# Neural Networks

# - Homework 3 -

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## 1 Exercises

### 1.1 Exercise 2.1

The delta rule described in the following equation:

 $\Delta w_{kj}(n) = \eta e_k(n) x_j(n)$ 

and Hebb's rule described in:

 $\Delta w_{kj}(n) = \eta y_k(n) x_j(n)$ 

represent the two different methods of learning. List the features that distinguish these two rules from each other.

Solution:

- Learning rule: delta rule is a part of error-correction learning, whereas Hebb's rule is a part of memory-based learning.
- Adjustment: in delta rule it is based on the difference between desired and actual output (the error) and in Hebb's rule on correlation, whether two neurons connected by synapse that is being considered are activated simultaneously or not.

#### 1.2 Exercise 2.10

Formulate the expression for the output  $y_j$  of neuron j in the network of Fig. 1, where:

 $x_i = i^{th}$  input signal,

 $w_{ii} = \text{synaptic weight from input i to neuron j},$ 

 $c_{kj}$  = weight of lateral connection from neuron k to neuron j,

 $y_j = \phi(v_j).$ 

What is the condition that would have to be satisfied for neuron j to be the winning neuron?

Solution:

$$y_j = \phi(\sum_{i=1}^4 w_{ji}x_i + \sum_{k=1, k \neq j}^3 c_{kj}y_k) = \phi(\sum_{i=1}^4 w_{ji}x_i + \sum_{k=1, k \neq j}^3 c_{kj}\phi(v_k))$$

, where  $k_1, k_2$  are numbers from 1 to 3 those are not equal j.

Neuron j is a winning neuron if  $v_i > v_k$  for all  $k \neq j$ .

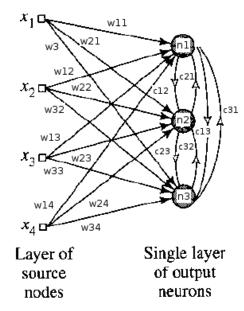
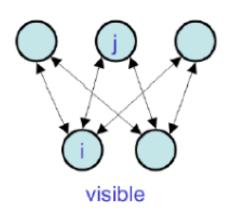


Figure 1: Competitive neural network with feedforward connections.

### 1.3 Exercise 4

A simple network is given below (from lecture slides). You have to update the weights once using Boltzmann learning for this network. Please do calculations by hand or by using MATLAB or Python. Use random numbers to initialise the weights. For training use a training set as [(0,1),(1,0)] or any training set of your choice.

Figure 2: NN from ex. 4. hidden



Because this NN is recurrent and has 5 neurons, weight matrix has size 5x5 with zeros on a main diagonal. The training sample is s = [1, -1, -1, -1, -1]. Initial weight matrix was filled with uniform[0, 1] numbers.

The weight update was done in matlab:

- 1 % Exercise 5 from the homework pdf.
- 2 %Update weights once with Boltzmann learning rule

```
3 %Authors P. Lukin, I. Vishniakou, E. Ovchinnikova
  %Initialization of a weight matrix. We have 5 neurons, so matrix
       is 5x5
w = zeros(5,5);
  w(1:2,3:5) = rand(2,3);
w(3:5,1:2) = rand(3,2);
  nu = 0.2;
  %Neuron's states
  s = -ones(1,5);
  %Training sample s1 =1 s2=10
  s(1) = 1;
  exps = s;
14
  % Clamped state
  %Energy change
  for i=3:5
18
       dE(i) = sum(w(1,:).*s);
19
  end
20
  %Flip
21
22
  for i=3:5
23
       r = rand(1,1);
24
       p = 1/(1+\exp(-dE(i)));
25
       if r>p
26
           s(i) = -s(i);
27
       end
  end
  sClamp = s;
30
31
  \% Free run state state
  s = \exp s;
33
  %Energy change
  for i=1:2
       dE(i) = sum(w(1,:).*s);
36
  end
37
  %Flip
38
39
  for i=1:2
40
       r = rand(1,1);
41
       p = 1/(1 + \exp(-dE(i)));
       if r>p
43
           s(i) = -s(i);
44
       end
45
  end
46
  sFree = s;
47
48
  %Weight update
```

```
for i=1:5
           for j=1:5
51
                 dw(\,i\;,j\,)\;=\;\underset{}{\mathtt{corrcoef}}\left(\,sClamp\left(\,i\,\right)\,,sFree\left(\,j\,\right)\,\right);
52
                  if isnan(dw(i,j))
53
                       dw(i, j) = 0;
54
                  \quad \text{end} \quad
55
           end
56
   end
   dw(1:5+1:5*5) = 0;
    'Weights update'
59
   dw
60
_{61} w =w+nu*dw
```