Intro to Neural Networks

BA865 – Mohannad Elhamod

Basic Linear Algebra

BOSTON UNIVERSITY

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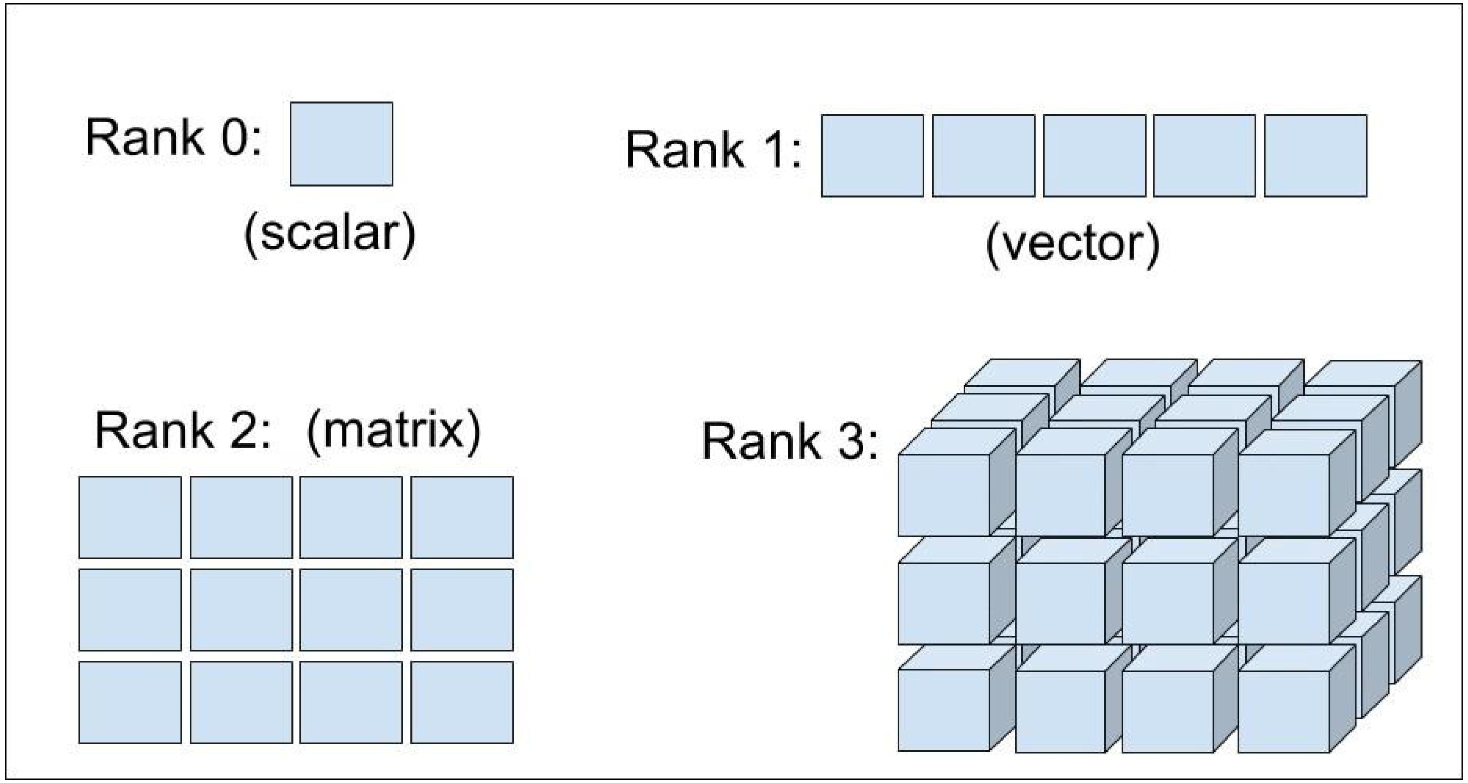
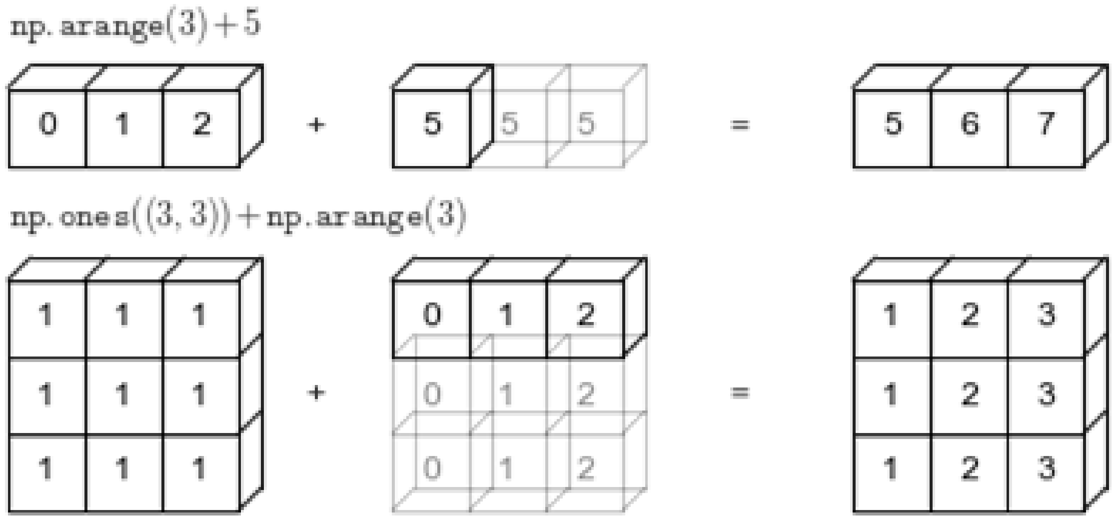
What is a Tensor?

- A tensor is a matrix of rank 3 or higher.
- Examples?
- You can think about it as "groups of vectors"



Addition

- Shape of the Two Tensors Needs to Conform.
- Sum Element-wise
  - Replicate B until it matches A's dimensions, then perform element-wise addition.
  - Each dimension can be replaced with a size of 1.



