☆ THE ART OF COMPUTER PROGRAMMING ☆

☆☆☆☆ ERRATA TO VOLUME 1 (after 2010) ☆☆☆☆

This document is a transcript of the notes that I have been making in my personal copy of *The Art of Computer Programming*, Volume 1 (third edition, 27th printing), since it was first printed in 2011. Previous errata are recorded in another file '`all1-pre.ps`'.

Four levels of updates — "errors," "amendments," "plans," and "improvements" — appear, indicated by four different typographic conventions:

▸**Page 666** line 1 _____ 04 Jul 1776

Technical or typographical errors (aka bugs) are the most critical items, so they are flagged with a '▸' preceding the page number. The date on which I first was told about the bug is shown; this is the effective date on which I paid the finder's fee. The necessary corrections are indicated in a straightforward way. If, for example, the book says 'n' where it should have said '$n+1$', the change is shown thus:

$n \mathrel{\not\searrow\!\!\!\rightarrow} n+1$

Page 666 line 2 _____ 14 Jul 1789

Amendments to the text appear in the same format as bugs, but without the '▸'. These are things I wish I had known about or thought of when I wrote the original text, so I added them later. The date is the date I drafted the new text.

Page 666 line 3 . . . . . 20 Nov 1917

Plans for the future represent a third kind of item. In such notes I sketched my intentions about things that I wasn't ready to flesh out further when I wrote them down. You can identify these items because they're written in slanted type, and preceded by a bunch of dots '. . . . . ' leading to the date on which I recorded the plan in my files.

Page 666 line 4 _____ 10 Jan 1938

The fourth and final category — indicated by page and line number in smaller, slanted type — consists of minor corrections or improvements that most readers don't want to know about, because they are so trivial. You wouldn't even be seeing these items if you hadn't specifically chosen to print the complete errata list in all its gory details. Are you sure you wanted to do that?

My shelves at home are bursting with preprints and reprints of significant research results that I want to digest and summarize, where appropriate, in the ultimate edition of Volume 1. I didn't do that in the third edition because I would surely have to do it over again later: New results continue to pour forth at a great rate, and I will have time to rewrite that volume only once. Volumes 4 and 5 need to be finished first. So I've put most of my effort so far into writing up those parts of the total picture that seem to have converged to their near-final

form. It follows, somewhat paradoxically, that the updates in this document are most current in the areas where there has been least activity.

On the other hand I do believe that the changes listed here bring Volume 1 completely up to date in two respects: (1) All of the research problems in the previous edition — i.e., all exercises that were rated 46 and above — have received new ratings of 45 or less whenever I learned of a solution; and in such cases, the answer now refers to that solution. (2) All of the historical information about pioneering developments has been amended whenever new details have come to my attention.

The ultimate, glorious, 100% perfect editions of Volumes 1–4A are works in progress. Please let me know of any improvements that you think I ought to make. Send your comments either by snail mail to D. E. Knuth, Computer Science, Gates Building 4B, Stanford University, Stanford CA 94305-9045, or by email to `taocp@cs.stanford.edu`. *(Use email for book suggestions only, please — all other correspondence is returned unread to the sender, or discarded, because I have no time to read ordinary email.) Although I'm working full time on Volume 4B these days, I will try to reply to all such messages within a year of receipt. Current news about The Art of Computer Programming is posted on*

`http://www-cs-faculty.stanford.edu/~knuth/taocp.html`

and updated regularly.

— Don Knuth, January 2011

What happened?
The subject took the bit in its teeth and ran away with it,
that's what happened.
I know now how Sir James Frazer felt when,
after setting out to dash off a brief monograph
on a single obscure rite, he found himself
in the embarrassing possession of
the 12 volumes of "The Golden Bough."

— WAVERLEY ROOT (1974)

☆☆☆☆☆ FUNDAMENTAL ALGORITHMS ☆☆☆☆☆

Last updated 26 January 2014

Most of these corrections have already been made in recent printings.

▸**Page 0** (on the back page of the dustcover, line 23) 10 Sep 2012

Whatever your backgound, ↜ Whatever your background,

Page xi new paragraph to follow line 20 25 Nov 2013

My efforts to extend and enhance these volumes have been enormously enhanced since 1980 by the wise guidance of Addison–Wesley's editor Peter Gordon. He has become not only my "publishing partner" but also a close friend, while continually nudging me to move in fruitful directions. Indeed, my interactions with dozens of Addison–Wesley people during more than three decades have been much better than any author deserves. The tireless support of managing editor John Fuller, whose meticulous attention to detail has maintained the highest standards of production quality in spite of frequent updates, has been particularly praiseworthy.

Page xvi line 16 16 Jun 2011

a *45* rating ↜ a *40* rating

Page xvi line 20 30 Jan 2013

creativity. ↜ creativity. All exercises with ratings of *46* or more are open problems for future research, rated according to the number of different attacks that they've resisted so far.

