# Image Denosing by Two Types of Non-Local Filter

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### *Introduction*

Denoising is the most fundamental issue in image processing, and has been attracted a huge amount of work. Several methods have been proposed to obtain the original image from an observation with different types of noise, a typical approach of which is utilizing the local correlation among image pixels, such as Gaussian smoothing model, the Total Variation minimization and the empirical Wiener filters [1-7]. The similarity among earlier methods both in spatial domain and in frequency domain is averaging. However, details and edges thus are degraded by unnecessary over-smoothing. Especially when it comes to impulse noise, linear filters perform not very well in noise suppression and edge preservation at the same time. Since it is inevitable to prevent noise superimposition because of data transfer error or coding error in analog-to-digital conversion image process. The need for studies on impulse noise removal by image processing is still considerable.

A plenty of the denoising methodologies and strategies build a model for the noise and/or for the original signal in a suitable subspace where the differences between them are accentuated based on the following observations: (a) different behaviors between the noise and clean signal shown in multi-resolution representation, (b) significant geometrical components of an image (edge) or time structures of a signal (sharp transitions) over-exceed noise information, especially at low resolutions. Hence, a bunch of research has involved the use of the wavelet transform for denoising because of its multi-resolution and energy compaction properties. The motivation is that the small wavelet coefficients in high-frequency bands that are more likely due to noise are thresholded, leaving the large wavelet coefficients which are more likely due to signal features. The influential works on signal denoising via wavelet thresholding or shrinkage attribute to Donoho and Johnstone [13].

An alternative to the wavelet-based denoising methods is the bilateral filter (BF) introduced by Tomasi and Manduchi [14] which considers both the spatial and the intensity information between a point and its neighboring points unlike the conventional linear filtering where only spatial information is considered. This is because the application of BF removes noise as well as some image details by spatial filtering without loss of edge information (range filtering).

Non-local means (NLM), first introduced by Buades *et al.* in [8], replaces the considered pixel by the weighted mean of all the pixels in the whole image or the surrounding neighborhoods. As the patch size reduces to one pixel, the NL means filter becomes equivalent to the BF.This filter can preserve image details textures and edges better in that it depends on the global self-redundancy of the images and evaluate the similarity in two pixels through the Gaussian weighted Euclidean distance.

In this report, the Non-Local Mean filter is first explained followed by the improved Non-Local Euclidean Medians [10] and B. K. Shreyamsha Kumar’s method [12]. The effectiveness of these methods is verified by a series experiments comparing to the conventional methods.

### *Non-Local Mean Algorithm*

The goal of image denoising is to remove the noise while retaining the important image features like edges, textures as much as possible. Extended the neighborhood filters to a wider class which they called it as non-local means (NL means), the NL-Means algorithm is defined simply by the formula:

$$NL[u](x) = \frac{1}{C(x)} \int_{\Omega} e^{\frac{-(G_a * |u(x+.)-u(y+.)|^2)(0)}{h^2}} u(y)dy \quad (3)$$

where $x \in \Omega$, C(x)= $\int_{\Omega} e^{\frac{-(G_a * |u(x+.)-u(y+.)|^2)(0)}{h^2}} u(y)dz$ is a normalizing constant. $G_a$is a Gaussian kernel and h acts as a filtering parameter. This formula amounts to say that the denoised value at x is a mean of the values of all points whose Gaussian neighborhood looks like the neighborhood of x.

Given a noisy image $u = (u_i)$, the denoised image $\hat{u} = (\hat{u}_i)$ at pixel $i$ is computed using formula

$$\hat{u}_i = \frac{\sum_i w_{ij} u_j}{\sum_i w_{ij}}, \quad (1)$$

where $w_{ij}$ is some weight (affinity) assigned to pixels $i$ and $j$. The sum in (1) is ideally performed over the whole image. In practice, however, one restricts $j$ to a geometric neighborhood of $i$, to a sufficiently large window of size and $S \times$ S [8].