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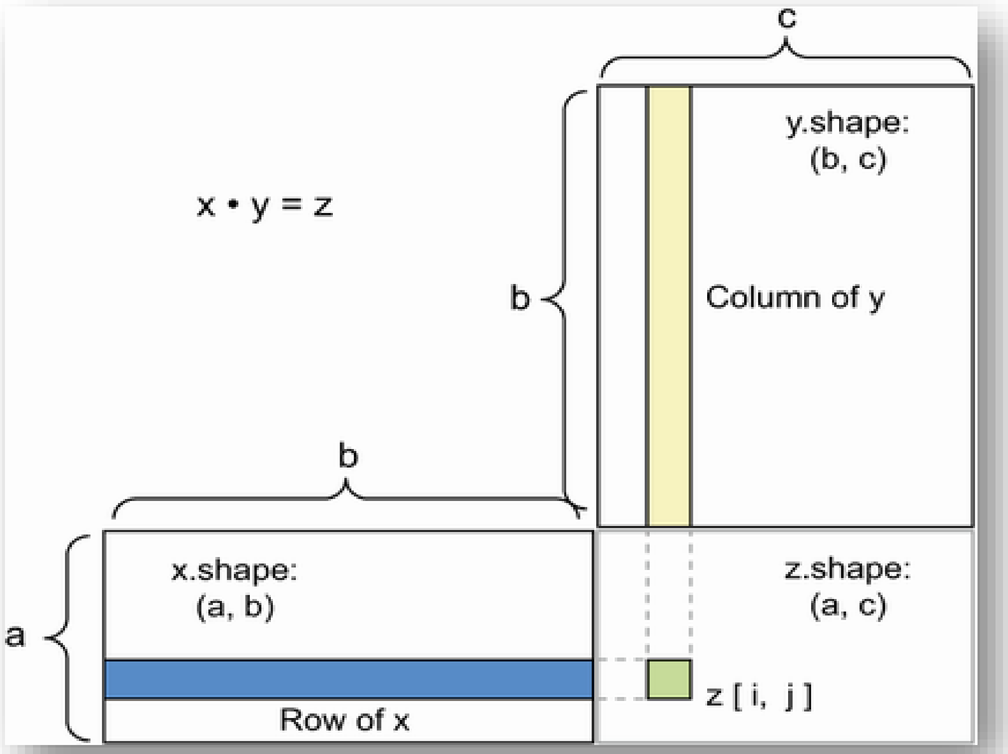
Multiplication (Dot Product)

- `x.shape[-1]` should equal `y.shape[0]`
- This is different from element-wise multiplication!



Linear Transformation

- Matrix multiplication is a “linear transformation”.
- What does that mean? Demo
  - The outputs are a linear combination of the inputs.
  - A line remains a line and relative positions are preserved (No warping!).
  - Reflection, scaling, rotation, and shear.
  - Not translation!

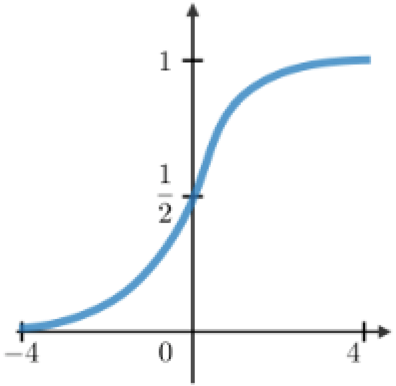
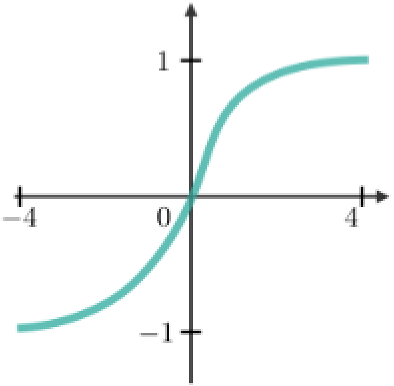
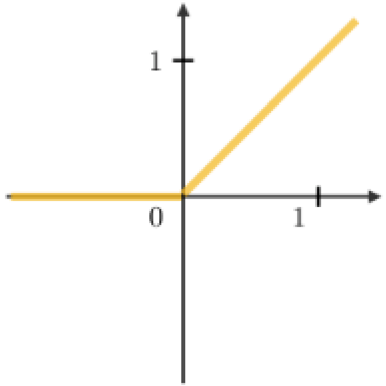
Enter Non-Linearities

What is a Non-Linearity?

- A non-linearity will lead to warping and “mangling”the space.
- Demo
- Examples:
  - Quadratic function.
  - Exponential function.
  - The step function.

Popular Examples

- [Demo]

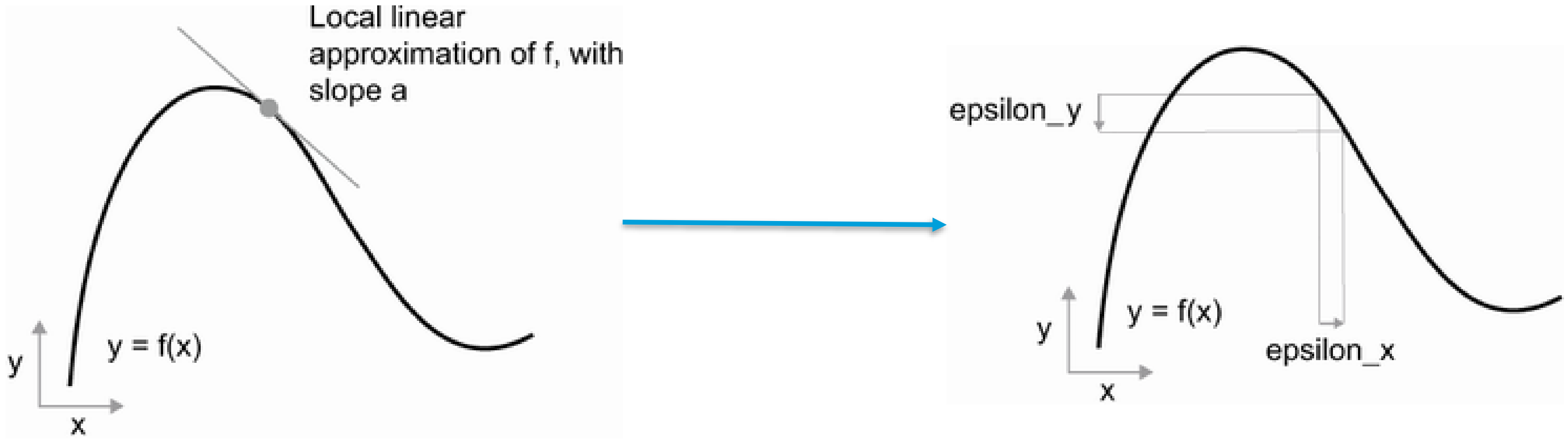
Sigmoid	Tanh	RELU
$g(z) = \frac{1}{1+e^{-z}}$	$g(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$	$g(z) = \max(0, z)$
		

