Ordered logistic regression with R

Ordered logit model

- The Ordered (or Ordinal) logit model (also ordered logistic regression or proportional odds model), is a regression model for ordinal dependent variables.
- For example, questions on a survey answered by a choice among "poor", "fair", "good", "very good", and "excellent".
- The purpose of the analysis is to see how well that response can be predicted by the responses to other questions, some of which may be quantitative

Ordered logistic regression with R

Ordered logit model

- It can be thought of as an extension of the logistic regression model that applies to dichotomous dependent variables, allowing for more than two (ordered) response categories.
- The model only applies to data that meet the proportional odds assumption

Ordered logistic regression with R

polr

- In this section we will use the polr command (from the MASS package) to estimate an ordered logistic regression model.
- The command name comes from **proportional odds logistic regression**, due to the the **proportional odds assumption** in the model.

Ordered logistic regression with R

polr

- polr uses the standard formula interface in R for specifying a regression model with outcome followed by predictors.
- We will also specify Hess=TRUE to have the model return the observed information matrix from optimization (called the Hessian) which is used to get standard errors.

Ordered logistic regression with R

```
## fit ordered logit model and store results ’m’
m <- polr(apply ~ pared +
          public + gpa, data = dat, Hess=TRUE)

## view a summary of the model
summary(m)
## Call:
## polr(formula = apply ~ pared +
             public + gpa, data = dat,
             Hess = TRUE)
```

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```
Coefficients:
         Value Std. Error t value
pared   1.0477      0.266   3.942
public -0.0588      0.298  -0.197
gpa     0.6159      0.261   2.363

Intercepts:
                            Value  Std. Error t value
unlikely|somewhat likely     2.204  0.780      2.827
somewhat likely|very likely  4.299  0.804      5.345
```

Ordered logistic regression with R

1. The “Call”, what type of model we ran, what options we specified, etc.
2. The usual regression output coefficient table including the value of each coefficient, standard errors, and $t-$value, which is simply the ratio of the coefficient to its standard error.
   (Remark: There is no significance test by default.)

Ordered logistic regression with R

3. We then have the estimates for the two intercepts (which are sometimes called **cutpoints**).
4. The intercepts indicate where the latent variable is cut to make the three groups that we observe in our data.

Ordered logistic regression with R

In the ordered logit model, there is an observed ordinal variable, Y. Y, in turn, is a function of another latent variable, Y*, that is not measured.

a. In the ordered logit model, there is a continuous, unmeasured latent variable Y*, whose values determine what the observed ordinal variable Y equals.
b. The continuous latent variable Y* has various threshold (or cutoff) points.

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Your value on the observed ordinal variable Y depends on whether or not you have crossed a particular threshold. For example, when M = 3

- Yi = 1 if Y*i is $\leq$ CP1
- Yi = 2 if CP1 $\leq$ Y*i $\leq$ CP2
- Yi = 3 id Y*i $\geq$ CP2

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- Note that this latent variable is continuous. In general, these are not used in the interpretation of the results.
- The cutpoints are closely related to thresholds, which are reported by other statistical packages.

Ordered logistic regression R

Model Diagnostics

- We see the residual deviance, -2 * Log Likelihood of the model as well as the AIC.
- Both the deviance and AIC are useful for model comparison.
- Of, course, some people are not satisfied without a $p-$value.
- One way to calculate a $p-$value in this case is by comparing the $t-$value against the standard normal distribution, like a $z-$test.

Ordered logistic regression R

- Of course this is only true with infinite degrees of freedom, but is reasonably approximated by large samples, becoming increasingly biased as sample size decreases.
- First we store the coefficient table, then calculate the $p-$values and combine back with the table.

Ordered logistic regression R

```
# store table
(ctable <- coef(summary(m)))
                              Value Std. Error t value
 pared                      1.04769     0.2658  3.9418
 public                    -0.05879     0.2979 -0.1974
 gpa                        0.61594     0.2606  2.3632
 unlikely|somewhat likely   2.20391     0.7795  2.8272
 somewhat likely|very likely 4.29936     0.8043  5.3453
```

Ordered Logistic regression R

```
# calculate and store p values
p <- pnorm(abs(ctable[, "t value"]),
      lower.tail = FALSE) * 2

