Neural Networks
- Homework 4 -

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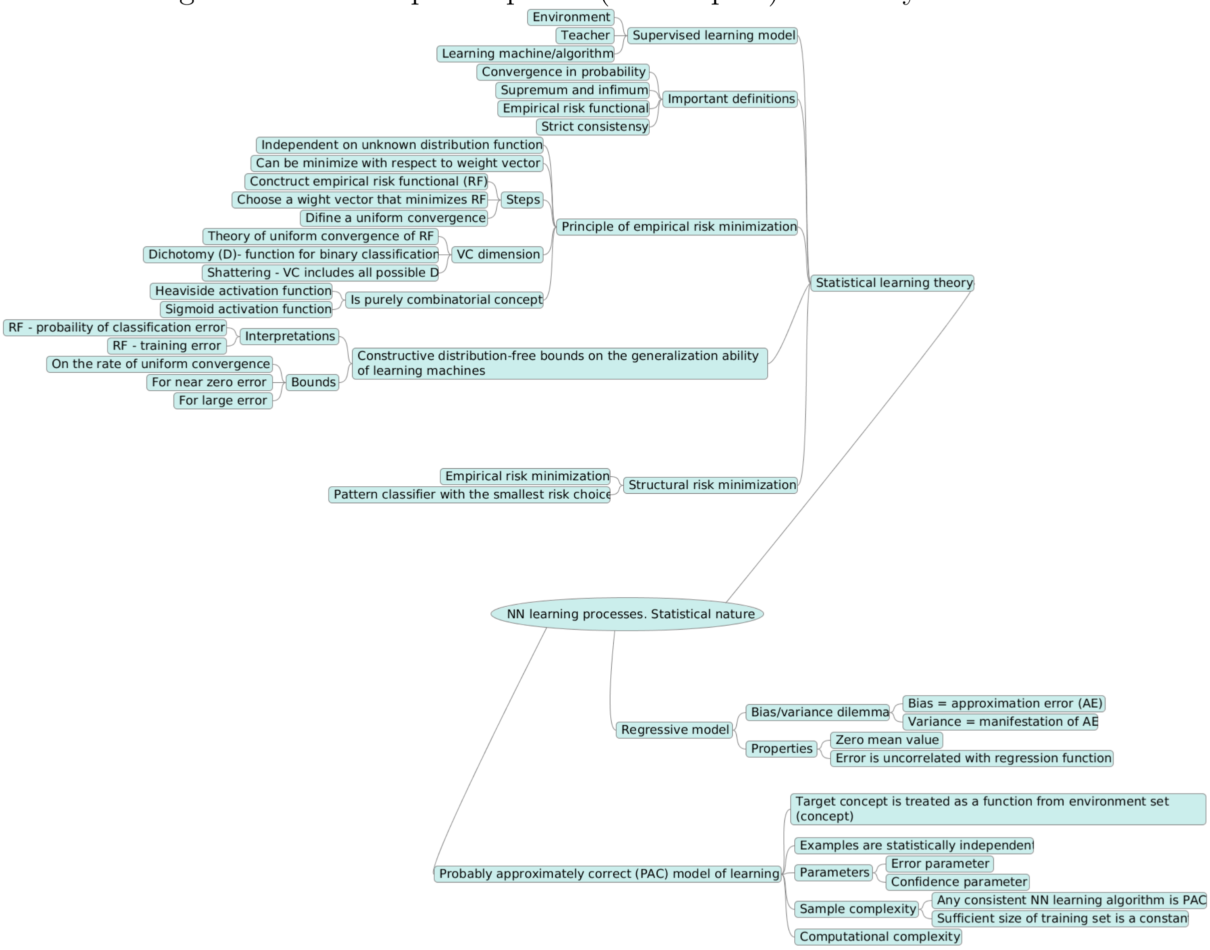
1 Mind map

Figure 1: Mind map. Chapter 2 (second part) from Haykins book.



2 Exercises

Exercise 2.1

Consider the space of instances X corresponding to all points in the x, y plane. Give the VC dimension of the following hypothesis spaces:

Solution:

a. H_r = the set of all rectangles in the x,y plane. i.e. $H_r = \{((a < x < b) \wedge (c < y < d))|a, b, c, d \in IR\}$

Since $H_r = \{((a < x < b) \wedge (c < y < d))|a, b, c, d \in IR\}$ we have non-rotatable rectangles only with horizontal and vertical edges those are not tight to the center of coordinate plane.
The shattering is possible in case if we can select a certain number of points so it will be possible to choose this number of points minus one without taking into account the last one. In Fig. 2 we show how we can shatter 4 points in all possible combinations. However, we can not choose the five points and select four of them without the fifth one. Fig. 3 shows that it is not possible to shatter 5 points. One can see that to define the fifth point we already have four defined points on a rectangle. Fifth point can be either inside or on the edge. Therefore, VC dimension is 4.

Figure 2: Four points shattered by a rectungular.

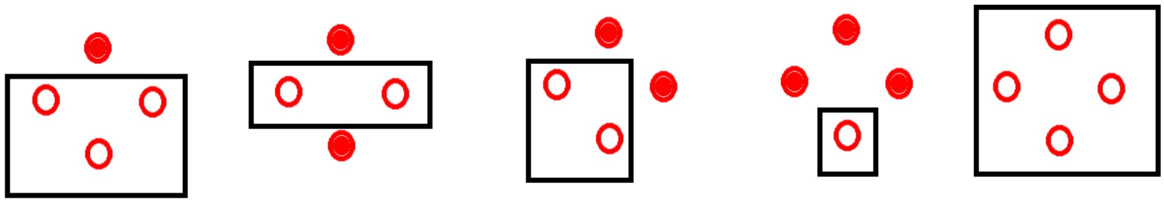
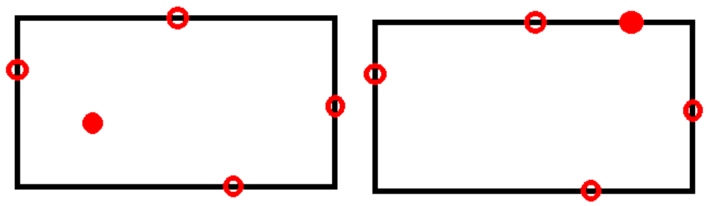


Figure 3: Five points not shattered by a rectungular.



b. H_c = the set of all circles in the x,y plane. Points inside the circle are classified as positive examples.