https://studymachinelearning.com

Basic Calculus

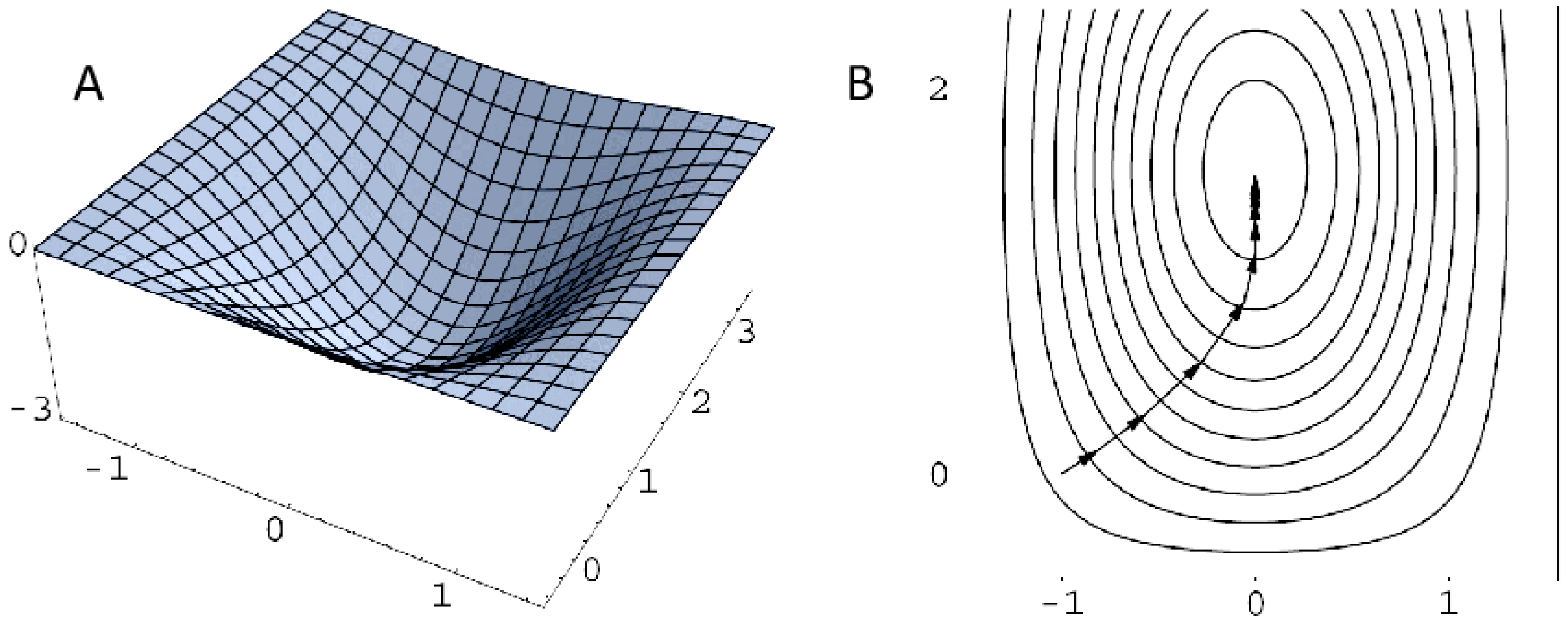
The Derivative

- It is the ***local*** rate of change
  - $$\lim_{\epsilon_x \to 0} \frac{\epsilon_y}{\epsilon_x} = \frac{\partial y}{\partial x}$$



