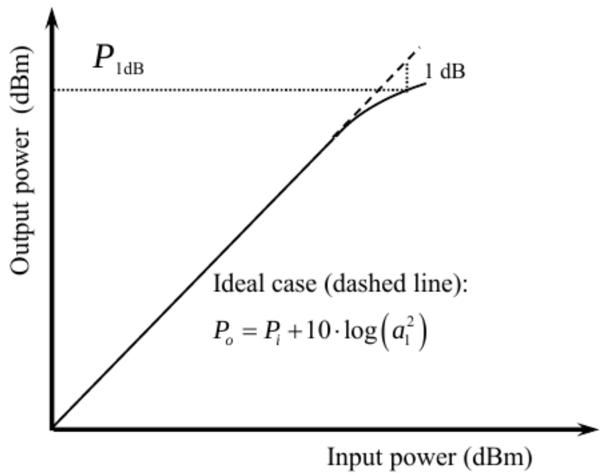


Figure 3.1: Graphical rappresentation of 1dB compression point

At P_{1dB} the output power is reduced of 1 dB with respect to the one given by the linear term:

$$P_{1dB} = 10log_{10}\left(\frac{a_1^3}{|a_3|}\right) + 0.62dBm \tag{3.18}$$

3.5 Third order Intercept Point

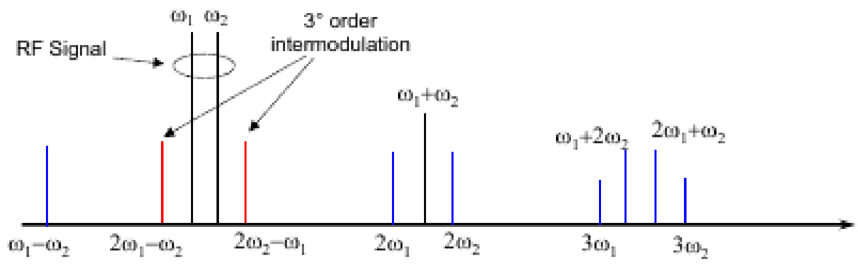


Figure 3.2: Third order intermodulation products in output spectrum

In Figure is rappresented a two-tone test and it's output highlighting the intermodulation product. A two-tone signal is the simplest rappresentation of an RF signal with a certain bandwidth.

We note as the intermodulation products at $(2\omega_1 - \omega_2)$ and $(2\omega_2 - \omega_1)$ are very close to the signal and considering for example phase noise of the LO it's clear that they can bring to SNR degradation.

From Figure we can derive:

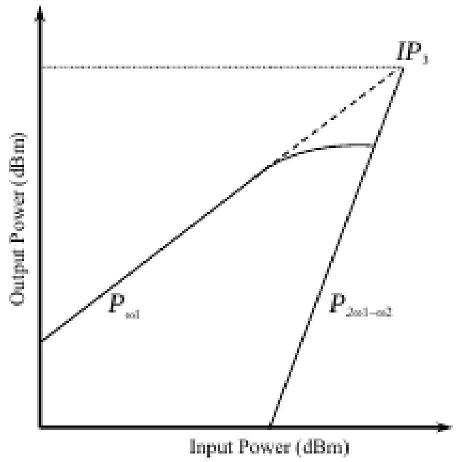


Figure 3.3: Third order intercept point graphical rappresentation

$$P_{\omega_1} = 10log\left(\left(\frac{a_1A + (9/4)a_3A^3}{\sqrt{2}}\right)^2 \frac{1000}{R}\right) \approx 10log\left(\left(\frac{a_1A}{\sqrt{2}}\right)^2 \frac{1000}{R}\right) \tag{3.19}$$

$$P_{2\omega_1-\omega_2} = 10log\left(\left(\frac{(3/4)a_3A^3}{\sqrt{2}}\right)^2 \frac{1000}{R}\right) \tag{3.20}$$

from which we can find IP_3:

$$IP_3 = 10log\left(\left(\frac{2}{3}\frac{a_1^3}{|a_3|}\right) \frac{1000}{R}\right) \tag{3.21}$$

$$IP_3 = 10log\frac{a_1^3}{|a_3|} + 11.25dBm \qquad (R = 50\Omega) \tag{3.22}$$

3.5.1 P_{1dB} and IP_3 Relationship

From IP_3 and P_{1dB} definition:

$$\begin{cases} IP_3 = 10log_{10}\left(\frac{a_1^3}{|a_3|}\right) + 11.25dBm \\ P_{1dB} = 10log_{10}\left(\frac{a_1^3}{|a_3|}\right) + 0.62dBm \end{cases}$$

