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The multicommodity network flow problem: state of the art classification, applications, and solution methods

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Abstract
Over the past decades, the Multicommodity Network Flow (MCNF) problem has grown popular in the academic literature and a growing number of researchers are interested in this field. It is a powerful operational research approach to tackle and solve many complicated problems, especially in transportation and telecommunication contexts. Yet, few literature reviews have made an effort to classify the existing articles accordingly. In this article, we present a taxonomic review of the MCNF literature published between 2000 and 2019. Based on an adapted version of an existing comprehensive taxonomy, we have classified 263 articles into two main categories of applications and solution methods. We have also analyzed the research interests in the MCNF literature. This classification is the first to categorize the articles into this level of detail. Results show that there are topics, which need to be addressed in future researches.

Keywords Multicommodity network flow · Literature review · Mixed-integer programming · Network optimization

1 Introduction

The multicommodity network flow (MCNF) problem has been considerably recognized in the transportation industry and communication networks. The importance of MCNF is motivated by the fact that although it is known as one of the large-scale, yet difficult, problems in the network optimization, it is considered as a cornerstone model in the network design with decomposable construction in the formulation

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(Shepherd and Zhang 2001; Ouorou and Mahey 2000). The multicommodity network flow problem was introduced by Ford and Fulkerson (1958) and Hu (1963). Since then, many researchers have attempted to extend the model by adding different aspects of the problem. Even for a very simple MCNF problem with continuous flows and linear costs, it would be practically very hard to find an optimal solution (Mahey and de Souza 2007). These difficulties arise from the complex nature of constraints structure and a large number of variables, which require an extensive computation time (Nagamochi and Ibaraki 1990). Nevertheless, massive achievements have been obtained in solving the model in the context of outstanding solution time and excellent optimal point, thanks to improvements in both algorithms and computation technology (McBride 1998).

Despite a rich body of literature, the lack of an extensive and comprehensive review has made the MCNF attractive to only a number of researchers such as the surveys that were conducted by Kennington (1978) and Assad (1978). The Kennington (1978) not only presented the impressive analysis of linear MCNF with different special formulation but also provided a good background of solution techniques with computational experiments. Likewise, Assad (1978) gave great definitions of both linear and nonlinear MCNF and proposed solution methods with applications. Afterward, Ouorou et al. (2000) published a paper in which solution methods were taken into consideration for nonlinear convex multicommodity networks. McBride (1998) briefly reviewed what has been done in the field of decomposition and simplex methods and claimed that the primal simplex method combined with the basis-partitioning approach is the most efficient way to solve the MCNF problem.

It is notable to mention that a large number of papers focused on the theoretical side of MCNF, to make it clear, the concentration was on providing relevant formulation and efficient algorithm as solution methods. In this context, earlier articles (Okamura and Seymour 1981; Erdös and Székely 1992; Okamura 1983; Schrijver 1991; Hurkens, Schrijver, and Tardos 1988; Truemper and Soun 1979; Chen and Chin 1992; Onaga and Kakusho 1971; Ali et al. 1985) have made a solid background in the literature. Maurras and Vaxes (1997) showed that the decomposition method could be used to solve multicommodity problems with jump constraints. To make the computation more efficient, Leighton et al. (1995) proposed a polynomial-time combinatorial approximation algorithm for the problem. Liu (1997) who bestowed different papers associated with MCNF introduced a dual steepest-edge method with two phases to solve capacitated MCNF. Gabrel et al. (1999) tried to solve MCNF with a general step cost function while the suggested exact solution method was based on the constraint generation approach and linear programming. Evans (1981) presented a heuristic-based approach to solving a multicommodity assignment problem with the concept of node-aggregation.

In addition to earlier papers, several articles (Seymour 2015; Hall et al. 2007; Srivastav and Stangier 2000; Fleischer and Skutella 2002) have been recently found that can be classified in the theoretical category. In this regard, Lee (2010) presented the integer multicommodity flow problem and suggested a linear programming model using polynomial variables and constraints. Guo and Niedermeier (2006) defined a multicommodity demand flow network as a capacitated tree with the aim of maximizing the profit of possible transported commodities. To solve the problem, the authors applied the fixed-parameter algorithm based on dynamic programming. Fakhri and Ghatee (2014) considered the concept of optimality conditions for fractional multicommodity flow problems. Since the model was formulated as fractional linear programming, the researchers proposed the duality of fractional MCNF.

Groß and Skutella (2015) focused on the time aspect in multicommodity flows to find the quickest flow to meet all demands. Meanwhile, the authors improved the speed-up through storage Wang and Kleinberg (2009). proposed an NP-complete problem named quadratic unconstrained binary optimization. The problem was modeled as a maximum multicommodity flow. Ghatee and Hashemi (2009) proposed a fuzzy concept in travel cost and travel demand for minimal cost multicommodity flow problem. According to each scheme, two associated algorithms were suggested. To achieve a reasonable path, the fuzzy shortest paths and K-shortest paths were used and for the second scheme, the Hukuhara difference was utilized. It is necessary to point out that Mendes et al. (1997) considered a linear multicommodity transportation problem with fuzzy constraint, costs, and objective function and introduced an algorithm based on the concept of comparison relation between fuzzy numbers.

The purpose of this article is to classify the academic literature on the MCNF. In this paper, as our main contribution, a comprehensive overview of the multicommodity network flow problem is presented. More importantly, we have cleared the ambiguity of the path of investigation in this area because we have exactly highlighted both applications and solution methods for the MCNF problems to help and inspire researchers to recognize which classification is required to be under more attention and concentration. We collected an extensive number of articles on multicommodity network flow problems from different journals available in various databases. Following that, we categorized the literature into two main groups of applications and solution methods.

The remainder of the paper is organized as follows. The scope of the survey is briefly explained in Sect. 2. A general overview of MCNF and the mathematical formulation is presented in Sect. 3. Applications of MCNF are categorized and described, in fine detail, in Sect. 4. Section 5 is devoted to different solution methods where they have been classified based on the nature of the algorithm. The paper is summarized and concluded in Sect. 6.

2 Scope of the survey

We surveyed the recent literature, published between 2000 and 2019, using a taxonomic framework. Only relevant articles published in English language journals are considered. We do not consider books, technical reports, and dissertations. To extract the most relevant literature and keep the number of articles manageable, we restrict our search to articles containing “multi-commodity” and “multicommodity network flow” in their title, abstract, and/or keywords. To complete the pile of references, we have also added articles addressed in the bibliographies of articles we had used in the first round. After careful reading of abstracts, methodologies, and conclusions, we considered the articles in our classification.

The search was limited to articles published by well-known publishers including Science Direct, Springer, JSTOR, and IEEE. Based on the SJR journal ranking scheme, all journals are classified into four categories, i.e. Q1, Q2, Q3, and Q4. Figure 1 illustrates the rank of journal and share of them in our surveys. As can be seen, Q1 journals published most of the surveyed articles. In addition, 27.76% of the surveyed articles have been published in conference proceedings.

The classification is done from two different perspectives. Firstly, we focused on the applications of the MCNF models in different contexts. Then, we classify papers based on the solution methods applied to the MCNF problems.

3 Multicommodity network flow problem

The multicommodity network flow problem is categorized as a complicated and significant problem in operation research, which inspired many transportations, production and communication researchers. This problem can be formulated as a linear programming model and can be solved in polynomial time (Ouorou and Mahey 2000). MCNF is described over a network where two or more commodities have to be transferred from specific origin nodes to destination nodes while each arc of the network has a specified capacity.

There are three main MCNF categories; the max MCNF problem, the max-concurrent MCNF problem, and the min-cost MCNF problem. The max MCNF problem is to maximize the sum of flows for all commodities between their origin and destination nodes. The max-concurrent problem is a kind of max MCNF problem, which maximizes the fraction of the satisfied demands for all commodities. The min-cost MCNF problem is to minimize the cost of flows such that all the demands are satisfied without violating the capacity constraint on all arcs.

From the context point of view, MCNF can be divided into two main categories: network routing and network design problems. Network routing problems include message routing in telecommunication, scheduling and routing in logistics and transportation and traffic equilibrium. Network design problems include designing a network in which the maximum load on each arc is minimized. In addition, the terminal layout problem is included in this category.

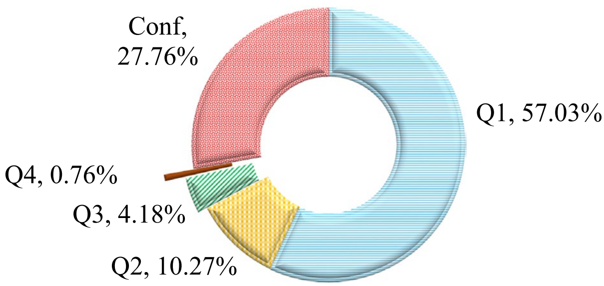


Fig. 1 The quality of the papers being surveyed

3.1 Multicommodity network flow formulation

Two types of formulation have been considered in the literature: the node-arc form and the arc-path form. The single commodity network flow (SCNF) formulation is extended to create MCNF node-arc form. On the other hand, several independent SCNF are connected to form MCNF problems. The arc-path form is proposed by (Tomlin 1966) and a network flow solution is divided into path flows and cycle flows and according to the assumption that no cycle has a negative cost; any arc flow vector can be expressed optimally by simple path flow.

3.1.1 The node-arc form

Let N be a non-empty set of n nodes, A denotes a non-empty set of all arcs and K be the non-empty set of all commodities which s_k and t_k display the origin and destination nodes in turn. Let u_{ij} be the capacity on the arc (i,j) of the network and c_{ij} represents the cost of a unit of flow on the arc (i,j).x_{ij}^k denotes the flow of commodity k on the arc (i,j), d^k represents the total demand for commodity k, and b_i^k is defined as the supply or demand of node i for commodity k (Wang 2003).

$$\min Z^*(x) = \sum_{k\in K} \sum_{(i,j)\in A} c_{ij}^k x_{ij}^k \tag{1}$$

$$\sum_{(i,j)\in A} x_{ij}^k - \sum_{(i,j)\in A} x_{ji}^k = b_i^k \quad \forall i \in N, \quad \forall k \in K \tag{2}$$

$$\sum_{k\in K} x_{ij}^k \le u_{ij} \forall (i.j) \in A \tag{3}$$

$$x_{ij}^k \ge 0 \forall (i,j) \in A, \quad \forall k \in K \tag{4}$$

where $b_i^k = d^K if i = s_k, b_i^k = -d^K \ if \ i = t_k,$ and $b_i^k = 0 \ if \ i \in N \backslash \{s_k, t_k\}$.

The objective function aims to minimize the total shipping cost. Constraint (2) is the supply/demand constraint. Constraint (3) captures the capacity or the bundle constraint, and constraint (4) is the non-negativity constraint (Schneur and Orlin 1998).

3.1.2 The arc-path form

Let P^k denote the set of all possible simple paths from s_k to t_k for each commodity k and each decision variable f_p is the flow on some path p and for the k th commodity in $p \in P^k$. Let d^k be the demand for commodity k and PC_p^c be the cost of path p using c_{ij}^k as the arc costs. δ_a^p is a binary indicator which equals to 1 if path p passes through the arc a, and 0 otherwise.

$$\min Z^*(f) = \sum_{k \in K} \sum_{p \in P^k} PC_p^c f_p \tag{5}$$

$$\sum_{p \in P^k} f_p = d^k \quad \forall k \in K \tag{6}$$

$$\sum_{k \in K} \sum_{p \in P^k} (\delta_a^p) f_p \le u_a \quad \forall a \in A \tag{7}$$

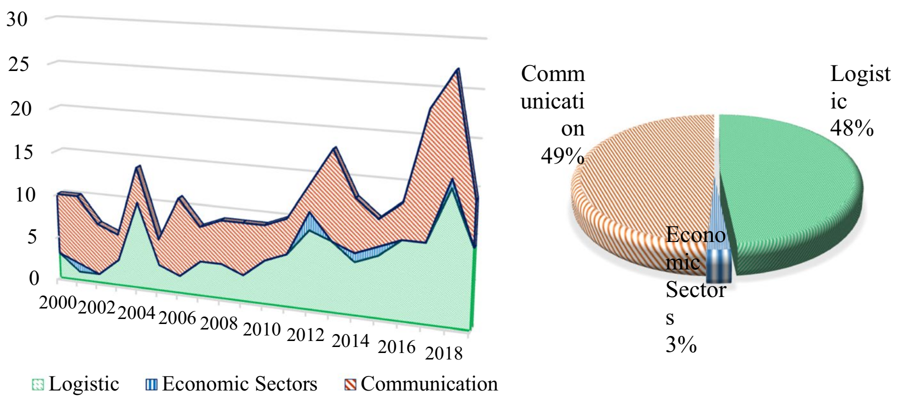
$$f_p \ge 0 \quad \forall p \in P^k, \quad \forall k \in K \tag{8}$$

Constraint (6) forces the total flow on all the paths connecting s_k to t_k for each commodity k must equal the demand d^k and constraint (7) states that the sum of the path flows passing through the arc is at most the capacity of the arc (Ahuja et al. 1993). Constraint (8) enforces non-negative values for f_p.

4 Applications of multicommodity network flow

The modeling capability of MCNF and its ability to distinguish between different commodities in the final solution has attracted many researchers to tackle problems using this modeling framework. The applications of MCNF, based on papers we reviewed, are extended in many areas from communication networks to logistics and energy. The reason why multicommodity network flow problems are noteworthy in operation research techniques is that MCNF can be applied for issues arises in the real world.

Figure 2b indicates that communication systems have used MCNF the most (49%). Logistics and transportation systems are the second important applications



(a) **(b)**

Fig. 2 **a** Trends in applications of MCNF, **b** the total number of articles in applications of MCNF. The close percentages prove that both subjects are active research areas. The trend in Fig. 2a shows that MCNF has been applied in this context, frequently. It is observed that the number of papers published in communication stood at seven in 2000 and decreased to three in 2003. The graph showed a fluctuation between 2003 until 2019 and reached its peak at 14 in 2017. The interest in using MCNF in modeling communication problems had a considerable decline in 2015 while after that a significant growth can be seen. As far as the logistic subject is concerned, the trend of articles in MCNF between 2000 until 2009 remained almost the same except for a boom in 2004 with 10 articles.

The comparison between the three graphs indicates that communication subject accounted for the most significant proportion especially in 2017 whereas using MCNF in the economic sector is far less admired between researchers. Given their powerful modeling potentials in capturing realistic operating conditions, the further applications of MCNF in transportation problems provide an opportunity for further research.

To make the review more usable and to stimulate researchers to use the MCNF modeling approach, we classified all papers in three main categories consisting of (1) logistic, transportation systems, and traffic engineering, (2) economic sectors and energy; and finally (3) communication and computer networks. Table 1 summarizes the papers devoted to each category. This classification focuses on the problem being tackled and a brief description of the fundamental tenets and objectives of the papers are discussed.

4.1 Logistics and transportation

The pressure of highly competitive markets effect on business operations, thereby a business tries to find an optimal way to produce and transfer values to customers. Business decision-makers decide how and when the products should be delivered such that the quality and quantity of them be satisfied cost-effectively. In this context, transportation models provide a powerful framework to help decision-makers (Amit and Amarpreet 2011). The role of transportation is significant in the supply chain and regardless of how much a business is large or small; the importance of transportation will not be diminished. Furthermore, traffic on roads derived from an increasing number of vehicles creates many mobility-related problems including congestion, air pollution, noise pollution and accidents (Zanjirani Farahani et al. 2013). Due to this fact, operations research played a remarkable role in planning and optimizing the transportation sector. A massive body of literature is available on the applications of multicommodity network flow problems applied to resolve transportation issues.

