$$B_j(x)m_{.j}(x)\sum_i A_i(x)m_{i.}(x)T_{ij} = m_{.j}(x) \tag{9.10}$$

$$B_j(x) = [\sum_i A_i(x)m_{i.}(x)T_{ij}]^{-1}$$

$$\sum_j A_i(x)B_j(x)m_{i.}(x)m_{.j}(x)T_{ij} = m_{i.}(x)$$

$$A_i(x)m_{i.}(x)\sum_i B_j(x)m_{.j}(x)T_{ij} = m_{i.}(x) \tag{9.11}$$

$$A_i(x) = [\sum_j B_j(x)m_{.j}(x)T_{ij}]^{-1}$$

$$\begin{aligned}&\sum_x A_i(x)B_j(x)m_{i.}(x)m_{.j}(x)T_{ij} = m_{ij}(.)\\ &T_{ij} = m(.)[\sum_x A_i(x)B_j(x)m_{i.}(x)m_{.j}(x)]^{-1}\end{aligned} \tag{9.12}$$

The above equations belong to the system of nonlinear equations. The system does not have an analytical solution and it must be solved by iterative procedures.

9.2.2. Quadratic Adjustment Approach

Another method for estimating the complete origin–destination migration flow matrix M with elements m_{ij} is the quadratic adjustment approach. For this procedure also we need the total number of migrants arriving in each region I_j, and departing from each region, O_i, in a two-region system. The problem is then to estimate the complete origin–destination migration flow matrix M with elements m_{ij}. The only difference between the entropy method and this method is that unlike the entropy method, the present method requires initial estimates of the elements m_{ij}.

Let us consider the matrix to be estimated as

$$\dot{M} = \begin{bmatrix} m_{11} & m_{21} \\ m_{12} & m_{22} \end{bmatrix}$$

where $I_1 = m_{.1}$, $O_1 = m_{1.}$, $I_2 = m_{.2}$, and $O_2 = m_{2.}$.

Suppose we also have a matrix M^0 of initial estimates, with elements m^0_{ij}, i.e.,

$$\dot{M} = \begin{bmatrix} \dot{m}_{11} & \dot{m}_{21} \\ \dot{m}_{12} & \dot{m}_{22} \end{bmatrix}$$

The initial estimates may be obtained from a variety of sources such as expert opinions, migration tables of earlier years, and so forth. Note that the column sums $m_{i.}$ and the row sums m_{j} of M need not be equal to the predefined number of departures $O_i = m_{i.}$, and arrivals $I_j = m_{.j}$. We have to adjust the elements of M so as to add up to the required totals. The sum constrained quadratic adjustment problem is as follows:

$$\textstyle\sum_j m_{ij} = O_i \qquad \text{for all } i$$

$$\textstyle\sum_j m_{ij} = I_j \qquad \text{for all } j$$

Based on these constraints, the task now is to adjust matrix $\dot{M}$ so that elements of $\dot{M}$ are as close as possible to the elements of M and the row sums and column sums add up to the predefined number of arrivals and departures. One such method that was recently used in the case of India is a biproportional adjustment approach (Nair, 1985). This method assumes that the elements of matrix M are biproportional to the elements of a guess matrix M. Then the elements (m_{ij}) of a matrix M can be as follows:

$$m_{ij} = r_{i.}s_{j.}M_{ij} \tag{9.13}$$

where m_{ij} and M_{ij} are elements of matrix $\dot{M}$ and M respectively; r_i and s_j are the row and column balancing factors. The values of r_i and s_j are estimated based on the constraints as follows:

$$\textstyle\sum_i m_{ij} = m_{.j} \qquad \text{for all } j$$

$$\textstyle\sum_j m_{ij} = m_{i.} \qquad \text{for all } i$$

$$\textstyle\sum_i m_{i.} = \sum_j m_{.j} = m_{..} \qquad \text{for all } i \text{ and } j$$

where $m_{i.}$ and $m_{.j}$ are the known row and column totals of the matrix M, and $M_{..}$ is the overall total of M. By summing (9.13) over all i, we get

$$\textstyle\sum_j m_{ij} = m_{.j} = s_j \sum_i r_i m_{ij}$$

or (9.14)

$$s_j = m_{ij}/(\textstyle\sum_i r_{i.}m_{ij})$$

or summing (9.13) over all j, we get

$$\sum_j m_{ij} = m_{i.} = r_i \sum_j s_{j.} m_{ij}$$

or (9.15)

$$r_i = m_{i.}/(\sum_j s_{j.} m_{ij})$$

To solve Eqs. (9.14) and (9.15), Stone (1962) suggested an iterative procedure which was stated by Nair (1985, p. 134) as follows:

Step 0: $K = 0$

$$r_i^0 = 1 \quad \text{fixed arbitrarily}$$

Step 1: $K = K + 1$

Calculate s_j:

$$s_j^{(k)} = \frac{m_{.j}}{(\sum_i r_i^{(k-1)} \dot{m}_{ij}^0)}$$

Step 2: Calculate r_i

$$r_i^{(k)} = \frac{m_{i.}}{(\sum_j s_j^{(k)} \dot{m}_{ij})}$$

Go to step 1 until convergence is reached for r_i and s_j values, or a stopping criterion is reached. For an empirical application and computer programming, see Nair (1985).

Apart from the aforementioned generalized estimation procedures and algorithms, there have been several notable developments specific to the field of multiregional demography. We shall focus on some of these in the following sections.

9.3. MODEL MIGRATION SCHEDULES

As indicated in Chapters 1 and 2, by and large, demographic phenomena exhibit regularities. The regularities observed in mortality schedules prompted Coale and Demeny (1966) to develop model life tables, Brass (1971) to develop simple models relating the survival probabilities of a population to a standard