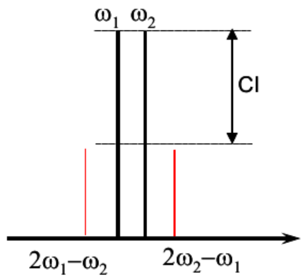
we get:

$$IP_3 = P_{1dB} + \Delta_p \qquad (\Delta_p = 10.63dB) \tag{3.23}$$

3.5.2 Mean Power

$Pm = (P_{\omega_1} + 3)$

3.5.3 Carrier to Intermodulation ratio (CI)



$$CI_{|dB} \approx 2IP_3 - 2P_m + 6 = 2(IP_3 - P_0) \qquad considering P_{\omega_1} \ll IP_3 \tag{3.24}$$

$Pm = (P_{\omega_1} + 3)$ the average power of the two-tone signal.

3.6 Power amplifiers

3.6.1 Output Power and non linearities

When assigning the output power of a PA, also the corresponding level of nonlinear distortion it must be specified. The 2-tone signal is often using as reference excitation. In this case it is usual to specify the maximum PEP power for which the CI is equal to 30dB:

$$PEP = P_m + 3 \qquad (2\ tones) \tag{3.25}$$

$$30 = 2IP_3 - 2(PEP - 3) + 6 \quad \Longrightarrow \quad PEP = IP_3 - 9 \tag{3.26}$$

from $IP_3 = P_{1dB} + \Delta_p$ we get:

$$PEP = P_{1dB} + (\Delta_p - 9) \qquad (for\ CI = 30dB) \tag{3.27}$$

3.6.2 Back-Off

The backoff of a PA is the difference in dB between the output power at P_{1dB} and the average output power. From the previous equations, it can be obtained:

$$BO \approx \frac{CI_{|dB}}{2} - \Delta_p - 3 \qquad (2\ tones) \tag{3.28}$$

3.6.3 Power Added Efficiency (PAE)

PAE in an important FoM for the ability of a PA to convert DC power to RF power:

$$PAE = 100 \frac{(RF\ Power\ at\ Load) - (RF\ Power\ at\ Input)}{DC\ power} \% \tag{3.29}$$

In the design process we must take into account that to increase linearity PAE should be decreased.

3.7 Cascaded Stages

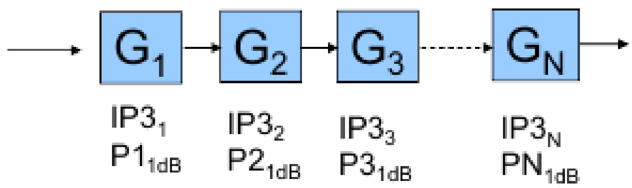


Figure 3.4: Example of a cascaded stage with third order non linearities.

3.7.1 Cascaded Stage IIP3

$$\left(\frac{1}{IIP3}\right)^2 = \left(\frac{1}{IIP3_1}\right)^2 + \left(\frac{G_1}{IIP3_2}\right)^2 + \left(\frac{G_1 G_2}{IIP3_3}\right)^2 + ... \left(\frac{G_1 G_2 ... G_{n-1}}{IIP3_n}\right)^2 \tag{3.30}$$

3.7.2 Cascaded 1dB Compression Point

$$\left(\frac{1}{P_{1dB}}\right)^2 = \left(\frac{1}{P_{1_{1dB}}}\right)^2 + \left(\frac{G_1}{P_{2_{1dB}}}\right)^2 + \left(\frac{G_1 G_2}{P_{3_{1dB}}}\right)^2 + ... \left(\frac{G_1 G_2 ... G_{n-1}}{P_{N_{1dB}}}\right)^2 \tag{3.31}$$