Erera et al. (2005) presented a model for managing tank containers. The decisions about container booking, allocating space to commodities, loading, and movement from depot to customers for providing demands and finally return to depot for cleaning and repositioning for next loading, are made by operators using an optimization model for integrated tank container management. The aim of the model, formulated as a deterministic multicommodity network flow on a time-expanded network, was

Table 1 summary of researches on the applications of MCNF

Applications	References
Logistic, transportation system and traffic engineering	Psaraftis (2011), Erera et al. (2005), Al-Khayyal and Hwang (2007), Noguera and Leirens (2010), Jin et al. (2009), Liu et al. (2008), Lin and Kwan (2013), Jeenanunta et al. (2011), Be´langer et al. (2006), Poh et al. (2004), Ghatee and Hashemi (2007), Koca and Yıldırım (2012), Mesquita et al. (2015), Jin (2012), Yaghini and Akhavan (2012), Maharjan and Matis (2012), Moz and Pato (2003), Moz and Pato (2004), Fleischer et al. (2004), Alumur et al. (2012), Jiang et al. (2014), Paraskevopoulos et al. (2016), Letchford and Salazar-González (2016), Karsten et al. (2015), Rudi et al. (2014), Quilliot et al. (2013), Frangioni and Gorgone (2014), Tanonkou et al. (2007), Alfaki and Haugland (2013), Joe et al. (2014), Tonneau et al. (2015), Powell and Carvalho (1997), Cortés et al. (2013), Ríos-Mercado et al. (2013), Haghani and Oh (1996), Shishegar et al. (2014), Cattaruzza et al. (2014), Letchford and Salazar-González (2015), Aringhieri and Cordone (2004), Cappanera and Gallo (2004), Caimi et al. (2011), Ayar and Yaman (2012), Canel et al. (2001), Qu et al. (2016), Pióro et al. (2016), Whitman et al. (2017), Mohammadi et al. (2017), McDiarmid et al. (2017), Sun et al. (2017a, b) Zhang et al. (2017a, b), Wright et al. (2017), Sun et al. (2017a, b), Jin (2018), Vahdani et al. (2018), Gao and Lee (2018), McCarter et al. (2018), Karimi and Bashiri (2018), Cariou et al. (2018), Balma et al. (2018), Zeng et al. (2018), Zhang et al. (2018a, b, c), Zhang et al. (2018a, b, c), Lee et al. (2019), Velichko et al. (2018), Hao et al. (2019), Jadin et al. (2019), Cheng et al. (2018), Ghasemi et al. (2019), Bevrani et al. (2019), Azadi Moghaddam Arani et al. (2019), Lu et al. (2019), Vaziri et al. (2019), Zhang et al. (2019)

Table 1 (continued)

Applications	References
Communication and computer networks	Ouorou and Mahey (2000), Ouorou et al. (2000), Ouorou et al. (2001), Wagner et al. (2007), Minoux (2001), Padmanabh and Roy (2006), Bazan and Jaseemuddin (2008), Madan et al. (2008), Hadjiat et al. (2000), Zantuti (2005), Leveque et al. (2006), Khuller et al. (1994), Robinson et al. (2006), Lin (2004), Lin (2001), Lin (2002), Lin (2007a, b), Lin (2007a, b), Sedeno-Noda et al. (2005), Shepherd and Zhang (2001), Ozdaglar and Bertsekas (2004), Hirai and Koichi (2011), Erez and Feder (2009), Lozovanu and Fonoberova (2006), Agarwal (2002), Kacprzak, et al. (2010), Rothfarb and Frisch (1970), Awerbuch and Khandekar (2009), Ward (1999), Wang and Kleinberg (2009), Mrad and Haouari (2008), Aldous et al. (2009), Masri et al. (2015), Levitin et al. (2013), Kabadurmus and Smith (2016), Even and Medina (2012), Queiroz and Humes (2001), Liu et al. (2013), Liu et al. (2014), Cheng and Sterbenz (2014), Gebremariam and Bikdash (2013), Wadie and Ashour (2013), Teyeb et al. (2014), Jiao and Dong (2016), Belijadid et al. (2013), Kollias and Nikolaidis (2015), Huang et al. (2014), Moungla et al. (2013), Chekuri et al. (2013), Jia, et al. (2014), Shitrit et al. (2014), Huerta and Hesselbach (2004), Kolar and Abu-Ghazaleh (2006), Kalantari and Shayman (2006), Kiwiel (2011), Castro and Frangioni (2000), McBride and Mamer (2001), Ouorou et al. (2001), Gupta et al. (2001), Albrecht (2001), Schauer and Raidl (2016), Zhang and Fan (2017), Jia et al. (2017), Xu et al. (2017), Brandt et al. (2017), Zhang et al. (2017a, b), Venkatadri et al. (2017), Layeb et al. (2017), Fortz et al. (2017), Balakrishnan et al. (2017), Ding (2017), Samani and Wang (2018), Sallam et al. (2018), Chien and Hung (2018), Zhang et al. (2018a, b, c), Bera et al. (2019)
Other economic sectors	Singh and Ahn (1978), Gautier and Granot (1995), Adhikari et al. (2012), Brotcorne et al. (2001), Manfren (2012), Shi et al. (2015), Chale-Gongora et al. (2014), Kuiteing et al. (2018)

to minimize transportation costs. Psaraftis (2011) tried to solve a multicommodity capacitated pickup and delivery for both single-vehicle and two-vehicle problems. A dynamic programming algorithm solution was proposed.

The concept of multicommodity flow is also used in the oil industry. In a pipeline network connecting oilfields, refineries, ports, and customers Noguera and Leirens (2010) applied an MCNF model to determine the optimal transportation of commodities such as gasoline, jet fuel, and diesel are transported. In this context, they developed a dynamic model of multicommodity pipeline network with interconnected pumps considering the dynamic effects of pressure, velocity and fluid density on the commodities transferred. Moreover, they extended a mixture model, which predicts the contaminated interface in each pipe section. Al-Khayyal and Hwang (2007) tried to address a problem that an oil company in Asia Pacific required to decide how to meet petrochemical products' demands for archipelago islands. It is noticeable that due to the difficulties such as scattered plants in each island and lack of transportation infrastructures, the company used ships to deliver separated categories of products. The proposed model determines the time that a ship, should carry the amount each commodity from supply ports to demand ports. It is also required for the model to contemplate the fact that the capacity of each port should be considered. To solve the problem, a mixed-integer linear program pickup and delivery model to minimize routing costs are developed.

The connection between logistics and inventory systems is an area where the concept of multicommodity flow could be applied. Jin et al. (2009) focused on increasing the efficiency of the inventory system by considering two crucial factors; the quantity of inventory and the inventory space. Authors integrated facility location and inventory decision problems to develop a multicommodity logistics network. Based on normally distributed deterministic demand of retailers the authors proposed a non-linear mixed-integer programming model to minimize the total cost of location, inventory, and transportation. A combined simulated annealing (CSA) algorithm was used to solve the problem. Koca and Yıldırım (2012) addressed a problem in which spare parts need to be transferred from a central depot to retailers to satisfy demands while total distribution costs are minimized considering the capacity of each vehicle. Based on the definition of the multicommodity distribution problem with fixed assignment costs, an integer linear programming formulation was developed. Hierarchical approach with two stages comprising a partial demand aggregation scheme at the first and disaggregation stage for the next level was used.

Liu et al. (Liu et al. 2008) considered a multicommodity network with a stochastic flow to capture uncertain features such as long transportation time, long-distance and numerous steps in the transportation process, aiming to determine successful transmission scheme and transportation cost. A multi-objective problem of flow allocation and control in a multicommodity stochastic-flow network was proposed and Genetic Algorithm was used to solve the model.

The driver rostering problem deals with scheduling schemes that determine a sequence of workdays and days off in the period for drivers. The purpose of the model is to deteriorate operating costs while boosting efficiency and service quality in accordance with roster balancing. An assignment/cover model and two different multicommodity problems were integrated into the form of mixed-integer linear programming (Mesquita et al. 2015) to tackle problems in urban transportation. Real-world data from two bus transit companies in Portugal were analyzed. The proposed model solved using a decompose-and-fix heuristic method. In this context, Moz and Pato (2003) developed an integer multicommodity flow for re-rostering the nurse schedule. Aringhieri and Cordone (2004) proposed a Lagrangian approach for the multilevel bottleneck assignment problem to model the crew roster. Cappanera and Gallo (2004) addressed the airline crew rostering problem and formulated the model as a 0–1 multicommodity flow problem.

Poh et al. (2004) considered a traffic engineering framework that automatically controls the routs for sustaining the highest traffic throughput. The maximum concurrent multicommodity flow was applied and the factors such as traffic demands, network topology and resources were used to obtain an optimum solution. Signal setting as a control parameter has a significant effect on the shortest path in traffic assignment problems, which boosts the level of service in the transportation systems. Ghatee and Hashemi (2007) proposed a decent direction algorithm for solving the minimum cost multicommodity flow model. Jin (2012) focused on the continuous multicommodity kinematic wave model to address the traffic flow problem in a road network. In another paper, Jin (2018) proposed a unifiable multicommodity kinematic wave traffic model considering the different speeds for commodities. Bevrani et al. (2019) improved the MCNF model with multi-modal transportation system model. The factors that distinguish this paper from others are different flow reduction functions and vehicle speed on each arc. The authors concluded that these factors have a great impact on the reliability and accuracy of MCNF models. A separable approach was applied to determine the optimal flow. Wright et al. (2017) formulated the multicommodity flow distribution for macroscopic traffic models based on the first-in-first-out constraint.

In addition to road transportation, MCNF can be applied in both rail and marine transportation systems. Concentrating on rail freight transportation, Yaghini and Akhavan (2012) presented an overview of the design of multicommodity networks (MCND) modeling and solution methods. They showed that decision-making in strategic, tactical and operational levels in rail freight transportation such as car blocking, train makeup planning, and empty car distribution can be successfully formulated using MCND. Karsten et al. (2015) applied a graph network in linear shipping network design. The number of rotations was connected in a way that vessels with diverse capacities and speed transported commodities to provide demands in each destination that is located in related rotations. They proposed a multicommodity network flow with transit time constraints to capture the problem and the column generation technique was used to find the solution.

Product distribution is an area where MCNF was applied. Tanonkou et al. (2007) considered a stochastic multicommodity distribution network design problem with random supply lead-time and random demands. The problem consisted of a single supplier serving a set of retailers through a set of distribution centers. Non-linear integer programming model was used to determine retailers to be served by a specific distribution center. The model was formulated to minimize the total location, shipment, and inventory costs while ensuring a given retailer service level. Because of the complexity of the problem, Lagrangian-relaxation was applied to find the solution. Quilliot et al. (2013) investigated shuttle fleet transportation, which transfers workers from home to their workplace. To achieve both minimum passengers' transportation time and minimum transportation costs, a multicommodity flow model with a dynamic network was proposed. A heuristic decomposition/aggregation process with a GRASP scheme was used to solve the model. Since different kinds of commodities can be sent on various paths over a mesh network, Cortés et al. (2013) developed a multicommodity flow distribution network with diversification constraints in which cost of arcs are considered as a concave function. An algorithm based on the Kuhn-Tucker method was proposed. The proposed model by Ríos-Mercado et al. (2013) considered a distribution problem for a bottled beverage company. The problem was modeled as a pickup and delivery vehicle routing problem, which dealt with features such as time windows on both customers and vehicles, multiple depots, multiple product delivery, split delivery, heterogeneous fleet, and dock capacity. A GRASP-based heuristic algorithm was proposed to solve the problem.

The combination of multicommodity flow with logistics queuing networks created a model with dynamic construction and time windows in which types of heterogeneous vehicles were considered differently as well as customers that made the formulation complicated to solve. Powell and Carvalho (1997) conducted equation methods including gradient approximation method, linear approximation, and multiplier adjustment procedure in solving multicommodity network queuing problems.

Multicommodity network flow was employed for the generalized pooling problem with bilinear constraints in a production system where raw materials are combined in tanks (pool) and thereafter are blended again to produce products (Alfaki and Haugland 2013). In another research, Boland et al. (2016) proposed an equivalent model with better computational results. Karimi and Bashiri (2018) formulated a multicommodity multimodal transportation network model to transfer commodities and raw materials by logistics hub network for intelligent manufacturing. They assumed that commodities provide a splittable flow and the capacity of hubs and vehicles is limited.

Material handling system (MHS) proved to be an important component in the production, distribution and transportation systems since it contributes to the total cost of the system. In an MHS, it is crucial to decide which equipment (vehicles or robots) should load which commodities and when they should convey products between different types of facilities. Combining the allocation of resources to tasks and then the assignment of execution start time to each task, Joe et al. Joe et al. (2014) applied a matrix-based discrete event controller to solve multicommodity network flow problems.

Managers make three important decisions, long-term strategies, tactical decisions and operational controlling. Tonneau et al. (2015) focused on the tactical decisions to formulate the transportation flow using multi-commodity, multi-flow problem. With the growth of concerns about global warming and the effect of transportation on this phenomenon, Rudi et al. (2014) concentrated on intermodal freight transport to reduce the factors such as greenhouse gas (GHG) emission and transportation costs as well as improvement in transit time. To formulate the problem as a capacitated multicommodity network flow model, the transport and transshipment of full truckloads were analyzed to minimize them, while considering the weighted and normalized criteria such as GHG emissions, cost, and transit time according to the tied in-transit capital and the distance traveled. Cariou et al. (2018) proposed an arc-flow based model based on a multicommodity pickup and delivery in shipping lines to maximize profit while considering CO2 and SOx emissions. A heuristic approach based on a genetic algorithm was used as a solution method.

Railways play a vital role in carrying passengers and commodities. In this context, Lin and Kwan (2013) focused on the train-scheduling problem. He formulated the problem as an integer fixed-charge multicommodity flow model with a two-phase approach. The model aimed to satisfy the demands while the number of train units and/or the operational costs are minimized. He applied branch-and-price to solve the problem. Azadi Moghaddam Arani et al. (2019) proposed an approach to assess the railway capacity, which is defined as the ability of a route to pass a maximum level of traffic flow. A multicommodity flow model considering time–space network formulation under line blockage was suggested to maximize the number of train paths.

Congestion, as the main source of delays in the freight rail transportation network, persuades (Paraskevopoulos et al. 2016) to propose the fixed-charged multicommodity network design model. The objective of the model was to minimize the operational and congestion costs considering the capacity constraints on both arcs and nodes. They also considered the flow of each type of commodity through appropriate arcs while alleviating capacity constraints on nodes. The problem was formulated as a mixed-integer second-order cone program and an evolutionary heuristic based on iterated local search and scatter search were applied.

Aviation is recognized as a fast and safe mode of transportation, which has created a great mutation in transportation infrastructure. MCNF also has been used in this field, Jeenanunta et al. (Jeenanunta et al. 2011) used a multicommodity network flow model to tackle aircraft routing and maintenance scheduling problem. The objective of the model was to minimize the total waiting time of fleets for maintenance checks. Be´langer et al. (2006) proposed a model for periodic fleet assignment problem with time windows. The commodities were sets of aircraft and flights. They used a non-linear integer multicommodity network flow formulation. To solve the problem, a branch-and-bound strategy was applied.

To achieve lower congestion in the airport environment and to reduce operating costs, such as fuel burn cost for aircraft taxies, Maharjan and Matis (2012) presented a binary multicommodity network flow to flight gate assignment problem. Gates were considered as commodities, which flow through a network to satisfy flight demands. The Decomposition method was used to solve the model.

The impact of rescuing people in natural disaster motivated Haghani and Oh (1996) to investigate how the managers can respond to emergencies to minimize the rate of loss of life and maximize the efficiency of the operation. Since the transportation of commodities in a disaster is one of the most vital issues, the authors decided to propose a complicated logistic model named multicommodity multimodal network flow problem with the time window. Two heuristic algorithms were used to solve the model. Shishegar et al. (2014) proposed a dynamic bi-objective, multi-commodity, multi-mode relief routing network. The model dealt with how the injured people should be transferred from the disaster area to medical centers and how the required commodity can be transported to help people. The objective is to minimize total transportation and inventory cost as well as unserved injuries. Vahdani et al. (2018) considered the allocation of distribution centers to transfer relief items to wounded people, vehicle routing and emergency roadway repair simultaneously when a disaster accrued. The problem was modeled as an integer nonlinear multi-objective, multi-period, multicommodity model. Two algorithms including non-dominated sorting genetic algorithm-II (NSGAII) and multi-objective particle swarm optimization (MOPSO) were proposed to solve the problem.

To better manage the rescue operations in disasters, Gao and Lee (2018) proposed a two-stage mixed-integer stochastic programming for the multicommodity redistribution process. The model considered three elements of uncertainty; supply, demand, and transportation network. Ghasemi et al. (2019) suggested an uncertain multi-objective multicommodity multi-period multi-vehicle location-allocation mixed-integer mathematical programming model when an earthquake disaster occurred. The model deals with the temporary care and accommodation centers' location and allocation of the affected areas to the located centers and hospitals, as well as the allocation of the distribution centers to temporary accommodation centers. The uncertainty of the model was based on the probabilistic scenario-based approach. The model was solved using epsilon-constraint, NSGA-II, and MMOPSO algorithms.

In the paper written by McCarter et al. (2018), the authors assessed the vulnerability of a multicommodity system when the network encounters disruptive events. The objective of the model is to reduce the costs while meeting the demands for each commodity. Genetic Algorithm was used to solve the proposed model. Whitman et al. (2017) focused on the vulnerability of multicommodity networks when it is under disruption. The authors compound a three-stage approach to measuring the vulnerability in the performance of the Swedish railway network. Firstly, MCNF was formulated based on minimizing the unmet demands and measuring the baseline network performance; secondly performing the MCNF while one link is removed from the network and evaluating the importance of the link and finally a TOPSIS method was used to rank the critical links.

Lu et al. (2019) proposed a multicommodity pickup-and-delivery traveling salesman problem and a randomized Tabu thresholding algorithm was used to solve the problem. Balma et al. (2018) provided a formulation based on MCNF for the asymmetric traveling salesman problem. Vaziri et al. (2019) proposed a vehicle routing problem with multicommodity multi-pickup and delivery problems based on carrier collaboration. The objectives of the model were to maximize the total profit and the fair sharing of profit among the carriers and minimize the travel time. A genetic algorithm was compared with the variable neighborhood search method for the problem. Letchford and Salazar-González (2015) proposed two MCNF formulations for the capacitated vehicle routing problem. In another paper, (Letchford and Salazar-González 2016) the authors improved the MCNF formulation of the sequential ordering problem by fixing variables and adding valid equations. Zhang et al. (2019) suggested a non-vehicle-index arc-flow formulation for a multicommodity many-to-many vehicle routing problem with simultaneous pickup and delivery considering fast fashion retailing company in Singapore. A meta-heuristic algorithm including adaptive memory programming techniques, the segment-based evaluation scheme, and advanced pool management method was combined to solve the problem. Cattaruzza et al. (2014) developed a multicommodity multi-trip vehicle routing problem with time windows in which incompatible commodities should be transported to customers to minimize the number of vehicles. An iterated local search was proposed to solve the problem.

Mohammadi et al. (2017) focused on a reliable hazardous material transportation network. A multi-modal multicommodity model based on hub location topology under uncertainties was proposed to minimize the risk of incidents. The problem was solved with a meta-heuristic algorithm named imperialist competitive algorithm. Sun et al. (2017a, b) applied generalized bounded rationality in a robust multicommodity network design problem for hazardous materials transportation. A cutting plane algorithm solved the problem.