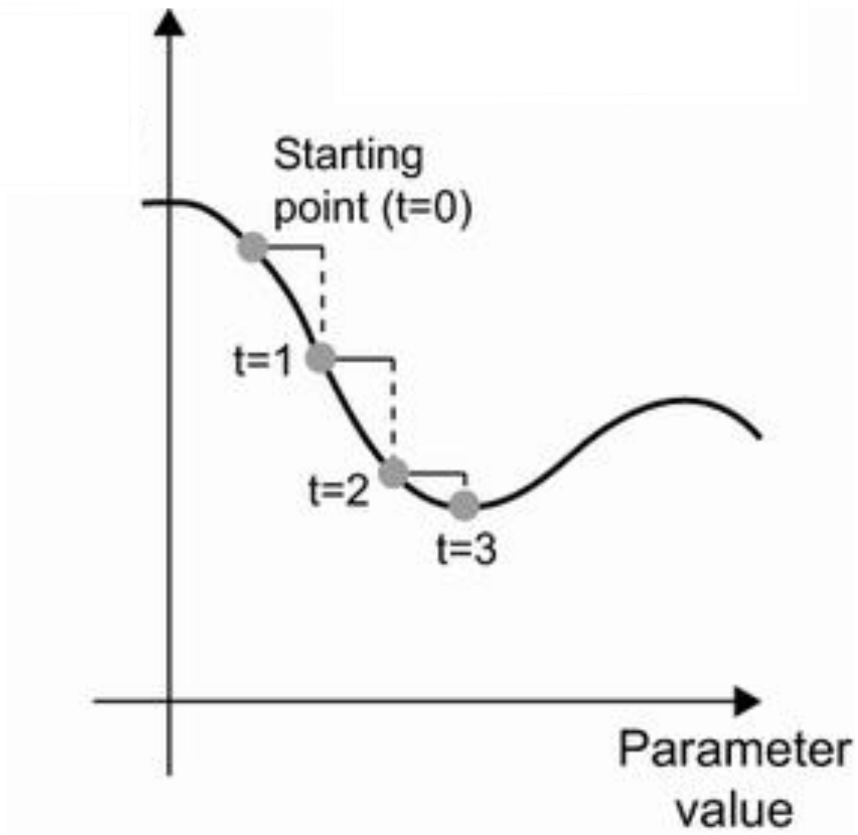
The Gradient

- The equivalent of the derivative in multi-dimensional space: $\nabla_{x,y}z$



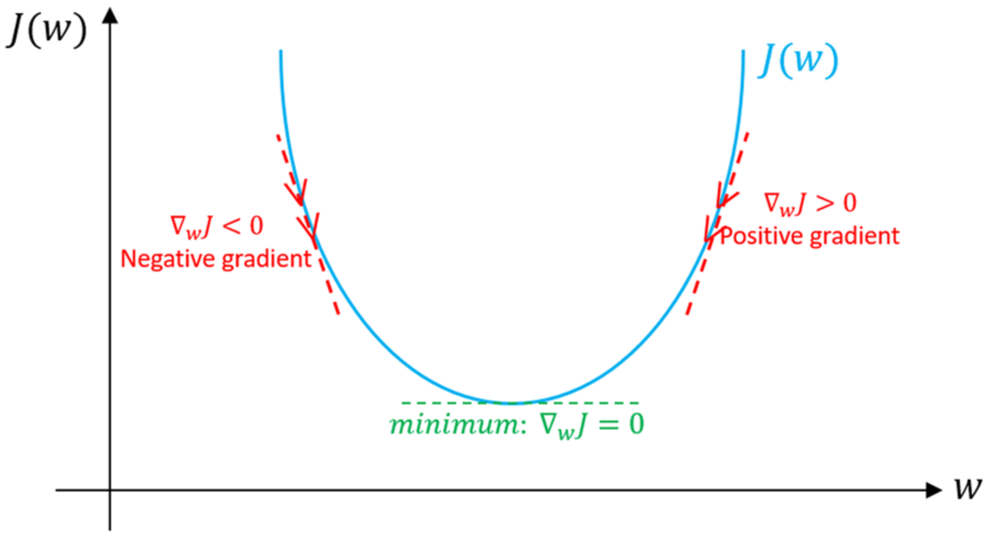
Gradient Descent

- Since the gradient points towards the steepest direction, following it (or going against it) will lead us to maximizing (or minimizing) the function's value.



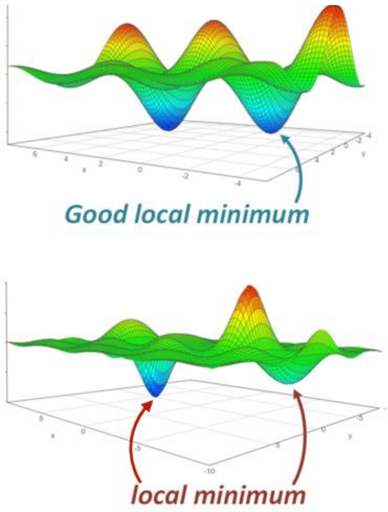
Optimization

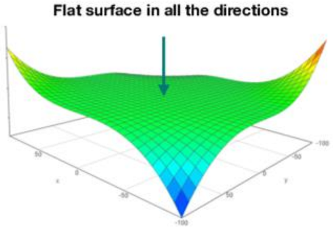
- By going against the direction of the gradient, you can minimize *an error function*. This is called *optimization*.
  - $\frac{\partial J}{\partial w}$ for one-dimensional parameter.
  - becomes $\nabla_w J$ when w is high-dimensional

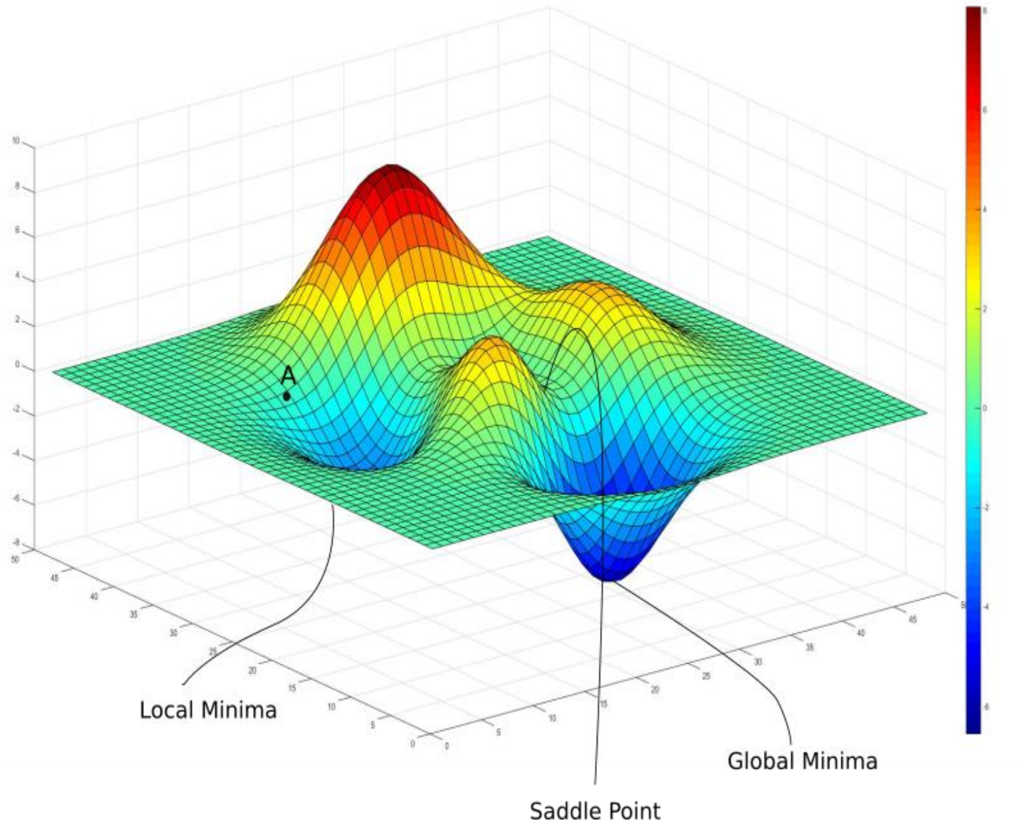


Optimization

- Can we always achieve lowest error?
- [Demo]







TechTalks

Chain Rule

- To get the gradient of a function of a function:
  - Chain (multiply) the gradient of the first in terms of the second, … until you reach the independent parameter.

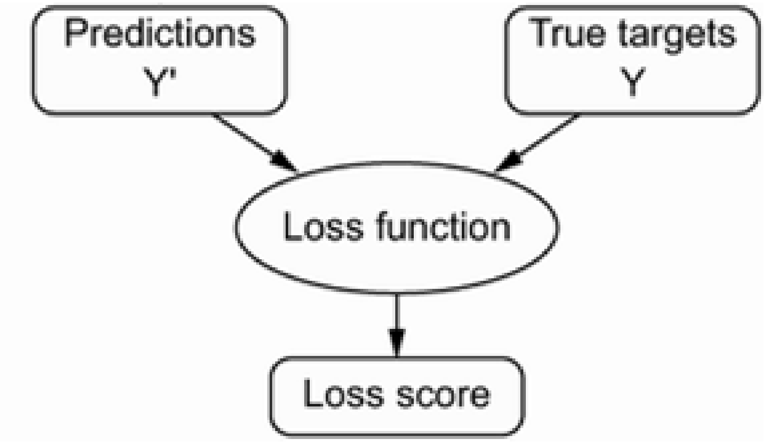
$$z = f(x,y) \qquad x = x(u,v)$$

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial u}$$

Calculating Error

Calculating Error

- The terms **error**, **loss**, and **cost** are almost used interchangeably.
- A loss is *generally* a measure of how “different” the ground truth is from the model’s predictions.

