High-Frequency Trading in a Limit Order Book

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Simulation Report by Anindita Basu

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Problem Setup

- Goal: Optimize a dealer's bid/ask quotes in a limit order book.
- Objective: Maximize expected exponential utility of terminal wealth.
- Two key risks:
  - Inventory risk due to mid-price diffusion.
  - Execution risk due to Poisson-arriving market orders.

Model Components

- Mid-price model: Arithmetic Brownian Motion (ABM).
- Arrival rates: $\lambda(\delta) = Ae^{-k\delta}$.
- Strategy types:
  1. Inventory-based (Avellaneda-Stoikov).
  2. Symmetric (centered around mid-price).

Effectiveness of Exponential Utility Risk Measure

- **Positive Correlation with Wealth:**
  Exponential utility increases monotonically with terminal wealth, encouraging profitability.
- **Asymmetric Risk Aversion:**
  Models risk aversion naturally via declining marginal utility — growth rate reduces for negative inventory.
- **Mathematical Tractability:**
  Leads to closed-form expressions for reservation price and optimal spreads via HJB reduction.
- **Cash-Independence of Reservation Price:**
  The optimal quoting strategy depends on inventory but *not* on the current cash position.

Reservation Price and Spread

$$r(s, q, t) = s - q\gamma\sigma^2(T - t)$$

$$\text{Spread} = \gamma\sigma^2(T - t) + \frac{2}{\gamma}\log\left(1 + \frac{\gamma}{k}\right)$$

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Inventory-aware strategy shifts reservation price to reduce terminal risk.

Simulation Setup

- 1000 Monte Carlo runs.
- Time steps: 240 (1s resolution over 4 minutes).
- Mid-price process: ABM, $\sigma = 2.0$.
- Parameters varied: $\gamma \in \{0.01, 0.1, 0.5\}$, k, σ.