Page 6 lines 1–3 13 Mar 2013

$\{m, n\}$ ↜ m and n [twice]
$\{n, r\}$ ↜ n and r [twice]

Page 8 lines −5 and −4 13 Mar 2013

Such a computational method ... it is also ↜ Every step of such a computational method is clearly effective, and experience shows that pattern-matching rules of this kind are also

▸**Page 17** line 26 15 Dec 2013

11 (1838) ↜ **12** (1838)

Page 18 clarification in exercise 4 10 Feb 2013

ϕ^{n-2}. ↜ ϕ^{n-2} for all positive integers n.

Page 19 clarification in exercise 5 line 1 05 Feb 2013

exact divisors ↜ positive integer divisors

Page 22 lines 1 and 3 after (8) 20 Jan 2013

10^x ↜ b^x [two places]

▸**Page 27** line 1 — 22 Jul 2011

$n > 1$ ↝ $n > 1$ and $n \neq e$.

Page 70 in exercise 25 — 07 Oct 2013

line 1: as in Eq. (30). ↝ as in Example 4 (see Eq. (30)).
line 5: the identity ↝ multiples of a special case of (34),

▸**Page 70** line 2 of exercise 25 — 07 Oct 2013

provided z is small enough ↝ provided that x is close enough to 1

Page 79 new exercise for Section 1.2.7 — 11 Feb 2013

25. [*M21*] Let $H_n^{(u,v)} = \sum_{1 \le j \le k \le n} 1/(j^u k^v)$. What are $H_n^{(0,v)}$ and $H_n^{(u,0)}$? Prove the general identity $H_n^{(u,v)} + H_n^{(v,u)} = H_n^{(u)} H_n^{(v)} + H_n^{(u+v)}$.

Page 104 new sentence preceding the exercises — 19 Jul 2012

[S. Bernstein had contributed key ideas in *Uchenye zapiski Nauchno-Issledovatel'skikh kafedr Ukrainy* **1** (1924), 38–48.]

Page 111 line above Fig. 12 — 08 Jul 2012

Petropolitanæ ↝ *Imperialis Petropolitanæ*

Page 114 line 7 — 26 Feb 2013

is less than ↝ is less in absolute value than

Page 116 line 2 of exercise 8 — 18 Feb 2012

$\binom{cn^2}{n}/c^n\binom{n^2}{n}$ ↝ $\binom{cn^2}{n}/(c^n\binom{n^2}{n})$

Page 144 line 2 — 29 Nov 2011

to zero. ↝ to zero, and the overflow toggle is cleared.

Page 151 line −11 — 29 Nov 2011

set to zero ↝ *set to positive zero*

Page 173 line 3 after the table — 15 Dec 2013

table. ↝ table, except that the unknown destination of e is represented there by ')' not '?'.

▸**Page 229** line 11 — 06 Feb 2011

Mechanization ↝ *Mechanisation*

Page 242 line 2 — 28 Apr 2012

top, front ↝ top, bottom, front

▸**Page 268** program line *71* — 30 Jun 2012

`QLINK(F)` ↝ `QLINK[F]`

Page 275 line 7 — 06 Sep 2012

list have ↝ list must have

Page 303 in the line before (13) — 29 Aug 2011

transformation: ↜ transformation (see M. H. Doolittle, *Report of the Superintendent of the U. S. Coast and Geodetic Survey* (1878), 115–120):

Page 331 rating of exercise 14 — 15 Aug 2012

22 ↜ *20*

▸**Page 339** replacement for line 17 — 14 Mar 2011

$$D(y) = 3\big(1/(x+1)\big) - \big(-(a(2x))/(x^2)^2\big), \tag{21}$$

Page 465 new quotation (to follow the one by Lehmer) — 18 Aug 2013

I must explain, to begin with,
that all the Trees, in this system, grow head-downwards:
the Root is at the top, *and the Branches are* below.
If it be objected that the name "Tree" is a misnomer, my answer
is that I am only following the example of all writers on Genealogy.
A Genealogical *tree* always *grows* downwards:
then why may not a Logical *"Tree" do likewise?*

— LEWIS CARROLL, in *Symbolic Logic* (1896)

Page 467 new line at end of answer 8 — 29 Apr 2013

Each iteration either decreases m or keeps m unchanged and decreases n.

Page 469 new lines at end of answer 8 — 26 Feb 2013

This construction can't make $d_k = 9$ for all $k > l$, because that could happen only if $(n + d_1/10 + \cdots + d_l/10^l + 1/10^l)^m \le u$.

Page 470 in step E1 of answer 28 — 28 Oct 2011

If $1-\epsilon \ldots k \leftarrow 1$. ↜ Set $x \leftarrow 1-\epsilon-x$, $y \leftarrow y_0$, and $k \leftarrow 1$, where $1-\epsilon$ is the largest possible value of x, and y_0 is the nearest approximation to $b^{1-\epsilon}$.

Page 472 new copy for end of answer 31 — 20 Oct 2011

Consequently we have $(\sum_{j=1}^n u_j)(\sum_{j=1}^n v_j) \le n\sum_{j=1}^n u_j v_j$ when $u_1 \le u_2 \le \cdots \le u_n$ and $v_1 \le v_2 \le \cdots \le v_n$, a result known as *Chebyshev*'s monotonic inequality. [See *Soobshch. mat. obshch. Khar'kovskom Univ.* **4**, 2 (1882), 93–98.]