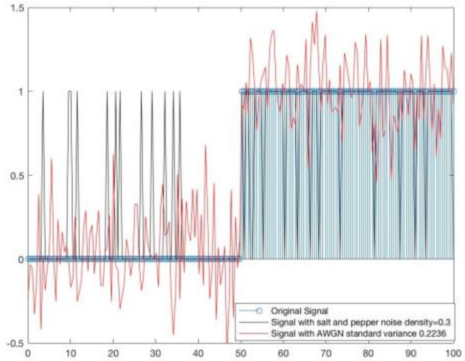
The central idea in [8] was to decide the weights using image patches centered around each pixel, namely as

$$w_{ij} = \exp\left( -\frac{1}{h^2} \left\| \boldsymbol{P}_i - \boldsymbol{P}_j \right\|^2 \right) \quad (2)$$

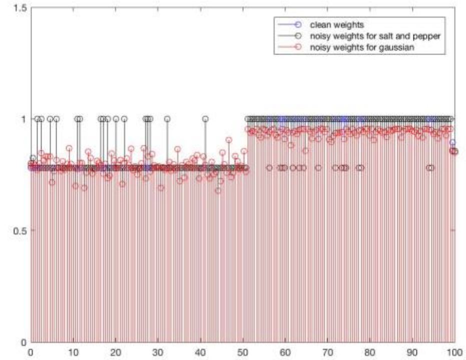
where $\boldsymbol{P}_i$ , $\boldsymbol{P}_j$ are the image patches of size $k \times k$ centered at pixels I and j. here, $\|\boldsymbol{P}\|$ is the Euclidean norm of patch **P** as a point in $\boldsymbol{R}^{k^2}$, and $h$ is a smoothing parameter. Pixels with similar neighborhoods are attributed larger weights while pixels whose neighborhoods look different gained less weights.

The main problem with the non-local processing is causing residual noise and artifacts for highly corrupted images. This problem is caused by improper calculation of the similarity between patches affected by too much noise.

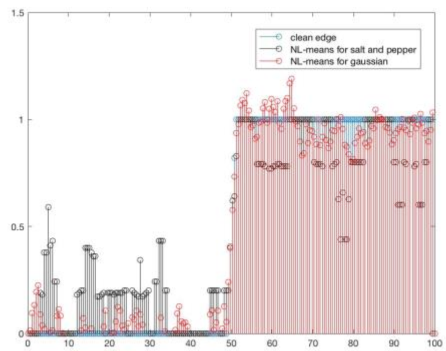
To explain such noise, consider below the particular case where the noise of an interested pixel is mixed with Gaussian noise and the salt and pepper noise. I choose a clean edge of unit height ranging from 1 to 100, including 200 samples. and add two types of noise (salt and pepper noise density=0.3, white Gaussian noise $\delta = 0.22$). Shown in Fig.1 (a). Select the reference pixel at location i = 150. The weights can be calculated by formula (2). The patches are 3-sampled (k = 3), search window is the whole edge (S = 200); Shown in Fig.1 (b). It can be clearly seen that, for Gaussian filter (red line), the weights are averaged. However, for salt and pepper noise(black line), some weights are wrongly calculated and reach extremely to 1 or 0. Further more, the Non-Local Means filtering is implemented as Fig.1 (c). Clearly, addictive noise is added expecially when noise type is salt and pepper.




(a)




(b)




(c)

Fig.1 (a)A clean edge and edge with salt and pepper noise(density = 0.3) and edge with Gaussian noise ($\delta = 0.22$) (b) weights (c)NL-means implements

Later K. N. Chaudhury and A. Singer *et al* proposed an improved non-local median filter weighting similarity by replacing the mean by the Euclidean median [10] to better search the original data. At the heart of non-local Euclidean medians filter(NLEM) is the observation that the median is more robust to outliers than the mean. However, this method use iteration which consumes a quite bit time.