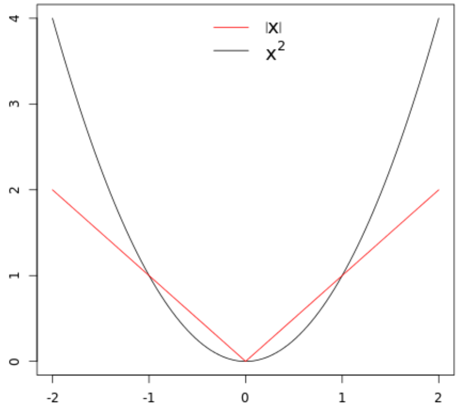


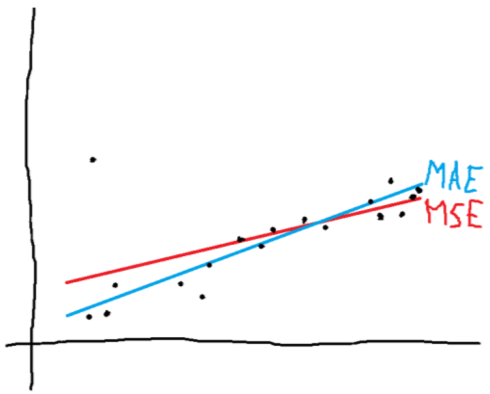
Types of Loss Functions

- Mean Absolute Error (MAE/ L1 loss)
  - Large and small differences are penalized proportionally.
- Mean Squared Error (MSE/ L2 loss)
  - Larger differences are penalized exponentially more than smaller ones

$$MAE = \frac{\sum_{i=1}^{n} | y_i - \hat{y}_i |}{n}$$

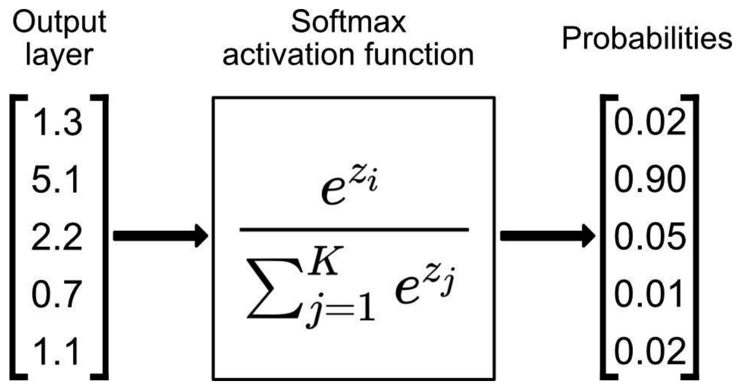
$$MSE = \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{n}$$



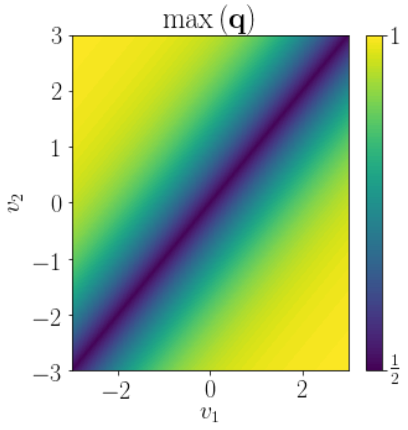


Obtaining Probabilities

- Probabilities occur frequently in the context of classification.
- Given an array of numbers, a ***softmax*** converts that array into a probability distribution
    - This is not the only way to convert an array into a probability distribution. However, it is the most popular way in deep learning. Why?



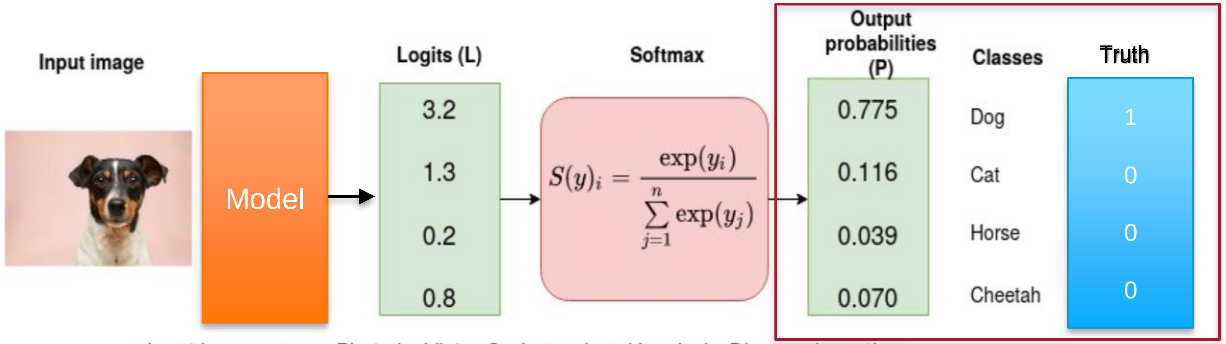
https://towardsdatascience.com/



https://charlielehman.github.io

Measuring Error in Probabilities

- We want to measure the error between the ground truth and a probability distribution.
- Given the output of some classification mode, you can turn it into probabilities using *softmax*, and then measure the error using ***cross-entropy***



Input image source: Photo by Victor Grabarczyk on Unsplash . Diagram by author.

$$H(P^*|\,P) = -\sum_i \underbrace{P^*(i)}_{\text{TRUE CLASS DISTRIBUTION}} \log \underbrace{P(i)}_{\text{PREDICTED CLASS DISTRIBUTION}}$$

The Perceptron

The Primordial Cell of Neural Networks

The building block: The Perceptron

- [Demo]()

Algorithm: Perceptron Learning Algorithm

$P \leftarrow inputs \quad with \quad label \quad 1;$
$N \leftarrow inputs \quad with \quad label \quad 0;$
Initialize $\mathbf{w}$ randomly;
while !*convergence* **do**