Simulated Quotes and Inventories for inventory strategy

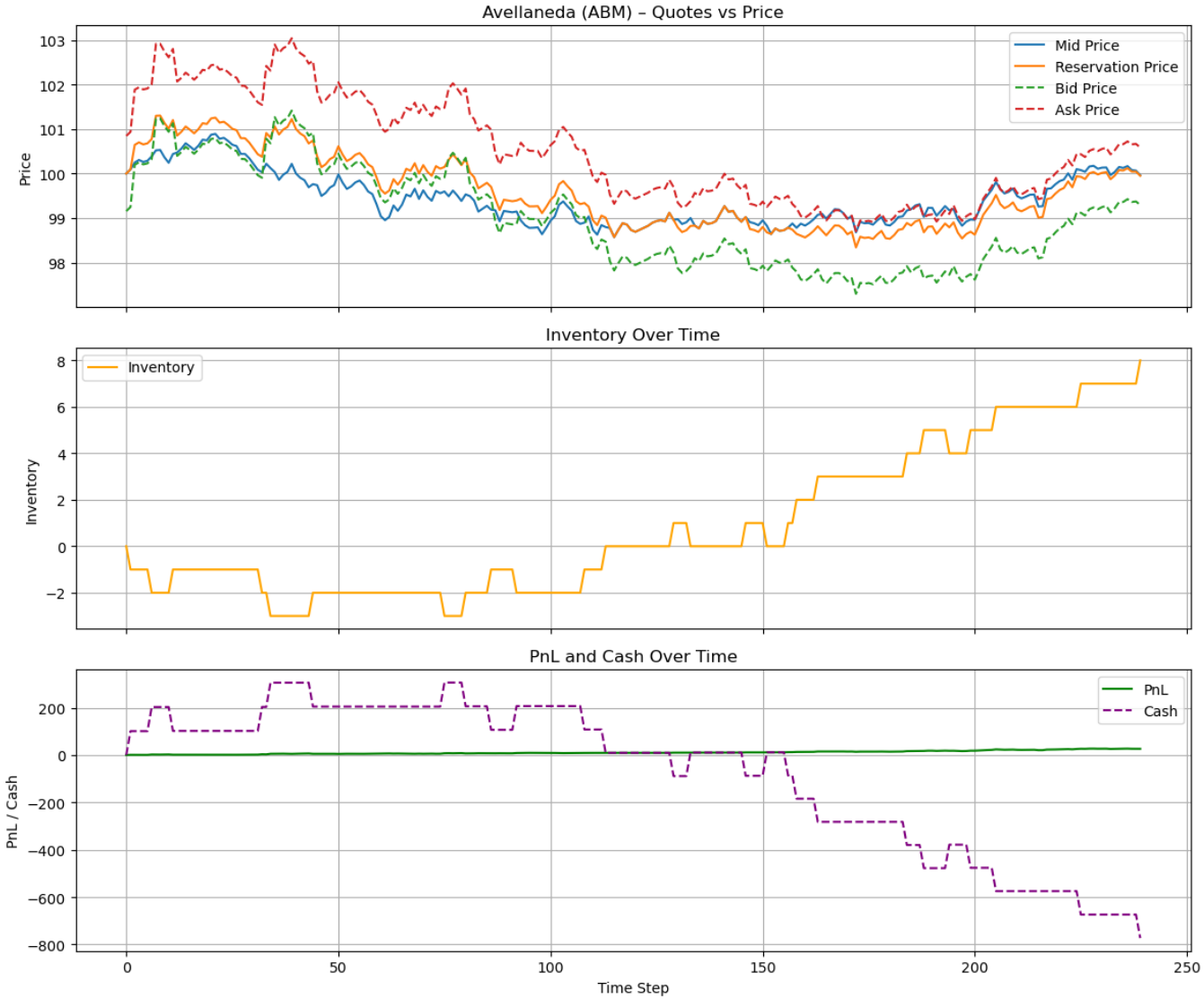

Figure: Quotes and inventory dynamics for one sample run (γ=0.1).

Simulated Quotes and Inventories for inventory strategy-contd.

- mid price and reservation price are different.
- their dynamics depend on inventory value.
- spread reduces over time.

