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Choice of routes in congested traffic networks: Experimental tests of the Braess Paradox

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Abstract

The Braess Paradox consists of showing that, in equilibrium, *adding* a new link that connects two routes running between a common origin and common destination may *raise* the travel cost for each network user. We report the results of two experiments designed to study whether the paradox is behaviorally realized in two simulated traffic networks that differ from each other in their topology. Both experiments include relatively large groups of participants who independently and repeatedly choose travel routes in one of two types of traffic networks, one with the added links and the other without them. Our results reject the hypothesis that the paradox is of marginal value and its force diminishes with experience. Rather, they strongly support the alternative hypothesis that with experience in traversing the networks financially motivated players converge to choosing the equilibrium routes in the network with added capacity despite sustaining a sharp decline in earnings.

JEL classification: C72; C92; D74

Keywords: Braess Paradox; Congested traffic networks; Route choice; Experiments

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valuable contribution to our paper. Professor Selten has made important suggestions about data analysis and pointed out additional equilibria that we had missed in our original manuscript. Additionally, the proofs of the assertions about the equilibrium solutions in Appendix B are mostly attributed to him. We acknowledge his contribution and thank him for it. We also thank Ryan O. Murphy and Maya Rosenblatt for their assistance in data collection.

1. Introduction

Transportation and communication networks are among the best examples of frequently used physical networks in which vertices (nodes) correspond to locations in space and edges (links) to connections between them. They provide the infrastructure to conduct much of our social and economic activities. It would seem rather natural to believe that increasing the capacity of an existing network or adding one or more new edges to traffic or communication networks would definitely not worsen and most likely improve efficiency. Braess (1968) has shattered this deeply entrenched belief by demonstrating that, paradoxically, adding a link that connects two alternative routes in parallel running between a common origin and common destination may *raise* the total travel cost of *all* the network users. This phenomenon, subsequently labeled the *Braess Paradox* (BP), has stimulated a rapidly growing body of research in transportation science, computer science, and applied probability. Researchers have attempted to classify networks in which the addition of one or more links could degrade network performance (Frank, 1981; Steinberg and Zangwill, 1983), discovered new paradoxes (Arnott et al., 1993; Cohen and Kelly, 1990; Dafermos and Nagurney, 1984; Fisk, 1979; Pas and Principio, 1997; Smith, 1978; Steinberg and Stone, 1988), proved that detecting the BP is algorithmically hard (Roughgarden, 2001), and quantified the degree of degradation in network performance due to unregulated traffic (Koutsoupias and Papadimitriu, 1999; Roughgarden and Tardos, 2002).

Steinberg and Zangwill concluded an early analysis of the BP with the claim that "under reasonable assumptions, Braess' Paradox is not a curious anomaly but in fact might occur quite frequently (1983, p. 317)." However, no direct empirical evidence has been forthcoming to buttress this claim. In a postscript to his exposition of the BP more than 35 years ago, Murchland (1970) remarked that Knödel had noted that major road investments in the center of the city of Stuttgart had failed to yield the benefits expected. This briefly mentioned case is the only example that has subsequently been cited by Kelly (1991), Roughgarden and Tardos (2002), and others. The New York Times also hinted at the counterintuitive consequences of road closures in an article with a provocative title that appeared on Christmas Day, 1990: "What if they closed 42nd street and nobody noticed?"

This sketchy and mostly anecdotal evidence is clearly insufficient to counter arguments about the importance and relevance of the BP to real transportation and communication problems. A claim can be made that the BP is no more than a theoretical curiosity, that it is too simple to model real-life traffic and communication networks, and that examples closer to the complexity of real life would prevent those kinds of paradoxes from being realized. Another argument is directed not so much against the realism of the model but against the assumed behavior of the network users. In formulating the BP, network users are viewed as independent "selfish" agents participating in a noncooperative game, where each agent wishes to choose a path from a common origin to a common destination that minimizes her travel cost. The paradox is based on an equilibrium analysis of a weighted traffic network both before and after one or more edges are added to it. But real network users, it may be claimed, may quickly learn to avoid traversing the new edges in an attempt to escape the adverse effects of the BP.

As empirical evidence about the occurrence of the BP is very difficult to come by, the approach that we pursue in the present study is to simulate simple traffic networks that are susceptible to the BP in the laboratory, have players choose routes before and after one or more edges are added to the original network, and find out whether systematic and replicable patterns of behavior emerge. If they do, it is of significant interest to determine whether they support the argument that the BP is of theoretical value only and its force, if at all evident, diminishes with experience or, alternatively, that they support the equilibrium analysis that gives rise to the BP.

Rooted in the methodology of experimental economics, this approach is not common in transportation research. We are familiar with only four experimental studies of the effects of traffic congestion that consider strategic interaction. The first is an experiment by Schneider and Weimann (2004) that was designed to test a simple model of bottleneck congestion on a single route in a rush-hour situation. The model was originally proposed by Arnott et al. (1990, 1993). The second is an experimental study by Gabuthy et al. (2004), who generalized the analysis of Arnott et al. (1990) to traffic networks with a single origin and single destination connected by two routes. The experimental evidence is mixed; Schneider and Weimann report evidence in support of equilibrium departure time in their first but not second experiment, whereas Gabuthy et al. report no support. The third study is due to Selten et al. (2004), who conducted laboratory experiments of a day-by-day route choice game with two parallel roads but no crossroad. Unlike the two studies by Gabuthy et al. and Schneider et al., the experiment by Selten et al. is not concerned with endogenous departure time but with route choice. They report aggregate road choices that are accounted for quite well by the Nash equilibrium predictions and large fluctuations around the mean choice frequencies that do not seem to diminish with experience. In a fourth study, Helbing (2004) repeated the experiments of Selten et al. with more iterations, and further tested additional experimental conditions in an attempt to better understand the reasons for the fairly large fluctuations around the mean choice frequencies. None of these studies is concerned with the counterintuitive implication of the BP that is the focus of the present study.

The rest of the paper is organized as follows. Section 2 introduces terminology and illustrates the BP in a network with the simplest possible form (called the *Minimal Critical Network* by Penchina, 1997). Section 3 reports the results of an experiment designed to examine route choice in the iterated Minimal Critical Network game. Section 4 extends the investigation to a topologically much richer network with three routes. The paradox in this more complex network is created by adding *two* new links to the original network, thereby giving rise to a total of *five* routes. Section 5 concludes.

2. The Braess Paradox

2.1. Notation and terminology

We consider networks with a common origin O and common destination D that are modeled as a directed graph $G = (V, E)$ with vertex (node) set V, edge (link) set E, and a set $K \subseteq V \times V$ of origin-destination (OD) pairs. We consider a finite, commonly known, and relatively small number of users, n, in contrast to the more common case discussed in the transportation literature that assumes infinitely many users. The traffic in the road network is described by the number f_{ij} of cars (users) moving along the edge (i, j) from vertex i to vertex j. The cost for each user of traversing from i to j along the link (i, j) when the flow on this link is f_{ij} is denoted by $c_{ij}(f_{ij})$. Travel costs are typically measured by time spent in travel or gasoline consumed. It is assumed that the cost of traveling on edge (i, j) at a given level of traffic is the same for all the

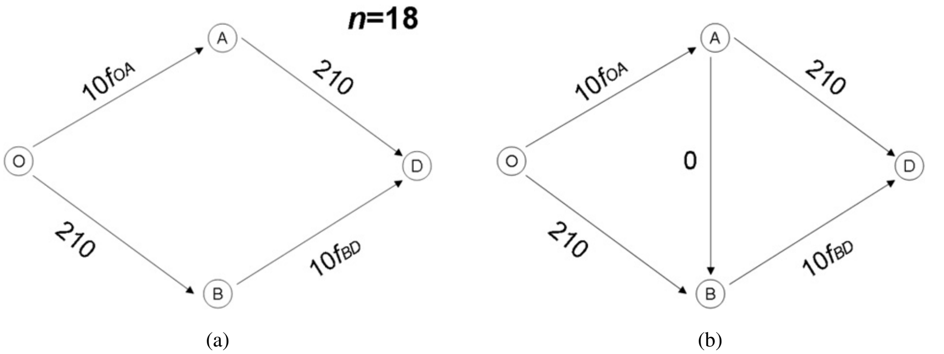

Fig. 1. Basic and augmented networks for Games 1A and 1B in Experiment 1.

users traversing this edge. Travel cost is the sum of the edge costs over all edges in the route (also called *path*) from O to D.

Each of the n users is assumed to independently seek a path that minimizes her travel cost. In equilibrium, the n users are distributed over one or more routes so that a unilateral change of path by any one user does not decrease the travel cost for that user, given that all other $n-1$ users do not change their routes. In both experiments, we consider road networks with affine costs, where for each edge $(i,j) \in E$, $c_{ij} = a_{ij} f_{ij} + b_{ij}$ for some $a_{ij}, b_{ij} \geqslant 0$. The *fixed* component b_{ij} can be interpreted as the minimum time to traverse edge (i,j) with no traffic, whereas the *variable* component a_{ij} corresponds to the effect of congestion. Affine cost functions are chosen because they provide good approximation to reality (Steinberg and Zangwill, 1983) and are most easily explained to the participants in network experiments. Moreover, almost all the papers, starting with Braess, have focused on this case.

The original game (hereafter called the *basic game*) has the same topology as the game studied by Braess but a different cost structure. It consists of a simple network with four vertices, four edges, and an anti-symmetric (Penchina, 1997) arrangement of the edges (Fig. 1a). One path (O–A–D) consists of an edge with a high fixed cost and low congestion cost starting at the origin followed by an edge with no fixed cost and high congestion cost. On the second path (O–B–D), the edges have identical costs to those of the first, but are arranged in a reverse order. In the *augmented game* (Fig. 1b), these edges are connected by a transversal edge (crossroad, bridge) of low fixed cost and low congestion cost connecting the end of the edge with no fixed cost in one path to the beginning of the edge with no fixed cost in the other path (edge (A–B) in Fig. 1b).

2.2. *Minimal critical network*

The essence of the BP comes from six qualitative properties (Penchina, 1997). The first three are necessary and sufficient for the BP to occur.

1. The network G must have both fixed and variable user costs.
2. The two paths in the basic network must have an opposite order of appearance of the edges dominated by fixed vs. variable costs.
3. The fixed cost on the "bridge" in the augmented network must be smaller than the difference in fixed costs between the edges dominated by fixed costs and those dominated by variable costs.

Three additional properties were proposed by Penchina to simplify the analysis:

4. Zero user cost on the bridge.
5. The two congested edges have identical linear variable user cost functions and zero fixed cost.
6. The two uncongested edges have identical fixed user cost functions and zero variable cost.

Networks satisfying these properties are called *Minimal Critical Networks* (Penchina, 1997). The one we study experimentally in Section 3 (Figs. 1a and 1b) has the cost functions: $c_{OA} = 10f_{OA}$, $c_{BD} = 10f_{BD}$, $c_{AD} = c_{OB} = 210$, and $c_{AB} = 0$. This cost structure satisfies all six properties above.

To illustrate the BP, assume the cost structure in Fig. 1 with $n = 18$. Consider first the basic network in Fig. 1a (Game 1A). There are $\frac{18!}{9!9!} = 48{,}620$ pure-strategy equilibria with 9 users traversing route $(O–A–D)$ and 9 others traversing route $(O–B–D)$. The cost for each user is $210 + 10 \times 9 = 300$. There is additional symmetric mixed-strategy equilibrium with an associated travel cost of 305 where each route is chosen with equal probability. The augmented network in Fig. 1b (Game 1B) has a unique pure-strategy equilibrium where all users choose the dominant strategy $(O–A–B–D)$ for travel cost of 360. The counterintuitive feature of this example is that the improvement of the network by adding a new link causes every user to be worse off by 20 percent of the original travel cost. Commenting on this effect, Cohen writes, "Adam Smith's Invisible Hand leads everyone astray (1988, p. 583)."

Even a stronger effect in this network is obtained with $n = 20$. In equilibrium, 10 users traverse route $(O–A–D)$ and 10 others route $(O–B–D)$ for a total individual travel cost of 310. When the cost-free edge $(A–B)$ is added (Fig. 1b), in equilibrium all 20 users traverse the route $(O–A–B–D)$ for total travel cost of 400, an increase of 29 percent. Assuming a linear cost structure and continuum of users, so that each user controls a negligible fraction of the overall traffic, Roughgarden and Tardos proved that the degradation of the network performance by the lack of central authority—dubbed the "price of anarchy" by Papadimitriu (2001)—cannot exceed $4/3$. As explained below, a major feature of our experimental design considerably enhances the adverse effect of the BP by subtracting the individual travel cost from a fixed endowment that assumes the *same value* in both Games 1A and 1B.

3. A two-route symmetric network with one additional edge

Experiment 1 has two major purposes. The first is to test for the occurrence of the BP in the Minimal Critical Network. The coordination problem faced by the $n = 18$ players in Game 1A is far from trivial as, under pure-strategy equilibrium play, they have to coordinate on one of multiple equilibria that are neither symmetric nor Pareto rankable. Because equilibrium play is only likely to be reached with considerable experience, Games 1A and 1B were each iterated 40 times.