# Combined table
(ctable <- cbind(ctable, "p value" = p))
                      Value Std. Error t value   p value
 pared              1.04769     0.2658  3.9418 8.087e-05
 public            -0.05879     0.2979 -0.1974 8.435e-01
 gpa                0.61594     0.2606  2.3632 1.812e-02
 unli..|some..      2.20391     0.7795  2.8272 4.696e-03
 some..|very..      4.29936     0.8043  5.3453 9.027e-08
```

Ordered Logistic Regression Examples of ordered logistic regression

- A marketing research firm wants to investigate what factors influence the size of soda (small, medium, large or extra large) that people order at a fast-food chain.
- These factors may include what type of sandwich is ordered (burger or chicken), whether or not fries are also ordered, and age of the consumer.
- While the outcome variable, size of soda, is obviously ordered, the difference between the various sizes is not consistent.
- The differece between small and medium is 10 ounces, between medium and large 8, and between large and extra large 12.

Ordered Logistic Regression Examples of ordered logistic regression

- A researcher is interested in what factors influence medaling in Olympic swimming.
- Relevant predictors include at training hours, diet, age, and popularity of swimming in the athlete's home country.
- The researcher believes that the distance between gold and silver is larger than the distance between silver and bronze.

Ordered Logistic Regression

- A study looks at factors that influence the decision of whether to apply to graduate school.
- College juniors are asked if they are unlikely, somewhat likely, or very likely to apply to graduate school.
- Hence, our outcome variable has three categories. Data on parental educational status, whether the undergraduate institution is public or private, and current GPA is also collected.
- The researchers have reason to believe that the "distances" between these three points are not equal.
- For example, the "distance" between *"unlikely"* and *"somewhat likely"* may be shorter than the distance between *"somewhat likely"* and *"very likely"*.

Ordered Logistic Regression Data Set - Graduate School Entry (ologit.csv)

- This hypothetical data set has a three level variable called **apply**, with levels *"unlikely"*, *"somewhat likely"*, and *"very likely"*, coded 1, 2, and 3, respectively, that we will use as our outcome variable.

Ordered Logistic Regression Predictors:

- pared , which is a 0/1 variable indicating whether at least one parent has a graduate degree;
- public , which is a 0/1 variable where 1 indicates that the undergraduate institution is public and 0 private,
- gpa , which is the student's grade point average.

```
            apply pared public  gpa
1     very likely     0      0 3.26
2 somewhat likely     1      0 3.21
3        unlikely     1      1 3.94
4 somewhat likely     0      0 2.81
5 somewhat likely     0      0 2.53
6        unlikely     0      1 2.59
```

Ordered Logistic Regression with R

Confidence Intervals

- We can also get confidence intervals for the parameter estimates.
- These can be obtained either by profiling the likelihood function or by using the standard errors and assuming a normal distribution.
- Note that profiled CIs are not symmetric (although they are usually close to symmetric).
- If the 95% CI does not cross 0, the parameter estimate is statistically significant.

Ordered Logistic Regression with R

```
(ci <- confint(m))
# default method gives profiled CIs

 Waiting for profiling to be done...
          2.5 %   97.5 %
 pared   0.5282   1.5722
 public -0.6522   0.5191
 gpa     0.1076   1.1309
```

Ordered Logistic Regression with R

```
# CIs assuming normality

confint.default(m)

            2.5 %  97.5 %
 pared     0.5268   1.569
 public   -0.6426   0.525
 gpa       0.1051   1.127
```

Ordered Logistic Regression with R

Confidence Intervals

- The CIs for both pared and gpa do not include 0; but the CI for public does.
- The estimates in the output are given in units of ordered logits, or ordered log odds.
- For pared, we would say that for a one unit increase in pared (i.e., going from 0 to 1), we expect a 1.05 increase in the expect value of apply on the log odds scale, given all of the other variables in the model are held constant.

Ordered Logistic Regression with R

Confidence Intervals

- For gpa, we would say that for a one unit increase in gpa, we would expect a 0.62 increase in the expected value of apply in the log odds scale, given that all of the other variables in the model are held constant.

Ordered Logistic Regression with R

- The coefficients from the model can be somewhat difficult to interpret because they are scaled in terms of logs.
- Another way to interpret logistic regression models is to convert the coefficients into odds ratios.
- To get the Odds Ratios and confidence intervals, we just exponentiate the estimates and confidence intervals.

Ordered Logistic Regression with R

Odds Ratios