Simulated Quotes and Inventories for symmetric strategy

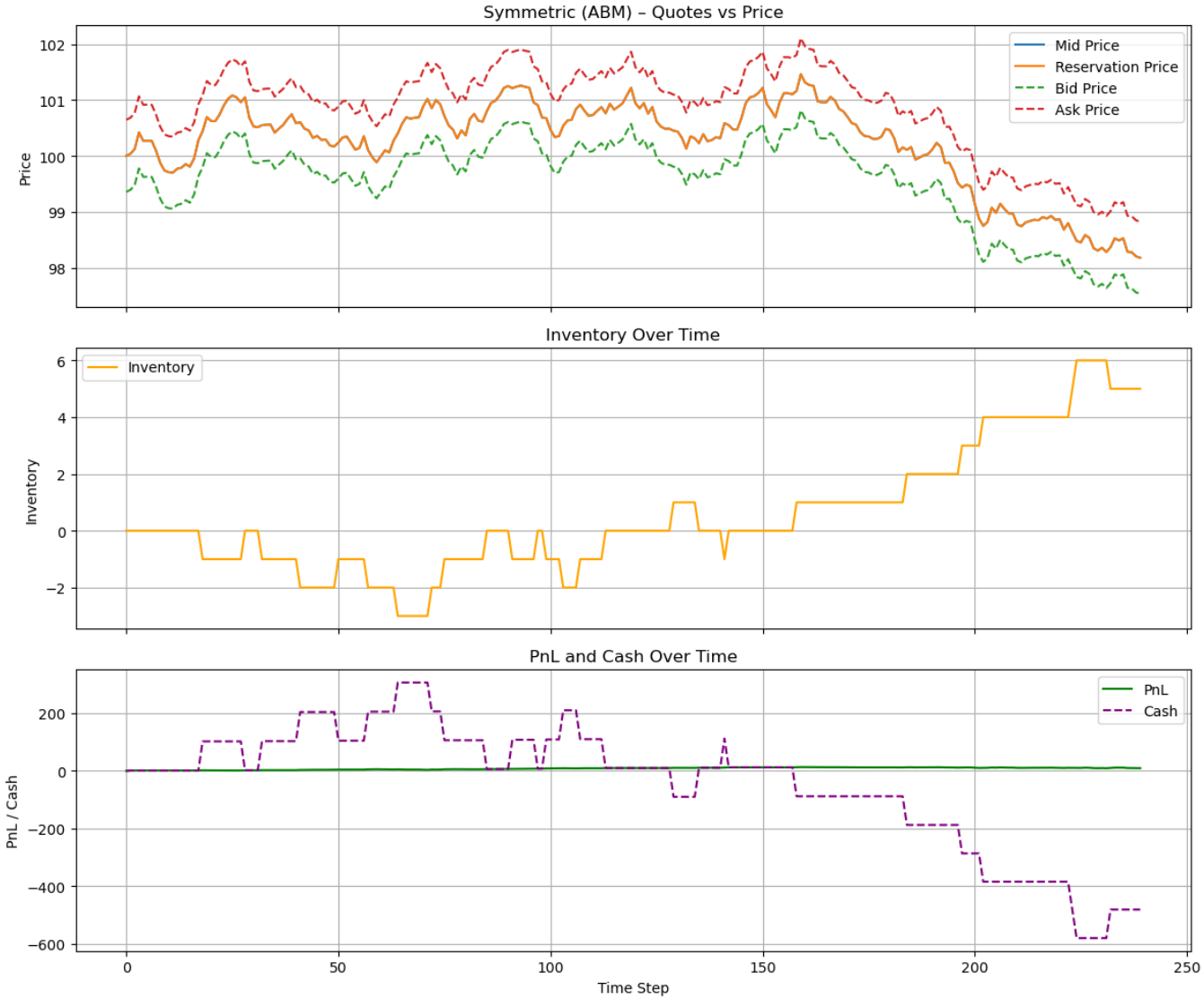

Figure: Quotes and inventory dynamics for one sample run (γ=0.1).

Simulated Quotes and Inventories for symmetric strategy-contd.

- mid price and reservation price overlap.
- spread is constant over time.