The counterintuitive feature of the BP is that the *addition* of a cost-free edge causes every user to be *worse off*. An alternative way of viewing this paradoxical result is to start with the augmented network in Game 1B, *delete* the cost-free edge $(A–B)$, and note that, in equilibrium, all users *benefit* from the degradation of the network. Phenomenologically, network users may perceive these two alternative formulations ("framings") of the BP—one in terms of loss and the other in terms of gain—quite differently. The second purpose of Experiment 1 is to compare these two alternative framings of the BP.

3.1. Method

3.1.1. Participants

The participants were 108 undergraduate and graduate students at the University of Arizona, who volunteered to participate in a computer-controlled experiment for payoff contingent on performance. Males and females participated in almost equal numbers. The participants were divided into 6 groups (sessions) of 18 members each. Three groups participated in Condition ADD and three others in Condition DELETE (see below). A session lasted about 90 minutes. Excluding a $5 show-up bonus, the mean payoff across the six sessions was $20.63.

3.1.2. Procedure

All six sessions were conducted at a large computerized laboratory with multiple terminals located in separate cubicles. Upon arrival at the laboratory, each player was asked to draw a marked chip from a bag that determined her seating. Players were then handed written instructions that they read at their own pace. Questions about the procedure were answered individually by the experimenter.

Each session was divided into two parts. Specific instructions were handed to the players at the beginning of each part. The instructions for Part I (see Appendix A) displayed the traffic network in Game 1A and explained the procedure for choosing one of the two routes in this game.[1] The instructions for Part II (see Appendix A) displayed the traffic network in Game 1B and explained the procedure for choosing one of the three routes in this game. Condition ADD was structured as follows. The players were first handed the instructions for Part I, and then played Game 1A for 40 identical trials without being instructed on Game 1B.[2] After completing Part I, the same players were handed a new set of instructions for Part II, and then asked to play Game 1B for 40 additional trials (a total of 80 trials for the session). Condition DELETE was the same with the exception that the order of presentation of Parts I and II was reversed.

The instructions for Part I graphically displayed the traffic network in Game 1A, explained the cost functions, and illustrated the computation of the travel cost for links with either variable or fixed costs. At the beginning of each trial, each player was given an endowment of 400 travel units (points). The payoff for the trial was computed separately for each player by subtracting her travel cost from her endowment. To choose one of the two routes—$(O–A–D)$ or $(O–B–D)$—the player had to click on the two links of this route and then press a "confirm" button. Clicking with the mouse on a link changed the link's color on the screen to indicate the player's choice. The player was then asked to verify her choice of route by clicking on a "Yes" button. After all the group members independently registered and subsequently verified their route choices, a new screen was displayed with information about the route chosen by the player, number of players choosing each of the two routes, and player's payoff for the trial.

The instructions for Part II displayed the network in Game 1B, explained the cost functions, and illustrated the computation of the travel cost for edges with either variable or fixed cost. Because Game 1B, unlike Game 1A, allows for negative externalities, these were explained in detail (see Appendix A).

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[1] The original subject instructions refer to Games 1A and 1B as Games 1 and 2, respectively. We have changed the labels in Appendix A to prevent confusion.

[2] The subjects were only instructed that they would play another game after completing Game 1A. No information about the topology and costs of Game 1B was given at that stage.

After Part II was completed, the players were paid their earnings in four randomly chosen trials from the forty trials in Part I and four additional trials randomly drawn from the forty trials completed in Part II. A randomly selected participant publicly drew the eight payoff trials at the end of the session. Points were accumulated across the eight payoff trials and converted to money at the exchange rate of 25 points = \$1.00. Players were paid their earnings individually and dismissed from the laboratory.

3.1.3. Main features of the design

Four major features of the design warrant brief discussion. First, no communication between players was possible. In accordance with the assumptions underlying the BP, the value of n was commonly known. Second, the experiment was conducted under full information; at the end of each trial the players were informed of the distribution of network users across all possible routes. This feature was introduced to facilitate learning over iterations of the stage game. Third, we opted for a within- rather than between-subject design so that the *same* players would experience the effect of adding (Condition ADD) or deleting (Condition DELETE) the cost-free edge (A–B). This design feature was introduced to compare the effects of the two alternative framings of the BP. Fourth, and perhaps most importantly, the same endowment of 400 travel units was assigned to each subject in both Games 1A and 1B. Under pure-strategy equilibrium play by all group members, this would have resulted in payoffs of 100 and 40 travel units per trial in Games 1A and 1B, respectively. One might expect—we certainly did—differences in behavior between the two experimental conditions. If the players in Condition ADD were to reach equilibrium in Game 1A, they would be expected to resist being drawn into choosing route (O–A–B–D) in Game 1B and thereby watch their payoff plummeting to 40 percent of their earlier earnings. In contrast, having no prior experience with Game 1A, players in Condition DELETE would be expected to converge quicker to route (O–A–B–D) in Game 1B and thereby increase their earnings by a factor of 2.5.

3.2. Results

This section proceeds as follows. We start by comparing the two conditions to each other. Not finding differences between the ADD and DELETE conditions, we combine the six sessions together. We then continue to test the implications of the BP on the aggregate level. We complete this section by discussing sequential dependencies and individual differences.

3.2.1. Differences between conditions

The purpose of this section is to test for differences between Conditions ADD and DELETE. If significant differences are found, then each condition has to be presented and analyzed separately. If not, then the data of the two conditions can be amalgamated. For this purpose, the following five statistics were computed for each of the six sessions. In Game 1A, we first counted the number of times (out of 40) that each player chose route (O–A–D) (the frequency of route (O–B–D) is obtained by subtraction from 40). We then computed the mean and standard deviation of route (O–A–D) choices across the members of each group (column 2 of Table 1). Table 1 (column 2) shows that the six means are very close to one another, ranging between 20.06 and 21.83. The corresponding standard deviations (ranging between 7.68 and 11.36) are relatively large suggesting considerable variation in route choice between players.

Since the network in Game 1B includes three rather than two routes, we computed the mean route choices for two of the three routes, namely, routes (O–A–D) and (O–B–D) (columns 4

Table 1
Mean values of summary statistics by game (Games 1A and 1B), session (1–3), and condition (ADD vs. DELETE) in Experiment 1

Statistic	Game 1A		Game 1B		
	Route (O–A–D) Mean No. of choices	Mean No. of switches by player	Route (O–A–D) Mean No. of choices	Route (O–B–D) Mean No. of choices	Mean No. of switches by player
Session 1	20.06	12.94	3.22	2.89	9.94
Cond. ADD	(8.02)	(5.14)	(2.64)	(2.45)	(7.41)
Session 2	21.83	11.56	4.56	3.39	10.78
Cond. ADD	(8.87)	(5.28)	(3.76)	(1.79)	(6.3)
Session 3	20.28	10.89	3.61	3.28	9.44
Cond. ADD	(10.75)	(7.28)	(1.94)	(2.56)	(5.23)
Session 1	20.22	11.33	3.39	3.67	9.39
Cond. DELETE	(11.36)	(7.22)	(2.97)	(3.63)	(6.61)
Session 2	20.06	11.72	4.06	2.72	8.78
Cond. DELETE	(11.10)	(8.37)	(5.58)	(1.96)	(6.48)
Session 3	20.11	14.78	4.11	3.28	11.28
Cond. DELETE	(7.68)	(5.20)	(3.77)	(2.61)	(9.29)

Standard deviations in parentheses.

and 5 of Table 1). Once again, the mean choice frequency for route (O–A–B–D) is obtained by subtraction from 40. Define a *switch* for some player i, if i chooses two different routes on trials t and $t+1$, $t = 1, 2, \ldots, 39$. Clearly, a player in Game 1A can only switch from one route to another, whereas in Game 1B she can switch to any of the two other routes. Columns 3 and 6 of Table 1 present the means and standard deviations of the number of switches per player (out of 39) for Games 1A and 1B, respectively. Taken together, the results displayed in all the five columns of Table 1 provide strong evidence of no differences between the two conditions (in fact, all six sessions are very similar).

In additional analyses, we computed similar statistics for each round. For example, we counted the number of players who chose route (O–A–D) in each round. We then computed the mean of these frequencies for each block of 10 consecutive rounds, and compared the six sessions to one another in terms of the trend, if any, across the four blocks. Once again, no difference between the two conditions is observed. The mean number of players (out of 18) who chose route (O–A–D) in Game 1B in Condition ADD decreased from 2.8 in block 1 to 0.4 in block 4 in session 1, from 4.1 to 0.7 in session 2, and from 3.2 to 0.6 in session 3. The corresponding means in Condition DELETE were (2.6, 0.4), (2.6, 1.1), and (3.0, 1.1). The results for route (O–B–D) in Game 1B were practically the same.

Recall that Condition ADD presented Game 1A in Part I, whereas Condition DELETE presented Game 1A in Part II. Having failed to detect any differences between the two conditions, the data from the six sessions can be combined. For the rest of the results section, we will present data for each condition separately as well as for both conditions together. In order to gain statistical power, most of the tests will be conducted on the six sessions combined.

3.2.2. Aggregate route choices

We proceed to test the implications of the BP. For each trial separately, we counted the number of players who chose route (O–A–D) in Game 1A. These frequencies were then averaged across players, trials, and sessions. Similar means were computed for route (O–B–D) in Game 1A and

Table 2
Means and standard deviations of number of players (out of 18) choosing each route by game (Games 1A and 1B) and condition (Condition ADD vs. DELETE) in Experiment 1

Condition	Game 1A (two routes)		Game 1B (three routes)		
	$(O–A–D)$	$(O–B–D)$	$(O–A–D)$	$(O–B–D)$	$(O–A–B–D)$
ADD	9.04	8.96	1.70	1.49	14.82
	(2.15)	(2.15)	(1.86)	(1.68)	(2.79)
DELETE	9.08	8.92	1.75	1.45	14.82
	(2.08)	(2.08)	(1.51)	(1.49)	(2.27)
BOTH	9.02	8.98	1.72	1.47	14.82
	(2.11)	(2.11)	(1.64)	(1.40)	(2.54)
Mixed-strategy equilibrium	9	9	0	0	18
	(2.12)	(2.12)			

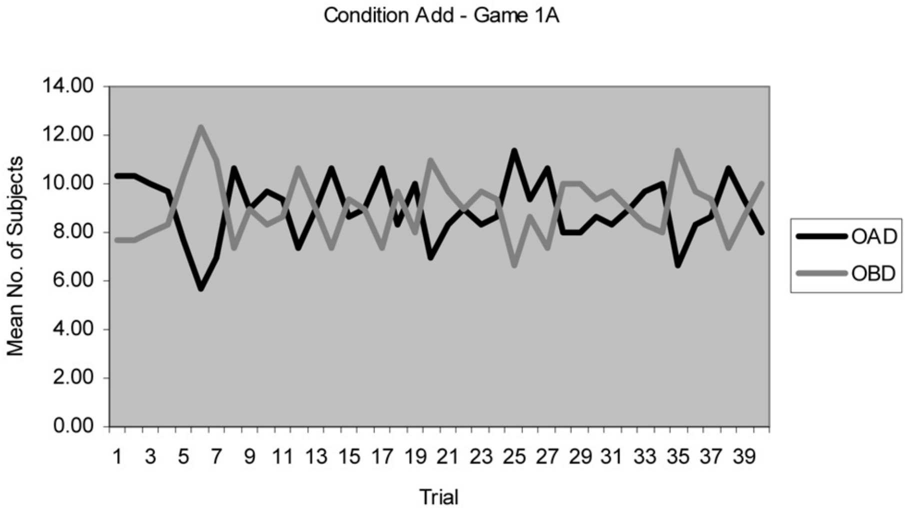
Standard deviations in parentheses.

for each of the three routes in Game 1B. Table 2 presents the means and standard deviations for each condition separately. The bottom row presents the equilibrium predictions. The standard deviations (2.12) in the bottom row refer to the symmetric mixed-strategy equilibrium where each player chooses routes $(O–A–D)$ and $(O–B–D)$ in Game 1A with equal probability.

