

POLITECNICO DI MILANO



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DIPARTIMENTO DI ELETTRONICA E INFORMAZIONE

RF Systems

NOTES ON EXERCISES
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Chapter 1

Antennas

1.1 Antennas Parameters

1.1.1 Directive Gain

The power radiated by an antenna depends on the direction. The *directive gain* toward a given direction is expressed as:

$$D(\theta, \phi) = \frac{\text{Radiation Intensity}}{\text{Isotropic Intensity}} = \frac{U(\theta, \phi)}{P_{rad}/4\pi} \quad (1.1)$$

1.1.2 Directivity function and D_M definition

There is a direction $(\theta_{MAX}, \phi_{MAX})$ where D is maximum. The directivity function is the function D normalized to D_M :

$$f(\theta, \phi) = \frac{D(\theta, \phi)}{D_M} \implies S_R(R, \theta, \phi) = \frac{P_{rad}}{4\pi R^2} D_M f(\theta, \phi) \quad (1.2)$$

D_M represents the ratio between the radiated power density in the direction where it is maximum divided by the power density obtained with an isotropic radiator:

$$D_M = \int_0^{2\pi} \int_0^{2\pi} f(\theta, \phi) \sin(\phi) d\theta d\phi \quad (1.3)$$

1.1.3 Radiated Power Density

It is usefull to evaluate the radiated power density as it can be used for an analysis of propagation of power towards the threedimensional space.

$$S_R = \frac{dP_{rad}}{ds} = \frac{1}{2} \text{Re}(E \times H^*) = \frac{1}{R^2} U(\theta, \phi) \quad (1.4)$$

Isotropic Radiation

In case of isotropic radiation the radiation intensity $U(\theta, \phi) = \frac{P_{rad}}{4\pi}$

Non Isotropic Radiation

In non isotropic cases we get that the radiation intensity is:

$$U(\theta, \phi) = \frac{P_{rad}}{4\pi} D(\theta, \phi) \quad (1.5)$$

The radiated power intensity results:

$$S_R(R, \theta, \phi) = \frac{P_{rad}}{4\pi R^2} D(\theta, \phi) \quad (1.6)$$

1.1.4 Fields Intensity

The radiated Power can be also related to the intensity of the electromagnetic fields:

$$S_R = \frac{1}{2} \frac{\sqrt{\epsilon_r}}{Z_0} |E|^2 = \frac{1}{2} \frac{Z_0}{\sqrt{\epsilon_r}} |H|^2 \quad (1.7)$$

The power density from a transmitting antenna (PT, G) at distance R along the direction of maximum radiation is given by:

$$S_R = \frac{P_T G}{4\pi R^2} \quad (1.8)$$

Then:

$$|E| = \frac{1}{R} \sqrt{\frac{Z_0 P_{ERP}}{2\pi \sqrt{\epsilon_r}}} \quad [V/m] \quad (1.9)$$

$$|H| = \frac{1}{R} \sqrt{\frac{\sqrt{\epsilon_r} P_{ERP}}{2\pi Z_0}} \quad [A/m] \quad (1.10)$$

1.1.5 Efficiency of Antennas

The impedance “seen” by the transmitter is determined by 2 contributors: Z_R ¹ and R_p ². The power dissipated by Z_R constitutes the radiated power³.

Assuming the reader aware of maximum power transfer theorem, neglecting the losses resistances because as stated before negligible in respect of the radiation impedance, we will assume impedance matching.

To understand how much of the electrical power is actually translated in radiated power the antenna efficiency can be defined as:

$$\eta = \frac{P_{rad}}{P_T} = \frac{Re\{Z_R\}}{Re\{Z_R\} + R_p} \quad (1.11)$$

1.1.6 Effective Radiated Power

The effective radiated power is simply defined as:

$$P_{ERP} = P_T G \quad (1.12)$$

¹radiation impedance

²loss resistance

³Typically $R_p \ll |Z_R|$

1.1.7 Gain of Antennas

Defined as:

$$G = \eta D_M \quad (1.13)$$

the gain is one of the fundamental parameters that describes an antenna.