Page 474 lines 3–5 of answer 43 — 26 Nov 2011

as in exercise 44 ... $(x_i - 1)$). ↜ by setting $x = 1$ in exercise 40 and obtaining $\prod_{k\ne i}(x_k - 1)/x_i \prod_{k\ne i}(x_k - x_i)$. After multiplying numerator and denominator by $x_i - 1$, we can sum on i by applying exercise 33 with $r = 0$ to the $n + 2$ numbers $\{0, 1, x_1, \ldots, x_n\}$.

Page 476 replacement for answer 4 — 22 Dec 2013

4. By part (f), $x \le \lceil x \rceil < x + 1$; hence $-x - 1 < -\lceil x \rceil \le -x$; use part (e).

Page 485 replacement for answer 22 — 10 Sep 2013

22. Assume that $n > 0$. The kth term is $r/(r-tk)$ times

$$\frac{1}{n!}\binom{n}{k}\prod_{0\le j<k}(r-tk-j)\prod_{0\le j<n-k}(n-1-r+tk-j)$$
$$=\frac{(-1)^{k-1}}{n!}\binom{n}{k}\prod_{0\le j<k}(-r+tk+j)\prod_{k\le j<n}(-r+tk+j)$$

and the two products give a polynomial of degree $n-1$ in k after division by $r-tk$. So the sum over k is zero by Eq. (34).

Page 485 lines 2 and 3 of answer 25 — 07 Oct 2013

(Alternatively ... $x = 1$.) We have ↝ When w is sufficiently small, we have

Page 491 replacement for last line of answer 67 — 18 Feb 2012

$\binom{n}{k} \ge (\frac{(n-k+1)e}{k})^k \frac{1}{ek}$, which is less memorable (but often sharper) than $\binom{n}{k} \ge (\frac{n}{k})^k$.

Page 493 new answer for section 1.2.7 — 11 Feb 2013

25. $H_n^{(0,v)} = \sum_{k=1}^n H_n^{(v)}$ and $H_n^{(u,0)} = H_n^{(u-1)}$; so the identity generalizes (8). [See L. Euler, *Novi Comment. Acad. Sci. Pet.* **20** (1775), 140–186, §2.]

▸**Page 503** last line of answer 3 — 05 Mar 2013

1937 ↝ 1927

Page 504 last line of answer 8 — 18 Feb 2012

$\binom{cn^2}{n}/c^n\binom{n^2}{n}$ ↝ $\binom{cn^2}{n}/(c^n\binom{n^2}{n})$

Page 514 replacement for line 5 of the program — 30 Apr 2013

```
3H      ENT2 9*8-8,1   Start at row 9.
```

▸**Page 514** replacement for lines 19 and 20 of the program — 10 Mar 2013

```
PHASE2 ENT3 9*8       At this point rA = min_j C(j)
3H     ENT2 0,3       Prepare to search a row.
```

▸**Page 552** in answer 12 — 30 Jun 2012

line 1: $29p$ ↝ $27p$
line 3: $\frac{3}{4}$ ↝ 78%

Page 570 new sentence to follow line 1 of answer 30 — 08 Oct 2013

Thus `LOC(T)` = `HEAD`, and `HEAD$` is the first node of the binary tree in symmetric order.

Page 570 line −3 of answer 30 — 08 Oct 2013

the algorithm of exercise 21 ↝ Algorithm U in answer 21

Page 624 through page 626 — 07 Apr 2012

[replace the notation $(R \Longrightarrow x;\ y)$ by $(R?\ x\colon\ y)$ in eleven places]

Page 628 new entries for Appendix C — 17 Apr 2011

Program 1.2.10M, 145, 186.
Program 1.4.3.1M, 204–211, 530.
Program 2.1A, 236.
Program 2.1B, 535.
Program 2.3.1S, 325.

▸**Page 629** in Appendix C — 17 Apr 2011

Algorithm 2.4B′, 606. ↯→ Algorithm 2.4B′, 605. Algorithm 2.4B″, 606.
Algorithm 2.5G, 613. ↯→ Algorithm 2.5G, 613–614.

Page 630 and following — 01 Jan 2011

Miscellaneous changes to the existing index of Volume 1 are collected here, including corrections and amendments to the old entries as well as new entries that are occasioned by the new material. Thus, the lines of the full index that have changed serve also as an index to the present document. However, when a correction or amendment has caused an old index entry to be deleted, the deletion is usually not indicated.

Bernstein, Sergei Natanovich (Бернштейн, Сергей Натанович), 104.
Carroll, Lewis (= Dodgson, Charles Lutwidge), 465.
Chebyshev (= Tschebyscheff), Pafnutii Lvovich (Чебышебъ, Пафнутій Львовичъ = Чебышев, Пафнутий Львович), inequalities, 98, 104, 472.
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Zedan, Hussein Saleh Mamoud (حسين صالح محمود زيدان), 576.