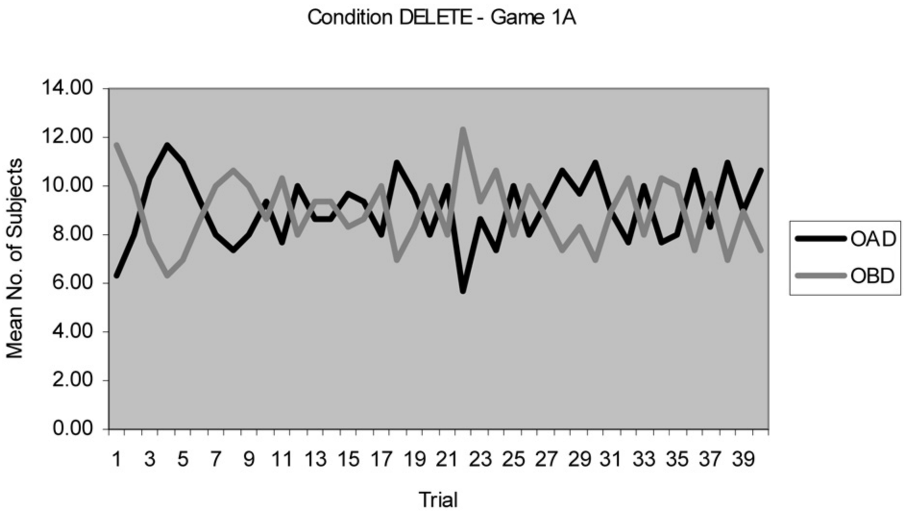
Three observations about Table 2 are in order. First, for each of the two conditions, it is evident (columns 2 and 3) that the two routes in Game 1A were chosen equally likely. Second, Table 2 shows that routes $(O–A–D)$ and $(O–B–D)$ in Game 1B were jointly chosen, on average, on 18 percent of the trials. A Wilcoxon signed ranks test shows that the number of players who chose each of the two original routes in Game 1B is significantly lower than in Game 1A ($z = −2.2$, $p < 0.05$, two-tailed test, for route $(O–A–D)$; $z = −2.2$, $p < 0.05$, two-tailed test, for route $(O–B–D)$). The third observation concerns the standard deviations reported for Game 1A. These seem to be inordinately close to the theoretical standard deviations under symmetric mixed-strategy equilibrium play. In fact, the symmetric mixed-strategy equilibrium accounts for the *aggregate* route choices in Game 1A almost perfectly. Under the symmetric mixed-strategy equilibrium, each player randomizes between the two routes with equal probabilities on each round. If this assumption holds, then the number of players traversing route $(O–A–D)$ in each round is sampled from a binomial distribution with $n = 18$ and $p = 0.5$. Therefore, we may calculate the expected distribution of this variable and compare it to the actual distribution in all rounds of all sessions. Our results show that we cannot reject the null hypothesis that the mixed-strategy equilibrium holds. This is supported by a chi-square test ($\chi^2(16) = 10.341$, $p = 0.848$).

3.2.3. Dynamics

Turning next to the dynamics of play, Figs. 2a and 2b display the mean number of players choosing each of the two routes in Game 1A. Consistent with Table 2, we observe no difference in the mean choice of the two routes. Each of the two means hovers over 9 but is seldom equal to 9. As shown above, there is no convergence to pure-strategy equilibrium. One possible hypothesis that we entertained is that subjects start by randomizing their choice of routes in Game 1A with equal probabilities, but switch to a pure-strategy equilibrium—with the *same* 9 subjects choosing each of the two routes—once they happen to stumble upon it. After all, why should a player deviate from equilibrium behavior and thereby increase her travel cost from 300 to 310? The results did not support our expectation. In fact, across all six sessions, we find only five runs of two consecutive $(9, 9)$ splits, and two runs of three consecutive $(9, 9)$ splits. Informal post-experimental interviews of some of the participants indicate the following reason for such



(a)



(b)

Fig. 2. Mean number of players choosing each route in Game 1A: Conditions ADD and DELETE.

deviations. If a $(9, 9)$ split is reached on trial t, a forward looking player might wish to deviate from route j to route k on trial $t + 1$, incurring a ten percent decrease in earnings, because of her expectation that two or more players would deviate from the more heavily chosen route k to route j on trial $t + 2$. If this happens, she may recuperate her loss by choosing route k also on trial $t + 2$. As we show below, this forward-looking "sophisticated" strategy did not pay off. A second reason (see below) is that some subjects do, indeed, randomize their strategies.

Turning next to Game 1B, the most critical finding of Experiment 1 is exhibited in Figs. 3a and 3b that display the mean number of players choosing each of the three routes in Game 1B. Fig. 3a shows the results for Condition ADD and Fig. 3b for Condition DELETE. In both conditions,

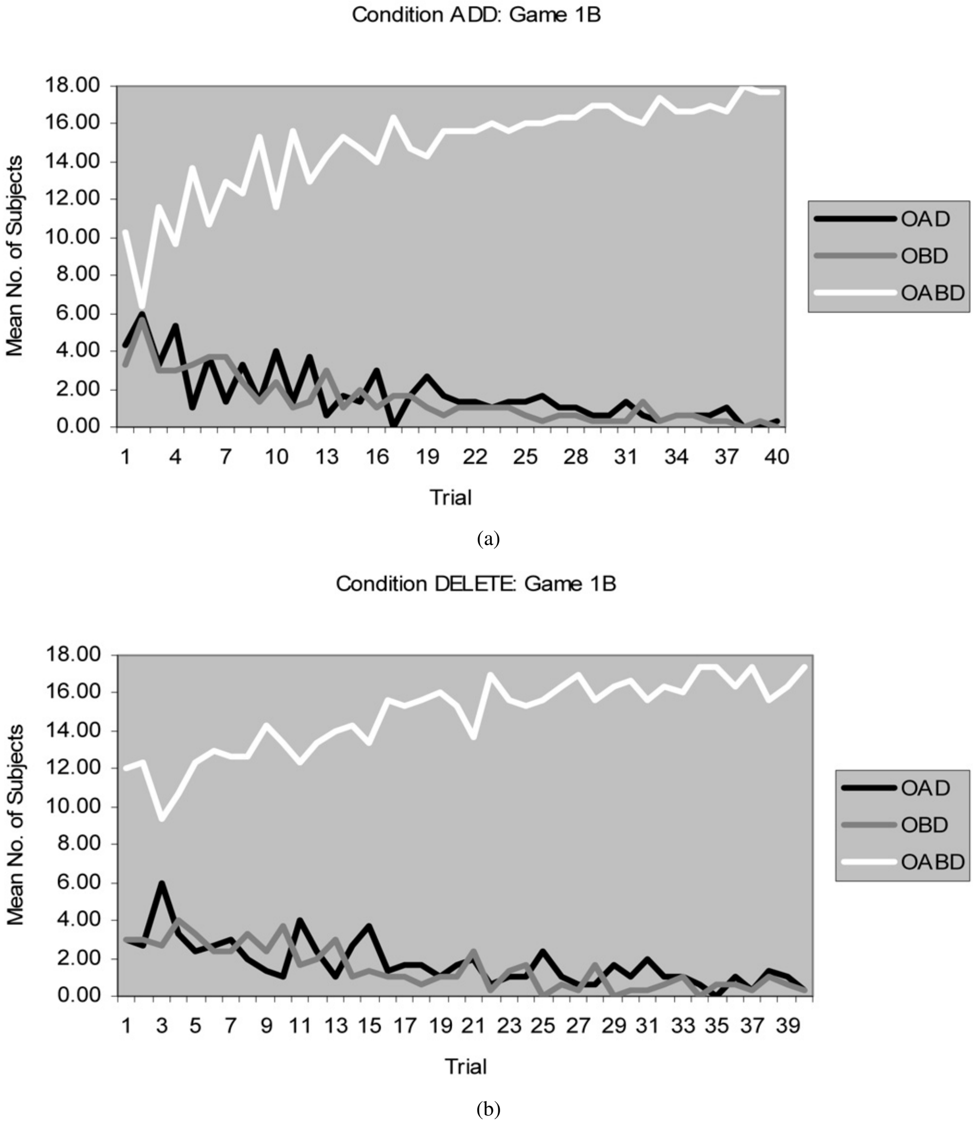


Fig. 3. Mean number of players choosing each route in Game 1B: Conditions ADD and DELETE.

almost 2/3 of the players already chose the dominant route (O–A–B–D) during the first five trials. With experience in traversing the traffic network in Game 1B, the frequencies of choice of the two dominated strategies converged to zero, and the frequency of choice of route (O–A–B–D) converged to 18. Figs. 3a and 3b show that all 40 trials were required to reach convergence. Results not reported here show that a few players struggled to avoid choosing the "bridge" in Game 1B. But with no communication possible, their signals were not heeded by the other players as the attraction of the Pareto deficient equilibrium strategy proved too strong to resist.

Convergence to pure-strategy equilibrium in Game 1B (see Appendix B) implies that deviations from equilibrium decline over time. To formally test this implication, we define deviation from equilibrium as the mean absolute difference between the expected and observed frequency of players choosing each route computed across all routes. Then, for each session separately, we computed a rank-order correlation (Kendall's tau) between the round number and the deviation score. In complete agreement with Fig. 2, three of the six correlations in Game 1A were negative, three positive, and none exceeded 0.28: −0.19, 0.27, 0.08, 0.16, −0.28, and −0.15, for sessions 1, 2, 3, 4, 5, and 6, respectively. In contrast, all the six correlations for Game 1B were negative, relatively high, and statistically significant: −0.62, −0.70, −0.61, −0.66, −0.63, and −0.51. On average, these correlations do not differ from zero. A Wilcoxon signed-ranks test reveals that the correlations are higher for Game 1A than for Game 1B ($z = -2.02$, $p < 0.05$, two-tailed test). The negative correlations for Game 1B are in complete agreement with the aggregate results depicted in Fig. 3 that display convergence to pure-strategy equilibrium.

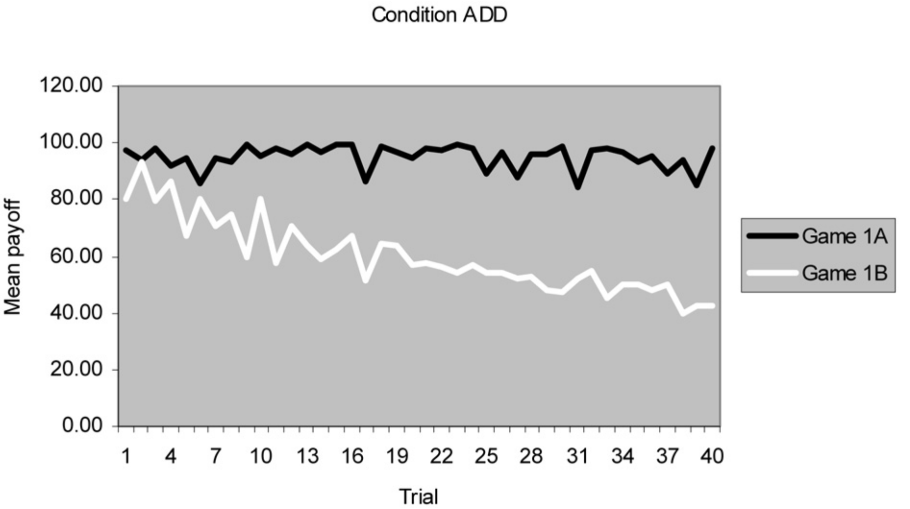
3.2.4. Aggregate payoffs

Denote by (f_j, f_k) the number of players choosing routes j and k in Game 1A, respectively. The mean payoff in Game 1A cannot exceed 100, and it decreases in the absolute difference $\Delta = | f_j - f_k|$. Thus, if $\Delta = 0$ (a $(9, 9)$ split), then the mean payoff is 100. To exemplify the effect of deviation from equilibrium on the payoff, mean payoffs for the $(10, 8)$, $(11, 7)$, and $(12, 6)$ splits are 98.8, 95.56, and 90, respectively. The expected payoff under mixed-strategy play is 95 and the associated standard deviation is 7.01. The mean payoff computed across all 54 players in sessions 1–3 of Condition ADD is 94.92. The corresponding mean for Condition DELETE is 95.21. Once again, we observe that the mixed-strategy equilibrium accounts for the mean Game 1A payoffs in both conditions extremely well.

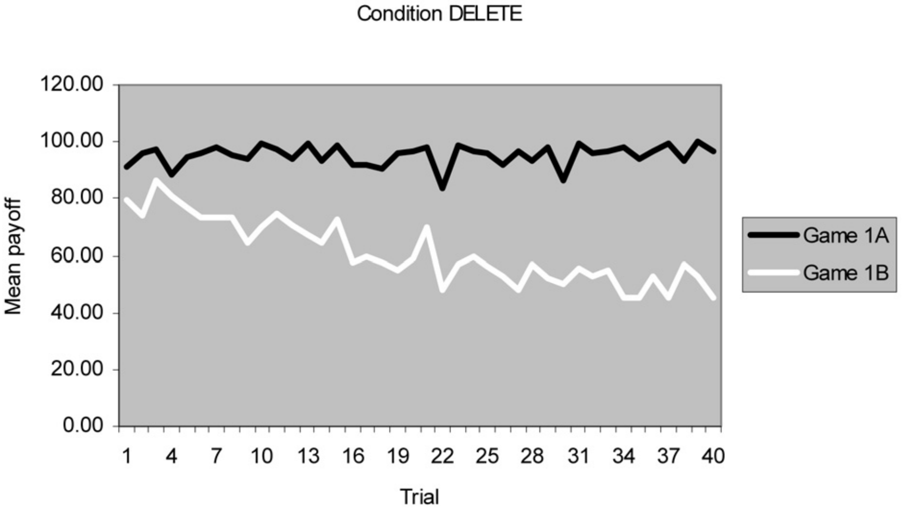
There is no mixed-strategy equilibrium for Game 1B. If all 18 players choose route (O–A–B–D), then each would earn 40 payoff units. Figs. 4a and 4b exhibit the running mean payoff by game for Conditions ADD and DELETE, respectively. The mean payoffs for Game 1B start around 80 in trial 1 in each condition and slowly decrease to about 40. Combining results across the two conditions and using the session as the unit of analysis, we compared the mean payoffs between Games 1A and 1B for each half of the session. The mean payoff for the first half (rounds 1–20) was significantly higher in Game 1A than the mean payoff for Game 1B ($z = 2.2$, $p < 0.05$) by the Wilcoxon signed-rank test. The corresponding result for the second half (rounds 21–40) was even more pronounced.