In practice, suppose the noisy image is $u = (u_i)$ denoised image is $\hat{u} = (\hat{u}_i)$ .

(1) First choose the smoothing parameter $h$, pixel neighborhood size $k$, and the search window $S$.

(2) Second extract patch $P_i$ for every pixel i, the size of $P_i$ is decided by $k$.

(3) For every pixel i, first compute $w_{ij} = \exp\left(-\frac{1}{h^2}\left\|\boldsymbol{P}_i - \boldsymbol{P}_j\right\|^2\right)$ for $j \in S(i)$.

Then compute $\hat{u}_i = \frac{\sum_i w_{ij} u_j}{\sum_i w_{ij}}$.

### *Non-Local Euclidean Median Algorithm*

The NLEM difference with NLM is the minimizer in step 3. The Euclidean mean is the minimizer of $\sum_j w_j \left\|\boldsymbol{P}-\boldsymbol{P}_j\right\|^2$ over all patches $\boldsymbol{P}$. The Euclidean median is minimizer of $\sum_j w_j \left\|\boldsymbol{P}-\boldsymbol{P}_j\right\|$ over all $\boldsymbol{P}$. Given the points $\boldsymbol{x_1}\ \boldsymbol{x_2}\ \ldots\ \boldsymbol{x_n} \in \boldsymbol{R^d}$ and weights $w_1\ w_2\ \ldots\ w_n$, it is required to find $\boldsymbol{x} \in \boldsymbol{R^d}$ that minimizes the convex cost $\sum_{j=1}^{n} w_j \left\|\boldsymbol{x}-\boldsymbol{x_j}\right\|$. There exists a literature on the computation of the Euclidean median. That is, based on the method of iteratively reweighted least squares (IRLS), the pixel gained an updated value as below:

$$x^{k+1} = \arg\min_{x\in R^d} \sum_{j=1}^{n} w_j \frac{\left\|x-x_j\right\|^2}{\left\|x^{(k)}-x_j\right\|} \tag{4}$$

and the minimizer is given by

$$x^{k+1} = \frac{\sum_j \mu_j^{(k)} \boldsymbol{x_j}}{\sum_j \mu_j^{(k)}} \tag{5}$$

where $\mu_j^{(k)} = w_j / \left\|\boldsymbol{x}^{(k)}-\boldsymbol{x_j}\right\|$. Starting with an intial guess, one keeps repeating this process until convergence. In practice, one needs to address the situation when $\boldsymbol{x}^{(k)}$gets close to some $\boldsymbol{x_j}$, which causes $\mu_j^{(k)}$ to blow up.

(1) First choose the smoothing parameter *h*, pixel neighborhood size *k*, and the search window *S*.

(2) Second extract patch $P_i$ for every pixel i, the size of $P_i$ is decided by *k*.

(3) For every pixel i, do

- (a) Set $w_{ij} = \exp\left(-\frac{1}{h^2}\left\|\boldsymbol{P}_i - \boldsymbol{P}_j\right\|^2\right)$ for $j \in S(i)$.
- (b) Find patch $\boldsymbol{P}$ that minimizes $\sum_{j=1}^{n} w_j \left\|\boldsymbol{x}-\boldsymbol{x_j}\right\|$.
- (c) Assign $\hat{u}_i$ the value of the center pixel in $\boldsymbol{P}$**.**

### *Image denoising based on non-local means filter and its method noise thresholding*

B. K. Shreyamsha Kumar *et al* proposed a method that combines non-local means filter and its method noise thresholding using wavelets to optimize the wrongly calculated pixels in non-local mean filter.

The application of NL means filter on the noisy image removes the noise and cleans the edges without losing too many fine structures and details. Even though the NL means filter is very effective in removing the noise at high SNR (with less noise) but as the noise increases, its performance deteriorates. This is because the similar local patches which are used to find the pixel weights are also noisy. At low SNR, the NL means filter not only removes the noise but at the same time it blurs the image thereby removing much of the image details. Consequently, the method noise will contain noise as well as image details along with some edges.