Zhang et al. (2017a, b) addressed the imbalance problem of the bicycle between user demands and availability of bikes in stations by a dynamic bicycle repositioning methodology. A multicommodity time–space network flow model was developed and solved using a heuristic algorithm.

4.2 Economic sector and energy

Technologies and methods always are changing in the economic sector because the role of economic fundamentals and principles are important and effective in countries prosperous. In the context of agriculture, Singh and Ahn (1978) developed a dynamic multi-crop programming model based on the multicommodity problem to analyze the economic and policy process for the development of the agricultural sector in Brazil. The problem was formulated as a recursive linear programming model.

Forest management comprises the decisions of when and how to plant trees and in which conditions and what time the trees can be cut and the amount of care that should be given to existing parks of trees. To tackle this problem Gautier and Granot (1995), formulated the forest management problem as a multicommodity network flow model with a convex cost function to achieve an economically efficient solution at a reasonable cost. Furthermore, to alter the parameters of the model, the authors measured the sensitivity of solutions.

Energy systems are needed in many aspects of our lives and create many challenges for governments Adhikari et al. (2012). applied the smart grid concept to capture the combination of issues like national energy security, climate-changing, pollution reduction, grid reliability. A multicommodity network flow model was used to manage dynamic energy to obtain a balance and coordination between model usefulness, accuracy, flexibility, solvability, and scalability in smart grid applications. In another article, Manfren (2012) developed a relationship between data mining techniques and multicommodity network flow models to manage dynamic energy.

The importance of a smart energy system is motivated by the fact that fossil energy consumption contributes to the emission of greenhouse gases, which need to be replaced by renewable energy sources. Shi et al. (2015) built a multicommodity mathematical model for smart energy systems. A greedy algorithm was proposed to decrease the complexity of solution and computation time.

The important role of the electric vehicle in reducing carbon dioxide is evident, but the utilization of them provokes concerns about battery capacity and limitation of travel over long distances Chale-Gongora et al. (2014). proposed a smart energy system for drivers to opt for the best paths regarding cost and time. The network was modeled using a multicommodity network flow model.

Brotcorne et al. (2001) considered a multicommodity transportation network with setting tolls on arcs for companies intended to maximize their revenues. A primal–dual heuristic algorithm solved the problem. Kuiteing et al. (2018) focused on a network-pricing problem when demand functions are nonlinear. Two solution methods involving exact and heuristic algorithms based on a bi-level approximation were considered to solve the mixed-integer programming based on MCNF.

4.3 Communication networks

In the light of development of technology, communication networks have inextricably entwined with our modern life, the application ranges of communication networks are scattered from daily used devices to complicated equipment in aircraft, radar scopes, computer systems, and telecommunication, and apparently the internet (Ahmed et al. 2017). Almost all the communication networks such as wireless networks and fiber optic networks can be formulated by MCNF. Wagner et al. (2007) used a multicommodity flow formulation as a generalization of the Steiner tree problem for designing the last mile in fiber-optic networks. Padmanabh and Roy (2006) modeled a routing protocol in wireless sensor network defined as a network of nodes with communication devices based on multicommodity flow algorithm. They proposed a search algorithm based on the golden ratio to solve the problem.

Massage routing problems in data networks drove Ouorou et al. (2000) to investigate the nonlinear convex multicommodity flow problem. After an extensive review of techniques for solving convex multicommodity models, numerical testing was applied to evaluate the performance of different solution techniques. In another paper, Ouorou et al. (2001) focused on network design and operations in telecommunications networks with congestion pricing. A bender decomposition method was proposed to solve the convex cost multicommodity flow problem with the capacity assignment and variable prices for the consumers.

Transferring messages through a telecommunication network requires an efficient routing algorithm to find an optimal path to meet the demands of users. Masri et al. (2015) proposed a multi-start variable neighborhood search method to solve the single path multicommodity flow problem. The model aimed to minimize the total time considering the limited capacity of resources with in-advanced reservations. Minoux (2001) proposed a generic model for discrete cost multicommodity for telecommunication networks. The author provided an overview of the exact algorithm and suggestions for future investigations.

Smart antennas in multi-hop wireless networks can escalate the spatial reuse while extending the transmission range and decreasing power consumption, which with increasing the capacity of the network can improve the performance of the network. To evaluate the network performance using the real smart antenna in comparison with applying the ideal model, Bazan and Jaseemuddin (2008) used a multicommodity flow problem with interference constraints in accordance with the generic interference model.

The capacitated non-simultaneous multicommodity flow assignment problem was captured in a wide area computer network by Zantuti (2005) to minimize the total average delay per packet according to the budget constraint. The problem embraced the selection of non-simultaneous routes with channel capacity. Khuller et al. (1994) tried to design a network named tree congestion problem with a constant degree. They formulated the problem as an MCNF problem and solved it using an approximation algorithm. Robinson et al. (2006) considered a stochastic network with the multi-criteria approach to evaluating flow disturbance in multicommodity or multiple origin–destination networks. Integrated with linear regression and multiple origin–destination networks, they formulated the problem as a multicommodity minimum-cost network flow model. Lin (2001) focused on the evaluation of multicommodity reliability for multicommodity maximum flow problem with stochastic edges. Afterward, he (2002) analyzed system performance in terms of system capacity in the multicommodity stochastic-flow network considering node failure and then decided to evaluate system reliability for the multicommodity stochastic-flow network (Lin 2004). Lin (2007a, b) considered the quality level of a supply–demand system measured by two indicators, system reliability, and system unreliability. A multicommodity limited-flow network (MLFN) with unreliable nodes and arcs subject to budget constraints was proposed to formulate the problem. He used a branch and bound algorithm to solve the problem. In another paper, Lin (2007a, b) proposed a performance index for the multicommodity stochastic-flow network with the concept of the budget constraint. Zhang and Fan (2017) focused on the interdiction effects of the changes on the budget for a multicommodity network flows.

Reliability analysis in the wireless mesh network can be done using a so-called Reliability Analytical Measurement (RAM) scheme to improve the quality of service. Samani and Wang et al. (2018) proposed two integer multicommodity flow problem formulations to maximize the number of streaming sessions and bandwidth provisioning for quality of service enhancement.

To calculate the impact of failures, Beljadid et al. (2013) proposed a multicommodity flow model. To find all efficient extreme points in the objective space, Sedeno-Noda et al. (Sedeno-Noda et al. 2005) proposed two-commodity minimum cost flow problem on an undirected network. A parametric network simplex method was used to solve the problem.

Minimum cost multicommodity flow is a common solution method in bandwidth allocation. Shepherd and Zhang (2001) applied a combinatorial algorithm to solve the problem in a ring network. The complexity of multi-flow networks motivated Hirai and Koichi (2011) to apply some reduction techniques to reduce the complexity of the problem to a facility location problem. The reduced problem, thus, solved using efficient solution methods.

Lozovanu and Fonoberova (2006) focused on a dynamic minimum cost multicommodity flow problems the network in which two factors, time-varying capacity and transit time, were considered for each edge. They used the time-expanded network algorithm to solve the problem. Agarwal (2002) applied a heuristic algorithm solution for the problem of designing a multicommodity network, where edges were capacitated a feasible flow of all traffic demands were also possible. Zhang et al. (2017a, b) proposed a two-level decomposition algorithm for the multicommodity multicast problem for traffic engineering. In another paper Zhang et al. (2018a, b, c) the authors developed a formulation for a multicast multicommodity flow model to minimize the maximum link utilization. The problem also was solved by the decomposition method.

In order to determine the volume capacity to transmitted data over a network, the Multicommodity Balancing Undirected Communication Bandwidth Trade

(BUCBT) exchange model was proposed in Kacprzak et al. (Kacprzak, et al. 2010), in which capacities of network links and demanded capacities between pairs of nodes were two types of commodities in the bandwidth market.

To rescue a network system from external attacks, it is required to evaluate the expected damage to the network Levitin et al. (2013). used multicommodity networks in which, the deprivation of any node is associated with the damage and the lack of the commodities. A Monte Carlo simulation approach was applied to find the solution.

The network design problems have received an ample amount of attention among academia. Kabadurmus and Smith (2016) focused on the development of the survivable multicommodity network design problem in two ways. Initially, the *k*-splittable flow was used with relays for each split path and each survivable path. Then, the capacitated edges were applied to make the formulation closer to reality. The exact and heuristic methods were integrated to obtain the optimum solution. To solve the well-known all-or-none problem in allocating bandwidth to satisfy service requests Chekuri et al. (2013), used a multicommodity modeling approach in which routable requests were considered as commodities which should be routed over a communication network Even and Medina (2012). also addressed the problem of routing a multicommodity flow in an online setting. The main purpose of the article was to propose a centralized, deterministic, all-or-none, non-preemptive online algorithm to satisfy high demands.

The importance of multi-radio multi-channel (MR-MC) network technique to improve the capacity of wireless networks motivates Liu et al. (2013) to investigate the resource allocation in the network. The combination of multicommodity flow with constraints derived from a multi-dimensional conflict graph provided optimization framework for assigning resources that in addition to minimizing energy consumption, could maintain the efficiency of the network with higher capacity. In another paper, Liu and et al. (2014) proposed Delayed Column Generation to solve the MR-MC network model.

When a natural disaster or regional challenges occur, it will affect telecommunication networks in a way that a series of nodes and links may be spoiled or disconnected. Various mechanisms have been analyzed to address the problem however Cheng and Sterbenz (2014), proposed a GeoDivRP routing algorithm that not only could generate distance *d*-separated paths but also optimized traffic allocation over the network. The problem was formulated as a multicommodity flow model to minimize the bounded-jitter in multiple paths. The importance of network vulnerability and survivability will be highlighted when a link or node failure occurs in the network. Survivability deals with preserving the system flow in case of unwanted events depending on the routing and congestion control planning. Gebremariam and Bikdash (2013) applied a multicommodity flow to minimize the transportation cost while commodity with utilizing *k* link-disjoint path met the demands flow rate. The multi-layer hierarchical ring network design arises in hierarchically structured networks that the requirement of survivability is substantial. The problem was formulated based on multicommodity flow (Schauer and Raidl 2016) to address the problem.

Since detailed routing between nets is complicated, NP-hard and should be analyzed in the context of alleviating design rule checking (DRC) violation, Jia et al. (2014) used multicommodity flow to address the problem. To reduce overall network delay while opting for the paths, load-balanced routing for wireless ad-hoc network propagates traffic load within the network that ensures minimum congestion for conveyance as well as created interference by neighboring nodes. In another paper, Jia et al. (2017) proposed a heuristic algorithm based on a concurrent detailed routing algorithm for MCNF.

Wadie and Ashour (2013) used a multicommodity flow formulation for multiple paths load-balanced routing scheme. The applied path length metric in the scheme lessens the storage capacity and consequently less power consumption while the quality of service in the network is increased. The allocation of virtual machines in a large-scale cloud system, which is constructed by data centers in an IP over Wavelength-Division-Multiplexing (WDM) within the backbone network, created a problem that was formulated by two approaches. Based on the hub location problem, Teyeb et al. (2014) proposed multicommodity flow modeling intending to minimize the traffic load between data centers. Min-cost multicommodity flow model was used by Jiao and Dong (2016) to formulate the problem of ordered escape routing within grid pin array to optimize total wire length and consider three various types of transformation comprising non-crossing, ordering and capacity transformation. Ward (1999) proposed minimum aggregate concave cost multicommodity uncapacitated strong-series–parallel network flow problem and a dynamic programming algorithm was used to solve the model.

Virtual network (VN) embedding deals with how the substrate resource could be used efficiently in which a set of virtual resources should be chosen to establish virtual networks for assuring the demand of users is met and optimal topology's constraints are satisfied. Huang et al. (2014) integrated virtual link mapping and virtual node mapping to formulate virtual network mapping according to the weighted multicommodity flow problem. The results show that better correlation was obtained between the required bandwidth of the virtual link and the residual bandwidth of the substrate link when cost weight value was allocated to substrate link according to two capacities encompass ratio of the needed capacities of the virtual link and the residual capacities of the substrate link.

Tracking people could be seen as a communication problem where different surveillance devices need to interchange tracking data. Shitrit et al. (2014) used it for basketball players, soccer players, and pedestrians. The problem was formulated by a multicommodity network flow problem considering the image-appearance constraints while persons could intersect each other's paths. The proposed model performed efficiently even when the clues were at distant time intervals.

To increase the WBAN longevity in the network topology Moungla et al. (2013), suggested dynamic communication range management based on a multicommodity flow model with the objectives of decreasing the sensor energy consumption and also providing the maximum global connectivity between nodes. The distributed dynamic topology control algorithm was proposed to obtain the objectives of the model.

Xu et al. (2017) applied the multicommodity network flow model using the time-expanded graph for data acquisition to optimize the satellite routing problem. The approximation method was used to solve the problem.

Brandt et al. (2017) improved the augmenting flows of a single commodity to multicommodity in software-defined networks. The authors proposed an efficient algorithm to solve the problem. Bera et al. (2019) proposed an adaptive flow-rule placement scheme in a software-defined network. Three phases including forwarding path selection in which a max-flow-min-cost optimization for multicommodity flows problem was formulated and a greedy heuristic approach was used to solve the problem. An integer linear programming problem was developed in the second phase named flow-rule placement. Finally, a rule redistribution algorithm was applied to accommodate a number of flows in the network. Baier et al. (2005) used approximation algorithms for a k-splittable multicommodity flow problem with the focus on imposing a limitation on the number of paths. Based on Baier's paper, Białoń (2017) presented a minimum-congestion k-splittable multicommodity network flow problem for a software-defined, circuit-switching network. Farrugia et al. (2018) proposed a multi-objective genetic algorithm to solve a mathematical model based on MCNF for Software Defined Network and in another paper (Farrugia et al. 2019) the authors improved the algorithm.

Sallam et al. (2018) focused on the Service Functions Chains (SFC) constraints in networks with combined PNFs and VNFs. The authors formulated the problem as a fractional multicommodity maximum flow problem. Chien and Hung (2018) presented an extended linear multicommodity multi-cost network model. An approximate algorithm was proposed to solve the problem Balakrishnan et al. (2017) formulated a minimum cost multicommodity network design with end-to-end service requirements model. The problem was solved by an optimization-based heuristic method.

Ding (2017) focused on uncertain minimum cost multicommodity flow problem and a decomposition-based algorithm was developed to solve an (α,β)-minimum cost multicommodity problem. Fortz et al. (2017) developed multicommodity flow problems with un-splittable flows and piecewise linear routing costs.

Layeb et al. (2017) proposed a novel formulation for the Discrete Cost Multicommodity Network Design Problem. Venkatadri et al. (2017) developed a multicommodity network problem based on the arc path formulation for multi-period cell formation problem.

5 Solution methods for MCNF problems

The investigation of concepts and modeling is not sufficient for a real problem to be solved, since finding an optimal solution is a motivation for an operations research scientist to tackle a problem. Based on our extensive review of the literature, it is evident that numerous techniques were applied to tackle multicommodity network flow problems, ranging from exact methods to heuristic and meta-heuristic approaches. In this section, and as shown in Fig. 3, we have categorized the solution

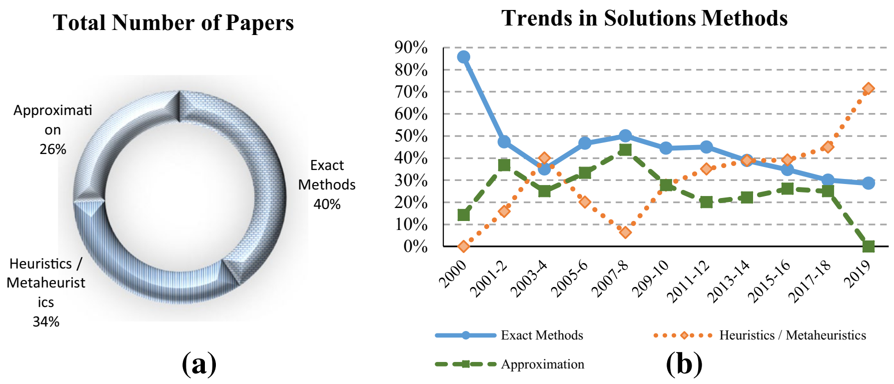


Fig. 3 **a** Total number of papers on solution methods, **b** trends in solution methods

methods in three main groups with subcategories, including (1) exact methods, (2) approximation techniques, and (3) heuristic approaches.

As illustrated in Fig. 3a the majority of papers (40%) used exact methods to solve the MCNF problems. Heuristic methods account for a considerable proportion while approximation was the less applied approach. Figure 3b shows how the number of papers categorized according to solution methods altered over 20 years. Exact methods experienced the largest amount in 2000, however, this upward trend suddenly broken and dropped to 35% in 2003–2004. It can be seen that the use of exact methods and Approximation was reduced for the next years. The growth of Heuristic and Metaheuristic methods could be observed during the period and it is predicted to rise in the future as the problems getting larger.

Three main categories were considered as solution methods for MCNF. Table 2 presents a reference to papers published in different subcategories. A brief description of each method is also presented.

5.1 Exact methods

Exact methods recognized as an approach to find the global optimum and accurate results for the optimization problems. Sub-categories including branching methods and decomposition-based methods were considered for exact methods solutions.