3.2.5. Switches

Selten et al. conducted an experiment on traffic networks quite similar to Game 1A. They studied a network with two parallel roads—a main road M and a side road S—connecting a common origin to a common destination. The cost functions were linear: $c_M = 6 + 2f_M$ and $c_S = 12 + 2f_S$, and the group size was $n = 18$. These cost functions result in multiple equilibria in which 12 players choose road M and 6 choose road S. Players chose their routes independently of one another in a stage game that was iterated 200 times. Similar to the results of Game 1A reported above, Selten et al. reported strong support for equilibrium play on the aggregate level. The mean number of players choosing route S in two different experimental conditions (that differed from each other in the outcome information at the end of each trial) were 5.98 and 6.06. Despite increasing the number of trials five-fold, they found no convergence to pure-strategy equilibrium. Rather, they observed considerable fluctuations around the means, not unlike the ones displayed in Figs. 2a and 2b. The standard deviations of the number of players choosing route S on any given trial assumed values between 1.53 and 1.94. Helbing (2004) reported similar results.



(a)



(b)

Fig. 4. Mean payoff by game and trial in Experiment 1: Conditions ADD and DELETE.

Neither Selten et al. nor Helbing invoked the mixed-strategy equilibrium solution to explain these substantial fluctuations. Rather, they attributed them to the multiplicity of equilibria. This may not fully explain the deviations from equilibrium once it has been reached that we reported above. We proposed above a "sophisticated" strategy where a few forward-looking players might have deviated from pure-strategy equilibrium play *deliberately* in the hope of exploiting this deviation in subsequent trials. Another possible reason may be grounded in the demand characteristics of the game. Some players may simply not believe that they are expected to stick to the same route once equilibrium is reached. Randomization of routes may account for the behavior of yet another fraction of the players. All of these reasons interact with the opportunity cost

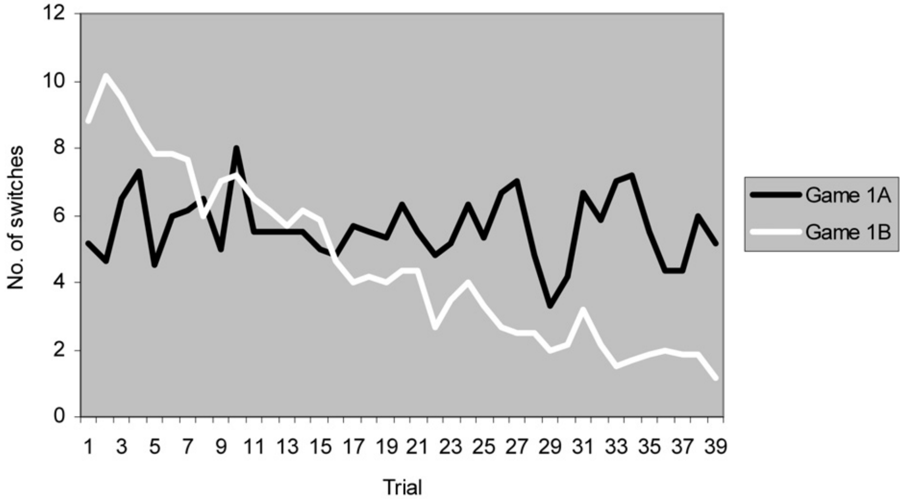


Fig. 5. Running mean number of switches by game in Experiment 1.

of a single deviation from a $(9, 9)$ split to $(10, 8)$ split that, as we reported above, reduces the individual earnings by only 10 percent.

Fig. 5 exhibits the running mean number (in steps of 5) of switches for Games 1A and 1B. The means for each trial are computed across all the 108 players. As players converge to choosing route $(O–A–B–D)$ in Game 1B, the number of switches converges to zero. Consistent with Figs. 3a and 3b, Fig. 5 shows that learning in Game 1B is slow; on average, the mean frequency of switches decreases by 1 every 5 trials. Fig. 5 further shows no downtrend in the mean number of switches in Game 1A. Our results are consistent with those reported by Selten et al. (2004, Fig. 3). Although there is a negative trend over the 200 trials in their experiment, their figure suggests no discernible trend in the first 40 trials. The mean number of switches per trial in Game 1A is about 6 (Fig. 5), whereas the mixed-strategy equilibrium yields an expected value of 9 switches. This difference is statistically significant ($z = 15.7$, $p < 0.01$). This is the first piece of evidence that rejects the symmetric mixed-strategy equilibrium. On the whole, players do switch their routes in Game 1A but not as frequently as predicted.

Is it beneficial to switch? To answer this question, we conducted two analyses. First, across all six sessions for each game separately, we correlated the individual frequency of switches ($\min = 0$, $\max = 39$) and the individual payoff across all the 108 players. Both correlations were negative and significant: -0.43 for Game 1A and -0.83 for Game 1B ($p < 0.05$ in both cases). To assess the magnitude of decrease in earnings associated with switching, we computed for each player the mean payoff across all the trials that were *preceded* by a switch and the mean payoff across all trials *not preceded* by a switch. This computation yielded two means for each player, $\text{pay}_{\text{switch}}$ and $\text{pay}_{\text{non-switch}}$ (pay_s and pay_{ns} for short). Players who switched less than four times were excluded from this analysis. Our results show that $\text{pay}_{\text{ns}} > \text{pay}_\text{s}$ for 58 of the 94 players in Game 1A and 76 of 87 players in Game 1B. Both results are significant ($p < 0.05$) by a sign test. On average, the players' mean earnings decreased by 2.86 in Game 1A and 11.72 in Game 1B as a result of switching.

The first analysis does not consider the possibility that a payoff following a switch may, indeed, be lower than the preceding payoff but nevertheless higher than it would have been without

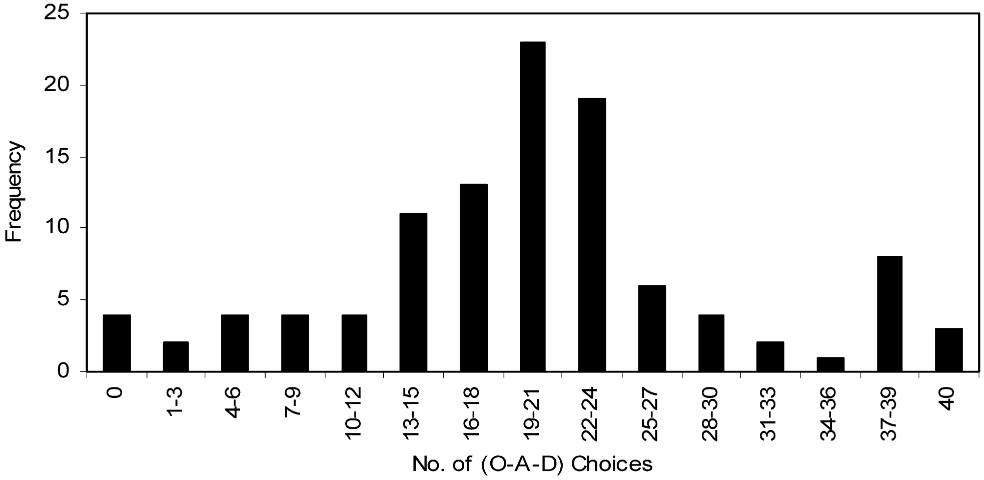


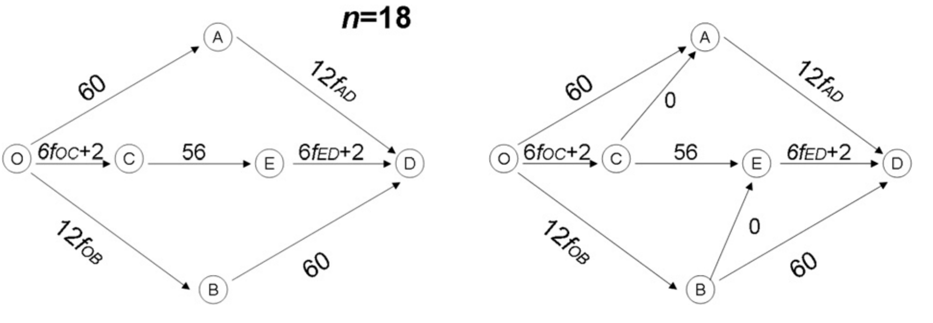
Fig. 6. Frequency distribution of number of players in Game 1A choosing route $(O–A–D)$.

the switch. Our second analysis checked this possibility. For each player across all the rounds in a given game, we computed the mean difference between her payoffs following a switch and the (counterfactual) payoffs she would have received without switching (assuming that the remaining players would play as they did). The mean differences for sessions 1, 2, 3, 4, 5, and 6 in Game 1A were −5.45, −1.25, −7.86, −1.84, 0.26, and 0.05 (the payoff following a switch is higher than the counterfactual at the 10% level, as shown by a Wilcoxon signed ranks one-tailed test, $z = -1.57$, $p < 0.10$). The corresponding means for Game 1B were −0.57, −1.98, −3.59, −3.14, 3.27, and −10.75 ($z = -1.36$, $p < 0.10$, one-tailed test). Taken together, both analyses suggest that, on average, switching was not beneficial.

3.2.6. Individual differences

Moving from aggregate to individual analyses, our results show that the symmetric mixed-strategy equilibrium does not account for all the *individual* route choices. Fig. 6 displays the frequency distribution of the number of players choosing route $(O–A–D)$ in Game 1A. Except of the two classes with a single frequency of 0 and 40, all the other frequencies are grouped in classes of 3 (1–3, 4–6, ..., 37–39). The mean and variance of this grouped frequency distribution are 20.06 and 91.1, respectively. The expected number and variance under mixed-strategy equilibrium play are 20 and 10, respectively. The observed frequency distribution differs significantly from the theoretical distribution in its variance (a 9:1 ratio) but not in its mean. The hypothesis that *all* players follow the mixed-strategy equilibrium with probability 0.5 of choosing route $(O–A–D)$ is flatly rejected. To illustrate that, Fig. 6 shows that 6 players chose route $(O–A–D)$ no more than 3 times and 11 other players chose it at least 37 times. The corresponding theoretical frequencies are essentially zero.

Fig. 6 suggests a mixture of player types, with a few players choosing the same route on almost all 40 trials and most others mixing their route choices although not necessarily with equal probabilities. For all the players who chose each of the two routes in Game 1A at least once (101 players), we conducted a run test to test the considerably weaker null hypothesis that each sequence of 40 route choices is generated by a Bernoulli process with fixed probability p $(0 < p < 1)$ that may differ from one player to another. This hypothesis was rejected for only 30



(a) (b)

Fig. 7. Basic and augmented networks for Games 2A and 2B in Experiment 2.

of the 101 players. The null hypothesis of random play in Game 1A cannot be rejected for about 70 percent of the players. The majority of the players seem to be mixing their choice of routes but not in equal proportions dictated by the mixed-strategy equilibrium.

3.2.7. Discussion

There are three major findings. First, the symmetric mixed-strategy equilibrium accounts for the choice data in all six sessions of Game 1A extremely well. However, there is no support for it on the individual level. Rather, most players do mix their route choices but not in the proportions implied by equilibrium play. Second, in support of the BP, when a cost-free edge is added to the basic game, players' route choices converge to the Pareto deficient equilibrium strategy that results in a 60 percent decrease in their earnings. Third, route choice and rate of learning are immune to alternative framings of the BP in terms of adding a cost-free edge to the basic game or deleting it from the augmented game. These findings are restricted to the Minimal Critical Network—the simplest traffic network in which the BP may be realized; they may not generalize to other networks with a different architecture. Experiment 2 was designed to extend the investigation to networks with a richer topology.

4. A three-route symmetric network with two additional edges

Fig. 7a exhibits the basic game (Game 2A) that we constructed for studying the BP in Experiment 2. Game 2A consists of a network with six vertices and three routes, namely, a two-edge route $(O–A–D)$, another two-edge route $(O–B–D)$, and a three-edge route $(O–C–E–D)$. The two routes $(O–A–D)$ and $(O–B–D)$ maintain the anti-symmetric arrangement of the fixed and variable travel costs as in Game 1A. Route $(O–C–E–D)$ has a single edge with only a fixed cost and two additional edges each with both variable and fixed costs. The augmented game in Experiment 2, Game 2B (Fig. 7b), includes *two* additional cost-free edges, namely, edge $(C–A)$ connecting nodes C and A, and edge $(B–E)$ connecting nodes B and E. These result in adding two new routes, namely, routes $(O–C–A–D)$ and $(O–B–E–D)$ for a total of five routes.

4.0.8. Equilibrium analysis

Call the three routes $(O–A–D)$, $(O–B–D)$, and $(O–C–E–D)$ the *old* routes, and the two additional routes $(O–C–A–D)$ and $(O–B–E–D)$ the *new* routes. Because of the symmetry of the payoffs in Games 2A and 2B, we only construct equilibria that maintain symmetry across the three old routes, the two new routes, or both. For example, we search for equilibria in Game 2B in which x players choose each of the two new routes, and $n-2x$ players each randomize across the three old routes with equal (1/3) probabilities. In another class of strategy profiles, we search for equilibria in Game 2B in which x players each randomize between the two new routes with equal (1/2) probabilities, and $(n-x)/3$ players choose each of the three old routes.

