Equation Intégrale de Von Karman

eq. couche limite : $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ (1)

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dP}{dx} + \nu\frac{\partial^2 u}{\partial y^2} \quad (2)$$

conditions limites : $y=0 \quad u=0,\ v=0$ (3)

$y \to \infty \quad u = U_e$ (4)

intégrons (1) $\quad v = -\int_0^y \frac{\partial u}{\partial x}\,dy + C$ (5)

condition limite (3) $\quad 0 = 0 + C \Rightarrow \underline{C=0}$

intégrons (2) avec substitution (5)

$$\int_0^\infty \Big[u\frac{\partial u}{\partial x} - \overbrace{\int_0^y \frac{\partial u}{\partial x}\,dy\,\frac{\partial u}{\partial y}}^{I} - U_e\frac{dU_e}{dx}\Big]dy = \int_0^\infty \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)dy \quad (6)$$

où la condition limite (4) a été utilisée ; en effet, l'équation (2) avec (4) devient $U_e\frac{dU_e}{dx} + v\cancel{\frac{\partial u}{\partial y}}^{0} = -\frac{1}{\rho}\frac{dP}{dx} + \nu\cancel{\frac{\partial^2 u}{\partial y^2}}^{0}$

ou $\quad U_e\frac{dU_e}{dx} = -\frac{1}{\rho}\frac{dP}{dx}$ } Bernoulli !

le terme I dans (6)

$$\int_0^\infty \underbrace{\int_0^y \frac{\partial u}{\partial x}\,dy}_{u}\ \underbrace{\frac{\partial u}{\partial y}\,dy}_{dv} = U_e\int_0^\infty \frac{\partial u}{\partial x}\,dy - \int_0^\infty u\frac{\partial u}{\partial x}\,dy \quad (7)$$

$$\int_0^\infty u\,dv = uv\Big|_0^\infty - \int_0^\infty v\,du$$

(7) dans (6) avec $\tau_w = \mu\frac{\partial u}{\partial y}\Big|_{y=0}$

$$\int_0^\infty \left[2u\frac{\partial u}{\partial x} - U_e \frac{\partial u}{\partial x} - U_e \frac{dU_e}{dx} \right] dy = -\frac{\tau_w}{\rho}$$

true(!) $-u\frac{\partial U_e}{\partial x} + u\frac{dU_e}{dx} = 0$

$$\int_0^\infty \frac{\partial}{\partial x}\left[u(u-U_e)\right] dy + \frac{dU_e}{dx}\int_0^\infty (u-U_e)\, dy = -\frac{\tau_w}{\rho}$$

$$\int_0^\infty \left[\frac{\partial}{\partial x} U_e^2 \frac{u}{U_e}\left(1-\frac{u}{U_e}\right)\right] dy + \frac{dU_e}{dx}\int_0^\infty U_e\left(1-\frac{u}{U_e}\right) dy = \frac{\tau_w}{\rho}$$

$$\frac{\partial}{\partial x}(U_e^2\theta) + U_e\delta^*\frac{dU_e}{dx} = \frac{\tau_w}{\rho}$$

si $H \equiv \dfrac{\delta^*}{\theta}$, $C_f \equiv \dfrac{\tau_w}{\frac{1}{2}\rho U_e^2}$

alors

$$\frac{\partial}{\partial x}(U_e^2\theta) + U_e H\theta\frac{dU_e}{dx} = U_e^2\frac{C_f}{2}$$

et

$$\underline{\frac{d\theta}{dx} + \theta(H+2)\frac{1}{U_e}\frac{dU_e}{dx} = \frac{C_f}{2}}$$