PnL Distribution Comparison

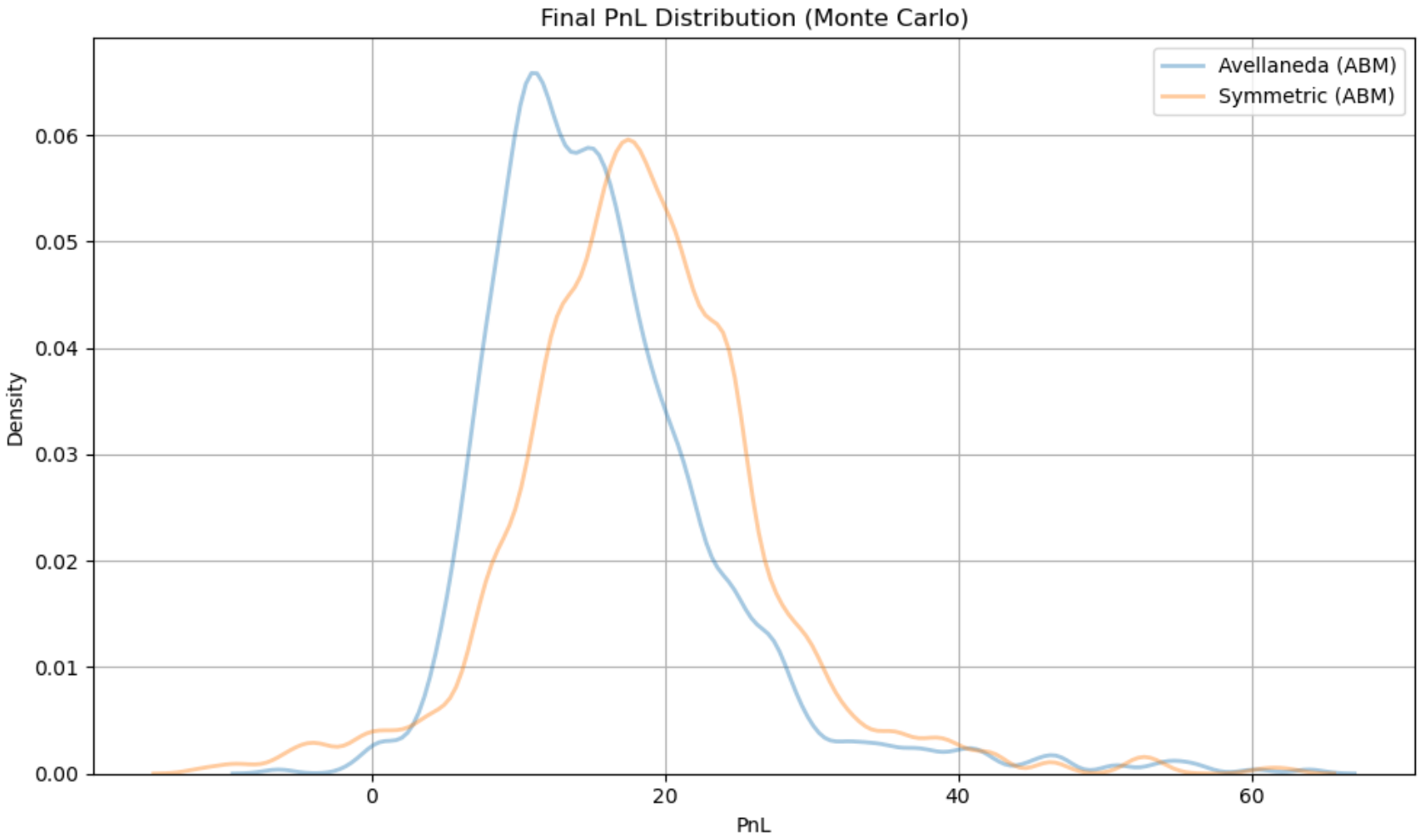

Figure: PnL histograms for inventory vs symmetric strategy (=0.1).

PnL Distribution Comparison - contd.

- lower profit values are less frequent for inventory strategy.

Final PnL Summary (1/2)

Strategy	γ	σ	k	**Mean**	**Std**	**Sharpe**
Avellaneda	0.01	1.0	1.0	27.11	6.13	4.42
Avellaneda	0.01	1.0	1.5	17.99	4.75	3.79
Avellaneda	0.01	2.0	1.0	26.61	9.09	2.93
Avellaneda	0.01	2.0	1.5	17.81	7.99	2.23
Avellaneda	0.10	1.0	1.0	27.63	6.51	4.25
Avellaneda	0.10	1.0	1.5	17.85	4.81	3.71
Avellaneda	0.10	2.0	1.0	25.28	9.49	2.66
Avellaneda	0.10	2.0	1.5	15.64	7.89	1.98
Avellaneda	0.50	1.0	1.0	28.28	9.20	3.07
Avellaneda	0.50	1.0	1.5	16.98	7.79	2.18
Avellaneda	0.50	2.0	1.0	18.96	12.77	1.49
Avellaneda	0.50	2.0	1.5	9.87	8.68	1.14

Final PnL Summary (2/2)

Strategy	γ	σ	k	**Mean**	**Std**	**Sharpe**
Symmetric	0.01	1.0	1.0	27.07	6.14	4.41
Symmetric	0.01	1.0	1.5	17.93	4.84	3.71
Symmetric	0.01	2.0	1.0	27.09	9.02	3.00
Symmetric	0.01	2.0	1.5	17.86	8.19	2.18
Symmetric	0.10	1.0	1.0	27.63	6.18	4.47
Symmetric	0.10	1.0	1.5	17.67	4.93	3.59
Symmetric	0.10	2.0	1.0	25.57	8.11	3.15
Symmetric	0.10	2.0	1.5	15.74	7.08	2.22
Symmetric	0.50	1.0	1.0	28.99	6.21	4.67
Symmetric	0.50	1.0	1.5	17.29	4.48	3.86
Symmetric	0.50	2.0	1.0	19.72	6.71	2.94
Symmetric	0.50	2.0	1.5	9.50	4.38	2.17

