* Three possibilities of linear system of equations :-

1- Has exactly one solution. } consistent
2- Has infinitely many solution. } consistent
3- Has no solution. inconsistent

<u>Consistent system:</u> system of equations that has at least one solution

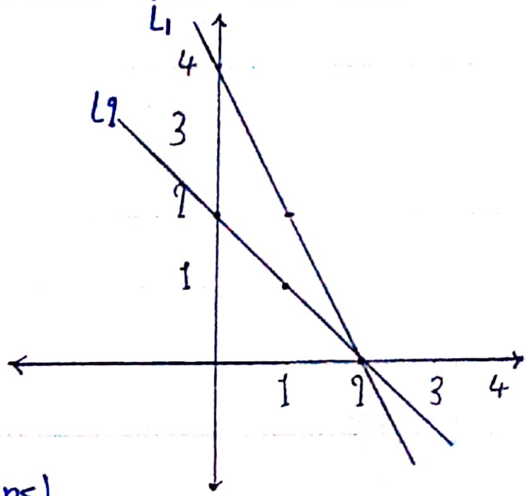
<u>Inconsistent system:</u> system of equations that has no solution.

① The line l_1 intersect line l_2 at only one point. (one solution)

$2x + y = 4$

$x + y = 2$

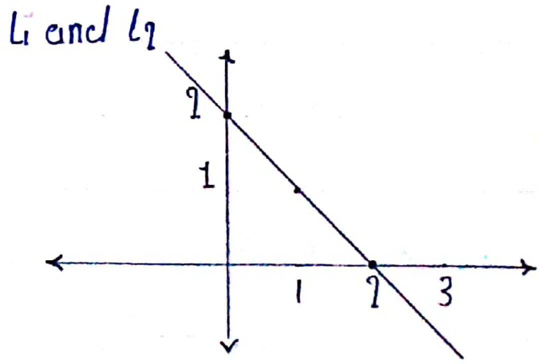
solution: $x = 2$, $y = 0$



② The line l_1 coincide line l_2. (Infinite solutions)

$2x + 2y = 4$

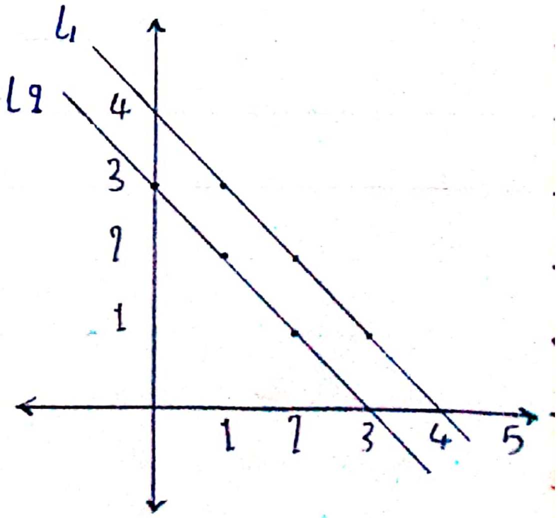
$x + y = 2$



③ The lines l_1, l_2 are parallel. (No solution)

$x + y = 4$

$2x + 2y = 6$



* How to determine if a system has no solution, or infinite solutions from Augmented matrix?

<u>A system has no solution.</u>

$$\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & 0 & 1 \end{array}\right] \qquad 0x_1 + 0x_2 = 1 \qquad 0 = 1$$

<u>A system has infinite solutions.</u>

$$\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right] \qquad x + y = 3$$

- <u>Example</u>:- solve by Gaussian Elimination.

$$x_1 - 2x_2 - 6x_3 = 12$$
$$2x_1 + 4x_2 + 12x_3 = -17$$
$$x_1 - 4x_2 - 12x_3 = 22$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -6 & 12 \\ 2 & 4 & 12 & -17 \\ 1 & -4 & -12 & 22 \end{array}\right] \quad \begin{array}{l} -2R_1 + R_2 \rightarrow R_2 \\ -R_1 + R_3 \rightarrow R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -6 & 12 \\ 0 & 8 & 24 & -41 \\ 0 & -2 & -6 & 10 \end{array}\right] \quad R_2/8 \rightarrow R_2$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -6 & 12 \\ 0 & 1 & 3 & -41/8 \\ 0 & -2 & -6 & 10 \end{array}\right] \quad 2R_2 + R_3 \rightarrow R_3$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -6 & 12 \\ 0 & 1 & 3 & -41/8 \\ 0 & 0 & 0 & -1/4 \end{array}\right] \quad \text{system has no solution}$$

- **Example:** solve by Gauss-Jordan

$$x + 2y - 3z + w = -2 \qquad -2 \qquad 2 - 2 - 3 + 1 = -2$$
$$3x - y - 2z - 4w = 1 \qquad 1 \qquad 6 + 1 - 2 - 4 = 1$$
$$2x + 3y - 5z + w = -3 \qquad -3 \qquad 4 - 3 - 5 + 1 = -3$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -3 & 1 & -2 \\ 3 & -1 & -2 & -4 & 1 \\ 2 & 3 & -5 & 1 & -3 \end{array}\right] \quad \begin{array}{l} -3R_1 + R_2 \to R_2 \\ -2R_1 + R_3 \to R_3 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -3 & 1 & -2 \\ 0 & -7 & 7 & -7 & 7 \\ 0 & -1 & 1 & -1 & 1 \end{array}\right] \quad R_2 / -7 \to R_2$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -3 & 1 & -2 \\ 0 & 1 & -1 & 1 & -1 \\ 0 & -1 & 1 & -1 & 1 \end{array}\right] \quad \begin{array}{l} -2R_2 + R_1 \to R_1 \\ R_2 + R_3 \to R_3 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & -1 & 0 \\ 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

has infitely many solutions

x, y leading variables

z, w free variables

$$x - z - w = 0 \qquad x = z + w$$
$$y - z + w = -1 \qquad y = z - w - 1$$

$x = 0$	$x = 2$
$y = -1$	$y = -1$
$z = 0$	$z = 1$
$w = 0$	$w = 1$

- **Example:** what condition that b_1, b_2 and b_3 should satisfy in order to solve the following system?

$$x_1 + x_2 + x_3 = b_1$$
$$x_1 + 2x_3 = b_2$$
$$2x_1 + x_2 + 3x_3 = b_3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & b_1 \\ 1 & 0 & 2 & b_2 \\ 2 & 1 & 3 & b_3 \end{array}\right] \begin{array}{l} -R_1 + R_2 \to R_2 \\ -2R_1 + R_3 \to R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & b_1 \\ 0 & -1 & 1 & b_2 - b_1 \\ 0 & -1 & 1 & b_3 - 2b_1 \end{array}\right] \quad -R_2 \to R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & b_1 \\ 0 & 1 & -1 & b_1 - b_2 \\ 0 & -1 & 1 & b_3 - 2b_1 \end{array}\right] \quad R_2 + R_3 \to R_3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & b_1 \\ 0 & 1 & -1 & b_1 - b_2 \\ 0 & 0 & 0 & b_3 - b_2 - b_1 \end{array}\right]$$

Condition

$$b_3 - b_2 - b_1 = 0$$
$$b_3 = b_2 + b_1$$

Note

1) If the number of equations is fewer than the number of variables, then the system has two possibilities: 1- No solution. 2- infinitely many solutions.

2) If the number of equations is greater than or equal to the number of variables, then the system has three possibilities: 1- Exactly one solution. 2- No solution. 3- Infinitely many solutions.