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Datenstrukturen & Algorithmen Programming Exercise 6 FS 13

In this exercise we are going to implement *Blum's algorithm* for median computation. Let x_1, \dots, x_n be a sequence of $n > 1$ elements. The algorithm finds the k -th smallest element by performing the following steps.

- 1) Divide the elements into $\lfloor \frac{n}{5} \rfloor$ groups of 5 elements each and at most one group containing the remaining $n \bmod 5$ elements.
- 2) For each of the above groups, find the median of each group by sorting the elements contained in it (for a group with 2 elements, the median is the smaller one, and for a group with 4 elements, the median is the 2nd-smallest one).
- 3) Recursively compute the median m among the above medians. This element is called the *median of medians*.
- 4) Use the partition step of quicksort to bring the element m to the correct position p_m in the sorted sequence. Then we have $p_m - 1$ elements on the left of m (with value at most m), and $n - p_m$ elements on the right of m (with value at least m).
- 5) If $k = p_m$, then we know that the pivot element is on the position we are looking for, and we return m . If $k < p_m$, then the k -th smallest element is located on the left of m , and we search recursively for the k -th smallest element among these $p_m - 1$ elements on the left. Otherwise, $k > p_m$, and we search recursively for the $(k - p_m)$ -th smallest element among the $n - p_m$ elements on the right.

Our final goal is to compute the median, i.e. the $\lceil n/2 \rceil$ -th element in the sorted sequence. For the sequence 3, 4, 2, 6, 4, 7, 1, the median is 4. We refer to the theory exercise for an example of the above algorithm.

Input The first line contains only the number t of test instances. After that, we have exactly one line per test instance containing the numbers n, x_1, \dots, x_n . While $n \in \mathbb{N}$, $1 \leq n \leq 1000$, describes the number of following integers, $x_i \in \mathbb{Z}$, $-10^8 \leq x_i \leq 10^8$ is the i -th number in the sequence.

Output For every test instance we output only one line. It contains the first sequence of medians of the groups of 5 elements, the first median of medians, and the overall median of the sequence.

Example

Input:

2

5 1 2 3 4 5

13 7 3 5 1 9 8 11 21 4 10 2 6 9

Output:

3 3 3

5 10 6 6 7

Hand-in: until Wednesday, 10th April 2013.