Face towards positive examples

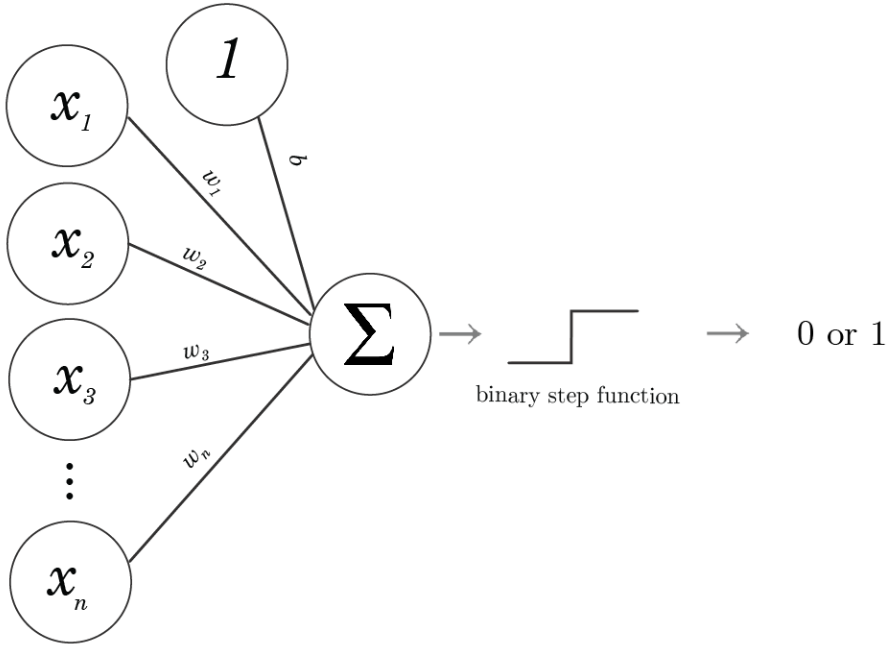
  Pick random $\mathbf{x} \in P \cup N$;
  if $\mathbf{x} \in P \quad and \quad \mathbf{w}.\mathbf{x} < 0$ **then**
    $\mathbf{w} = \mathbf{w} + \mathbf{x}$;
  end

Face away from negative examples

  if $\mathbf{x} \in N \quad and \quad \mathbf{w}.\mathbf{x} \geq 0$ **then**
    $\mathbf{w} = \mathbf{w} - \mathbf{x}$;
  end

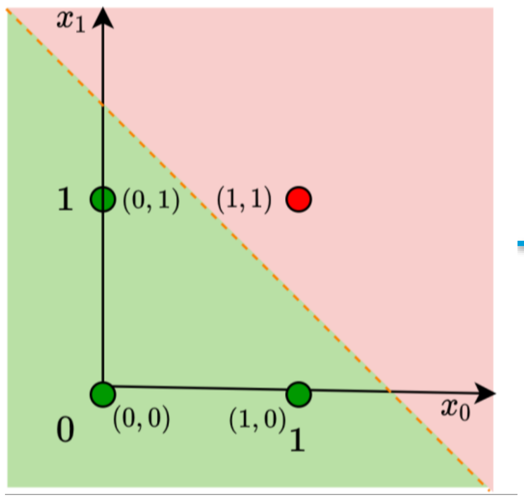
end
//the algorithm converges when all the inputs are classified correctly

Figure courtesy of Akshay L Chandra

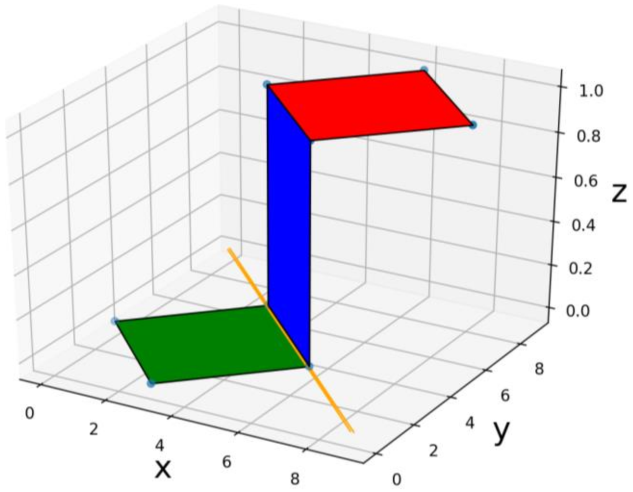


The building block: The Perceptron

- Does the perceptron remind you of something?
  - The perceptron uses a step function. Logistic Regression uses a sigmoid.
  - [Demo]



Perceptron - Top view



Perceptron - 3D view

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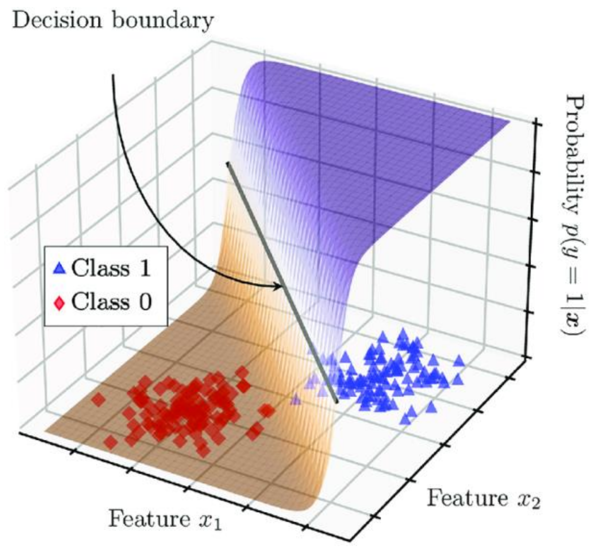