The wavelet thresholding adds power to the proposed method as noise components can be eliminated better in detail sub-bands of method noise. The framework of this proposed method is as below the figure.

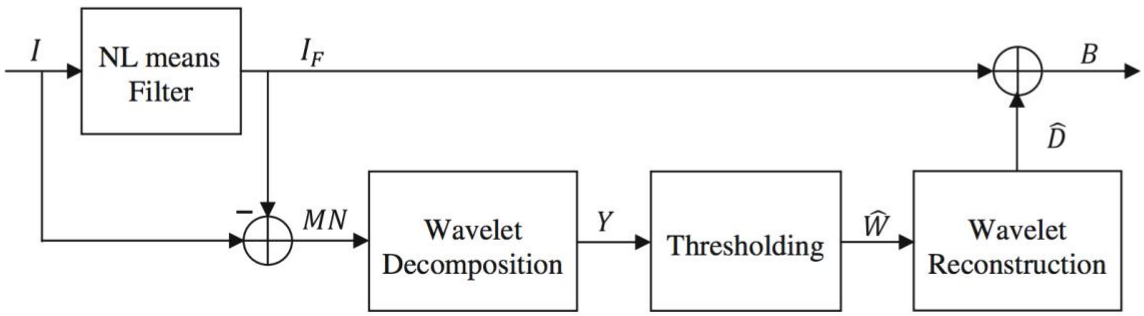



Fig.2 The framework of a method that combines non-local means filter and its method noise thresholding using wavelets

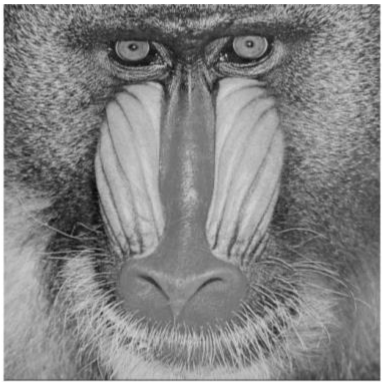
## *Experiments*

Now I present the results of some denoising experiments. For all experiments, the picture size is $512 \times 512$, $S$ = 21, $k$ = 7, $h = 5$. The input images are corrupted by a simulated Gaussian white noise with zero mean and five different standard deviations σ ∈ [10, 20, 30, 40, 50].

Experiments includes Gaussian filter, Median filter, Bilateral filter, Non-Local means filter, Non-local means with threshold. The original pictures (a) Lena (b) Barbara (c) Baboon (d) Cameraman are shown below.

(a)

(b)

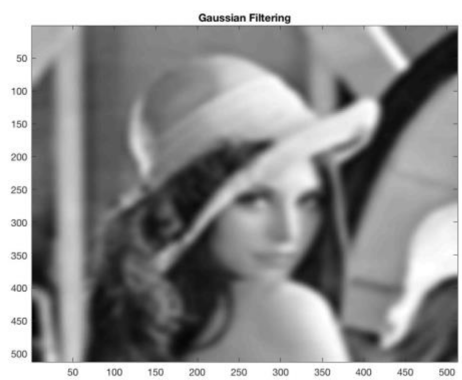


(c) (d)

Fig.3 original pictures (a) Lena (b) Barbara (c) Baboon (d) Cameraman

The image quality is measured by visual inspection as there is no generally accepted objective way to judge the image quality of a denoised image. There are two criteria that are used widely in the literature: (1) visibility of the artifacts and (2) preservation of edge details.