1.1.8 Received Power

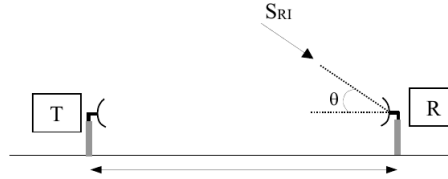


Figure 1.1: Simplified scheme of a Radio Link with an additional power density from a non optimal direction

Given the power density (S_R)⁴ and the direction of arrival (θ) of an incident radiation the received power can be calculated as:

$$P_r = A_e S_R f(\theta) \quad (1.14)$$

where:

- A_e is the **effective Area** of the antenna
- $f(\theta)$ is the directivity function

1.1.9 Effective Area

$$\frac{G}{A_e} = \frac{4\pi}{\lambda^2} \quad (1.15)$$

1.1.10 Equivalent length

The equivalent length can be related to the effective area using the following formula:

1.2 Radio Link

1.2.1 Link Budget

Supposing to have a Radio Link as in Figure 1.2 between 2 Antennas at a certain relative distance R the link budget may be evaluated by means of the **Friis equation** which at the end leads to a simple analytical expression for the receiving power⁵:

$$P_{r|dB} = P_{t|dB} - 10 \log \frac{4\pi R}{\lambda} + G_{t|dB} + G_{r|dB} \quad (1.16)$$

in which:

⁴magnitude of the Poynting vector

⁵which is the logarithmic form of the Friis equation

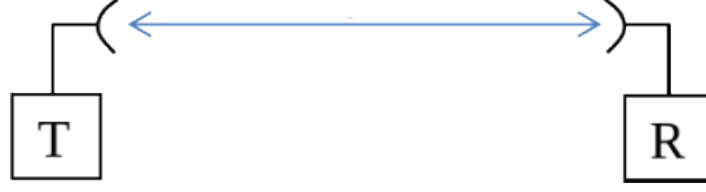


Figure 1.2: Simplified scheme of a Radio Link

- $P_{r|dB}$ is the effective power received by the antenna.
- $P_{t|dB}$ is the overall transmitted power of the TX side.
- $G_{r|dB}$ is the gain of the receiving antenna.
- $G_{t|dB}$ is the gain of the transmitting antenna.

while the term $20 \log \frac{\lambda}{4\pi R}$ stands for the free space losses. Note from a practical point of view there as at least a couple of DoF in the design flow to obtain a certain receiving power.

We may also report the linear form of the **Friis equation**:

$$P_r = P_t \left(\frac{\lambda}{4\pi L} \right)^2 G_T G_R \quad (1.17)$$

1.2.2 Attenuators' Noise

In any RF receiver we are going to have at least some losses between the antenna and the LNA. Sometimes also in other sections of the receiver path attenuation couldn't be avoided. We may evaluate the additional noise introduced by an attenuator A_{f1} for example the one in Figure 1.3 as:

$$T_{f1} = T_0 (10^{\frac{A_{f1}}{10}} - 1) \quad (1.18)$$

1.2.3 Equivalent Temperature

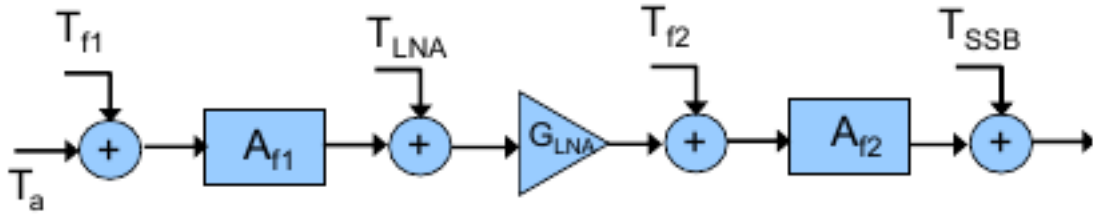


Figure 1.3: General structure of a noisy receiving chain

Supposing to have a receiving chain as in Figure⁶ 1.3 we may want to represent the system with noiseless components and an input referred noise considering all contribution. This could

⁶note that here A_x stands for an attenuation $= \frac{1}{Gain}$ and G stands for a Gain

be very usefull beacuse using this model the Signal-to-Noise Ratio evalutation turns out to be extremly simple, while we can just compare the receiving power with the overall noise directly at the input port.