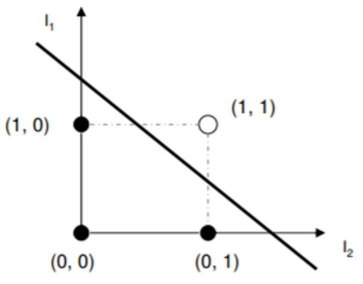
Logistic Regression - 3D view

https://www.researchgate.net

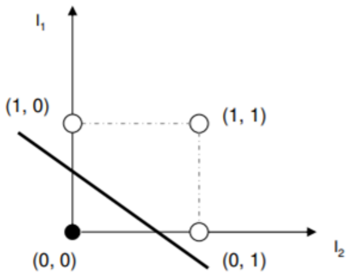
The building block: The Perceptron

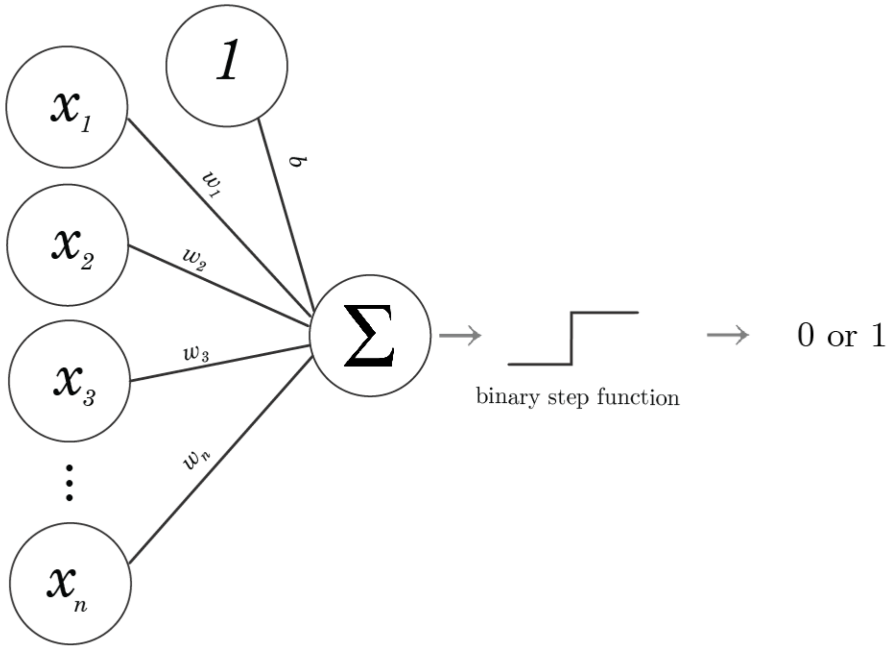
- Works well for linearly separable cases.

AND		
I_1	I_2	out
0	0	0
0	1	0
1	0	0
1	1	1



OR		
I_1	I_2	out
0	0	0
0	1	1
1	0	1
1	1	1





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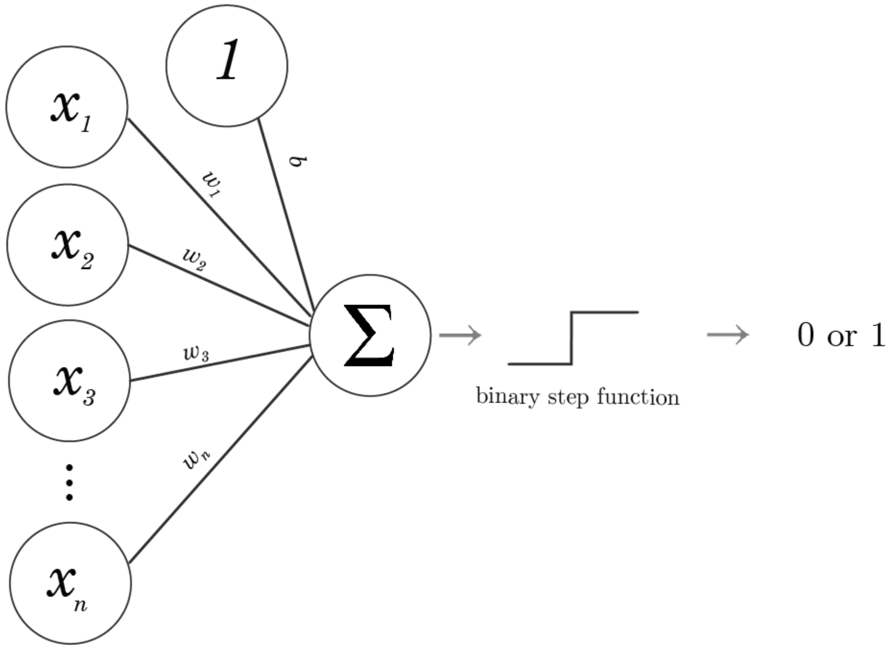
The building block: The Perceptron

- Can we work around the “linear separability” issue”?