The denoised images of Lena with $\sigma = 10$ by different methods are shown in Fig.4


(a) (b)



(c) (d)

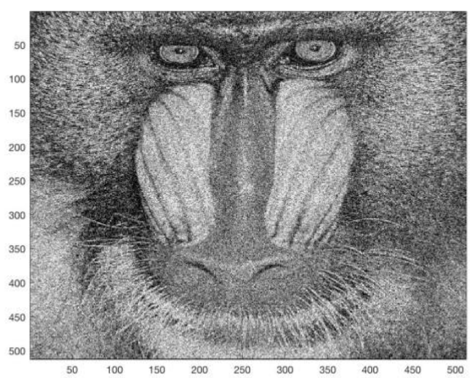

(e)


(f)

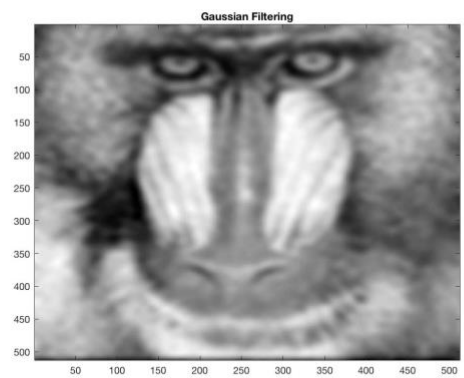
Fig.4 (a) noisy picture with σ = 10 (b) Gaussian filter (c) Median filter (d) Bilateral filter (e) NL-means filter (f) NL-means and thresholding

It is known that the BF removes noise by domain filtering and retains the edges by range filtering, but this is at the cost of image details. It also has the effect of flattening the gray levels in an image considerably resulting in a cartoon-like appearance. This is observed in (d) of Figs.4.

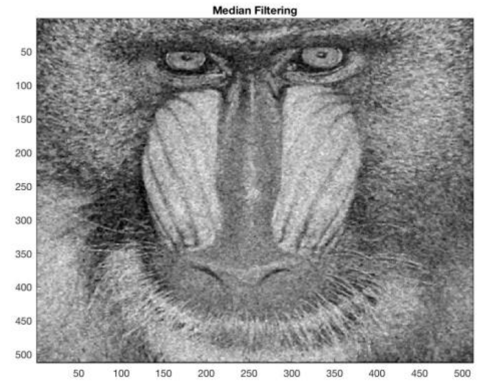
The denoised images of Baboon with σ = 40 by different methods are shown in Fig.5



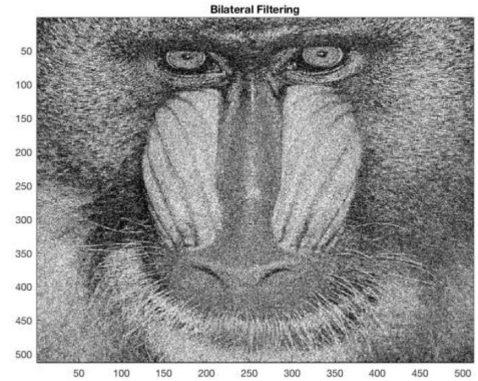
(a)



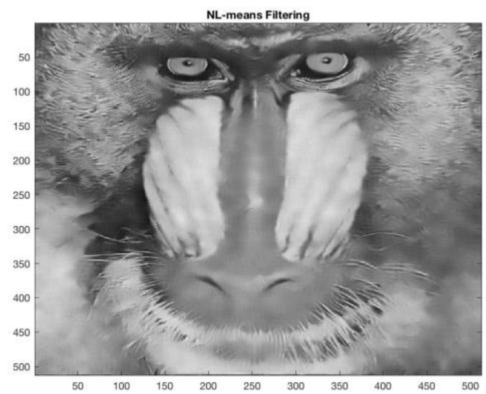

(b)