5.1.1 Branching methods

In solving combinatorial optimization problems, branching the solution space as a tree, and then searching a tree is a well-known approach to finding the optimal solution. Branch and bound algorithm is a technique, which is widely used for solving large-scale NP-hard combinatorial and discrete optimization problems. The algorithm searches the complete solution space by imposing bound on the feasible region to find the optimal solution. Selection of the node to progress, bound calculation and branching are the three main components of iterations in the branch and bound algorithm (Clausen 1999). In this context, Sarubbi et al.

Table 2 Summary of researches on the solution methods of MCNF

Solution methods		References
Exact methods	Branching methods	Sarubbi et al. (2008), Cao and Uebe (1993), Foulds (1981), Canel, et al. (2001), Barnhart et al. (2000), Brunetta et al (2000), Truffot et al. (2005), Baldacci et al. (2004), Gamst et al. (2010), Lin (2006), Lim and Smith (2007), Castro and Nabona (1996), Gamst and Petersen (2012), Castro (2000), Castro and Frangioni (2000), Castro (2003), Gendron and Larose (2014), Lin (2002), Chouman et al. (2016), Akyüz et al. (2018), Boccia et al. (2018), Bartolini and Schneider (2018), Mejri et al. (2019)
	Decomposition based methods	Yeh (2011), Çakır (2009), Geoffrion and Graves (1974), Ferland et al. (1978), Mamer and McBride (2000), McBride and Mamer (2001), Hadjiat et al. (2000), Detlefsen and Wallace (2002), Baier et al. (2005), Pierre-Olivier et al. (2013), Ciappina et al. (2012), Fakhri and Ghatee (2016), Morabito et al. (2014), Marla et al. (2014), Shetty and Muthukrishnan (1990), Lawphongpanich (2000), Schneur and Orlin (1998), Chardaire and Lisser (2002), Bektaş et al. (2010), Yeh (2008), Machado et al. (2010), Barnhart et al. (1995), Katayama et al. (2009), Aneja and Nair (1982), Frangioni and Gendron (2009), Moradi et al. (2015), Lei et al. (2013), Cappanera and Frangioni (2003), Holmberg and Yuan (2003), Hernández-Pérez and Salazar-González (2009), Costa et al. (2009), Ouorou et al. (2001), Dai et al. (2016), Hossain et al. (2017), Dorneles et al. (2017), Oğuz et al. (2018), Rahmaniani et al. (2018)
Approximation		Gupta et al. (2001), Kumar et al. (2002), Dragan et al. (2002), Awerbuch and Leighton (1993), Fleischer (2000), Larsson and Yuan (2004), Tao (2002), Cheng et al. (2008), Moitra (2009), Grandoni and Rothvoß (2011), Akyuz et al. (2012), Retvari et al. (2004), Crainic et al. (2001), Holmberg et al. (2008), Li et al. (2009), Topaloglu and Powell (2006), Sarubbi and Mateus (2007), Thiongane et al. (2015), Thanh et al.(2010), Nace et al. (2008), Hajiaghayi and Leighton (2006), Choi and Choi (2006), Sami, et al. (2014), Eickmeyer and Kawarabayashi (2013), Zhong et al. (2013), Gouveia (1996), Papadimitriou et al. (2015), Albrecht (2001), Gabrel et al. (2003), Ouaja and Richards (2004), Fleischer and Sethuraman (2005), Babonneau et al. (2006), Gendron and Gouveia (2016), Weibin et al. (2017), Białoń (2017), Charikar et al. (2018)

Table 2 (continued)

Solution methods		References
Heuristic / metaheuristics	Genetic algorithm	Nasiri et al. (2010), Ghatee (2011), Momeni and Sarmadi (2016), Alavidoost et al. (2018), Farrugia et al. (2018), Masri et al. (2019), Farrugia et al. (2019)
	Simulated annealing	Yaghini et al. (2012), Moshref-Javadi and Lee (2016), Wei et al. (2014)
	Tabu Search	Cabrera and Toledo (2010), Zaleta and Socarrás (2004), Crainica et al. (2006), Li et al. (2016), Vu et al. (2013), Ghamlouche et al. (2003), Ghamlouche et al. (2004), Gendron et al. (2003a, b), Amrani et al. (2011)
	Other heuristics	Pirkul and Jayaraman (1998), Shi et al. (2004), Kennington and Shalaby (1977), Crainic et al. (2004), Xue, Li, and Xue (2012), Achterberg and Raack (2010), Gendron et al. (2003a, b), Rodrıguez-Martın and Salazar-Gonzalez (2011), Gamst (2013), Ríos-Mercado et al. (2013), Paraskevopoulos et al. (2015), Mollah et al. (2017), Myung (2001), Myung (2006), Quilliot et al. (2013), Alvarez et al. (2005), Aloise and Ribeiro (2011), Brotcorne, Labbé, Marcotte, and Savard (2001), Agarwal (2002), Munguía, et al. (2017), Bin Obaid and Trafalis (2020), Garg and Upadhyaya (2018), Katayama (2019)

(2008) used the cut-and-bound algorithm for solving a class of multicommodity traveling salesman problem. In the algorithm, fixed and variable costs were allocated to each arc and each commodity, respectively. The proposed mixed-integer linear programming formulation captures the cost structure, the quality of service to customers, delivery priorities and risks.

Foulds (1981) proposed a solution based on branch and bound procedure for the multicommodity flow network design problem. The capacitated multicommodity p-median transportation problem for container terminal management was presented and solved using a heuristic branch-and-bound algorithm. Akyüz et al. (2018) combined two branch and bound algorithm consisting of the partitioning of the allocation space and partitioning of the location space to solve a multicommodity capacitated multi-facility Weber problem.

Canel et al. (2001) proposed an algorithm for capacitated multicommodity multi-period (dynamic) multi-stage facility location problems. The model aimed to minimize the total transportation costs and opening facilities costs. The algorithm using the branch and bound was suggested to find the optimal solution.

The Multicommodity k-splittable flow problem (MCKFP) deals with the number of distinct commodities, which should be conveyed to different destinations with the objective of a minimum possible cost. The comparison between various formulations of the model solved based on the branch and price algorithm was provided by Gamest et al. (2010). A two-index model with associated branching strategy was compared to the three-index model proposed by Truffot et al. (2005). The main goal of the paper was to present the branch-and-price algorithm for the two-index model and a heuristic approach for a three-index model. It was proved that the two-index algorithm outweighs the three-index one. Barnhart et al. (2000) considered constrained version of linear Multicommodity flow problem, named origin–destination integer multicommodity flow problem. The problem was solved using column generation technique, branch and price and cut algorithm.

A branch and cut algorithm was used by Brunetta et al. (2000) to provide an exact solution for an integer multicommodity flow problem. The problem was formulated to present several classes of inequalities extended using so-called lifting procedures. As part of the solution approach, multi-handle comb inequalities were introduced for the polyhedral structure of the solution space.

The problem in which attackers damage arcs or nodes of a network system to create delays or followers' profit drop is called interdiction problem. Authors Lim and Smith (2007) formulated network interdiction on a multicommodity network flow problem and implemented two procedures consist of district approach which is defined when some arcs are entirely abolished and continuous approach when the capacity of arcs are declined. For the first procedure, a partitioning algorithm was proposed and for the next stage, the heuristic-based approach was generalized for a better optimal solution Lin (2006). focused on a new performance index for a multicommodity stochastic flow network with capacity and cost attributes on each arc. The system capacity was defined as a vector and a probability associated with the proposed performance index was determined in a way that the system capacity is less than or equal to a given pattern based on the budget constraint. The minimal cuts algorithm was used to solve the problem.

The primal partitioning was taken into consideration as a separating strategy for MCNF with linear or nonlinear objective function by Castro and Nabona (1996). Gamst and Petersen (2012) defined the multicommodity k-splittable maximum flow problem as maximizing the amount of routed flow through a capacitated network such that each commodity uses at most k paths and the capacities are satisfied. Two solution methods based on Dantzing-Wolfe decomposition were proposed. The methods had different branching schemes; they forbid sub-paths in the first stage and force and forbid the use of certain paths in the next stage.

A simulation–optimization approach was suggested by Mejri et al. (2019) to solve the cost MCNF problem with random demands. The solution method combined a cut-generation procedure along with a column generation algorithm and a Monte Carlo simulation approach to provide more efficiency.

Boccia et al. (2018) proposed a multicommodity location routing problem considering the flow interception approach. A branch-and-cut algorithm was applied to solve the flow intercepting facility location-routing problem. Bartolini and Schneider (2018) developed a two-commodity flow formulation for the capacitated truck-and-trailer routing problem in which some valid inequalities were proposed to improve the formulation. A branch and cut algorithm was used to solve the problem. Castro (2000) formulated a multicommodity network flow problem and proposed a specialized interior-point algorithm to solve the problem. In another paper Castro and Frangioni (2000), improved the interior-point algorithm using a parallel implementation.

5.1.2 Decomposition methods

Decomposition methods were developed by George Dantzing and Wolfe (1960). This solution procedure is useful for solving the substructure of an integer program because some linear programming problems have a special structure that can be divided into sub problems. A common approach for exposing the substructure is to relax a set of complicating constraints (Ralphs and Galati 2011). The Decomposition method is efficient for MCNF problems. Yeh (2011) considered the weighted multicommodity multistate unreliable network (WMMUN) which is an extension of the multistate network and include multistate unreliable components with different weight capacities. Yeh used the D-minimal path (D-MP) for calculating the direct WMMUN reliability and applied a sequential decomposition to find all real D-MP. Nevertheless, in (2008) he also had proposed an algorithm for WMMUN and mentioned its disadvantages in 2011.

The accurate investigation of the supply chain requires the precise analysis of parameters of the system that is hard to be achieved. However, distributions planning in this regard help decision-makers to address the problem efficiently. Considering this fact Çakır (2009), presented a multicommodity multi-mode distribution-planning model solved by the primal decomposition algorithm. Geoffrion and Graves (1974) implemented Benders decomposition for multicommodity capacitated problems. The model was applied for distribution systems of Hunt-Wesson foods firm.

Ferland et al. (1978) considered a multicommodity network flow problem in which the capacity of some arcs increased by integer numbers within a fixed budget. The model allocated optimal budget such that the sum of the flows of all the commodities maximize. Due to the large size of the problem, the Benders decomposition was used to divide the problem into the smaller one. Two main components were carried out; the generation of budget allocations and the evaluation of the objective for allocations. The Rosen's primal partitioning and non-linear duality theory were applied.

Mamer and McBride (2000) developed the pricing strategy for solving large-scale linear programs based on the simplex method. The idea of the procedures was obtained by a column-generation approach of Dantzing-Wolfe decomposition, which solves a relaxed sub-problem to identify basic columns. Two sets of multicommodity flow problems were used to test the approach; the routing telecommunications traffic and the logistics problems.

Pierre-Olivier et al. (2013) presented the maximum concurrent flow problem (MCF) which is one the most difficult type of multicommodity network flow problems. The model was solved by the decomposition approach with the contribution of generating trees in a convenient way instead of paths. Since all commodities had the same source, the authors decided to solve the problem by a combinatorial polynomial-time algorithm. Afterward, the efficiency of the path and tree approaches and combinatorial polynomial-time algorithm with the tree-based approach were compared.

Inaccurate information, as well as uncertainty in the graph structure and the parameters, led to integrating the theory of fuzzy sets with multicommodity network flow problems. In this regard Ciappina et al. (2012), focused on Dantzing-Wolf decomposition to solve the linear programming model with fuzzy costs.

Fakhri and Ghatee (2016) proposed a new strategy for partitioning in bender decomposition named Bender-based branch and bound algorithm. The solution method not only was useful for fixed-charge multicommodity network design but also was a tool for solving a wide range of mixed-integer nonlinear programming problems. Oğuz et al. (2018) presented a multicommodity network flow formulation for a stochastic and deterministic restricted facility location problems with unknown destinations constructed on a discretization of the plane. A bender decomposition algorithm solved the model. Rahmaniani et al. (2018) used a Bender decomposition method for the multicommodity capacitated network design problem with demand uncertainty.

The usage of the multicommodity flow problem in an open queuing network (OQN) can be found in the paper of Morabito et al. (2014). The system works according to Jackson networks in which the arrival and service process of the commodities are considered Poisson. The steady-state performance measures such as average delays and waiting time in the queue were focused on this model. On the other hand, the network routing algorithm and network decomposition method were integrated to solve the problem.

For obtaining an optimal solution Marla et al. (2014) divided the problem into a routing master problem and scheduling sub-problems by decomposition approach. The proposed model named Commodity Routing with Time Windows under Uncertainty was formulated according to multicommodity network flow with time windows. Stochastic nature of supplies and demands of commodities;

available capacities of the network links; and travel and service times on the network were the factors that determined the uncertainty in the model.

Shetty and Muthukrishnan (1990) applied the multicommodity network model for parallel processing defined as a technology that enhanced the computation speed in the computer industry. The Resource-directive decomposition approach was taken into account for solving the model. The generation of an iteration of Dantzing-Wolfe decomposition led to the effectual optimal solution in the paper of Lawphongpanich (2000).

The simplex method is a common algorithm for solving linear programs, in this context Chardaire and Lisser (2002), proposed the direct methods including specialization of the primal simplex algorithm and the dual affine scaling algorithm for a non-oriented multicommodity network flow problem. Dantzig-Wolf decomposition method was implemented and compared with the analytic center cutting plane method. Detlefsen and Wallace (2002) developed the Simplex algorithm based on the inverse of the basis matrix for a multicommodity network flow problems.

The transportation problems arisen in petrochemical industries about production and distribution encouraged Machado et al. (2010) to formulate the compound multicommodity model as a multicommodity network flow problem. The proposed model decreased the number of proportionality constraints using the primal–dual simplex algorithm due to a large number of coupling constraints led to an enormous size matrix.

The Column generation based on the Dantzing-Wolfe decomposition approach is efficient for solving large linear programs. Finding a column with a negative reduced cost under an optimal dual solution to the constraints of the linear program is different in each application of column generation (Sankaran 1995). In this method, the best subset of columns is selected by the master problem and is solved using a MIP formulation of the problem (Jacquet-Lagreze and Lebbar 1999). The Column generation is used for solving the MCNF problems. Message routing problems in communication industries create a network that each message has a specific origin and a specific destination node with a capacity restriction on arcs. The problem can be modeled as a minimum-cost multicommodity network flow problem Barnhart et al. (1995). used a cycle-based multicommodity formulation and column generation techniques to reduce the network costs in message routing problems.

Frangioni and Gendron (2009) presented 0–1 reformulation based on the multiple-choice model for the multicommodity capacitated network design (MCND) problem. Two cutting-plane algorithms with a different base, however, with the same lower bound were compared and finally for a better solution the column-and-row generation approach was applied. Aneja and Nair (1982) presented a multicommodity network flow problem to maximize the expected value of multicommodity in a network and the flows on each arc experience a probabilistic loss rate. The algorithm considered for solving the problem involved column generation.

Moradi et al. (2015) considered the bi-objective multicommodity minimum cost flow. Bi-objective simplex method and Dantzing-wolf decomposition were integrated to develop the column generation algorithm. The combination of column generation and row generation were applied in a capacity scaling heuristic to solve a multicommodity capacitated network design problem in the paper of Katayama et al. (2009).

Flight delays or cancellations in airlines can trigger difficulties in the field of passengers and airport services costs. These disruptions occur due to various reasons such as mechanical repairs of aircraft or inappropriate weather. To recover the schedules of aircraft Quansheng and Peng (2013), used multicommodity flow to model recovery network. The column generation technique was applied to solve the problem. The capacity of an airport network system with changeable local resource requirements is a challenging issue that was considered in Hossain et al (2017) paper. The authors proposed a heuristic algorithm based on the Hill-Climber algorithm to solve a multicommodity network flow problem. Dorneles et al. (2017) proposed a multicommodity flow model for the high school timetabling problem. The problem was solved using column generation algorithm.

5.2 Approximation methods

Sami et al. (2014) proposed a multicommodity network flow (MCNF) problem with a discrete event in which the model achieved the dispatching and scheduling policy considering the minimization of time as the cost. The researchers connected time-weighted automata modeling with MCNF for formulating the behavior of the vehicle. A polynomial-time approximation based algorithm solved the model.

Cheng et al. (2008) considered the maximum flow problem with a minimized maximum of met rates that is the type of bi-criteria multicommodity flow problem. A pseudo-polynomial time approximation algorithm was applied to solving the problem Awerbuch and Leighton (1993). proposed $(1 + \varepsilon)$-approximation algorithm for the multicommodity flow problem based on a simple edge balancing technique. Fleischer (2000) also presented an algorithm for the maximum multicommodity flow, which describes fully polynomial-time approximation schemes.

Kumar et al. (2002) considered a multicommodity buy-at-bulk network design named rent-or-buy problem. The authors implemented a simple primal–dual algorithm and then proposed the constant-factor approximation algorithm for network design with multiple commodities. Grandoni and Rothvoß (2011) improved an approximation approach for multicommodity connected facility location. Akyuz et al. (2012) proposed the multicommodity extension of the capacitated multi-facility Weber problem. They implemented location-allocation and discrete approximation heuristics using different strategies to solve the problem.

Since the Lagrangian-based methods have proved their efficiency in solving difficult formulation models Crainic et al. (2001) considered two Lagrangian relaxations named shortest path relaxation and Knapsack relaxation for multicommodity capacitated fixed-charge network design problems. Calibrating and comparing bundle and sub-gradient methods for optimization of Lagrangian dual and comprehensive method behavior analysis was the substantial aim of the paper. Retvari et al. (2004) introduced a new Lagrangian-relaxation methodology for solving the minimum cost multicommodity flow problem. The algorithm was used in the field of OSPF traffic and able to give an initial feasible solution Thanh et al. (2010). used a heuristics algorithm based on linear relaxation which combined with correction procedures for solving the multi-period, multi-mixed integer linear program for designing and planning both strategic and tactical decisions.