We have identified two classes of equilibria in Game 2A.

1. Equilibria in which exactly 6 players choose each of the 3 old routes (see Appendix B). There are $18!/(6!6!6!) = 17{,}153{,}156$ such equilibria that are asymmetric each resulting in travel cost of 132.
2. A single mixed-strategy equilibrium in which each player independently randomizes across the 3 routes with equal (1/3) probabilities. Solving for the expected travel cost (ETC) from the equation

$$\mathrm{ETC} = \sum_{j=0}^{17}\big[60 + 12(j+1)\big]\binom{17}{j}(1/3)^j(2/3)^{17-j} = 140.$$

In a similar way, we constructed four classes of equilibria for Game 2B.

1. Pure-strategy equilibria in which exactly 9 players choose each of the 2 new routes (see Appendix B). There are $18!/(9!9!) = 48{,}620$ such equilibria that are asymmetric and only involve pure strategies. Each results in travel cost of 164.
2. A single mixed-strategy equilibrium in which each of the 18 players independently randomizes over the two new routes with equal (1/2) probabilities. The expected travel cost is calculated from

$$\mathrm{ETC} = \sum_{j=0}^{17}\big[12(j+1) + 6(j+1) + 2\big]\binom{17}{j}(1/2)^j(1/2)^{17-j} = 173.$$

3. Asymmetric mixed-strategy equilibria in which 15 players independently choose each of the 2 new routes with equal (1/2) probabilities, and each of the 3 old routes is chosen by a single player. There are $18!/(15!3!) = 816$ equilibria in this class. The expected travel costs for players randomizing between the 2 new routes is 162 and for the players choosing an old route it is 164.
4. Asymmetric mixed-strategy equilibria in which 8 players choose route $(O\text{–}C\text{–}A\text{–}D)$, 8 players choose route $(O\text{–}B\text{–}E\text{–}D)$, a single player chooses one of the 3 old routes, and yet another player randomizes between the 2 remaining old routes with equal (1/2) probabilities. There are $3 \times 18!/(8!8!) = 11{,}814{,}600$ equilibria in this class. In each of them 17 players choose pure strategies and only a single player randomizes. The expected travel cost for the 16 players who choose one of the 2 new routes is 161, and for the 2 players who choose the old routes it is 168.

Under all classes of equilibria that we have identified, the cost of travel increases when two new routes are added to Game 2A. This is the BP. Under the pure-strategy equilibrium, cost of travel increases from 132 to 164 by 24 percent. Under the symmetric mixed-strategy equilibrium, expected cost of travel also increases by 24 percent from 140 to 173.

Before proceeding with the experimental results, a comparison of the network games in Experiments 1 and 2 is in order. Three major differences between these two experiments underscore the generality of the BP. First, the two networks in Experiment 2 have a richer topology than the networks in Experiment 1, presenting the player with a considerably more difficult choice problem. Game 2A has three routes, two with two links each and one with three links. If deviating from any given route in Game 2A, a player has two alternative routes to choose from rather than a single alternative route in Game 1A. In Game 2B, a player has to choose one of five routes compared to one of three routes in Game 1B, which is strictly dominant. Even if she decides not to traverse any of the three old routes, she still has to choose between the two new routes (O–C–A–D) and (O–B–E–D). Second, we chose the endowments in such a way that players in Experiment 2 could end up the game with negative payoffs. The endowments in Games 2A and 2B were fixed at 196. It is easy to verify that, given this endowment, if 12 or more players were to choose any of the three routes in Game 2A, each of them would end up with negative payoffs. In contrast, with a 400 travel-point endowment in Experiment 1, no distribution of network users across the two routes in Game 1A would have resulted in negative payoffs. Third, and most importantly, in equilibrium all 18 players in Game 1B choose the strictly dominant route (O–A–B–D); the two old routes in Game 1A are abandoned. In contrast, as shown above, there are multiple equilibria in Game 2B that yield a very difficult coordination game. Even more importantly, some of these (classes 3 and 4 above) allow the old routes to be chosen, in equilibrium, by a small fraction of the players. As we show below, this is exactly what happened in Experiment 2.

4.1. Method

4.1.1. Participants

One hundred and eight undergraduate and graduate students at the University of Arizona participated in Experiment 2. All of them volunteered to take part in a group decision making experiment for payoff contingent on performance, and none of them had participated in Experiment 1. Male and female players participated in about equal numbers. The players were divided into 6 groups (sessions) of 18 members each. Because Experiment 1 yielded no significant differences between conditions ADD and DELETE, only Condition ADD was implemented with Game 2A presented in Part I of the experiment and Game 2B in Part II. A session lasted 100–110 minutes. Excluding a $5 show-up bonus, the mean payoff was $16.

4.1.2. Procedure

The procedure was the same as in Condition ADD of Experiment 1 with two major exceptions. First, because we expected slower learning than in Experiment 1, the number of trials in each part of the experiment was doubled from 40 to 80. Second, the endowment for each of the 160 trials in Games 2A and 2B was set at the same value of 196. This resulted in individual earnings under pure-strategy equilibrium play of $196 - 132 = 64$ and $196 - 164 = 32$ travel units in Games 2A and 2B, respectively, and a corresponding 2:1 ratio of equilibrium payoffs (compared to a ratio of 2.5:1 in Experiment 1). The information provided to the player at the end of each trial, the number of payoff trials, the payoff associated with each route, and the conversion rate of travel units to dollars were the same as in Experiment 1.

4.2. Results

The organization of this section follows the Results section in Experiment 1. We first test for session effects, then test for the implications of the BP on the aggregate level, and finally conclude this section with the study of switches and individual differences.

4.2.1. Session effects

We computed the following statistics for each of the three sessions of Experiment 2:

- The number of times (out of 80) that each player chose routes (O–A–D), (O–B–D), and (O–C–E–D) in Game 2A.
- The number of times (out of 80) that each player chose routes (O–A–D), (O–B–D), (O–C–E–D), (O–C–A–D), and (O–B–E–D) in Game 2B.
- The frequency of switches per player (out of 79) in Games 2A and 2B.

Table 3 presents the means and standard deviations of these statistics. The results are presented by game (Game 2A in the upper panel and Game 2B in the lower panel). Within each game, the results are presented by session. Despite the relatively small number of sessions for each game, several hypotheses are testable. If the BP has an effect, then the original three routes in Game 2A should be chosen less frequently in Game 2B. For each of the three routes (O–A–D), (O–B–D), and (O–C–E–D), we used the Wilcoxon signed-ranks test to test the null hypothesis that the frequencies of choices in the six sessions were drawn from the same distribution in both games. This hypothesis was decisively rejected in each case ($z = -2.2$, $p < 0.05$). Table 3 shows that, on average, each of the three original routes was chosen approximately three times more often in Game 2A than in Game 2B.

The equilibrium solution implies that each of the three routes in Game 2A should be chosen with equal frequencies. Since the three routes are not independent of each other, we chose two of them for comparison, namely, routes (O–A–D) and (O–C–E–D). We used the Wilcoxon matched-pairs signed-ranks test to compare the frequencies for the six sessions. The null hypothesis of equal frequencies of choice of the two routes could not be rejected ($z = -1.15$, $p = 0.25$). Note that the mean frequencies for these two routes in Game 2A, namely, 26.42 and 26.86, are almost identical. We then repeated the same test for the frequencies of choice of these same two routes in Game 2B (respective means are 9.24 and 9.14). Once again, the null hypothesis could not be rejected ($z = 0$, $p = 1$). The equilibrium solutions for Game 2B imply that the two new routes (O–C–A–D) and (O–B–E–D) should be chosen with equal frequencies (respective means are 26.12 and 27.25). Once again, the Wilcoxon matched-pairs signed-ranks test could not reject this null hypothesis ($z = -1.57$, $p = 0.12$). Taken together, the results of all these tests are in agreement with equilibrium play.

Finally, we used the Mann-Whitney U test to compare the number of switches per session between Games 2A and 2B (see right-hand columns of the two panels in Table 3). Despite the difference in number of routes between the two games (3 in Game 2A vs. 5 in Game 2B), the null hypothesis of equal frequency of switches between routes could not be rejected ($z = -1.57$, $p = 0.12$).

4.2.2. Aggregate route choice

For each trial separately, we counted the number of players (out of 18) who chose routes (O–A–D), (O–B–D), and (O–C–E–D) in Game 2A, and then averaged the results across trials

Table 3
Mean values of summary statistics by game (Games 2A and 2B) and by session (1–6) in Experiment 2

Game 2A

Statistic	Route (O–A–D) Mean No. of choices	Route (O–B–D) Mean No. of choices	Route (O–C–E–D) Mean No. of choices	Number of switches by player
Session 1	26.94	26.72	26.33	24.50
	(17.22)	(23.51)	(21.50)	(13.17)
Session 2	26.60	25.61	27.78	38.94
	(11.58)	(14.48)	(16.61)	(24.05)
Session 3	25.39	27.67	26.94	35.44
	(10.47)	(21.60)	(14.48)	(19.91)
Session 4	26.67	26.28	27.06	35.11
	(15.92)	(11.61)	(15.26)	(15.17)
Session 5	26.44	26.78	26.78	42.28
	(16.24)	(9.77)	(13.77)	(26.36)
Session 6	26.50	27.22	26.28	35.06
	(19.37)	(9.59)	(16.95)	(13.40)
Mean Freq.	26.42	26.71	26.86	35.22

Game 2B

Statistic	Route (O–A–D) Mean No. of choices	Route (O–B–D) Mean No. of choices	Route (O–C–E–D) Mean No. of choices	Route (O–C–A–D) Mean No. of choices	Route (O–B–E–D) Mean No. of choices	Number of switches by player
Session 1	8.11	8.06	9.33	26.33	28.17	28.89
	(7.40)	(11.06)	(16.21)	(25.35)	(26.10)	(23.46)
Session 2	8.89	9.11	9.00	25.83	27.17	43.83
	(6.77)	(8.63)	(7.40)	(15.75)	(17.00)	(24.75)
Session 3	10.67	10.28	10.56	24.67	23.83	43.89
	(6.76)	(8.80)	(8.33)	(20.57)	(20.2)	(21.76)
Session 4	12.94	9.89	11.00	21.78	24.39	41.28
	(16.43)	(6.31)	(7.78)	(13.37)	(18.58)	(16.62)
Session 5	6.11	5.72	6.94	30.94	30.28	40.22
	(5.87)	(5.93)	(6.17)	(20.49)	(21.25)	(15.06)
Session 6	8.72	6.44	8.00	27.17	29.67	34.06
	(17.99)	(5.34)	(8.18)	(19.49)	(21.85)	(13.08)
Mean Freq.	9.24	8.25	9.14	26.12	27.25	38.69

Standard deviations in parentheses.

and sessions. Similar means were computed for each of the five routes in Game 2B. Table 4 (row 2) presents the means and standard deviations. Rows 3–6 display the expected frequencies of route choice and the standard deviations under the equilibrium solutions. Under the pure-strategy equilibrium, 6 players choose each of the 3 routes in Game 2A and 9 players choose each of the two new routes in Game 2B. No variability is predicted. The symmetric mixed-strategy equilibria yield the same expected frequencies, and in addition allow for considerable variability. Class 3 of equilibria yield expected frequencies of 7.5 for each new route with associated standard deviations of 1.94. In contrast, class 4 equilibria allow for no variability in the choice of the new routes.

The results for Game 2A support the major finding for Game 1A in Experiment 1. On average, the three old routes were chosen equally likely; the means for routes (O–A–D), (O–B–D), and

Table 4
Observed and predicted means and standard deviations of number of players choosing each route by game (Games 2A and 2B) in Experiment 2

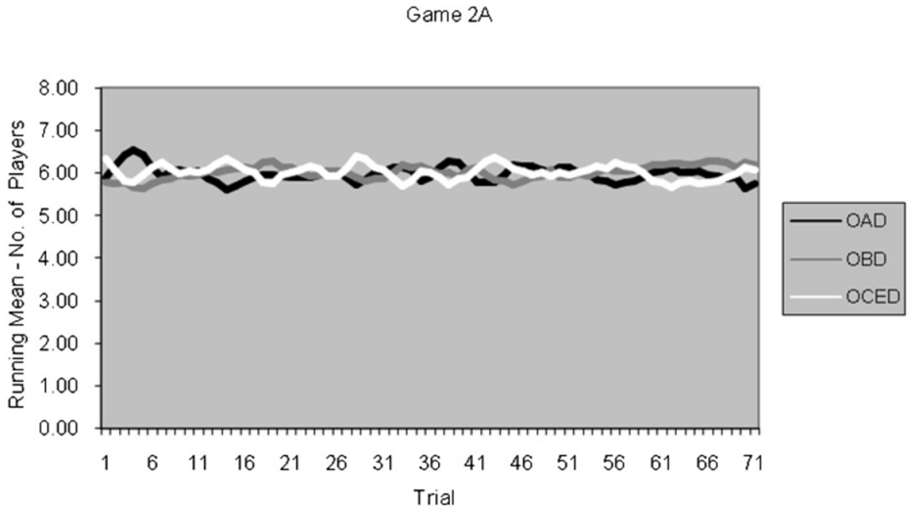
Statistic	Game 2A			Game 2B				
	Route (O–A–D)	Route (O–B–D)	Route (O–C–E–D)	Route (O–A–D)	Route (O–B–D)	Route (O–C–E–D)	Route (O–C–A–D)	Route (O–B–E–D)
Observed	5.95 (1.86)	6.01 (1.83)	6.04 (1.83)	2.08 (1.29)	1.86 (1.89)	2.06 (1.23)	5.88 (1.88)	6.13 (1.85)
Equilib. (Class 1)	6 (0)	6 (0)	6 (0)	0	0	0	9 (0)	9 (0)
Equilib. (Class 2)	6 (2.00)	6 (2.00)	6 (2.00)	0	0	0	9 (2.12)	9 (2.12)
Equilib. (Class 3)				1 (0)	1 (0)	1 (0)	7.5 (1.94)	7.5 (1.94)
Equilib. (Class 4)				0.67 (0.33)	0.67 (0.33)	0.67 (0.33)	8 (0)	8 (0)

Standard deviations in parentheses.