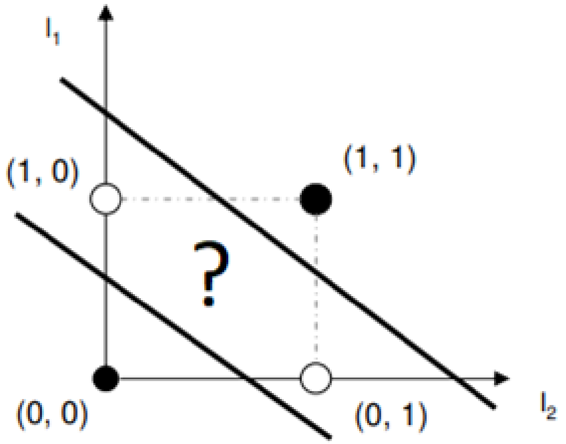


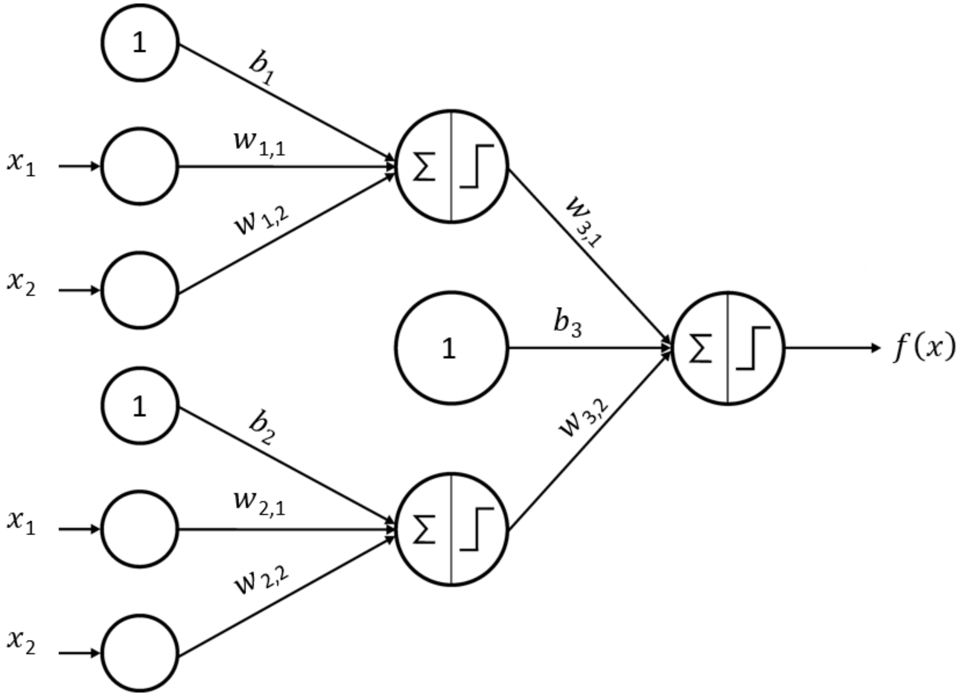
Adam Dhala

Power in Numbers: Multiple Perceptrons

- Demo

XOR		
I_1	I_2	out
0	0	0
0	1	1
1	0	1
1	1	0

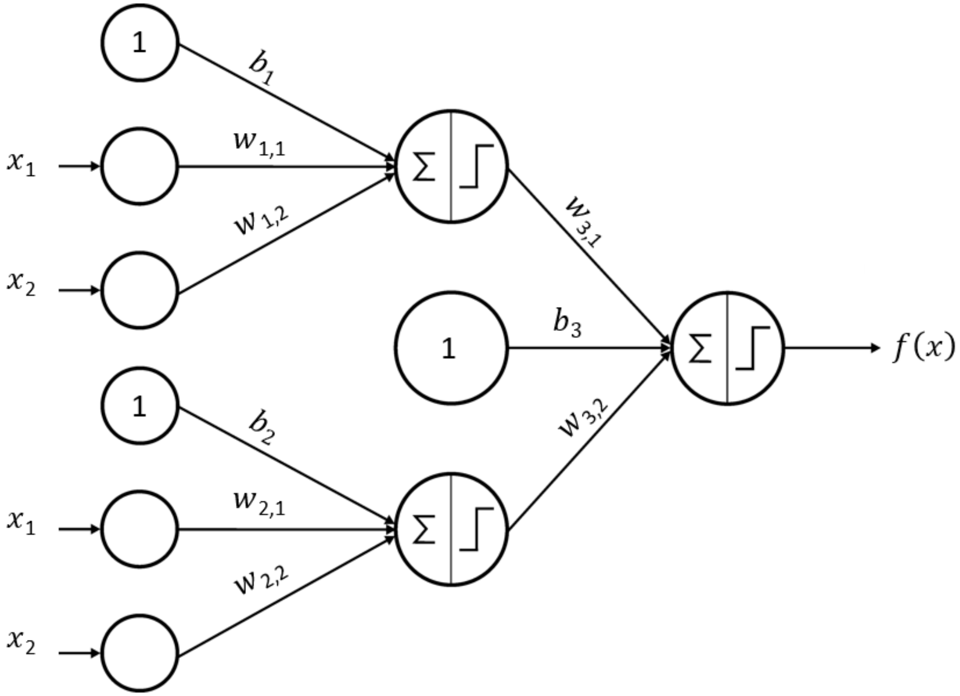




western-neuralnets.ca

Power in Numbers: Multiple Perceptrons

The Multi-Layer Perceptron (MLP) was born!



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More Layers!

- In theory, a single hidden layer is sufficient to learn any function. In practice, however, networks with more layers are easier to optimize than networks larger width.
- The more layers/nodes a model has, the higher its capacity is, making it able learn more complex decision boundaries.
- Demo

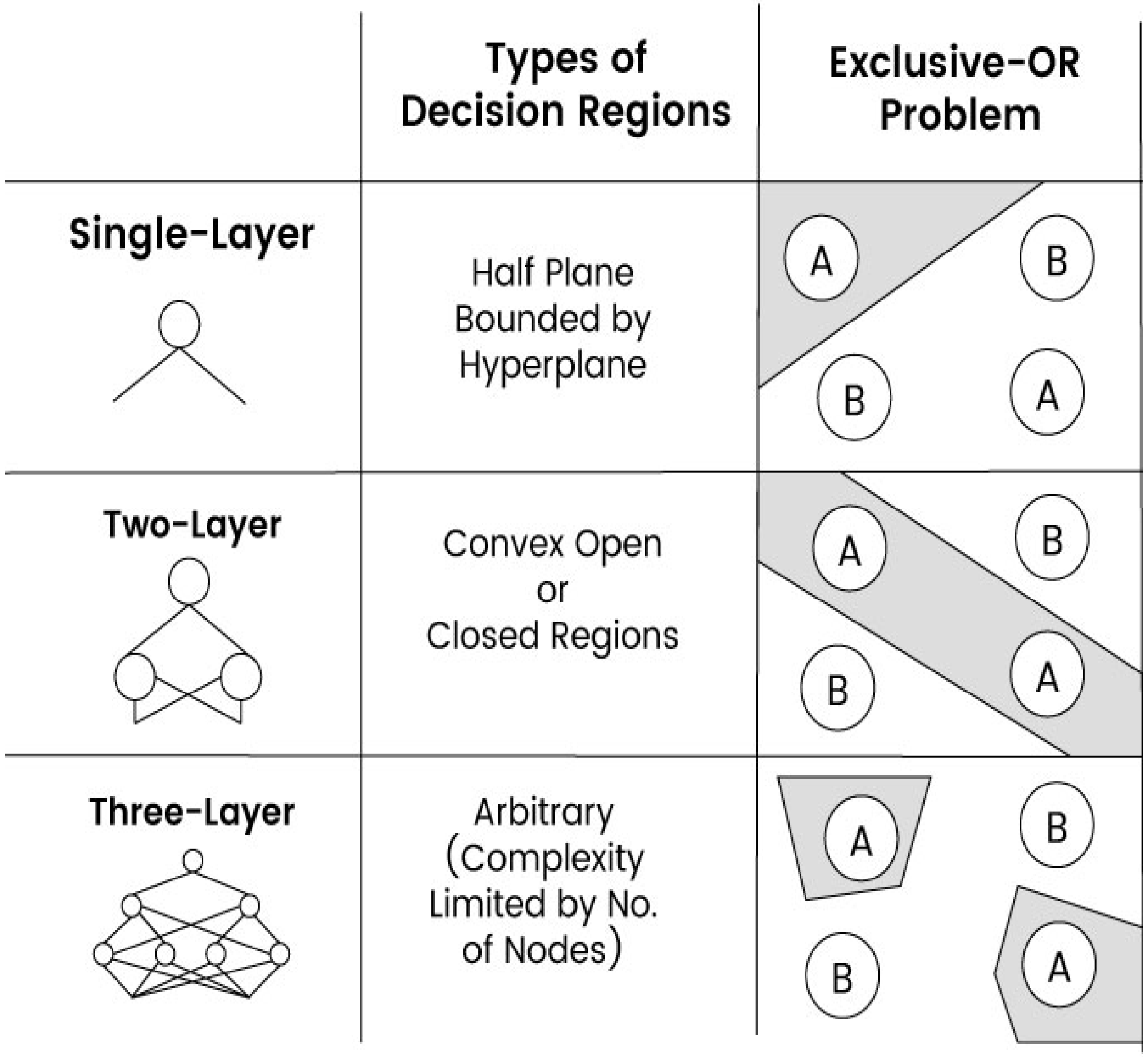

Figure courtesy of Very Engineering Team