```
exp(coef(m))
  pared public    gpa
 2.8511 0.9429 1.8514
```

```
 # Odds Ratios and CIs
exp(cbind(OR = coef(m), ci))
            OR  2.5 % 97.5 %
 pared  2.8511 1.6958  4.817
 public 0.9429 0.5209  1.681
 gpa    1.8514 1.1136  3.098
```

Ordered Logistic Regression with R

- These coefficients are called **proportional odds ratios** and we would interpret these pretty much as we would odds ratios from a binary logistic regression.
- For pared, we would say that for a one unit increase in parental education, i.e., going from 0 (*Low*) to 1 (*High*), the odds of "*very likely*" applying versus "*somewhat likely*" or "*unlikely*" applying combined are 2.85 greater, given that all of the other variables in the model are held constant.

Ordered Logistic Regression with R

- Similarly, the odds "*very likely*" or "*somewhat likely*" applying versus "*unlikely*" applying is 2.85 times greater, given that all of the other variables in the model are held constant.
- For gpa (and other continuous variables), the interpretation is that when a student's gpa moves 1 unit, the odds of moving from "*unlikely*" applying to "*somewhat likely*" or "*very likely*" applying (or from the lower and middle categories to the high category) are multiplied by 1.85.

Ordinal Logistic Regression with R

Assumption of Proportional Odds

- One of the assumptions underlying ordinal logistic regression is that the relationship between each pair of outcome groups is the same.
- In other words, ordinal logistic regression assumes that the coefficients that describe the relationship between, say, the lowest versus all higher categories of the response variable are the same as those that describe the relationship between the next lowest category and all higher categories, etc.

Ordinal Logistic Regression with R

Testing the Assumption

- Because the relationship between all pairs of groups is the same, there is only one set of coefficients.
- If this was not the case, we would need different sets of coefficients in the model to describe the relationship between each pair of outcome groups.
- Thus, in order to asses the appropriateness of our model, we need to evaluate whether the proportional odds assumption is tenable.

Ordinal Logistic Regression with R

Testing the Assumption

- Statistical tests to do this are available in some software packages.
- However, these tests have been criticized for having a tendency to reject the null hypothesis (that the sets of coefficients are the same), and hence, indicate that there the parallel slopes assumption does not hold, in cases where the assumption does hold *(Harrell 2001 p. 335)*.
- Currently R to perform any of the tests commonly used to test the parallel slopes assumption.

Ordinal Logistic Regression with R

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- Harrell does recommend a graphical method for assessing the parallel slopes assumption.
- The values displayed in this graph are essentially (linear) predictions from a logit model, used to model the probability that y is greater than or equal to a given value (for each level of y), using one predictor (x) variable at a time.

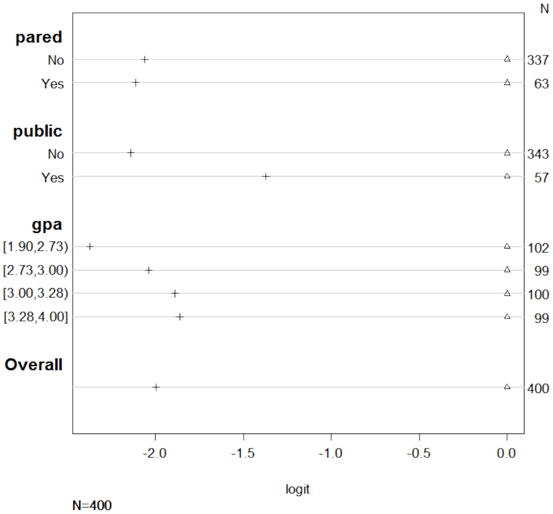

N
pared
No
Yes
337
63
public
No
Yes
343
57
gpa
[1.90,2.73)
[2.73,3.00)
[3.00,3.28)
[3.28,4.00]
102
99
100
99
Overall
400
-2.0
-1.5
-1.0
-0.5
0.0
logit
N=400

Figure:

Ordered Logistic regression R

- Turning our attention to the predictions with public as a predictor variable, we see that when public is set to "*No*" the difference in predictions for apply greater than or equal to two, versus apply greater than or equal to three is about 2.14 $(-0.204 - (-2.345) = 2.141)$.
- When public is set to "yes" the difference between the coefficients is about 1.37. $(-0.175 - (-1.547) = 1.372)$.

Ordered Logistic regression R

- The differences in the distance between the two sets of coefficients (2.14 vs. 1.37) may suggest that the parallel slopes assumption does not hold for the predictor public.
- That would indicate that the effect of attending a public versus private school is different for the transition from "*unlikely*" to "*somewhat likely*" and "*somewhat likely*" to "*very likely*."

Ordered logistic regression R

- If the proportional odds assumption holds, for each predictor variable, distance between the symbols for each set of categories of the dependent variable, should remain similar.
- To help demonstrate this, we normalized all the first set of coefficients to be zero so there is a common reference point.

Ordered logistic regression R

- Looking at the coefficients for the variable pared we see that the distance between the two sets of coefficients is similar.
- In contrast, the distances between the estimates for public are different (i.e. the markers are much further apart on the second line than on the first), suggesting that the proportional odds assumption may not hold.

Ordered logistic regression R

