Numerical Simulation of the Avellaneda-Stoikov Market Making Model

Model Parameters

Let the simulation use the following parameters:

$$
\begin{aligned}
S_0 &= 100.00 \quad \text{(initial mid-price)} \\
\sigma &= 2.0 \quad \text{(volatility)} \\
\gamma &= 0.1 \quad \text{(risk aversion)} \\
A &= 140 \quad \text{(base arrival rate)} \\
k &= 1.5 \quad \text{(order sensitivity)} \\
T &= 1.0 \quad \text{(time horizon)} \\
\Delta t &= \frac{1}{240} \quad \text{(time step)}
\end{aligned}
$$

Reservation Price

At time t, with inventory q_t, the reservation price is:

$$r_t = S_t - q_t \cdot \gamma \cdot \sigma^2 \cdot (T - t)$$

Optimal Spread and Quote Distances

The optimal bid-ask spread is:

$$\text{spread}_t = \gamma\sigma^2(T - t) + \frac{2}{\gamma} \ln\left(1 + \frac{\gamma}{k}\right)$$

Assuming symmetric quotes around r_t, we define:

$$\delta_t = \frac{\text{spread}_t}{2}$$

Bid and Ask Quotes

$$
\begin{aligned}
p_t^b &= r_t - \delta_t \\
p_t^a &= r_t + \delta_t
\end{aligned}
$$

Order Arrival Intensities

$$
\begin{aligned}
\lambda_t^b &= A \cdot e^{-k \cdot \delta_t^b}, \quad \delta_t^b = S_t - p_t^b \\
\lambda_t^a &= A \cdot e^{-k \cdot \delta_t^a}, \quad \delta_t^a = p_t^a - S_t
\end{aligned}
$$

Execution Probabilities

$$P(\text{fill bid}) = 1 - \exp\left(-\lambda_t^b \cdot \Delta t\right)$$
$$P(\text{fill ask}) = 1 - \exp\left(-\lambda_t^a \cdot \Delta t\right)$$

State Updates

If bid fills:

$$q_{t+\Delta t} = q_t + 1, \quad X_{t+\Delta t} = X_t - p_t^b$$

If ask fills:

$$q_{t+\Delta t} = q_t - 1, \quad X_{t+\Delta t} = X_t + p_t^a$$

Mid-Price Evolution

Mid-price evolves as:

$$S_{t+\Delta t} = S_t + \sigma \cdot \sqrt{\Delta t} \cdot \xi_t, \quad \xi_t \sim \mathcal{N}(0,1)$$

Terminal P&L

At time T:

$$\text{PnL} = X_T + q_T \cdot S_T$$