Mandatory Assignment (Option A)

INF 170: Modeling and Optimization

Fall 2019

We consider a shipping company, operating either within tramp shipping or industrial shipping, which operates a heterogeneous fleet of ships. The ships may have different capacities, service speeds and cost structures, and due to previous assignments, they become available for new assignments at different times and different locations. In addition, some ships may be prevented from carrying certain cargoes due to incompatibilities.

If a cargo is assigned to a ship, the ship must load the cargo in its corresponding loading port and later unload the cargo in its unloading port. All loading and unloading operations must be performed within a time interval that is specific to that operation for a given cargo. Ship capacities must not be violated. The shipping company may choose to use spot charter to transport a given cargo, instead of carrying it using one of its own ships.

Below is a mathematical formulation of the industrial and tramp routing and scheduling problem, where the set of ships is denoted by V, and the capacity of ship $v \in V$ is K_v. If we let i denote a cargo with the relevant amount of Q_i, there is a node i corresponding to the loading port and a node $i+n$ corresponding to the unloading port, with n being the number of cargoes. The set of loading nodes is denoted by N^P and the set of unloading nodes is denoted by N^D.The set of nodes that can be visited by ship v is N_v and this set includes an origin node $o(v)$ and an artificial destination node $d(v)$. The set of arcs that ship v can traverse is A_v. We also introduce the shorthand $N_v^P = N^P \cap N_v$ for loading nodes that can be visited by ship v and $N_v^D = N^D \cap N_v$ for unloading nodes that can be visited by ship v.

Each node has a time window $[\underline{T}_i, \bar{T}_i]$. The cost of sailing from i to j using ship v is C_{ijv} and the associated travel time is T_{ijv}. The time at which service begins at node i using ship v is t_{iv} and l_{iv} is the total load on board after completing service at node i using ship v. The variables x_{ijv} are binary flow variables, indicating whether ship v moves directly from node i to node j. Binary variables y_i indicate whether cargo i is transported by the available vessel fleet. If the cargo is not transported with the available vessel fleet, a cost C_i^S incurred. For industrial shipping, this corresponds to the cost of using spot charter to transport the cargo. For tramp shipping the cost represents either the loss of revenue from not being able to carry an optional cargo or the cost of using spot charter to transport a mandatory contract cargo. The mathematical formulation of the problem becomes as follows.

$$Min \qquad Z = \sum_{v \in V} \sum_{(i,j) \in A_v} C_{ijv} x_{ijv} + \sum_{i \in N^p} C_i^S y_i \tag{1}$$

subject to

$$\sum_{v \in V} \sum_{(i,j) \in A_v} x_{ijv} + y_i = 1, \qquad i \in N^P \tag{2}$$

$$\sum_{j \in N_v \neq o(v)} x_{o(v)jv} = 1, \qquad v \in V \tag{3}$$

$$\sum_{(i,j) \in A_v} x_{ijv} - \sum_{(j,i) \in A_v} x_{jiv} = 0, \qquad v \in V, i \in N_v \backslash \{o(v), d(v)\} \tag{4}$$

$$\sum_{j \in N_v \neq d(v)} x_{jd(v)v} = 1, \qquad v \in V \tag{5}$$

$$l_{iv} + Q_j - l_{jv} \leq K_v(1 - x_{ijv}), \qquad v \in V, j \in N_v^P, (i,j) \in A_v \tag{6}$$

$$l_{iv} - Q_j - l_{(n+j)v} \leq K_v(1 - x_{i(n+j)v}), \qquad v \in V, j \in N_v^P, (i, n+j) \in A_v \tag{7}$$

$$0 \leq l_{iv} \leq K_v, \qquad v \in V, i \in N_v^P \tag{8}$$

$$t_{iv} + T_{ijv} - t_{jv} \leq (\bar{T}_i + T_{ijv})(1 - x_{ijv}), \qquad v \in V, (i,j) \in A_v \tag{9}$$

$$\sum_{(i,j) \in A_v} x_{ijv} - \sum_{(n+i,j) \in A_v} x_{(n+i)jv} = 0, \qquad v \in V, i \in N_v^P \tag{10}$$

$$t_{iv} + T_{i(n+i)v} - t_{(n+i)v} \leq 0, \qquad v \in V, i \in N_v^P \tag{11}$$

$$\underline{T}_i \leq t_{iv} \leq \bar{T}_i, \qquad v \in V, i \in N_v \tag{12}$$

$$y_i \in \{0,1\}, \qquad i \in N^p \tag{13}$$

$$x_{ijv} \in \{0,1\}, \qquad v \in V, (i,j) \in A_v \tag{14}$$

Formulate the model in AMPL and find the optimal solution using the attached data file as an instance.