The concentration on multicommodity network flow problem in which fixed cost on paths and arc capacity constraints have been integrated provide a challenging model that is problematic to solve. The application of this model could be seen for the empty freight car distribution process in trains in the paper of Holmberg et al. (2008). The Lagrangian based heuristic was used to obtain a solution. Papadimitriou et al. (2015) used a Lagrangian approach for the capacitated multicommodity fixed-charge network design.

With the advent of 3-D stacking of silicon layers in system-on-chips, the interconnect problem was reduced considerably. The presented model by Zhong et al. (2013) was formulated with the min-cost multicommodity flow to allocate pins and Through-Silicon Vias for decreasing the wire length and consequently improving delay and power consumption. The model was solved by Lagrangian relaxation.

Li et al. (2009) presented the integrated model consists of the capacitated plant location problem and the multicommodity minimum cost flow problem. To solve the mixed-integer linear model, a Lagrangian-based approach was used to achieve the lower and upper bounds of the model. Given upper bounds, the proposed Tabu search provided a feasible solution.

Sarubbi and Mateus (2007) presented the multicommodity traveling salesman problem (MTSP). In this problem, a salesman travels to each node to make a tour while the objective is to satisfy all the demands of different commodities such that minimize the sum of the fixed and variable costs. For solving the problem, they used a Lagrangian-based heuristic approach.

The Terminal layout problem (TLP) deals with discovering which routes are optimal between terminals and central nodes. Two approaches of formulation consist of directed and undirected multicommodity flow were compared for TLP in which hop constraint was taken into consideration. Hop constraints in Minimal Spanning Tree impose a restriction on edges between nodes and have a strong relationship with the maximum delay transmission time in the network. Since these models were too large, the researchers took advantage of Lagrangian relaxation to solve the problem in Gouveia's (1996) paper.

Three mathematical programming models were formulated for multicommodity capacitated fixed-charge network design problem with non-bifurcated flows and hop constraints by Thiongane et al. (2015), which comprises the classical arc-based formulation, path-based formulation, and hop-indexed model. In the paper, lower bounds in the bifurcated and non-bifurcated cases, and the capacitated and un-capacitated cases were compared and the relationship between them was investigated. Lagrangian relaxations, linear programming (LP) relaxations, and partial relaxations of the integrality constraints were used for these models. Finally, the Lagrangian relaxation approach proved its efficiency in computing time and bound quality Choi and Choi (2006). used linear relaxation for the 0–1 multicommodity minimum cost network flow problem.

Nace et al. (2008) focused on the Max–Min Fair multicommodity flows in a capacitated network and provided a theoretical analysis of this problem. They considered the lexicographically minimum and maximum load problem and proposed a polynomial approach to solving the problem. Two applications of a telecommunication network, routing, and load balancing were applied in their paper.

Charikar et al. (2018) considered MCNF to model the placing of a middle-boxes problem in communication networks as well as the traffic flows on the links and processing resources on the nodes. Approximation algorithms were used to solve the problem. Albrecht (2001) developed an algorithm based on an approximation approach to solving the MCNF problem. Tao (2002) proposed a difference of convex functions approach to solving the multicommodity network optimization with step increasing cost functions.

The comparison of algorithms including column generation and Lagrangian relaxation were discussed by Weibin et al. (2017) to solve the multicommodity network flow problem. The column generation algorithm proved its superiority in results.

5.3 Heuristic methods

The heuristic approach is considered as a method, which is based on experience or judgment and the solution cannot give a guarantee of optimality. The popularity of heuristic methods has derived from the reasons that finding the optimal solution for complicated models takes a long time and sometimes is impossible. Additionally, sometimes different solutions are needed to compare for achieving the final solution (Aickelin and Clark 2011). Many MCNF problems were solved by heuristic procedures that we categorized them in four parts (1) Genetic algorithm, (2) Simulated Annealing, (3) Tabu Search (4), and Other heuristics.

5.3.1 Genetic algorithm

The Genetic Algorithm is based on the process of natural evolution. The properties of the population of individuals alter or mutate and a selection scheme chooses the new generation called good members because of their higher proportion of the characteristics. These members mix with other good members to present a near-optimum solution (Khouja et al. 1998). Alavidoost et al. combined (2018) three algorithms consist of a Non-dominated Sorting Genetic Algorithm (NSGA-II), Non-dominated Ranking Genetic Algorithm (NRGA), and Pareto Envelope-based Selection Algorithm (PESA-II) to solve mixed-integer nonlinear programming for multicommodity tri-echelon model. To compare the proposed multi-objective algorithms, Taguchi Method has been applied and the results showed that NSGA-II and NRGA have the same ability while PESA-II has the better CPU time. Masri et al. (2019) considered a multi-objective single-path multicommodity flow problem. Three algorithms including MGA based on NSGAII, multi-objective variable neighborhood search, and hybrid meta-heuristic HMGA were used to solve the problem.

Nasiri et al. (2010) proposed a multi-period multicommodity location-allocation problem. A genetic algorithm was used to solve the problem Ghatee (2011). used the multicommodity network flow problem for minimizing delay and congestion and increasing the quality of service for channels. He proposed a cooperative algorithm, which consists of path enumeration, and a hybrid genetic algorithm.

5.3.2 Simulated annealing

Simulated Annealing (SA) is an effective technique to find a global optimum in the large discrete search space for optimization problems. Yaghini et al. (2012) developed a solution method for node-arc formulation of the capacitated fixed-charge multicommodity network design problem. The hybrid algorithm comprising a Simplex method and simulated annealing were combined to solve the problem. Moshref-Javadi and Lee (2016) proposed a hybrid heuristic algorithm including the simulated annealing and the variable neighborhood search to solve the customer-centric multi-commodity vehicle routing problem with split delivery. The problem determines the decision on the vehicle routes as well as the number of commodities, while the aim was to minimize the total waiting time of customers.

Wei et al. (2014) highlighted the multi-source single path multicommodity flow problem comprising path determination and bandwidth allocation. The first part deals with the concept that the information requests assumed as commodities and must be conveyed in a single path between origin and destination in telecommunication networks and the second one assigns radio frequencies to commodities due to the limitations for bandwidth in each channel. A simulated annealing approach was used to solve the problem.

5.3.3 Tabu search

Tabu search is a meta-heuristic method, which solves combinatorial problems and develops a local heuristic search approach to find a solution space over the local optimum. In this context (Zaleta and Socarrás 2004), Cabrera and Toledo (2010) considered the distribution network design model and proposed the Tabu search solution for multicommodity capacitated network flow problem. Zaleta and Socarrás (2004) developed the fixed-charge capacitated multicommodity network design problem and formulated a model according to a mixed integer programming problem. Tabu Search was used to solve the model. Crainica et al. (2006) focused on the capacitated multicommodity network design problem. The multilevel cooperative Tabu search based on the principles of local interactions was applied to address the problem.

Li et al. (2016) presented a designed balanced capacitated multicommodity network design model with heterogeneous vehicles. A Tabu search method was applied to solve the arc-based mixed programming formulation.

A Tabu search approach and path relinking procedure combined to solve Capacitated Multicommodity Fixed-cost Network Design problem with Design-Balance constraints by Vu et al. (2013). Ghamlouche et al. (2003) developed the fixed-charge capacitated multicommodity network design problem considering the new cycle-based neighborhood structures in which Tabu search method was used to evaluate the quality of neighborhoods. Gendron et al. (2003a, b) combined a Tabu search heuristic with slope scaling to solve a multicommodity location problem with balancing requirements.

5.3.4 Other heuristics

In addition to well-known heuristic methods, researchers have developed other heuristics to provide an optimal solution more efficiently. Pirkul and Jayaraman (1998) discussed the distribution network strategic design problem in which the multiple products distributed from plants with warehouses to the customer according to their different demands. They formulated the multi-commodity, multi-plant capacitated facility location problem to minimize fixed costs. A Lagrangian relaxation heuristic was suggested to solve the problem.

Shi et al. (2004) investigated on supply chain strategy consists of optimal distribution location and allocation of customers. They especially focused on warehouses location according to multicommodity capacitated facility location problem. The meta-heuristic and mixed-integer programming tools (branch and cut) were combined as a nested partitions (NP) framework to solve the problem. Kennington and Shalaby (1977) found a solution for multicommodity problems based on heuristics techniques. They presented a resource-directive approach and used a sub-gradient optimization approach for reallocating the resources.

Xue et al. (2012) built an optimization model for the generic multicommodity logistics network design with an impact on the importance of logistics and distribution logistics network. The model aimed to minimize the total cost using Baumel-Wolf network distribution heuristic.

The suggested heuristic approach by Rodrıguez-Martın and Salazar-Gonzalez (2011) is a MIP-based greedy randomized adaptive search procedure (GRASP) that each iteration can be divided into two segments comprising obtaining a feasible solution and thereafter improvement of the first part. The initial solution was achieved by a heuristic method based on the nearest neighborhood and then, the improvement was constructed according to the modification of the branch-and-cut algorithm. The combination of mathematical programming and meta-heuristic techniques was used for an extension of the traveling salesman problem (TSP) known as the multicommodity one-to-one pickup-and-delivery traveling salesman problem. Katayama (2019) proposed a greedy heuristic algorithm for the capacitated MCNF based on ink-rerouting and partial link-rerouting heuristics to reduce the computation time.

Gendron et al. (2003a, b) combined two approaches named Slope Scaling and Variable Neighborhood Descent to propose a parallel heuristic framework for solving the multicommodity capacitated location problem with balancing requirements. The model addressed the problem of depots location while considering how to provide the demand for empty containers with the minimum total cost of depots.

The focus of Gamest (2013) was to suggest a heuristic approach for multicommodity k-splittable maximum flow problem. The evolutionary algorithm presented by Paraskevopoulos and et al. (2015) for the fixed-charge capacitated multicommodity network design problem was based on three main contributions. It considered an efficient iterated local search using cycle-based neighborhood operators while utilized a creative perturbation strategy based on the ejection chain and secondly it employed a scatter search as an improved method.

Max–min fairness (MMF) is an approach in the modeling of network performance for creating rate allocation and resource management. The set of flows is labeled with max–min rates in a way that the rates of flows should not violate the assigned rate and only the larger flows can be alleviated. Mollah et al. (2017) presented two algorithms according to MMF for multicommodity flows in fat-tree topology. In this context, Bin Obaid and Trafalis (2020) proposed the MMF approach for multicommodity networks using goal programming technique. Two objectives were considered for the linear mode including the maximization of the flow and minimization of the difference inflow.

In Myung's (2006) paper two problems consist of multicommodity flow problems (MFP) and the integer multicommodity flow problem (IMFP) on cycle graph was introduced. The developed algorithm for MFP was according to the proposed approach for solving the ring loading problem in the author's previous paper (Myung et al. 1997). Another time-linear algorithm for the IMFP also was similar to the paper published in (Myung 2001).

Garg and Upadhyaya (2018) combined a Hybrid algorithm based on Multi-protocol label switching and profile classes to solve the traffic demands in multicommodity network flow problems. Munguía et al. (Munguía, et al. 2017) developed a parallel local search approach for the Fixed Charge Multicommodity Network Flow problem applied for shared memory parallel systems and distributed memory systems.

6 Discussion and conclusions

This paper classifies 263 articles, which used the multicommodity network flow model and have been published between 2000 and 2019. The resulting classification tables enable future researchers to find relevant literature. Additionally, the classification allowed analyzing which applications and solution algorithms are the most popular.

In this section, some insights on which application and solution methods of multicommodity network flow problems appeared together. Table 3 shows that transportation problems seem to be solved more often using exact methods especially branching (16.67%). The problems in which communication networks were addressed, heuristic methods proposed by authors indicate a higher percentage (12.37%), and branching methods were the next largest proportion (10.75%). Although heuristics and meta-heuristics methods are the most frequently used methods in researches (it is highlighted using the bold font), the classical meta-heuristic methods such as the Genetic Algorithm and Simulated Annealing were less applied in the literature and authors tend to suggest novel algorithms to solve the MCNF problems.

The survey indicates that the multicommodity network flow literature consists of a rather wide range of problems, mainly focused on communication and computer networks. It also indicates that transportation systems are going to be an active research domain of the applications of MCNF. Researchers have paid more attention

Table 3 Summary of solution methods used in different applications

Combination of application and solution methods	Exact methods		Approximation (%)	Heuristic/metaheuristics			
	branching methods (%)	Decomposition based methods (%)		Genetic algorithm (%)	Simulated Annealing (%)	Tabu search (%)	Other heuristics (%)
Logistic, transportation system and traffic engineering	16.67	10.75	11.83	4.84	1.08	3.23	10.75
Communication and computer networks	10.75	9.68	2.15	1.08	1.08	1.61	12.37
Other economic sectors	0.00	0.54	0.54	0.00	0.00	0.00	1.08

to the MCNF modeling framework that includes real-life characteristics and assumptions, thereby making their models more realistic and their solution approaches more applicable in practice. However, real-life characteristics are often considered either individually or with a limited number of other characteristics. Therefore, future research could focus on even *richer* problems by simultaneously considering multiple real-life characteristics, and developing efficient solution methods to solve the problems. On the other hand, many researchers still propose highly problem-tailored solution methods, which are not directly applicable to other variants of the problem. The classification reveals that only a few articles propose general optimization algorithms. The further development of such general solution approaches seems highly worthwhile.

References

Achterberg T, Raack C (2010) The MCF-separator: detecting and exploiting multicommodity flow structures in MIPs. Math Program Comput 2(2):125–165

Adhikari RS, Aste N, Manfren M (2012) Multicommodity network flow models for dynamic energy management – Smart Grid applications. Energy Procedia 14:1374–1379

Agarwal YK (2002) Design of capacitated multicommodity networks with multiple facilities. Oper Res 50(2):333–344

Ahmed W, Hasan O, Pervez U, Qadir J (2017) Reliability modeling and analysis of communication networks. J Netw Comput Applcations 78:191–215

Ahuja RK, Magnanti TL, Orlin JB (1993) Network flows: theory, algorithms, and applications. Prentice-Hall, NewJersey

Aickelin U, Clark A (2011) Heuristic optimisation. J Oper Res Soc 62(2):251–252

Akyuz MH, Oncan T, Altınel IK (2012) Efficient approximate solution methods for the multicommodity capacitated multi-facility Weber problem. Comput Oper Res 39(2):225–237

Akyüz M, Öncan T, Altınel (2018) Branch and bound algorithms for solving the Multicommodity capacitated multi-facility Weber problem. Ann Oper Res 279:1–42

Alavidoost M, Tarimoradi M, Zarandi M (2018) Bi-objective mixed-integer nonlinear programming for multicommodity tri-echelon supply chain networks. J Intell Manuf 29(4):809–826

Albrecht C (2001) Global routing by new approximation algorithms for multicommodity flow. IEEE Trans Comput Aided Des Integr Circuits Syst 20(5):622–632

Aldous D, McDiarmid C, Scott A (2009) Uniform multicommodity flow through the complete graph with random edge-capacities. Oper Res Lett 37(5):299–302

Alfaki M, Haugland D (2013) A Multicommodity flow formulation for the generalized pooling problem. J Global Optim 56(3):917–937

Ali I, Charnes A, Tiantai S (1985) Karmarkar's projective algorithm: a null space-variant for multicommodity generalized networks. Acta Math Appl Sin 2(2):168–190

Al-Khayyal F, Hwang S-J (2007) Inventory constrained maritime routing and scheduling for multicommodity liquid bulk, Part I: applications and model. Eur J Oper Res 176(1):106–130

Aloise D, Ribeiro C (2011) Adaptive memory in multistart heuristics for multicommodity network design. J Heuristics 17(2):153–179

Alumur S, Nickel S, Saldanha-da-Gama F, Verter V (2012) Multi-period reverse logistics network design. Eur J Oper Res 220(1):67–78

Alvarez A, González-Velarde J, De-Alba K (2005) GRASP embedded scatter search for the multicommodity capacitated network design problem. J Heuristics 11(3):233–257

Amit K, Amarpreet K (2011) Application of classical transportation methods for solving fuzzy transportation problems. J Transp Syst Eng Inf Technol 11(5):68–80

Amrani H, Martel A, Zufferey N, Makeeva P (2011) A variable neighborhood search heuristic for the design of multicommodity production-distribution networks with alternative facility configurations. OR Spectr 33(4):989–1007