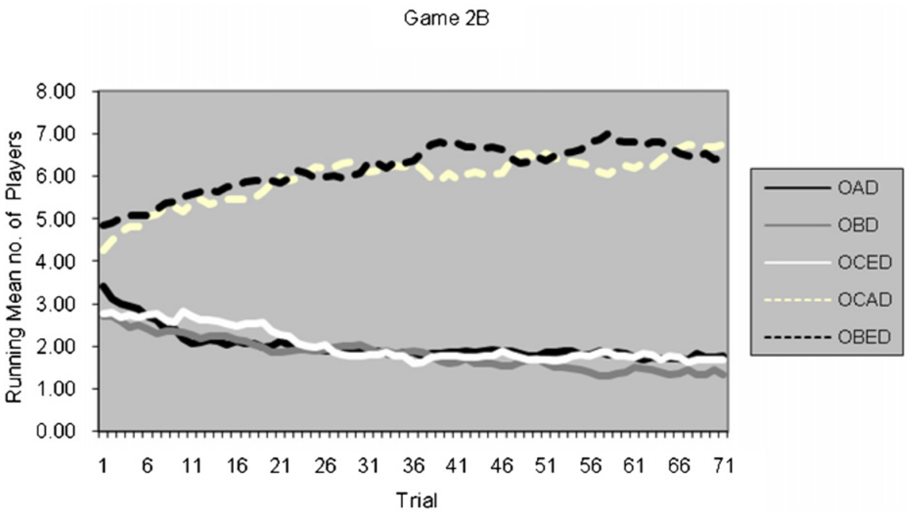
(O–C–E–D) are 5.95, 6.01, and 6.04, respectively. There are fluctuations around the means that again, similarly to Experiment 1 (see Table 2), are captured rather well by the symmetric mixed-strategy equilibrium. The three old routes in Game 2B were chosen by 33 percent of the trials. This percentage greatly exceeds the percentages under the equilibria in all four classes. However, again as in Experiment 1, the frequency of choice of the old routes decreases with experience. We turn next to examination of these learning trends.

Figs. 8a and 8b display the mean number of players choosing each of the three routes in Game 2A and each of the five routes in Game 2B. To reduce variability and better exhibit the trends across trials, we present the running means in steps of 10 rather than the individual means for each trial. Consider first Fig. 8a. It is evident that the mean route choices in Game 2A hover around 6 from the very first trials. We observe no differences in choice frequency among the three routes, nor discern any learning trends across the 80 trials. Also similar to Game 1A, we observe no tendency to maintain the same choices until the end of the session once a (6, 6, 6) split has been obtained. Out of a total of 720 trials across the six sessions in Game 2A, 24 ended up in a (6, 6, 6) split, and these were evenly spaced across the sessions. However, there was only a single run of two consecutive (6, 6, 6) splits, and no run of three or more (6, 6, 6) splits. Pure-strategy equilibria were seldom reached and almost never maintained.

Results pertaining to the major hypothesis of the present study are displayed in Fig. 8b that plots the running means of route choice in steps of 10. The two new routes (O–C–A–D) and (O–B–E–D) are already separated from the three non-equilibrium routes in the first few trials. This separation increases across trials. Fig. 8b shows that the players started choosing the two new routes on the early trials, that learning was faster in the first 40 trials than in the remaining 40 trials, that the two new routes were chosen, on average, in roughly equal proportions, and that the three old routes were also chosen in roughly equal proportions. Table 5 shows the means and standard deviations of choice of each of the five routes in the last ten trials in Game 2B. The statistics are presented by session and then across all six sessions in the bottom row of the table. Comparison of Tables 4 and 5 shows that the equilibria in class 3 best account for the observed means. In particular, they allow for a choice of the old routes by a significant minority of the players. However, they do not account for the variability of choice of the old routes, and they overestimate the observed mean choices of the new routes.



(a)



(b)

Fig. 8. Running mean number of players choosing each route in Games 2A and 2B.

4.2.3. Switches

Fig. 9 displays the running mean number of switches for Games 2A and 2B. For both games, there is clear downtrend in the mean number of switches. The mean number of switches in Game 2A declines from about 9 on the first five trials or so to about 7 at the end of the session. The corresponding values for Game 2B are 12 and 8. A player who adheres to the symmetric mixed-strategy equilibrium in Game 2A should switch her route from one trial to another with probability 2/3. This would result, on average, in 12 players switching their routes on any given trial. Similarly to Experiment 1, our results show that players switched their routes in Game 2A but not as frequently as predicted.

Table 5
Means and standard deviations of number of players choosing each route in the last ten rounds of Game 2B in Experiment 2

	Route $(O–A–D)$	Route $(O–B–D)$	Route $(O–C–E–D)$	Route $(O–C–A–D)$	Route $(O–B–E–D)$
Session 1	2.20	0.90	1.90	6.10	6.90
	(0.40)	(1.04)	(0.94)	(1.01)	(1.04)
Session 2	1.10	1.80	1.40	7.30	6.40
	(0.94)	(1.60)	(0.66)	(1.10)	(1.56)
Session 3	2.70	1.50	2.20	6.10	5.50
	(0.64)	(1.02)	(1.17)	(1.04)	(1.28)
Session 4	2.40	1.80	2.30	6.00	5.50
	(1.02)	(0.98)	(1.55)	(1.67)	(1.43)
Session 5	0.67	0.80	1.00	8.40	7.00
	(0.67)	(1.08)	(0.77)	(1.56)	(1.79)
Session 6	1.40	1.20	1.40	6.50	7.50
	(0.66)	(0.87)	(0.80)	(1.43)	(1.43)
Across sessions	1.76	1.33	1.70	6.73	6.47
	(1.01)	(1.16)	(1.08)	(1.54)	(1.53)

Standard deviations in parentheses.

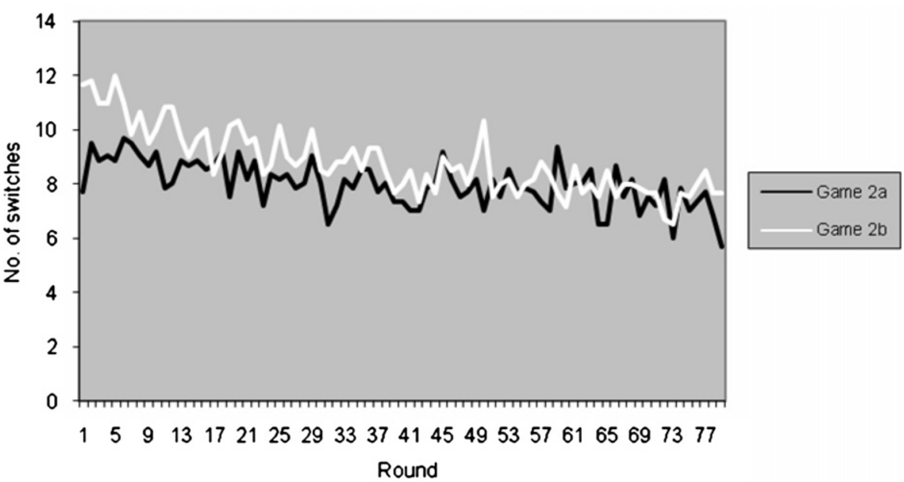


Fig. 9. Running mean number of switches by game in Experiment 2.

Is it beneficial to switch? To answer this question, we conducted the same two analyses as in Experiment 1. First, for each session separately, we counted for each player the number of switches (min $= 0$, max $= 79$) in Game 2A and correlated it with her payoff in Game 2A. The correlations are all negative and significant: -0.82, -0.49, -0.70, -0.46, -0.76, and -0.81, for sessions 1–6, respectively. We computed the same correlation between number of switches and individual payoff in Game 2B. Again, all six correlations are negative and significant: -0.56, -0.68, -0.59, -0.62, -0.71, and -0.78 for sessions 1–6, respectively. It is clear that players who switch often earn, on average, less than people who switch less often.

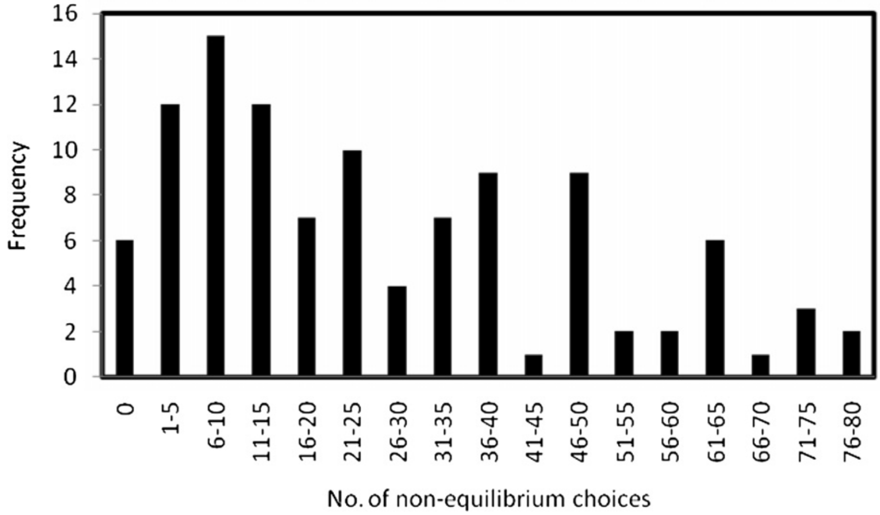


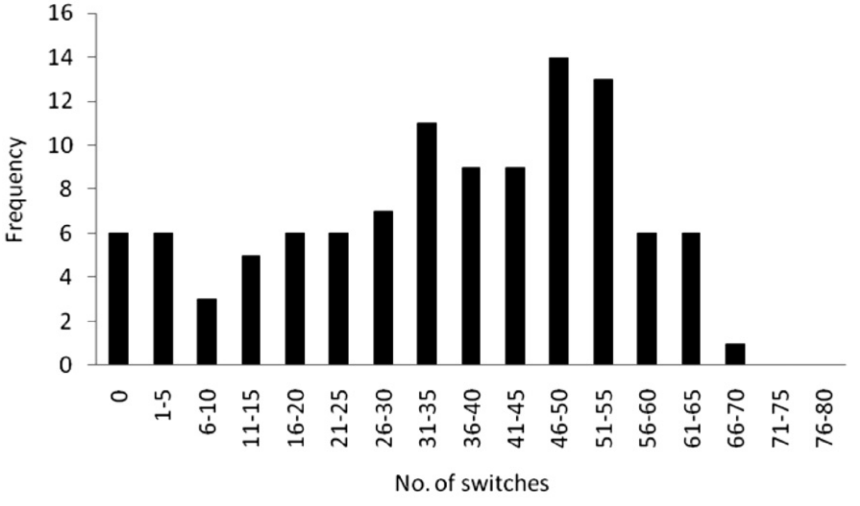
Fig. 10. Frequency distribution of number of players in Game 2B choosing the old routes.

Second, for each player separately and across all rounds, we computed the mean difference between payoffs in the trials following a switch and the payoffs a player would have received were she not to switch. The mean differences for sessions 1–6 in Game 2A were −1.17, −0.38, −1.73, −1.03, −5.05, and −1.87. The corresponding means for the six sessions in Game 2B were −1.58, −3.49, −1.90, −4.77, −0.70, and −2.85. This provides additional evidence in support of the conclusion that, on average, switching routes too often is not beneficial.