```
plot(s, which=1:3, pch=1:3,
    xlab=’logit’, main=’ ’,
    xlim=range(s[,3:4]))
```

Ordered logistic regression R

- Once we are done assessing whether the assumptions of our model hold, we can obtain predicted probabilities, which are usually easier to understand than either the coefficients or the odds ratios.
- For example, we can vary gpa for each level of pared and public and calculate the probability of being in each category of apply.
- We do this by creating a new dataset of all the values to use for prediction.

Ordered logistic regression R

```
newdat <- data.frame(
  pared = rep(0:1, 200),
  public = rep(0:1, each = 200),
  gpa = rep(seq(from = 1.9, to = 4,
      length.out = 100), 4))

newdat <- cbind(newdat, predict(m,
   newdat, type = "probs"))
```

Ordered logistic regression R

```
# Show first few rows
head(newdat)
  pared public   gpa unlikely somewhat likely very likely
1     0      0 1.900   0.7376          0.2205     0.04192
2     1      0 1.921   0.4932          0.3946     0.11221
3     0      0 1.942   0.7325          0.2245     0.04299
4     1      0 1.964   0.4867          0.3985     0.11484
5     0      0 1.985   0.7274          0.2285     0.04407
6     1      0 2.006   0.4802          0.4023     0.11753
```

Ordered logistic regression R

- Now we can reshape the data long with the reshape2 package and plot all of the predicted probabilities for the different conditions.
- We plot the predicted probilities, connected with a line, coloured by level of the outcome, apply, and facetted by level of pared and public.
- We also use a custom label function, to add clearer labels showing what each column and row of the plot represent.

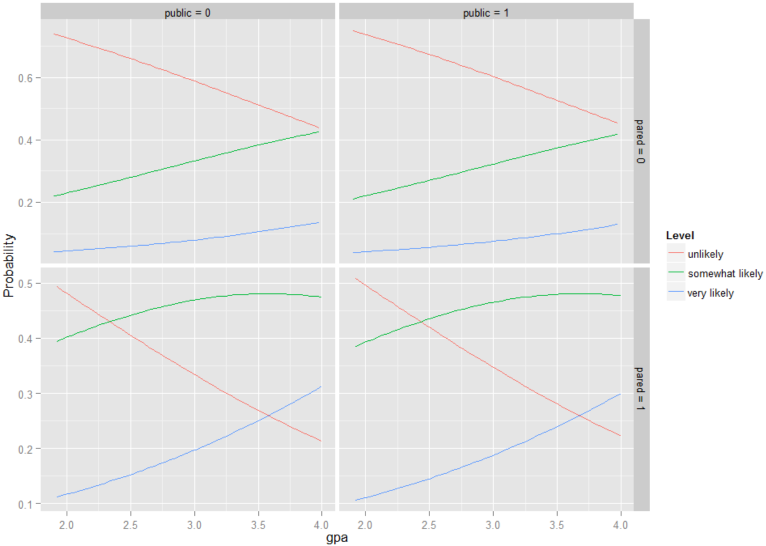
Ordered logistic regression R

```
library(reshape2)

lnewdat <- melt(newdat,
  id.vars = c("pared", "public", "gpa"),
  variable.name = "Level",
  value.name="Probability")
```

Ordered logistic regression R

```
%## view first few rows
%head(lnewdat)
%##   pared public   gpa    Level Probability
%## 1     0      0 1.900 unlikely      0.7376
%## 2     1      0 1.921 unlikely      0.4932
%## 3     0      0 1.942 unlikely      0.7325
%## 4     1      0 1.964 unlikely      0.4867
%## 5     0      0 1.985 unlikely      0.7274
%## 6     1      0 2.006 unlikely      0.4802
```


public = 0
public = 1
pared = 0
pared = 1
Probability
0.6
0.4
0.2
0.5
0.4
0.3
0.2
0.1
2.0
2.5
3.0
3.5
4.0
gpa
Level
unlikely
somewhat likely
very likely

Ordered logistic regression R

Things to consider

Perfect prediction: Perfect prediction means that one value of a predictor variable is associated with only one value of the response variable.

Pseudo-R-squared:

There is no exact analog of the R-squared found in OLS. There are many versions of pseudo-R-squares.

Diagnostics:

Doing diagnostics for non-linear models is difficult, and ordered logit/probit models are even more difficult than binary models.

Ordered logistic regression R

Sample size:
Both ordered logistic and ordered probit, using maximum likelihood estimates, require sufficient sample size.

Empty cells or small cells:
You should check for empty or small cells by doing a crosstab between categorical predictors and the outcome variable. If a cell has very few cases, the model may become unstable or it might not run at all.