To get an $T_{equivalent}$ we have to refer at the input any additional noise by dividing it by the Gain from to input to the point where the noise is injected.

At the end for the example in figure we get:

$$T_{sys} = T_{eq} = T_a + T_{f1} + T_{LNA}A_{f1} + T_{f2}\frac{A_{f1}}{G_{LNA}} + T_{SSB}\frac{A_{f1}A_{f2}}{G_{LNA}} \quad (1.19)$$

where:

- T_{f1} is the noise introduced by the first attenuator
- T_{LNA} is the noise introduced by the LNA
- T_{f2} is the noise introduced by the second attenuator
- T_{SSB} is the noise introduced by the mizer

With the same concept we may find the *Output Referred Noise* T_{out} by simply applyng the relative transfer function at each noise to the ouput:

$$T_{out} = T_a\frac{G_{LNA}}{A_{f1}A_{f2}} + T_{f1}\frac{G_{LNA}}{A_{f1}A_{f2}} + T_{LNA}\frac{G_{LNA}}{A_{f2}} + T_{f2}\frac{1}{A_{f2}} + T_{SSB} \quad (1.20)$$

and to get T_{eq} we may divide T_{out} by the noiseless gain $\frac{G_{LNA}}{A_{f1}A_{f2}}$

$$T_{eq} = \frac{T_{out}}{\frac{G_{LNA}}{A_{f1}A_{f2}}} \quad (1.21)$$

after some algebra it's clear that the two different approaches are absolutely equivalent.

1.2.4 Noise Figure

The Noise Figure is very usefull as it gives an immediate evaluation of the overall degration that a system gives to the SNR.

Noise Figure is usually calculate diving the overall noise of the system by the noise given by the source; this operation is usually done at the output but can be done in every point of the system.

1.2.5 Noise figue and equivalent temperature relationship

$$T_{eq} = T_0(10^{\frac{NF_{eq}}{10}} - 1) \quad (1.22)$$

where T_0 it's usually the room temperature considered $290^\circ C$.

1.2.6 Datarate Limitation

Given the modulation and demodulation scheme⁷, the bandwidth (B) and the minimum SNR required the maximum possible datarate (R) is limited by noise.

Known the equivalent noise temperature ($T_{eq} = T_{sys}$) e can write:

$$SNR = \frac{P_r}{KT_{sys}B} = \left(\frac{E_b}{N_0} \right) \left(\frac{R}{B} \right) \quad (1.23)$$

DRAFT

⁷Given the mod/demod scheme the energy transmitter per bit is fixed

Chapter 2

Receivers

2.1 Image

The problem of the image rises during the demodulation process when the RF signal is downconverted to an Intermediate Frequency (IF) because of the mixer by means of the LO downconverts to IF both the RF signal and the Image which any signal at the same opposite distance¹ from the LO with respect to RF signal.

Example

Let's report an example for clarification²:

- $f_{RF} = 12\text{GHz}$
- $f_{IF} = 140\text{MHz}$

we know that the local oscillator frequency (f_{LO}) is above the signal band.
Find the frequency of the image is quite simple:

$$f_{IF} = |f_{RF} - f_{LO}| \quad (2.1)$$

from this we are able to find f_{LO} and then:

$$f_{IM} = f_{LO} - f_{IF} = f_{RF} - 2f_{IF} \quad (2.2)$$

2.2 Cascaded Noise Figure

The evaluation of the Noise figure of cascaded stage could be performed with this formula:

$$NF_{eq} = NF_1 + \frac{NF_2 - 1}{G_1} + \frac{NF_3 - 1}{G_1 G_2} \quad (2.3)$$

¹Note that this distance is by definition IF

²1st February 2017 exercise 1 question a)