Sensitivity to Market Depth and Volatility

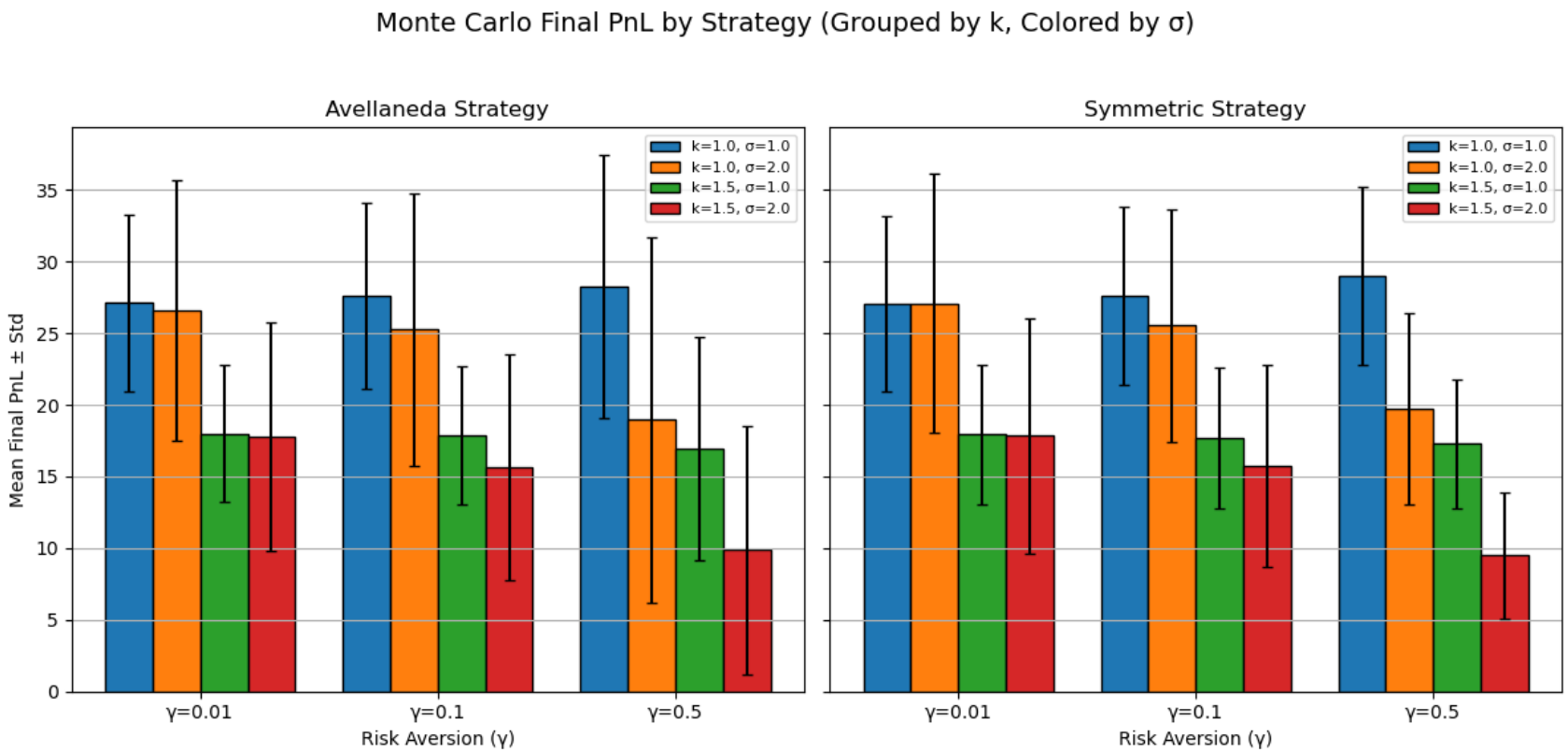

Figure: Monte Carlo means and std deviations for varied k, σ.

Conclusion

- Inventory-based strategy has lower PnL variance.
- Symmetric strategy earns more, but with higher risk.
- Inventory control is most beneficial under high γ or long horizons.