Aneja Y, Nair K (1982) Multicommodity network flows with probabilistic losses. Manag Sci 28(9):1080–1086
Aringhieri R, Cordone R (2004) The multicommodity multilevel bottleneck assignment problem. Electron Notes Discrete Math 17:35–40
Assad AA (1978) multicommodity network flows—a survey. Networks 8(1):37–91
Awerbuch B, Khandekar R (2009) Greedy distributed optimization of multicommodity flows. Distrib Comput 21(5):317–329
Awerbuch B, Leighton T (1993) A simple local-control approximation algorithm for multicommodity flow. In: 34th annual symposium on foundations of computer science. IEEE, pp 459–468
Ayar B, Yaman H (2012) An intermodal multicommodity routing problem with scheduled services. Comput Optim Appl 53(1):131–153
Arani Azadi Moghaddam A, Jolai F, Nasiri M (2019) A Multicommodity network flow model for railway capacity optimization in case of line blockage. Int J Rail Transp 7:1–24
Babonneau F, Du Merle O, Vial J-P (2006) Solving large-scale linear multicommodity flow problems with an active set strategy and proximal-ACCPM. Oper Res 54(1):184–197
Baier G, Köhler E, Skutella M (2005) The k-splittable flow problem. Algorithmica 42(3–4):231–248
Balakrishnan A, Li G, Mirchandani P (2017) Optimal network design with end-to-end service requirements. Oper Res 65(3):729–750
Baldacci R, Hadjiconstantinou E, Mingozzi A (2004) An exact algorithm for the capacitated vehicle routing problem based on a two-commodity network flow formulation. Oper Res 52(5):723–738
Barnhart C, Hane CA, Vance PH (2000) Using Branch-and-price-and-cut to solve origin-destination integer multicommodity flow problems. Oper Res 48(2):318–326
Balma A, Salem S, Mrad M, Ladhari T (2018) Strong Multicommodity flow formulations for the asymmetric traveling salesman problem. Eur J Oper Res 27:72–79
Barnhart C, Hane C, Johnson E, Sigismondi G (1995) A column generation and partitioning approach for multicommodity flow problems. Telecommun Syst 3(3):239–258
Bartolini E, Schneider M (2018) A two-commodity flow formulation for the capacitated truck-and-trailer routing problem. Discrete Appl Math 275:3–18
Bazan O, Jaseemuddin M (2008) Multicommodity flow problem for multi-hop wireless networks with realistic smart antenna model. In: International conference on research in networking. Springer, Berlin, pp 922–929
Bélanger N, Desaulniers G, Soumis F, Desrosiers J (2006) Periodic airline fleet assignment with time windows, spacing constraints, and time-dependent revenues. Eur J Oper Res 175(3):1754–1766
Bektaş T, Chouman M, Crainic T (2010) Lagrangean-based decomposition algorithms for multicommodity network design problems with penalized constraints. Netw Int J 55(3):171–180
Beljadid A, Hafid A, Boushaba M (2013) Reliability analytical measurement to design wireless mesh networks. In: IEEE international conference on advanced networks and telecommunications systems (ANTS). IEEE, Kattankulathur, India, pp 1–6
Bera S, Misra S, Jamalipour A (2019) Flowstat: adaptive flow-rule placement for per-flow statistics in SDN. IEEE J Sel Areas Commun 37(3):530–539
Bevrani B, Burdett RL, Bhaskar A, Yarlagadda PK (2019) A multi commodity flow model incorporating flow reduction functions. Flexible Serv Manuf J. https://doi.org/10.1007/s10696-019-09349-4
Białoń P (2017) A randomized rounding approach to a k-splittable multicommodity flow problem with lower path flow bounds affording solution quality guarantees. Telecommun Syst 64(3):525–542
Bin Obaid HS, Trafalis TB (2020) An approximation to max min fairness in multi commodity networks. Comput Manag Sci 17:65–77
Boccia M, Crainic T, Sforza A, Sterle C (2018) Multicommodity location-routing: flow intercepting formulation and branch-and-cut algorithm. Comput Oper Res 89:94–112
Boland N, Kalinowski T, Rigterink F (2016) New Multicommodity flow formulations for the pooling problem. J Global Optim 66(4):669–710
Brandt S, Foerster K-T, Wattenhofer R (2017) Augmenting flows for the consistent migration of multicommodity single-destination flows in SDNS. Pervasive Mobile Comput 36:134–150
Brotcorne L, Labbé M, Marcotte P, Savard G (2001) A bilevel model for toll optimization on a multicommodity transportation network. Transp Sci 35(4):345–358
Brunetta L, Conforti M, Fischetti M (2000) A polyhedral approach to an integer multicommodity flow problem. Discrete Appl Math 101(1–3):13–36

Cabrera G, Toledo C (2010) A Tabu search algorithm with a probabilistic neighbor selection criterion for capacitated multicommodity network flow problem. In: International conference on technologies and application of artificial intelligence. IEEE, Hsinchu, Taiwan, pp 1–5
Caimi G, Chudak F, Fuchsberger M, Laumanns M, Zenklusen R (2011) A new resource-constrained multicommodity flow model for conflict-free train routing and scheduling. Transp Sci 45(2):212–227
Çakır O (2009) Benders decomposition applied to multi-commodity, multi-mode distribution planning. Expert Syst Appl 36(4):8212–8217
Canel C, Khumawala BM, Law J, Loh A (2001) An algorithm for the capacitated, multicommodity multiperiod facility location problem. Comput Oper Res 28(5):411–427
Cao B, Uebe G (1993) An algorithm for solving capacitated multicommodity p-median transportation problems. Oper Res Soc 44(3):259–269
Cappanera P, Frangioni A (2003) Symmetric and asymmetric parallelization of a cost-decomposition algorithm for multicommodity flow problems. INFORMS J Comput 15(4):369–384
Cappanera P, Gallo G (2004) A multicommodity flow approach to the crew rostering problem. Oper Res 52(4):583–596
Cariou P, Cheaitou A, Larbi R, Hamdan S (2018) Liner shipping network design with emission control areas: a genetic algorithm-based approach. Transp Res Part D: Transp Environ 63:604–621
Castro J (2000) A specialized interior-point algorithm for multicommodity network flows. SIAM J Optim 10(3):852–877
Castro J (2003) Solving difficult multicommodity problems with a specialized interior-point algorithm. Ann Oper Res 124(1–4):35–48
Castro J, Nabona N (1996) An implementation of linear and nonlinear multicommodity network flows. Eur J Oper Res 92(1):37–53
Castro J Frangioni A (2000) A parallel implementation of an interior-point algorithm for multicommodity network flows. In: International conference on vector and parallel processing. Springer, Berlin, pp 301–315
Cattaruzza D, Absi N, Feillet D, Vigo D (2014) An iterated local search for the multicommodity multitrip vehicle routing problem with time windows. Comput Oper Res 51:257–267
Chale-Gongora HG, Weck O, Doufene A, Ishimatsu T, Krob D (2014) Planning an Itinerary for an electric vehicle. In: IEEE international energy conference (ENERGYCON). IEEE, Cavtat, Croatia, pp 1385–1391
Chardaire P, Lisser A (2002) Simplex and interior point specialized algorithms for solving nonoriented multicommodity flow problems. Oper Res 50(2):260–276
Charikar M, Naamad Y, Rexford J, Zou X (2018) Multicommodity flow with in-network processing. In: International symposium on algorithmic aspects of cloud computing. Springer, Cham, pp 73–101
Chekuri C, Khanna S, Shepherd FB (2013) The all-or-nothing multicommodity flow problem. SIAM J Comput 42(4):1467–1493
Chen YL, Chin YH (1992) Multicommodity network flows with safety considerations. Oper Res 40(1):S48–S55
Cheng Y, Sterbenz JP (2014) Geodiverse routing with path jitter requirement under regional challenges. In: 6th international workshop on reliable networks design and modeling (RNDM). IEEE, Barcelona, Spain, pp 179–186
Cheng C, Tang H, Zhao C (2008) Approximation algorithm for a bicriteria multicommodity flow problem. In: IEEE international conference on service operations and logistics, and informatics. IEEE, Beijing, China, pp 1756–1760
Cheng X, Tang K, Li X (2018) New multicommodity flow formulations for the generalized pooling problem. IFAC-PapersOnLine 51(18):162–167
Chien T, Hung H (2018) Extended linear multicommodity multi-cost network and maximal flow limited cost problems. Int J Comput Netw Commun 10:79–93
Choi D-S, Choi I-C (2006) On the effectiveness of the linear programming relaxation of the 0–1 multicommodity minimum cost network flow problem. In: International computing and combinatorics conference. Springer, Berlin, Heidelberg, pp 517–526
Chouman M, Crainic T, Gendron B (2016) Commodity representations and cut-set-based inequalities for multicommodity capacitated fixed-charge network design. Transp Sci 51(2):650–667
Ciappina JR, Yamakami A, Silva RC (2012) Decomposition's Dantzig–Wolfe applied to fuzzy multicommodity flow problems. Comput Oper Res 39(12):3394–3407
Clausen J (1999) Branch and bound algorithms principles and examples. University of Copenhagen, Department of Computer Science, pp 1–30

Cortés P, Muñuzuri J, Guadix J, Onieva L (2013) An optimal algorithm for the demand routing problem in multicommodity flow distribution networks with diversification constraints and concave costs. Int J Prod Econ 146(1):313–324

Costa A, Cordeau J-F, Gendron B (2009) Benders, metric and cutset inequalities for multicommodity capacitated network design. Comput Optim Appl 42(3):371–392

Crainic TG, Frangioni A, Gendron B (2001) Bundle-based relaxation methods for multicommodity capacitated fixed-charge network design. Discrete Appl Math 112(1–3):73–99

Crainic T, Gendron B, Hernu G (2004) A slope scaling/Lagrangean perturbation heuristic with long-term memory for multicommodity capacitated fixed-charge network design. J Heuristics 10(5):525–545

Crainica TG, Li Y, Toulouse M (2006) A first multilevel cooperative algorithm for capacitated multicommodity network design. Comput Oper Res 33(9):2602–2622

Dai W, Sun X, Wandelt S (2016) Finding feasible solutions for multicommodity flow problems. In: 35th Chinese control conference (CCC). IEEE, Chengdu, China, pp 2878–2883

Dantzing G, Wolfe P (1960) Decomposition principle for linear programs. Oper Res 8(1):101–111

Detlefsen N, Wallace S (2002) The simplex algorithm for multicommodity networks. Netw Int J 39(1):15–28

Ding S (2017) Uncertain minimum cost multicommodity flow problem. Soft Comput 21(1):223–231

Dorneles Á, de Araújo O, Buriol L (2017) A column generation approach to high school timetabling modeled as a multicommodity flow problem. Eur J Oper Res 256(3):685–695

Dragan F, Kahng A, Mandoiu I, Muddu S, Zelikovsky A (2002) Provably good global buffering by generalized multiterminal multicommodity flow approximation. IEEE Trans Comput Aided Des Integr Circuits Syst 21(3):263–274

Eickmeyer K, Kawarabayashi K-I (2013) Approximating multi commodity network design on graphs of bounded pathwidth and bounded degree. In: International symposium on algorithmic game theory. Springer, Berlin, pp 134–145

Erdös PL, Székely LA (1992) Evolutionary trees: an integer multicommodity max-flow - min-cut theorem. Adv Appl Math 13(4):375–389

Erera AL, Morales JC, Savelsbergh M (2005) Global intermodal tank container management for the chemical industry. Transp Res Part E: Logist Transp Rev 41(6):551–566

Erez E, Feder M (2009) Improving the multicommodity flow rates with network codes for two sources. IEEE J Sel Areas Commun 27(5):814–824

Evans JR (1981) The multicommodity assignment problem: a network aggregation heuristic. Comput Math Appl 7(2):187–194

Even G, Medina M (2012) Online multicommodity flow with high demands. In: International workshop on approximation and online algorithms. Springer, Berlin, pp 16–29

Fakhri A, Ghatee M (2014) Fractional multicommodity flow problem: duality and optimality conditions. Appl Math Model 38(7–8):2151–2162

Fakhri A, Ghatee M (2016) Application of Benders decomposition method in solution of a fixed-charge multicommodity network design problem avoiding congestion. Appl Math Model 40(13–14):6468–6476

Farrugia N, Briffa J, Buttigieg V (2018) An evolutionary multipath routing algorithm using SDN. In: 9th international conference on the network of the future (NOF). IEEE, Poznan, Poland, pp 1–8

Farrugia N, Briffa JA, Buttigieg V (2019) Solving the multicommodity flow problem using a multi-objective genetic algorithm. In: IEEE congress on evolutionary computation (CEC). IEEE, Wellington, New Zealand, pp 2816–2823

Ferland JA, Girard A, Lafond L (1978) Multicommodity flow problem with variable arcs capacities. The J Oper Res Soc 29(5):459–467

Fleischer LK (2000) Approximating fractional multicommodity flow independent of the number of commodities. SIAM J Discrete Math 13(4):505–520

Fleischer L, Skutella M (2002) The quickest multicommodity flow problem. In: International conference on integer programming and combinatorial optimization. Springer, Berlin, pp 36–53

Fleischer L, Sethuraman J (2005) Efficient algorithms for separated continuous linear programs: the multicommodity flow problem with holding costs and extensions. Math of Oper Res 30(4):916–938

Fleischer L, Jain K, Mahdian MM (2004) Tolls for heterogeneous selfish users in multicommodity networks and generalized congestion games. In: 45th annual IEEE symposium on foundations of computer science. IEEE, Rome, Italy, pp 277–285

Ford L Jr, Fulkerson D (1958) A suggested computation for maximal multicommodity network flows. Manag Sci 5(1):97–101

Fortz B, Gouveia L, Joyce-Moniz M (2017) Models for the piecewise linear unsplittable multicommodity flow problems. Eur J Oper Res 261(1):30–42

Foulds LR (1981) A multicommodity flow network design problem. Transp Res Part B: Methodol 15(4):273–283

Frangioni A, Gendron B (2009) 0–1 reformulations of the multicommodity capacitated network design problem. Discrete Appl Math 157(6):1229–1241

Frangioni A, Gorgone E (2014) Bundle methods for sum-functions with “easy” components: applications to multicommodity network design. Math Program 145(1–2):133–161

Gabrel V, Knippel A, Minoux M (1999) Exact solution of multicommodity network optimization problems with general step cost functions. Oper Res Lett 25(1):15–23

Gabrel V, Knippel A, Minoux M (2003) A comparison of heuristics for the discrete cost multicommodity network optimization problem. J Heuristics 9(5):429–445

Gamst M (2013) A local search heuristic for the multicommodity k-splittable maximum flow problem. Optim Lett 8(3):919–937

Gamst M, Petersen B (2012) Comparing branch-and-price algorithms for the multicommodity k-splittable maximum flow problem. Eur J Oper Res 217(2):278–286

Gamst M, Jensen PN, Pisinger D, Plum C (2010) Two-and three-index formulations of the minimum cost multicommodity k-splittable flow problem. Eur J Oper Res 202(1):82–89

Gao X, Lee GM (2018) A stochastic programming model for multicommodity redistribution planning in disaster response. In: IFIP international conference on advances in production management system. Springer, Cham, pp 67–78

Garg P, Upadhyaya S (2018) Hybrid multi-commodity-based widest disjoint path algorithm (HMBWDP). In: Next-generation networks. Springer, Singapore, pp 333–340

Gautier A, Granot F (1995) forest management: a multicommodity flow formulation and sensitivity analysis. Manag Sci 41(10):1654–1668

Gebremariam W, Bikdash M (2013) LP-based flow rate control and modeling of capacity collapse propagation over long links. In: Proceedings of IEEE southeastcon. IEEE, Jacksonville, FL, USA, pp 1–6

Gendron B, Larose M (2014) Branch-and-price-and-cut for large-scale multicommodity capacitated fixed-charge network design. EURO J Comput Optim 2(1–2):55–75

Gendron B, Gouveia L (2016) Reformulations by discretization for piecewise linear integer multicommodity network flow problems. Transp Sci 51(2):629–649

Gendron B, Potvin J-Y, Soriano P (2003a) A parallel hybrid heuristic for the multicommodity capacitated location problem with balancing requirements. Parallel Comput 29(5):591–606

Gendron B, Potvin J-Y, Soriano P (2003b) A tabu search with slope scaling for the multicommodity capacitated location problem with balancing requirements. Ann Oper Res 122(1–4):193–217

Geoffrion AM, Graves GW (1974) Multicommodity distribution system design by benders decomposition. Manage Sci 20(5):822–844

Ghamlouche I, Crainic TG, Gendreau M (2003) Cycle-based neighbourhoods for fixed-charge capacitated multicommodity network design. Oper Res 51(4):655–667

Ghamlouche I, Crainic T, Gendreau M (2004) Path relinking, cycle-based neighbourhoods and capacitated multicommodity network design. Ann Oper Res 131(1–4):109–133

Ghasemi P, Khalili-Damghani K, Hafezolkotob A, Raissi S (2019) Uncertain multi-objective Multicommodity multi-period multi-vehicle location-allocation model for earthquake evacuation planning. Appl Math Comput 350:105–132

Ghatee M (2011) QoS-based cooperative algorithm for integral multicommodity flow problem. Comput Commun 34(7):835–846

Ghatee M, Hashemi S (2007) Descent direction algorithm with multicommodity flow problem for signal optimization and traffic assignment jointly. Appl Math Comput 188(1):555–566

Ghatee M, Hashemi M (2009) Some concepts of the fuzzy multicommodity flow problem and their application in fuzzy network design. Math Comput Modell 49(5–6):1030–1043

Gouveia L (1996) Multicommodity flow models for spanning trees with hop constraints. Eur J Oper Res 95(1):178–190

Grandoni F, Rothvoß T (2011) Approximation algorithms for single and multicommodity multi-commodity. In: International conference on integer programming and combinatorial optimization. Springer, Berlin, pp 248–260

Groß M, Skutella M (2015) A tight bound on the speed-up through storage for quickest Multicommodity flows. Oper Res Lett 43(1):93–95