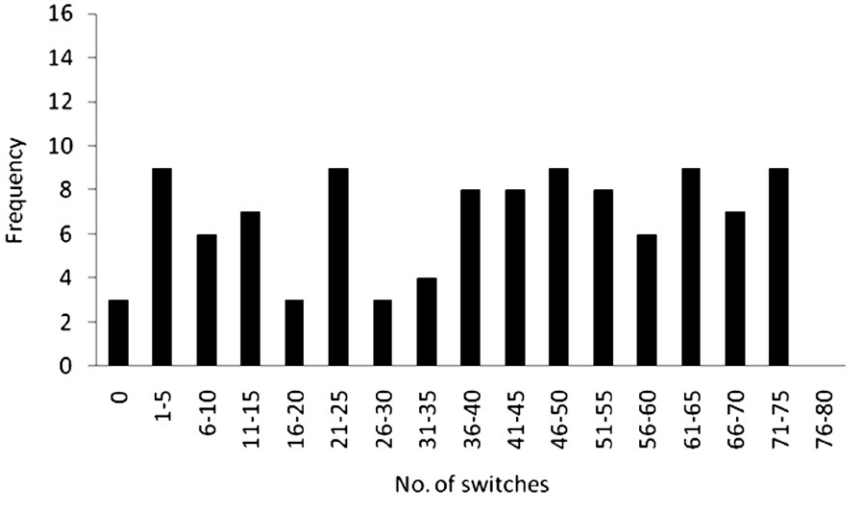
Under mixed-strategy equilibrium play, the frequencies of switches in Games 2A and 2B are not correlated. To test this implication, we computed the correlation between the individual number of switches in Games 2A and 2B for each session separately. The correlations were positive and quite high: 0.76, 0.74, 0.68, 0.85, 0.73 and 0.89 for sessions 1–6, respectively. All of them are highly significant ($p < 0.01$). We conclude from this analysis that the propensity to switch is unrestricted by the topology of the network. Players who switch more often in one network are likely to switch more often in another.

4.2.4. Individual differences

In our last analysis, we focus on individual differences in route choice and number of switches. Turning first to route choice, Table 4 shows that the three old routes in Game 2B were chosen together, on average, by 6 of the 18 players. Table 4 does not tell us whether all players chose the old routes in Games 2B with roughly equal proportions or some players chose these routes considerably more often than others. Fig. 10 answers this question by exhibiting the frequency distribution of the number of players choosing the old routes in Game 2B. It shows that 6 players (out of 104) never chose the old routes across all 80 trials, and a total of 33 players chose the old routes no more than 10 times. In contrast, 26 players (about 24 percent) chose the old routes in more than 50 percent of the trials and 5 of these 15 players chose them on at least 71 trials. The individual frequencies of the old route choices range all the way from 0 to 80 trials, suggesting that whatever conclusions we may draw with regard to the realization of the BP in the population may not apply to most of the individual players.



(a)



(b)

Fig. 11. Frequency distribution of number of individual switches in Games 2A and 2B.

Turning next to the individual differences in the frequency of switches, Figs. 11a and 11b display the frequency distributions of individual number of switches for Games 2A and 2B, respectively. Once again, we observe individual differences that cover almost the entire range from 0 to 79 switches. We cannot tell whether the propensity to switch reflects boredom with the task, randomization, exploration, deliberate attempts to change group behavior in order to exploit non-equilibrium play on subsequent trails, or some combination of these factors.

4.2.5. Discussion

The results of Experiment 2 generalize the main findings of Experiment 1 to a considerably richer topology, where players have to choose one of three routes in the basic game and one of five routes in the augmented game. The symmetric mixed-strategy equilibrium accounts surprisingly well for the aggregate behavior of the players in all six sessions of the basic game. Asymmetric mixed-strategy equilibria in which most of the players randomize over the two new routes and a few players choose the old routes provide a first-order approximation to the asymptotic mean route choices in Game 2B. However, in both games there is no support for either pure- or mixed-strategy equilibrium on the individual level. Simultaneously accounting for the considerable individual differences in route choice and frequency of switching and the orderly and predictable behavior on the aggregate level remains a challenge for future research.

5. General discussion and conclusions

Like the finitely iterated Prisoner's Dilemma and the Centipede games, the BP dramatically illustrates the counterintuitive implications of the equilibrium analysis. Unlike these two games, the counterintuitive results are obtained by comparing to each other networks with different capacities and not by invoking backward induction that presupposes common knowledge of rationality. Arguments have been raised (Cohen, 1988) against the relevance of the BP to real situations. Similar arguments have been raised, in turn, against the relevance of the Prisoner's Dilemma and Centipede games. According to these criticisms, the surprises in these abstract games and their counterintuitive implications arise from the many respects in which they differ from reality rather than from those aspects they share with it (Cohen, 1988). In defending the study of the finitely iterated Prisoner's Dilemma game, Axelrod wrote: "It is the very complexity of reality which makes the analysis of an abstract interaction so helpful as an aid to understanding" (1984, p. 19). We contend that the same argument applies here as well, and that the experimental analysis of the BP makes an additional contribution to the study of the adverse effects of negative externalities in large groups.

The present findings should best be evaluated in light of the main features of the experimental design. Two are particularly critical: the within-subject design and the provision of the same endowment for the basic and augmented games. Both were introduced in order to test the BP under the most stringent conditions, where the *same* users traverse the network either with or without the additional links and where the addition (deletion) of one or more links results in loss (gain) in the equilibrium payoff that far exceeds the $4/3$ upper limit on the "price of anarchy" for linear cost functions set by Roughgarden and Tardos (2002). The equilibrium analysis underlying the BP shows that, for certain parameter values (edge cost functions, number of players), adding a new edge to a transportation network results in increased travel costs for all network users.

Our experimental findings fall into three main groups. First, the equilibrium solution accounts very accurately for the mean route choices in the two basic games in Experiments 1 and 2. However, convergence to equilibrium play is not reached as players continue to switch routes over iterations of the stage game resulting in fluctuations around the means that do not diminish with experience. Our analysis of individual data suggests a mixture of player types, some choosing the same route over most iterations, some exhibiting the opposite behavior by mixing their routes although not necessarily with the equilibrium probabilities, and some deliberately upsetting any equilibrium in pure strategies that is temporarily reached in an attempt to exploit deviations from equilibrium on later trials. A more detailed analysis of individual types and sequential dependencies is relegated to future studies.

The second major finding is strong support for the equilibrium analysis underlying the BP. When the same players shift from playing the basic game to playing the augmented game, their choice of routes changes in the direction of equilibrium play despite the substantial drop in their payoff. These results are tempered by the topology of the network. Convergence to equilibrium in the Minimal Critical Network (Experiment 1) is achieved after 40 trials. In contrast, route choice in Game 2B moves in the direction of one of the equilibrium solutions but convergence is not reached after 80 trials. These findings suggest additional studies that manipulate the topological properties of the network (e.g., the cardinality of the sets V and E, number and placement of the additional edges in the augmented game, and degree of asymmetry in edge costs) and vary the number of iterations and number of players.

The third major finding is that the results are not affected by alternative framings of the BP in terms of gains or losses. This finding would seem to suggest that the effects of gains and losses with respect to a status quo point, so prominent in the study of individual decision behavior, might not extend to interactive decisions in a large class of noncooperative n-person games. This, too, is a topic for future research.

Appendix A

Instructions for Game 1A

Welcome to an experiment on route selection in traffic networks. During this experiment you'll be asked to make many decisions about route selection in a traffic network game. Your payoff will depend on the decisions you make as well as the decisions made by the other participants. A research foundation has contributed the funds to support this research.

Please read these instructions now. In case you have any questions, please raise your hand and the experimenter will come to answer them.

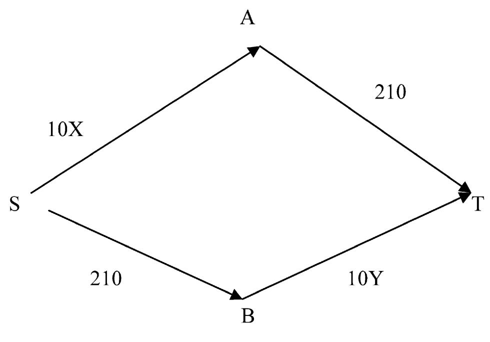
Please note that hereafter communication between the participants is strictly prohibited. If they communicate with one another by any shape or form, the experiment will be canceled.

The traffic network game

There are 18 participants in this experiment, including yourself, who will be asked to serve as drivers and choose a route to travel in two traffic network games that are described below. The two games will differ from one another. Below we present the instructions for Game 1A. The ones for Game 1B will be presented later after you complete Game 1. You will play Game 1A for 40 identical rounds.

Description of Game 1

Consider the very simple traffic network exhibited in a diagram form on the next page. Each driver is required to choose one of two routes in order to travel from the starting point, denoted by S, to the final destination, denoted by T. There are two alternative routes and they denoted in the diagram by either $[S–A–B]$ or $[S–B–T]$.



Travel is always costly in terms of the time needed to complete a segment of the road, tolls, fuel etc. The travel costs are written near each segment of the route you choose. For example, if you choose route $[S–A–T]$, you will be charged a total cost of $10X + 210$ where X indicates the *number* of participants who choose segment $[S–A]$ to travel from S to T plus a fixed cost of 210 for traveling on segment $[B–T]$. Similarly, if you choose route $[S–B–T]$, you will be charged a total travel cost of $210 + 10Y$, where Y indicates the *number* of participants who choose the segment $[B–T]$ to drive from S to T. Please note that the cost charged for segments $[S–A]$ and $[B–T]$ depends on the number of drivers choosing them. In contrast, the cost charged for traveling on segments $[A–T]$ and $[S–B]$ is fixed at 210 and does not depend on the number of drivers choosing them. All the drivers make their route choices independently of one another and leave point S at the same time.

Example. If you happen to be the only driver who chooses route $[S–A–T]$, and all other 17 drivers choose route $S–B–T$, then your travel cost from point S to point T is equal to $(10 \times 1) + 210 = 220$. If, on another round, you and 2 more drivers choose route $[S–B–T]$ and 15 other drivers choose route $[S–A–T]$, then your travel cost for that round will be $210 + (10 \times 3) = 240$.

At the beginning of *each round*, you will receive an *endowment* of **400** points. Your payoff for each round will be determined by subtracting your travel cost from your endowment. To continue the previous example, if your travel cost for the round is 210, your payoff will be $400 - 210 = 190$ points. If it is 230, then your payoff for that round will be $400 - 230 = 170$ points.

At the end of each round, you will be informed of the number of drivers who chose each route and your payoff for that round. All 40 rounds in Game 1 have exactly the same structure.

Procedure. At the beginning of each round, the computer terminal in front of you will exhibit the above diagram with two routes, namely, $[S–A–T]$ and $[S–B–T]$. Then, you will be asked to choose which of the two routes to travel. To choose a route, simply *click on that route*. For example, if you choose route $[S–A–T]$, then click once on segment $[S–A]$ and once on segment $[A–T]$. Clicking on those two segments will change their color to indicate your choice of route. If you decide to change your route, you can click on the two segments of the alternative route. Once you have chosen your route and pressed the “Confirm” button, the computer will ask you to verify your choice. You do so by clicking on the “OK” button.

After choosing a route, you will be presented with a wait message until all 18 drivers have made their decisions. Once each driver chooses one of the two routes, the computer will present you with a screen that includes the following information:

– The route you have chosen.
– The number of drivers who chose route $[S–A–T]$.
– The number of drivers who chose route $[S–B–T]$.
– Your payoff for that round.

Once you have completed playing 40 rounds of Game 1, the computer will inform you that Game 1 is over and Game 2 is about to start. Then, a new set of instructions for Game 2 will be distributed to you.

Payments. At the end of the session you will be paid for 4 rounds randomly selected from the 40 rounds in Game 1 and 4 more rounds randomly selected from the 40 rounds in Game 2. The choice of the payment rounds will be made publicly by drawing 4 cards from a pack of 40 cards numbered from 1 through 40. Once these 8 rounds are chosen, they will be recorded by the experimenter, the payment of each subject will be computed according to his/her earnings on these rounds, and the computer will inform you of your total points and the resulting earnings for the session.

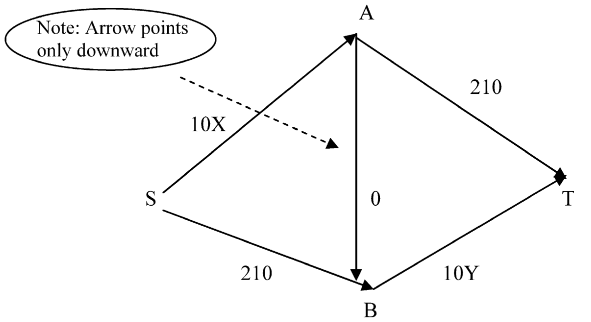
Points will be converted into money at the rate 25 points = \$1.00.

All the decisions will only be made by clicking the “mouse.” Therefore, please do not touch the keyboard.

Please place the instructions on the table in front of you to indicate that you have completed reading them. Game 1 will begin within shortly.

Instructions for Game 1B

Game 1B is very similar to Game 1A with the exception that we have added another segment from point A to point B with *zero* travel cost. As a result, in choosing a travel route in this new traffic network, now you have to choose one of *three* routes, namely route [S–A–B], route [S–B–T], or route [S–A–B–T]. Similarly to Game 1, you have to choose a single travel route. The traffic network for Game 1B is displayed below.



Travel costs are computed exactly as in Game 1A. If you choose route [S–A–T], you will be charged a total travel cost of $(10X + 210)$, where X indicates the number of drivers who chose the segment [S–A] to travel from S to T via either route [S–A–T] *or* route [S–A–B–T]. Similarly, if you choose route [S–B–T], you will be charged a total travel cost of $(210 + 10Y)$ where Y indicates the number of drivers who chose the segment [B–T] to travel from S to T via either route [S–B–T] *or* route [S–A–B–T]. If you choose route S–A–B–T, you have to spend a total travel cost of $(10X + 0 + 10Y)$ where X indicates the number of drivers who chose the segment $[S - A]$ to drive from S to T, and Y indicates the number of drivers who chose the segment [B–T] to drive from S to T.