A circle can be defined by 3 points, so the VC is 3. In Fig. 4 we show 3 points shattering and in Fig. 5 that 4 points cannot be shattered.

Figure 4: Four points shattered by a rectungular.

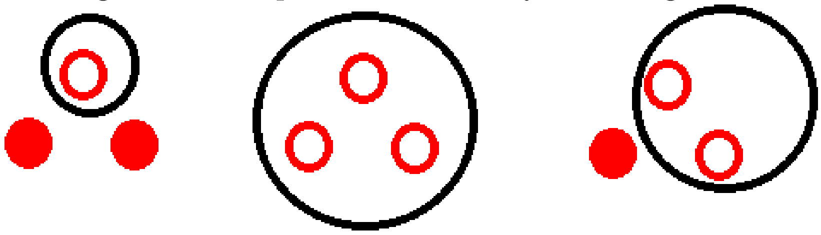
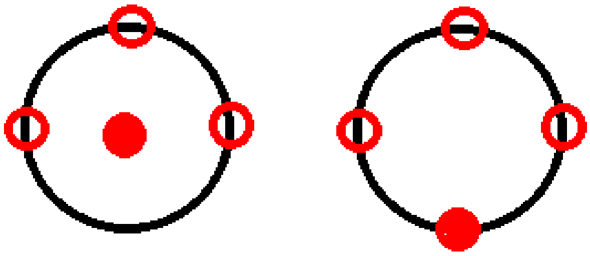


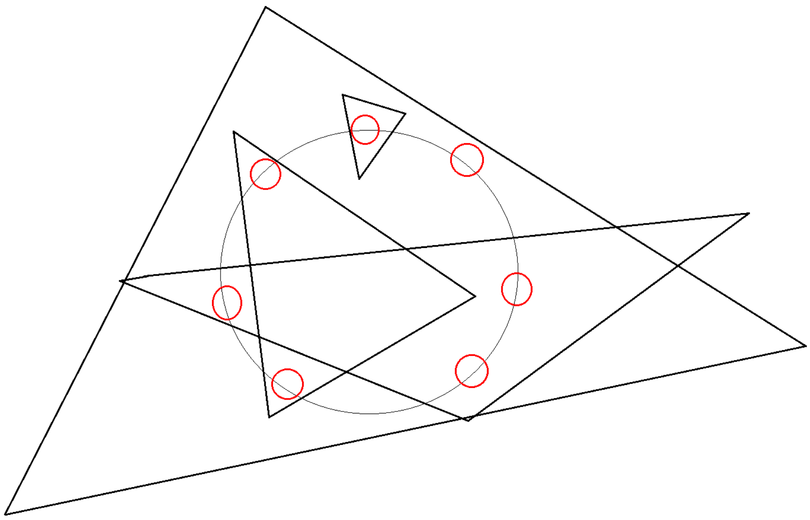
Figure 5: Five points not shattered by a rectungular.



c. H_t = the set of all triangle in the x,y plane. Points inside the triangle are classified as positive examples.

In this case we can always cut 3 points or less of the certain type, because the triangle has three vertices that can be easily seen when we put the points on a circle (see Fig. 6). So, it's not a problem to divide 3 from 4 as well as the less number of point and the all cases will be taken into account. In case of 8 points we can only divide 3 points, so the case 4-4 is missing. That is why VC is 7.

Figure 6: Four points shattered by a rectungular.



Exercise 2.2

Definition: consistent learner

- A learner is consistent if it outputs hypotheses that perfectly fit the training data, whenever possible. It is quite reasonable to ask that a learning algorithm be consistent, given that we typically prefer a hypothesis that fits the training data over one that does not.

  Task:

- Write a consistent learner for H_r from last Exercise (i.e. $H_r = \{((a < x < b) \wedge (c < y < d))|a, b, c, d \in IR\}$). Generate a variety of target concept rectangles at random, corresponding to different rectangles in the plane. Generate random examples of each of these target concepts, based on a uniform distribution of instances within the rectangle from (0,0) to (100, 100).

Plot the generalization error as a function of the number of training examples, m. On the same graph, plot the theoretical relationship between e and m, for d = .95. Does theory fit experiment?

Solution:

Generalization error ($\hat{\epsilon}\epsilon(h)$) is given as the following:

for each sample $Z_i = 1\{h_i(x) \neq c(x)\}$ (Bernoully random variable):

$$\hat{\epsilon} = \frac{1}{m}\sum_{j=1}^{m} Z_i$$

Probability that the version space with respect to H_r and S (a sequence of training examples $m \geqslant 1$) is not ϵ-exhausted (with respect to c) is less than $|H|e^{-\epsilon m}$, where ϵ is an error. $0 < \epsilon < 0.5$, probability $0 < \delta < 0.5$.

$\epsilon \geqslant \frac{1}{m}(lnH + ln\frac{1}{\delta})$, so:
$\epsilon \geqslant \frac{1}{m}(ln4 + ln\frac{1}{(1-0.95)})$