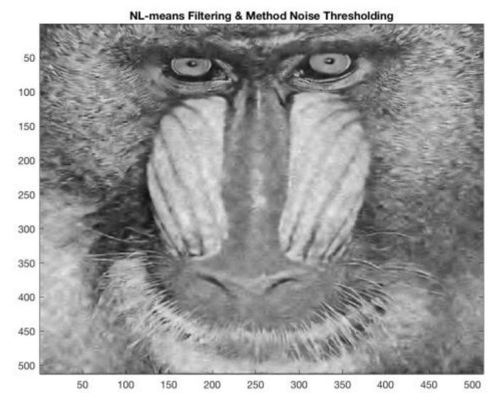

(c)




(d)




(e)




(f)

Fig.5 (a) noisy picture with σ = 40 (b) Gaussian filter (c) Median filter (d) Bilateral filter (e) NL-means filter (f) NL-means and thresholding

It is observed from Figs. 5. that the denoised images by GT MT even BT still have some amount of noise with blocking artifacts and able to retain some of the details at low SNR. The application of NL means filter removes the details to some extent and blurs the image at low SNR as the similar local patches used to find pixel weights are noisy.

The denoised images of Barbara with σ = 50 by different methods are shown in Fig.6

Fig.6 (a) noisy picture with σ = 50 (b) Gaussian filter (c) Median filter (d) Bilateral filter (e) NL-means filter (f) NL-means and thresholding

The performances of the proposed methods are measured quantitatively using the mean square error(MSE)

$$MSE = \frac{1}{mn}\sum_{i=0}^{m-1}\sum_{j=0}^{n-1}[I(i,j)-K(i,j)]^2 \quad (6)$$

| σ | 10 | 20 | 30 | 40 | 50 | 10 | 20 | 30 | 40 | 50 |
|---|---|---|---|---|---|---|---|---|---|---|
| input | Lena | | | | | Barbara | | | | |
| GT | 254.946 | 256.257 | 259.936 | 267.434 | 280.026 | 545.561 | 548.663 | 555.966 | 568.3665 | 587.011 |
| MT | 40.073 | 94.4623 | 181.417 | 301.191 | 453.055 | 265.466 | 323.779 | 411.321 | 529.113 | 676.878 |
| BT | 43.822 | 248.045 | 679.907 | 1292.1 | 2033.22 | 60.632 | 274.720 | 692.323 | 1279.3 | 1992.1 |
| NLM | **23.232** | 42.656 | 65.209 | 93.553 | **126.479** | 39.930 | 72.692 | 116.366 | 173.497 | 242.305 |
| NLFMT | 22.837 | **42.158** | **65.143** | **93.498** | 126.734 | **39.198** | **71.818** | **114.295** | **169.711** | **235.371** |
| input | Baboon | | | | | Cameraman | | | | |
| GT | 702.909 | 703.899 | 705.488 | 711.287 | 729.298 | 375.644 | 383.094 | 394.320 | 429.332 | 447.095 |
| MT | 312.301 | 372.192 | 461.398 | 647.533 | 720.221 | 34.078 | 86.893 | 154.324 | 300.398 | 418.156 |
| BT | 72.669 | 296.055 | 706.221 | 1309.2 | 2053.2 | 36.721 | 221.228 | 899.425 | 1234.32 | 1912.3e |
| NLM | 84.076 | 187.0486 | 223.244 | 287.244 | 310.256 | 20.009 | 44.356 | 73.245 | **112.356** | **151.280** |
| NLFMT | **74.164** | **177.266** | **221.332** | **279.412** | **307.233** | **19.963** | **44.075** | **68.084** | 114.259 | 159.794 |

Table.1 MSE for all filters

It is observed from the Table 1 that the denoised images by NLFMT has the smallest MSE than other methods in most cases. The MSE of NLM and NLFMT is very close. Weird, BT blow up easily.

### *Conclusions*

In this report, the amalgamation of NL means filter and its method noise thresholding using wavelet has been implemented. The performance of the NL filters is compared with Gaussian filter, Median filter, and Bilateral filter. Through experiments conducted on standard images, it was found that the NLM method has improved the results of denoising approach in performance in terms of visual quality and MSE.

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