Note that, unlike Game 1, drivers who choose route [S–A–T] and route [S–A–B–T] *share the segment* [S–A]. Similarly, drivers who choose routes [S–B–T] and [S–A–B–T] *share the segment $[B - T]$.*

Example. Supposing that you choose route [S–A–B–T], 3 other drivers choose route [S–A–T], and 14 additional drivers choose route [S–B–T]. Then, your total travel cost for that period is equal to $(10 \times 4) + 0 + (10 \times 15) = 190$. Note that in this example, 4 drivers (including you) traveled on the segment [S–A] and 15 drivers (again, including you) traveled the segment [B–T] to go from S to T. Each of the 3 drivers choosing route [S–A–T] will be charged a travel cost of $(10 \times 4) + (210) = 250$, and each of the 14 drivers choosing the route [S–B–T] will be charged a travel cost of $(210) + (10 \times 15) = 360$.

Similarly to Game 1A, at the beginning of each round you will receive an endowment of 400 points. Your payoff for each round will be determined by subtracting your total travel cost from your endowment for that round. The information you receive at the end of each round is the same as in Game 1. In particular, at the end of each round the computer will display:

– The route you have chosen.
– The number of drivers who chose route [S–A–T].
– The number of drivers who chose route [S–B–T].
– The number of drivers who chose route [S–A–B–T].
– Your payoff for that round.

Payoffs will be determined exactly as in Game 1A (4 payment rounds randomly drawn out of 40).

Thank you for your participation in this experiment.

Appendix B. Proofs

The proofs presented in this appendix are mostly attributed to Professor Reinhard Selten. The authors wish to thank him again for his invaluable contribution.

In this appendix we prove three assertions about the equilibrium strategies in Games 1B (Fig. 1b), 2A (Fig. 7a), and 2B (Fig. 7b).

Assertion 1. *For every player* $i = 1, 2, \ldots, 18$ *of Game* 1B (*Fig.* 1b)*, choosing the route* $(O–A–B–D)$ *is a strictly dominant strategy.*

Proof. Each player i has three strategies $k = 1, 2$, and 3 standing for choosing routes $(O–A–D)$, $(O–B–D)$, and $(O–A–B–D)$, respectively. Consider some fixed and arbitrary player j, and for each k let g_k denote the number of other players choosing route k. Because $n = 18$, it follows that $g_1 + g_2 + g_3 = 17$.

Let C_k denote the cost of player j as a function of g_1, g_2, and g_3, if she plays strategy k. C_k is the sum of costs of the edges of route k. We have (see Fig. 1b):

$$C_1 = 10(1 + g_1 + g_3) + 210,$$
$$C_2 = 10(1 + g_2 + g_3) + 210,$$
$$C_3 = 10(2 + g_1 + g_2 + 2g_3).$$

To prove Assertion 1, it is sufficient to show that $C_1 > C_3$ and $C_2 > C_3$ for any g_1, g_2, and g_3. Subtracting C_1 from C_3 and C_2 from C_3, we have

$$C_3 - C_1 = 10(1 + g_2 + g_3) - 210,$$
$$C_3 - C_2 = 10(1 + g_1 + g_3) - 210.$$

Because $g_1 + g_2 + g_3 = 17$, the expressions $1 + g_2 + g_3$ and $1 + g_1 + g_3$ are at most 18. We obtain

$$C_3 - C_1 \leqslant 10(18 - 21) = -30,$$
$$C_3 - C_2 \leqslant 10(18 - 21) = -30.$$

Thus, regardless of the strategies chosen by the other 17 players, player j's cost for choosing $k = 3$ is at least 30 less than the cost of playing strategies 1 or 2. Since j is arbitrary, this conclusion holds for any player i. Moreover, because strategy $(O–A–B–D)$ is strictly dominant for each player, it is unique. □

Assertion 2. *At every pure-strategy equilibrium of Game* 2A (*Fig.* 7a)*,* 6 *players choose route* $(O–A–B)$*,* 6 *choose route* $(O–B–D)$*, and 6 more choose route* $(O–C–E–D)$*.*

Proof. Each player i has three strategies $k = 1, 2$, and 3 standing for choosing routes $(O–A–D)$, $(O–B–D)$, and $(O–C–E–D)$, respectively. Let g_k denote the number of players choosing route k and C_k the cost associated with choosing this strategy, given g_1, g_2, and g_3. We have $g_1 + g_2 + g_3 = 18$.

The proof is by contradiction. Because the three routes 1, 2, and 3 have the same cost function $(60 + 12g_k)$, assume without loss of generality that $g_1 \geqslant g_2 \geqslant g_3$.

In choosing their routes, one of the following four cases holds:

(h1) $g_1 = g_2 = g_3$.
(h2) $g_1 > g_2 = g_3$.
(h3) $g_1 = g_2 > g_3$.
(h4) $g_1 > g_2 > g_3$.

Assume (h2). This implies that $g_1 \geqslant 8$ and $g_2 = g_3 \leqslant 5$. Consequently, $C_1 \geqslant 156$ and $C_2 = C_3 \leqslant 120$. Then, any player can switch from strategy 1 to strategy 2 or 3 and incur a smaller cost of at most 132. Next, assume (h3). This implies that $g_1 = g_2 \geqslant 7$ and $g_3 \leqslant 4$. Consequently, $C_1 = C_2 \geqslant 144$ and $C_3 \leqslant 108$. Then, if any of the two players switch from strategies 1 and 2 to strategy 3 their cost will decrease to at most 132, which is smaller than their current cost of 144. Finally, assume (h4). This implies that $g_1 \geqslant 7$ and $g_3 \leqslant 5$. Consequently, $C_1 \geqslant 144$ and $C_3 \leqslant 120$. Then, once again, a player switching from strategy 1 to strategy 3 will end up with a lower cost that is at most 132. Consequently, a pure-strategy combination that includes (h2), (h3), or (h4) cannot be an equilibrium. The same argument applies to any permutation of the strategies 1, 2, and 3. This establishes the uniqueness of the solution in (h1). □

Assertion 3. *At every pure-strategy equilibrium of Game* 2B (*Fig.* 7b)*,* 9 *players choose strategy* $(O–C–A–D)$ *and* 9 *others choose strategy* $(O–B–E–D)$.

Proof. Each of the 18 players has five strategies $k = 1, 2, 3, 4$, and 5, that stand for choosing routes $(O–A–D)$, $(O–B–D)$, $(O–C–E–D)$, $(O–C–A–D)$, and $(O–B–E–D)$, respectively. For any k, denote by f_k the number of players choosing the pure strategy k and by C_k the cost of choosing this same strategy, given f_1, f_2, f_3, f_4, and f_5. Similarly, for every edge XY, denote by f_{XY} the number of players traversing this edge and by C_{XY} the cost associated with this edge.

We assume that a pure strategy is always played. The proof includes four steps in which equations (a), (b), (c), and (d) below are established in this order:

(a) $f_1 = 0$,
(b) $f_2 = 0$,
(c) $f_3 = 0$,
(d) $f_4 = f_5 = 9$.

Each of these equations is derived by an indirect proof.

I. We first establish that (a) holds. To do so, we assume (a1), derive conditions (a2) through (a8), and then conclude that (a1) cannot be a part of a pure-strategy equilibrium.

(a1) $f_1 > 0$
(a2) $f_{OC} \geqslant 9$,
(a3) $f_{OB} \leqslant 8$,
(a4) $f_3 = 0$,
(a5) $f_4 \geqslant 9$,
(a6) $f_{AD} \geqslant 10$,
(a7) $C_1 \geqslant 180$,
(a8) $C_5 \leqslant 146$.

Because (a1) is an equilibrium strategy, we must have $2 + 6(f_{OC} + 1) \geqslant 60$; otherwise, a player choosing strategy 1 will have an incentive to switch to strategy 4. The smallest integer satisfying this inequality for f_{OC} is 9, resulting in (a2). Since at least one player chooses strategy 1 and at least 9 players choose one of strategies 3 or 4, there can be at most 8 players traversing edge $(O–B)$. This yields (a3). A player choosing strategy 3 has an incentive to replace $(O–C–E)$ by $(O–B–E)$, thereby switching to strategy 5, since $112 \leqslant 6f_{OC} + 58$ in view of (a2), and $12(f_{OB} + 1) \leqslant 108$ by (a3). This yields (a4). Therefore, all the players who go through edge

(O–C) must choose strategy 4. This yields (a5). Since the routes of at least one player choosing strategy 1 and of at least 9 players choosing strategy 4 go through (A–D), we must have (a6) and, therefore, (a7). Inequality (a8) is a consequence of (a3) and (a4). Switching from strategy 1 to strategy 5 increases f_{OB} to $f_{OB}+1$ and f_{ED} to $f_{ED}+1$, and consequently increases the maximum of C_5 by 18 from 146 to 164. Note, however, that it is still the case that $164 < 180$. This switch is profitable contrary to (a1). Therefore, (a) must hold in equilibrium.

II. Turning next to equation (b), we begin with assuming (b1), derive conditions (b2) through (b8), and conclude that a strategy combination with (b1) cannot be a part of a pure-strategy equilibrium.

(b1) $f_2 > 0$,
(b2) $f_{ED} \geqslant 9$,
(b3) $f_{AD} \leqslant 8$,
(b4) $f_3 = 0$,
(b5) $f_5 \geqslant 9$,
(b6) $f_{OB} \geqslant 10$,
(b7) $C_2 \geqslant 180$,
(b8) $C_4 \leqslant 146$.

The proof is analogous to that of equation (a). If $f_{ED} < 9$, then a player choosing strategy 2 would have an incentive to switch to strategy 5. This yields condition (b2). Since at least 10 players choose strategies that do not include the edge (A–D), we have (b3). In view of $12(f_{AD}+1) \leqslant 108$, a player choosing strategy 3 is better off replacing (C–E–D) by (C–A–D). This yields (b4). In view of $f_1 = f_3 = 0$, it follows from (b2) that (b5) is satisfied. And in view of $f_{OB} = f_2 + f_5$, it also follows from (b1) that (b6) is satisfied. Together with (b2), this yields (b7). Finally, inequality (b8) is a consequence of (b3) and (b4). In view of conditions (b7) and (b8), a player benefits by switching from strategy 2 to strategy 4. Therefore, a strategy combination that includes (b1) cannot be a part of a pure-strategy equilibrium and (b) holds.

III. Turning next to equation (c), we begin by assuming (c1) and proceed to the derivation of conclusions (c2), (c3) and (c4).

(c1) $f_3 > 0$,
(c2) $\min[f_4, f_5] \leqslant 8$,
(c3) $\min[C_1, C_2] \leqslant 156$,
(c4) $C_3 = 168 + 6f_3 \geqslant 174$.

Conditions (a) and (b) imply that $f_1 = f_2 = 0$. Therefore, $f_3 + f_4 + f_5 = 18$ holds. It follows by (c1) that $f_4 + f_5$ is equal to at most 17. This yields (c2). Since $f_1 = f_2 = 0$, we have $f_{AD} = f_4$ and $f_{OB} = f_5$. It follows by (c2) that (c3) holds. Therefore, $C_3 = 60 + 6(f_3 + f_4) + 6(f_3 + f_5)$. Since $f_3 + f_4 + f_5 = 18$, this yields $C_3 = 168 + 6f_3$. In view of (c1), f_3 must be equal to at least 1. Therefore, (c4) holds. If a player who chooses strategy 3 switches to either strategy 1 or strategy 2, then either f_{AD} or f_{OB}, respectively, is increased by 1 and, as a result of choosing either strategy 1 or 3, the cost of the new strategy of this player is increased by 12. The new cost for the more favorable of the two strategies 1 and 2 is, therefore, at most 168. Thus, a player choosing strategy 3 benefits from switching to this more favorable strategy. Consequently, we conclude that a pure-strategy combination that includes (c1) cannot be an equilibrium.

IV. It remains to prove (d). To do so, we assume (d1) and then derive conditions (d2), (d3), and (d4).

(d1) $f_4 \neq f_5$,
(d2) $\min[f_4, f_5] \leqslant 8$,
(d3) $\min[C_4, C_5] \leqslant 146$,
(d4) $\max[C_4, C_5] \geqslant 182$.

In view of (a), (b), and (c) that together establish that $f_1 = f_2 = f_3 = 0$, we have $f_4 + f_5 = 18$. Together with (d1), this establishes (d2). Moreover, we have $f_{OC} = f_{AD} = f_4$ and $f_{OB} = f_{ED} = f_5$. Therefore, C_4 and C_5 are as follows:

$$C_4 = 2 + 18 f_4,$$
$$C_5 = 2 + 18 f_5.$$

In view of (d2), this yields (d3). It follows by (d2) and the equality $f_4 + f_5 = 18$ that $\max[f_4, f_5]$ is at least 10. Therefore, the formulas for C_4 and C_5 also yield (d4). If a player switches from the less favorable of the two strategies 4 and 5 to the more favorable one, the cost of this new strategy increases by 18 to at most 164. Therefore, this switch is beneficial. Consequently, a pure-strategy combination that includes (d1) cannot be a part of a pure-strategy equilibrium. This completes the four steps of the proof of Assertion 3. □

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