Guo J, Niedermeier R (2006) A fixed-parameter tractability result for multicommodity demand flow in trees. Inf Process Lett 97(3):109–114
Gupta A, Kleinberg J, Kumar A, Rastogi R, Yener B (2001) Provisioning a virtual private network: a network design problem for multicommodity flow. In; Proceedings of the thirty-third annual ACM symposium on theory of computing, pp 389–398
Hadjiat M, Maurras J-F, Vaxes Y (2000) A primal partitioning approach for single and non-simultaneous multicommodity flow problems. Eur J Oper Res 123(2):382–393
Haghani A, Oh S-C (1996) Formulation and solution of a multi-commodity, multi-modal network flow model for disaster relief operations. Transp Res Part A: Policy Pract 30(3):231–250
Hajiaghayi M, Leighton T (2006) On the max-flow min-cut ratio for directed multicommodity flows. Theoret Comput Sci 352(1–3):318–321
Hall A, Hippler S, Skutella M (2007) Multicommodity flows over time: efficient algorithms and complexity. Theoret Comput Sci 379(3):387–404
Hao J, Shi Y, Sun H, Sheng M, Li J (2019) Rerouting based congestion control in data center networks. In: IEEE international conference on communications workshops (ICC workshops). IEEE, Shanghai, China, pp 1–6
Hernández-Pérez H, Salazar-González J-J (2009) The Multicommodity one-to-one pickup-and-delivery traveling salesman problem. Eur J Oper Res 196(3):987–995
Hirai H, Koichi S (2011) On duality and fractionality of multicommodity flows in directed networks. Discrete Optim 8(3):428–445
Holmberg K, Yuan D (2003) A multicommodity network-flow problem with side constraints on paths solved by column generation. INFORMS J Comput 15(1):42–57
Holmberg K, Joborn M, Melin K (2008) Lagrangian based heuristics for the multicommodity network flow problem with fixed costs on paths. Eur J Oper Res 188(1):101–108
Hossain M, Alam S, Abbass H (2017) A dynamic multicommodity flow optimization algorithm for estimating airport network capacity. Air Traffic Manag Syst II:205–220
Hu TC (1963) Multicommodity network flows. Oper Res 11(3):344–360
Huang T, Lv B, Liu J (2014) Work in progress: a novel virtual network mapping algorithm with weighted multicommodity flow. In: 9th international conference on communications and networking in China (CHINACOM). IEEE, Maoming, China, pp 166–169
Huerta M, Hesselbach X (2004) Application of the theory of the multicommodity for the flows distribution in MPLS networks. In: The 13th IEEE workshop on local and metropolitan area network. IEEE, Mill Valley, CA, USA, pp 119–124
Hurkens C, Schrijver A, Tardos É (1988) On fractional multicommodity flows and distance functions. Discrete Math 73(1–2):99–109
Jacquet-Lagreze E, Lebbar M (1999) A column generation model for a scheduling problem with maintenance constraints. Electron Notes Discrete Math 1:1–12
Jadin M, Aubry F, Schaus P, Bonaventure O (2019) CG4SR: near optimal traffic engineering for segment routing with column generation. In: IEEE INFOCOM 2019—IEEE conference on computer communications. IEEE, Paris, France, pp 1333–1341
Jeenanunta C, Kasemsontitum B, Noichawee T (2011) A multicommodity flow approach for aircraft routing and maintenance problem. In: IEEE international conference on quality and reliability. IEEE, Bangkok, Thailand, pp 150–155
Jia X, Cai Y, Zhou Q, Chen G, Li Z, Li Z (2014) MCFRoute: a detailed router based on multicommodity flow method. In: IEEE/ACM international conference on computer-aided design (ICCAD). IEEE, San Jose, CA, USA, pp 397–404
Jia X, Cai Y, Zhou Q, Yu B (2017) A multicommodity flow based detailed router with efficient. IEEE Trans Comput Aided Des Integr Circuits Syst 37(1):217–230
Jiang Y, Zhang X, Rong Y, Zhang Z (2014) A multimodal location and routing model for hazardous materials transportation based on Multicommodity flow model. Procedia-So Behav Sci 138:791–799
Jiao F, Dong S (2016) Ordered escape routing for grid pin array based on min-cost multicommodity flow. In: 21st Asia and South Pacific design automation conference (ASP-DAC). IEEE, Macau, China, pp 384–389
Jin W-L (2012) A kinematic wave theory of Multicommodity network traffic flow. Transp Res Part B: Methodol 46(8):1000–1022
Jin W-L (2018) Unifiable multicommodity kinematic wave model. Transp Res Part B: Methodol 117:639–659

Jin Q, Feng S, Li-xin M, Gui-jun T (2009) April) Optimal model and algorithm for multicommodity logistics network design considering stochastic demand and inventory control. Syst Eng Theory Pract 29(4):176–183
Joe YY, Gan OP, Lewis FL (2014) Multicommodity flow dynamic resource assignment and matrix-based job dispatching for multi-relay transfer in complex material handling systems (MHS). J Intell Manuf 25(4):681–697
Kabadurmus O, Smith AE (2016) Multicommodity k-splittable survivable network design problems with relays. Telecommun Syst 62(1):123–133
Kacprzak P, Kaleta M, Kołtyś K, Pałka P, Pieńkosz K, Toczyłowski E, Żółtowska I (2010) Multicommodity exchange model for trading bandwidth in undirected networks. In: 14th international telecommunication network strategy and planning symposium (NETWORKS). IEEE, Warsaw, Poland, pp 1–5
Kalantari M, Shayman M (2006) Routing in multicommodity sensor networks based on partial differential equations. In: 40th annual conference on information sciences and systems. IEEE, Princeton, NJ, USA, pp 402–406
Karimi B, Bashiri M (2018) Designing a multicommodity multimodal splittable supply chain network by logistic hubs for intelligent manufacturing. Proc Manuf 17:1058–1064
Karsten CV, Pisinger D, Ropke S, Brouer BD (2015) The time-constrained multicommodity network flow problem and its application to liner shipping network design. Transp Res Part E: Logist Transp Rev 76:122–138
Katayama N (2019) A combined fast greedy heuristic for the capacitated multicommodity network design problem. J Oper Res Soc 70:1–14
Katayama N, Chen M, Kubo M (2009) A capacity scaling heuristic for the multicommodity capacitated network design problem. J Comput Appl Math 232(1):90–101
Kennington JL (1978) A survey of linear cost multicommodity network flows. Oper Res 26(2):209–236
Kennington J, Shalaby M (1977) An Effective subgradient procedure for minimal cost multicommodity flow problems. Manage Sci 23(9):994–1004
Khouja M, Michalewicz Z, Wilmot M (1998) The use of genetic algorithms to solve the economic lot size scheduling problem. Eur J Oper Res 110(3):509–524
Khuller S, Raghavachari B, Young N (1994) Designing multicommodity flow trees. Inf Process Lett 50(1):49–55
Kiwiel K (2011) An alternating linearization bundle method for convex optimization and nonlinear multicommodity flow problems. Math Program 130(1):59–84
Koca E, Yıldırım E (2012) A hierarchical solution approach for a multicommodity distribution problem under a special cost structure. Comput Oper Res 39(11):2612–2624
Kolar V, Abu-Ghazaleh N (2006) A multicommodity flow approach for globally aware routing in multihop wireless networks. In: Fourth annual IEEE international conference on pervasive computing and communications (PERCOM'06). IEEE, Pisa, Italy, pp 313–317
Kollias A, Nikolaidis I (2015) Seasonally aware routing for thermoelectric energy harvesting wireless sensor networks. In: International conference on smart cities and green ICT systems (SMARTGREENS). IEEE, Lisbon, Portugal, pp 1–11
Kuiteing A, Marcotte P, Savard G (2018) Pricing and revenue maximization over a multicommodity transportation network: the nonlinear demand case. Comput Optim Appl 71(3):641–671
Kumar A, Gupta A, Roughgarden T (2002) A constant-factor approximation algorithm for the multicommodity rent-or-buy problem. In: The 43rd annual IEEE symposium on foundations of computer science. IEEE, Vancouver, BC, Canada, pp 333–342
Larsson T, Yuan D (2004) An augmented lagrangian algorithm for large scale multicommodity routing. Comput Optim Appl 27(2):187–215
Lawphongpanich S (2000) Simplicial with truncated Dantzig-Wolfe decomposition for nonlinear multicommodity network flow problems with side constraints. Oper Res Lett 26(1):33–41
Layeb SB, Heni R, Balma A (2017) Compact MILP models for the discrete cost multicommodity network design problem. In: International conference on engineering & MIS (ICEMIS). IEEE, Monastir, Tunisia, pp 1–7
Lee K (2010) Integral polyhedra related to integer multicommodity flows on a cycle. Discrete Appl Math 158(3):235–238
Lee S, Younis M, Murali A, Lee M (2019) Dynamic local vehicular flow optimization using real-time traffic conditions at multiple road intersections. IEEE Access 7:28137–28157

Lei Q, Jiang D, Zhao P, Ma T (2013) Research on the disrupted airline scheduling. In: 10th international conference on service systems and service management IEEE. IEEE, Hong Kong, China, pp 332–336

Leighton T, Makedon F, Plotkin S, Stein C, Tardos É, Tragoudas S (1995) Fast approximation algorithms for multicommodity flow problems. J Comput Syst Sci 50(2):228–243

Letchford A, Salazar-González J-J (2015) Stronger Multicommodity flow formulations of the capacitated vehicle routing problem. Eur J Oper Res 244:730–738

Letchford A, Salazar-González J-J (2016) Stronger Multicommodity flow formulations of the (capacitated) sequential ordering problem. Eur J Oper Res 251(1):74–84

Leveque O, Madan R, Shah D (2006) Uniform multicommodity flow in wireless networks with gaussian fading channels. In: IEEE international symposium on information theory. IEEE, Seattle, WA, USA, pp 1846–1850

Levitin G, Gertsbakh I, Shpungin Y (2013) Evaluating the damage associated with intentional supply deprivation in Multicommodity network. Reliab Eng Syst Saf 119:11–17

Li J, Chu F, Prins C (2009) Lower and upper bounds for a capacitated plant location problem with multicommodity flow. Comput Oper Res 36(11):3019–3030

Li X, Wei K, Aneja Y, Tian P (2016) Design-balanced capacitated multicommodity network design with heterogeneous assets. Omega 67:145–159

Lim C, Smith JC (2007) Algorithms for discrete and continuous multicommodity flow network interdiction problems. IIE Trans 39(1):15–26

Lin Y-K (2001) Study on the multicommodity reliability of a capacitated-flow network. Comput Math Appl 42(1–2):255–264

Lin Y-K (2002) Study on the system capacity for a multicommodity stochastic-flow network with node failure. Reliab Eng Syst Saf 78:57–62

Lin Y-K (2004) An algorithm to evaluate the system reliability for multicommodity case under cost constraint. Comput Math Appl 48(5):805–812

Lin Y-K (2006) A simple algorithm to generate all (d, B)-MCs of a multicommodity stochastic-flow network. Reliab Eng Syst Saf 91(8):923–929

Lin Y-K (2007a) On a multicommodity stochastic-flow network with unreliable nodes subject to budget constraint. Eur J Oper Res 176(1):347–360

Lin Y-K (2007b) System reliability of a limited-flow network in multicommodity case. IEEE Trans Reliab 56(1):17–25

Lin Z, Kwan R (2013) An integer fixed-charge multicommodity flow (FCMF) model for train unit scheduling. Electron Notes Discrete Math 41:165–172

Liu C-M (1997) Network dual steepest-edge methods for solving capacitated multicommodity network problems. Comput Ind Eng 33:697–700

Liu Q, Zhao Q, Zang W (2008) Study on multi-objective optimization of flow allocation in a multicommodity stochastic-flow network with unreliable nodes. J Appl Math Comput 28(1–2):185–198

Liu L, Cao X, ChengY (2013) Energy-aware optimal resource allocation in MR-MC wireless networks. In: IEEE global communications conference (GLOBECOM). IEEE, Atlanta, GA, USA, pp 4865–4870

Liu L, Cao X, Cheng Y, Du L, Song W, Wang Y (2014) Energy-efficient capacity optimization in wireless networks. In: IEEE conference on computer communications. IEEE, Toronto, ON, Canada, pp 1384–1392

Lozovanu D, Fonoberova M (2006) Optimal dynamic multicommodity flows in networks. Electron Notes Discrete Math 25:93–100

Lu Y, Benlic U, Wu Q (2019) A population algorithm based on randomized tabu thresholding for the multicommodity pickup-and-delivery traveling salesman problem. Comput Oper Res 101:285–297

Machado C, Mayerle S, Trevisan V (2010) A linea model for compound multicommodity network flow problems. Comput Oper Res 37(6):1075–1086

Madan R, Shah D, Lévêque O (2008) Product multicommodity flow in wireless networks. IEEE Trans Inf Theory 54(4):1460–1476

Maharjan B, Matis TI (2012) Multicommodity flow network model of the flight gate assignment problem. Comput Ind Eng 63(4):1135–1144

Mahey P, de Souza MC (2007) Local optimality conditions for multicommodity flow problems with separable piecewise convex costs. Oper Res Lett 35(2):221–226

Mamer JW, McBride RD (2000) A decomposition-based pricing procedure for large-scale linear programs: an application to the linear multicommodity flow problem. Manag Sci 46(5):693–709

Manfren M (2012) Multicommodity network flow models for dynamic energy management—mathematical formulation. Energy Proc 14:1380–1385
Marla L, Barnhart C, Biyani V (2014) A decomposition approach for commodity pickup and delivery with time-windows under uncertainty. J Sched 17(5):489–506
Masri H, Krichen S, Guitouni A (2015) A multi-start variable neighborhood search for solving the single path multicommodity flow problem. Appl Math Comput 251:132–142
Masri H, Krichen S, Guitouni A (2019) Metaheuristics for solving the bi-objective single-path multicommodity communication flow problem. Int Trans Oper Res 26(2):589–614
Maurras J-F, Vaxes Y (1997) Multicommodity network flow with jump constraints. Discrete Math 165:481–486
McBride RD (1998) advances in solving the multicommodity-flow problem. Interfaces 28(2):32–41
McBride R, Mamer J (2001) Solving the undirected multicommodity flow problem using a shortest path-based pricing algorithm. Netw Int J 38(4):181–188
McCarter M, Barker K, Johansson J, Ramirez-Marquez E, J. (2018) A bi-objective formulation for robust defense strategies in Multicommodity networks. Reliab Eng Systm Saf 176:154–161
McDiarmid C, Scott A, Withers P (2017) Uniform multicommodity flows in the hypercube with random edge-capacities. Random Struct Algorithms 50(3):437–463
Mejri I, Layeb S, Haouari M, Zeghal Mansour F (2019) A simulation-optimization approach for the stochastic discrete cost multicommodity flow problem. Eng Optim 52:1–20
Mendes RR, Yamakami A, Alves IC (1997) Modelling and solving a class of fuzzy multicommodity transportation problem. In: Proceedings of the sixth IEEE international conference on IEEE fuzzy systems, vol 1, pp 313-318
Mesquita M, Moz M, Paias A, Pato M (2015) A decompose-and-fix heuristic based on Multicommodity flow models for driver rostering with days-off pattern. Eur J Oper Res 245(2):423–437
Minoux M (2001) Discrete cost multicommodity network optimization problems and exact solution methods. Ann Oper Res 106(1–4):19–46
Mohammadi M, Jula P, Tavakkoli-Moghaddam R (2017) Design of a reliable multi-modal multicommodity model for hazardous materials transportation under uncertainty. Eur J Oper Res 257(3):792–809
Moitra A (2009) Approximation algorithms for multicommodity-type problems with guarantees independent of the graph size. In: 50th annual IEEE symposium on foundations of computer science. IEEE, Atlanta, GA, USA, pp 3–12
Mollah MA, Yuan X, Pakin S, Lang M (2017) Rapid calculation of max-min fair rates for multicommodity flows in fat-tree networks. IEEE Trans Parallel Distrib Syst 29(1):156–168
Momeni M, Sarmadi M (2016) A genetic algorithm based on relaxation induced neighborhood search in a local branching framework for capacitated multicommodity network design. Netw Spatial Econ 16(2):447–468
Morabito R, de Souza MC, Vazquez M (2014) Approximate decomposition methods for the analysis of multicommodity flow routing in generalized queuing networks. Eur J Oper Res 232(3):618–629
Moradi S, Raitha A, Ehrgott M (2015) Abi-objective column generation algorithm for the Multicommodity minimum cost flow problem. Eur J Oper Res 244(2):1–10
Moshref-Javadi M, Lee S (2016) The customer-centric, multicommodity vehicle routing problem with split delivery. Expert Syst Appl 56:335–348
Moungla H, Touati N, Mehaoua A (2013) Efficient heterogeneous communication range management for dynamic WBAN topology routing. In: First international symposium on future information and communication technologies for ubiquitous healthcare (Ubi-HealthTech). IEEE, Jinhua, China, pp 1–5
Moz M, Pato M (2003) An integer multicommodity flow model applied to the rerostering of nurse schedules. Ann Oper Res 119(1–4):285–301
Moz M, Pato M (2004) Solving the problem of rerostering nurse schedules with hard constraints: new multicommodity flow models. Ann Oper Res 128(1–4):179–197
Mrad M, Haouari M (2008) Optimal solution of the discrete cost multicommodity network design problem. Appl Math Comput 204(2):745–753
Munguía L-M, Ahmed S, Bader D, Nemhauser G, Goel V, Shao Y (2017) A parallel local search framework for the fixed-charge multicommodity network flow problem. Comput Oper Res 77:44–57
Myung Y-S (2001) An efficient algorithm for the ring loading problem with integer demand splitting. SIAM J Discrete Math 14(3):291–298
Myung Y-S (2006) Multicommodity flows in cycle graphs. Discrete Appl Math 154(11):1615–1621

Myung Y-S, Kim H-G, Tcha D-W (1997) Optimal load balancing on SONET bidirectional rings. Oper Res 45(1):148–152
Nace D, Doan LN, Klopfenstein O, Bashllari A (2008) Max–min fairness in multicommodity flows. Comput Oper Res 35(2):557–573
Nagamochi H, Ibaraki T (1990) Multicommodity flows in certain planar directed networks. Discrete Appl Math 27(1–2):125–145
Nasiri GR, Davoudpour H, Movahedi Y (2010) A genetic algorithm approach for the multi-commodity, multi-period distribution planning in a supply chain network design. In: International conference on swarm, evolutionary, and memetic computing. Springer, Berlin, pp 494–505
Noguera JF, Leirens S (2010) Modelling and simulation of a multicommodity pipeline network. In: IEEE ANDESCON. IEEE, Bogota, Colombia pp 1–6
Oğuz M, Bektaş T, Bennell J (2018) Multicommodity flows and Benders decomposition for restricted continuous location problems. Eur J Oper Res 266(3):851–863
Okamura H (1983) Multicommodity flows in graphs. Discrete Appl Math 6(1):55–62
Okamura H, Seymour PD (1981) Multicommodity flows in planar graphs. J Comb Theory Ser B 31(1):75–81
Onaga K, Kakusho O (1971) On feasibility conditions of multicommodity flows in networks. IEEE Trans Circuit Theory 18(4):425–429
Ouaja W, Richards B (2004) A hybrid multicommodity routing algorithm for traffic engineering. Netw Int J 43(3):125–140
Ouorou A, Mahey P (2000) A minimum mean cycle cancelling method for nonlinear multicommodity flow problems. Eur J Oper Res 121(3):532–548
Ouorou A, Mahey P, Vial J-P (2000) A survey of algorithms for convex multicommodity flow problems. Manag Sci 46(1):126–147
Ouorou A, Luna H, Mahey P (2001) Multicommodity network expansion under elastic demands. Optim Eng 2(3):277–292
Ozdaglar, A. E., & Bertsekas, D. P. (2004). Optimal solution of integer multicommodity flow problems with application in optical networks. In: Frontiers in global optimization, pp 411–435
Padmanabh K, Roy R (2006) Multicommodity flow based maximum lifetime routing in wireless sensor network. In: 12th international conference on parallel and distributed systems—(ICPADS'06). IEEE, Minneapolis, MN, USA, pp 1–8
Papadimitriou D, Fortz B, Gorgone E (2015) Lagrangian relaxation for the time-dependent combined network design and routing problem. In: IEEE international conference on communications (ICC). IEEE, London, UK, pp 6030–6036
Paraskevopoulos DC, Gürel S, Bektas T (2016) The congested multicommodity network design problem. Transp Res Part E: Logist Transp Rev 85:166–187
Paraskevopoulos DC, Bektas T, Crainic TG, Potts CN (2015) A cycle-based evolutionary algorithm for the Þxed-charge capacitated multicommodity network design problem. Eur J Oper Res 253(2):1–35
Pierre-Olivier B, Walid B-A, Eric G (2013) A new model for multicommodity flow problems, and a strongly polynomial algorithm for single-source maximum concurrent flow. Electron Notes Discrete Math 41:311–318
Pióro M, Fouquet Y, Nace D, Poss M (2016) Optimizing flow thinning protection in multicommodity networks with variable link capacity. Oper Res 64(2):273–289
Pirkul H, Jayaraman V (1998) a multi-commodity, multi-plant, capacitated facility location problem: formulation and efficient heuristic solution. Comput Oper Res 25(10):869–878
Poh T-V, Jiang J, Reed MJ (2004) Multicommodity flow optimization in support for packet-switched network traffic engineering. In: International workshop on IP operations and management. IEEE, Beijing, China, pp 23–28
Powell WB, Carvalho TA (1997) Dynamic control of multicommodity fleet management problems. Eur J Oper Res 98(3):522–541
Psaraftis HN (2011) A multi-commodity, capacitated pickup, and delivery problem: the single and two-vehicle cases. Eur J Oper Res 215(3):572–580
Qu Y, Bektaş T, Bennell J (2016) Sustainability SI: multimode multicommodity network design model for intermodal freight transportation with transfer and emission costs. Netw Spatial Econ 16(1):303–329
Quansheng L, Peng Z (2013) Research on the disrupted airline scheduling. In: IEEE 10th international conference on service systems and service management (ICSSSM), pp 332–336

Queiroz M, Humes C (2001) The projected pairwise multicommodity flow polyhedron. Appl Math Lett 14(4):443–448
Quilliot A, Liberalino H, Bernay B (2013) Scale multicommodity flow handling on dynamic networks. In: International conference on large-scale scientific computing. Springer, Berlin, pp 297–304
Rahmaniani R, Crainic T, Gendreau M, Rei W (2018) Accelerating the Benders decomposition method: application to stochastic network design problems. SIAM J Optim 28(1):875–903
Ralphs TK, Galati MV (2011) Decomposition methods for integer programming. In: Cochran JJ, Cox LA, Keskinocak P, Kharoufeh JP, Smith JC (eds) Wiley encyclopedia of operations research and management science
Retvari G, Biro JJ, Cinkler T (2004) A novel lagrangian-relaxation to the minimum cost multicommodity flow problem and its application to OSPF traffic engineering. In: Ninth international symposium on computers and communications. IEEE, Alexandria, Egypt, pp 957–962
Ríos-Mercado RZ, López-Pérez JF, Castrillón-Escobar A (2013) A GRASP for a multi-depot multicommodity pickup and delivery problem with time windows and heterogeneous fleet in the bottled beverage industry. In: International conference on computational logistics. Springer, Berlin, pp 143–157
Robinson AR, Chan Y, Dietz DC (2006) Detecting a security disturbance in multi commodity stochastic networks. Telecommun Syst 31(1):11–27
Rodrıguez-Martın I, Salazar-Gonzalez JJ (2011) The multicommodity one-to-one pickup-and-delivery traveling salesman problem: a metaheuristic. In: International conference on network optimization. Springer, Berlin, pp 401–405
Rothfarb B, Frisch I (1970) Common terminal multicommodity flow with a concave objective function. SIAM J Appl Math 18(2):489–502
Rudi A, Frohling M, Zimmer K, Schultmann F (2014) Freight transportation planning considering carbon emissions and in-transit holding costs: a capacitated Multicommodity network flow model. EURO J Transp Logist 5(2):1–38
Sallam G, Gupta G, Li B, Ji B (2018) Shortest path and maximum flow problems under service function chaining constraints. In: IEEE conference on computer communications IEEE INFOCOM 2018, pp 2132–2140
Samani A, Wang M (2018) MaxStream: SDN-based flow maximization for video streaming with QoS enhancement. In: IEEE 43rd conference on local computer networks (LCN), pp 287–290
Sami S, Lin L, Shehabinia A, Su R, Chong C, Jeon S (2014) Discrete-event based vehicle dispatching and scheduling in multicommodity transportations. In: 13th international conference on control automation robotics & vision (ICARCV). IEEE, Singapore, pp 169–174
Sankaran J (1995) Column generation applied to linear programs in course registration. Eur J Oper Res 87(2):328–342
Sarubbi J, Mateus GR (2007) Model and algorithms for the multicommodity traveling salesman problem. In: 7th international conference on hybrid intelligent systems. IEEE, Kaiserlautern, Germany, pp 113–119
Sarubbi J, Miranda G, Luna HP, Mateus G (2008) A cut-and-branch algorithm for the multicommodity traveling salesman problem. In: IEEE international conference on service operations and logistics, and informatics. IEEE, Beijing, China, pp 1806–1811
Schneur RR, Orlin JB (1998) A scaling algorithm for multicommodity flow problems. Oper Res 46(2):231–246
Schauer C, Raidl GR (2016) A multicommodity flow-based model for multi-layer hierarchical ring network design. Electron Notes Discrete Math 52:189–196
Schrijver A (1991) Short proofs on multicommodity flows and cuts. J Comb Theory Ser B 53(1):32–39
Sedeno-Noda A, Gonzalez-Martın C, Gutierrez J (2005) The bi-objective undirected two-commodity minimum cost flow problem. Eur J Oper Res 164(1):89–103
Seymour P (2015) Criticality for multicommodity flows. J Comb Theory Ser B 110:136–179
Shepherd B, Zhang L (2001) A cycle augmentation algorithm for minimum cost multicommodity flows on a ring. Discrete Appl Math 110(2–3):301–315
Shetty B, Muthukrishnan R (1990) A Parallel projection for the multicommodity network model. J Oper Res Soc 41(9):837–842
Shi L, Meyer R, Bozbay M, Miller A (2004) A nested partitions framework for solving large-scale multicommodity facility location problems. J Syst Sci Syst Eng 13(2):158–179
Shi H, Blaauwbroek N, Nguyen P, Kamphuis R, (. (2015) Energy management in multicommodity smart energy systems with a greedy approach. Appl Energy 167:385–396

Shishehgar S, Tavakkoli-Moghaddam R, Siadat A, Mohammadi M (2014) Design of a dynamic bi-objective relief routing network in the earthquake response phase. In: IEEE international conference on industrial engineering and engineering management. IEEE, Bandar Sunway, Malaysia, pp 325–329
Shitrit HB, Berclaz J, Fleuret F, Fua P (2014) Multicommodity network flow for tracking multiple people. IEEE Trans Pattern Anal Mach Intell 36(8):1614–1627
Singh I, Ahn C (1978) A dynamic multicommodity model of the agricultural sector. Eur Econ Rev 11(2):155–179
Srivastav A, Stangier P (2000) On complexity, representation, and approximation of integral multicommodity flows. Discrete Appl Math 99(1–3):183–208
Sun C, Wallace S, Luo L (2017a) Stochastic multicommodity network design: the quality of deterministic solutions. Oper Res Lett 45(3):266–268
Sun L, Karwan M, Kwon C (2017b) Generalized bounded rationality and robust multicommodity network design. Oper Res 66(1):42–57
Tanonkou G, Benyoucef L, Xie X (2007) Design of multicommodity distribution network with random demands and supply lead-times. In: IEEE international conference on automation science and engineering. IEEE, Scottsdale, AZ, USA, pp 698–703
Tao PD (2002) DC programming approach for multicommodity network optimization problems with step increasing cost functions. J Global Optim 22(1–4):205–232
Teyeb H, Balma A, Hadj-Alouane N, Tata S (2014) Optimal virtual machine placement in large-scale cloud systems. In: IEEE 7th international conference on cloud computing. IEEE, Anchorage, AK, USA pp 424–431
Thanh PN, Péton O, Bostel N (2010) A linear relaxation-based heuristic approach for logistics network design. Comput Ind Eng 59(4):964–975
Thiongane B, Cordeau J-F, Gendron B (2015) Formulations for the nonbifurcated hop-constrained multicommodity capacitated fixed-charge network design problem. Comput Oper Res 53:1–8
Tomlin J (1966) Minimum-cost multicommodity network flows. Oper Res 14(1):45–51
Tonneau Q, Bostel N, Dejax P, Hospital R, Mülhauptz V, Yeung T (2015) Multimodal multi-flow problem with transformation: application to waste supply chain. In: International conference on industrial engineering and systems management (IESM). IEEE, Seville, Spain, pp 876–882
Topaloglu H, Powell W (2006) Dynamic-programming approximations for stochastic time-staged integer multicommodity-flow problems. INFORMS J Comput 18(1):31–42
Truemper K, Soun Y (1979) Minimal forbidden subgraphs of unimodular multicommodity networks. Math Oper Res 4(4):379–389
Truffot J, Duhamel C, Mahey P (2005) Using branch-and-price to solve multicommodity k-splittable flow problems. In: Proceedings of international network optimization conference (INOC)
Vahdani B, Veysmoradi D, Shekari N, Mousavi S (2018) Multi-objective, multi-period location-routing model to distribute relief after earthquake by considering emergency roadway repair. Neural Comput Appl 30(3):835–854
Vaziri S, Etebari F, Vahdani B (2019) Development and optimization of a horizontal carrier collaboration vehicle routing model with multicommodity request allocation. J Clean Prod 224:492–505
Velichko A, Gribova V, Fedori L (2018) Simulation software for multicommodity flows model of interregional trade. In: 3rd Russian-Pacific conference on computer technology and applications (RPC). IEEE, Vladivostok, Russia, pp 1–5
Venkatadri U, Elaskari S, Kurdi R (2017) A Multicommodity network flow-based formulation for the multi-period cell formation problem. Int J Adv Manuf Technol 91(1–4):175–187
Vu DM, Crainic TG, Toulouse M (2013) A three-phase metaheuristic for capacitated multicommodity fixed-cost network design with design-balance constraints. J Heuristics 19(5):757–795
Wadie CS, Ashour M (2013) Multicommodity flow, multiple paths load balanced routing in ad-hoc networks. In: 15th international conference on advanced communications technology (ICACT). IEEE, PyeongChang, South Korea, pp 1128–1133
Wagner D, Raidl G, Pferschy U, Mutzel P, Bachhiesl P (2007) A multicommodity flow approach for the design of the last mile in real-world fiber optic networks. In: Operations research proceedings. Springer, Berlin, Heidelberg, pp 197–202
Wang D, Kleinberg R (2009) Analyzing quadratic unconstrained binary optimization problems via multicommodity flows. Discrete Appl Math 157(18):3746–3753
Wang IL (2003) Shortest paths and multicommodity network flows (doctoral dissertation). School of Industrial and Systems Engineering, Georgia Institute of Technology
Ward J (1999) Minimum-aggregate-concave-cost multicommodity flows in strong-series-parallel networks. Math Oper Res 24(1):106–129

Wei K, Li X, Lin S, Yue C (2014) A simulated annealing based heuristic for the multi-source single-path Multicommodity network flow problem. In: 11th international conference on service systems and service management (ICSSSM). IEEE, Beijing, China, pp 1–6
Weibin D, Zhang J, Xiaoqian S (2017) On solving Multicommodity flow problems: an experimental evaluation. Chin J Aeronaut 30(4):1481–1492
Whitman M, Barker K, Johansson J, Darayi M (2017) Component importance for multicommodity networks: application in the swedish railway. Comput Ind Eng 112:274–288
Wright M, Gomes G, Horowitz R, Kurzhanskiy A (2017) On node models for high-dimensional road networks. Transp Res Part B: Methodol 105:212–234
Xu X, Zhang Y, Lu J (2017) Routing optimization of small satellite networks based on multicommodity flow. In: International conference on machine learning and intelligent communications. Springer, Cham, pp 355–363
Xue D, Li Z, Xue N (2012) Multicommodity logistics network design based on heuristic algorithm. In: Advances in electronic engineering, communication and management, vol 1. Springer, Berlin, Heidelberg, pp 75–80
Yaghini M, Akhavan R (2012) Multicommodity network design problem in rail freight transportation planning. Proc-Soc Behav Sci 43:728–739
Yaghini M, Momeni M, Sarmadi M (2012) A Simplex-based simulated annealing algorithm for node-arc capacitated multicommodity network design. Appl Soft Comput 12(9):2997–3003
Yeh W-C (2008) A simple minimal path method for estimating the weighted multicommodity multistate unreliable networks reliability. Reliab Eng Syst Saf 93(1):125–136
Yeh W-C (2011) A sequential decomposition method for estimating flow in a multi-commodity multistate network. IEEE Trans Reliab 60(3):612–621
Zaleta NC, Socarrás AM (2004) Tabu Search-based algorithm for capacitated multicommodity network design problem. In: 14th international conference on electronics, communications and computers. IEEE, Veracruz, Mexico, pp 1–5
Zanjirani Farahani R, Miandoabchi E, Szeto W, Rashidi H (2013) A review of urban transportation network design problems. Eur J Oper Res 229(2):281–302
Zantuti AF (2005) The capacity and non-simultaneously multicommodity flow problem in wide area network and data flow management. In: 18th international conference on systems engineering (ICSEng'05). IEEE, Las Vegas, NV, USA, pp 76–80
Zeng Z, Meng L, Hong X (2018) Integrated optimization of rolling stock and crew rescheduling for high-speed railway. In: International conference on intelligent rail transportation (ICIRT). IEEE, Singapore, pp 1–5
Zhang P, Fan N (2017) Analysis of budget for interdiction on multicommodity network flows. J Global Optim 67(3):495–525
Zhang D, Yu C, Desai J, Lau HY, Srivathsan S (2017a) A time-space network flow approach to dynamic repositioning in bicycle sharing systems. Transp Res part B: Methodol 103:188–207
Zhang J, Zhang X, Sun M (2017) Two-level decomposition for Multicommodity multicast traffic engineering. In: IEEE 36th international performance computing and communications conference (IPCCC). IEEE, San Diego, CA, USA, pp 1–2
Zhang J, Zhang X, Sun M, Yang C (2018) Minimizing the maximum link utilization in multicast multicommodity flow networks. IEEE Commun Lett 22(7):1478–1481
Zhang T, Li H, Zhang S, Wang P, Li J (2018) STAG-based dynamic two-commodity maximum flow algorithm for time-varying networks. In: 10th international conference on wireless communications and signal processing (WCSP). IEEE, Hangzhou, China, pp 1–5
Zhang T, Li H, Li J, Zhan S, Shen H (2018) A dynamic combined flow algorithm for the two-commodity max-flow problem over delay-tolerant networks. IEEE Trans Wireless Commun 17(12):7879–7893
Zhang Z, Cheang B, Li C, Lim A (2019) Multicommodity demand fulfillment via simultaneous pickup and delivery for a fast-fashion retailer. Comput Oper Res 103:81–96
Zhong W, Chen S, Geng Y, Yoshimura T (2013) Lagrangian relaxation based pin assignment and through-silicon via planning for 3-D SoCs. In: 10th international conference on ASIC. IEEE, Shenzhen, China, pp 1–4