PHYS 108: Problem Set #6

Tuesday, November 5, 2024 3:09 PM

1) a. $A = 31.2\ mm^2$

$L = 85.5\ cm$

$I = 115\ A$

$n = 8.47 \times 10^{28}\ m^{-3}$

$$v_{drift} = \frac{I}{nAe}$$

$$= \frac{115\ A}{(8.47 \times 10^{28}\ m^{-3})(31.2\ mm^2)(1.6 \times 10^{-19}\ C)}$$

$$= \frac{115\ A}{(8.47 \times 10^{28}\ m^{-3})(31.2 \times 10^{-6}\ m^2)(1.6 \times 10^{-19}\ C)}$$

$$= \frac{115}{422822.4} \cdot \frac{A \cdot m}{C}$$

$$= 2.7198 \times 10^{-4}\ m/s$$

$$t = \frac{L}{v_{drift}}$$

$$= \frac{85.5\ cm}{2.7198 \times 10^{-4}\ m/s}$$

$$= \frac{0.855\ m}{2.7198 \times 10^{-4}\ m/s}$$

$$= 3143.59\ s$$

b.

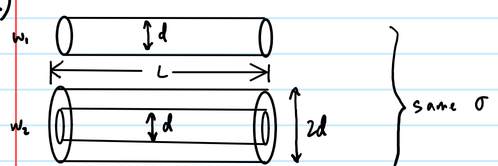
$$J = \frac{I}{A}$$

$$= \frac{115\ A}{31.2\ mm^2}$$

$$= \frac{115\ A}{31.2 \times 10^{-6}\ m^2}$$

$$= 3.6859 \times 10^6\ \frac{A}{m^2}$$

2)



$$R = \frac{L}{\sigma A}$$

$$R_{w_1} = \frac{L}{\sigma(\pi(d/2)^2)} = \frac{L}{\sigma(d^2\pi/4)} = \left(\frac{L}{\sigma}\right)\left(\frac{4}{d^2\pi}\right)$$

$$R_{w_2} = \frac{L}{\sigma(\pi(2d/2)^2 - \pi(d/2)^2)} = \left(\frac{L}{\sigma}\right)\left(\frac{1}{d^2\pi - d^2\pi/4}\right) = \left(\frac{L}{\sigma}\right)\left(\frac{1}{3\pi d^2/4}\right) = \left(\frac{L}{\sigma}\right)\left(\frac{4}{3\pi d^2}\right)$$

$$\cancel{\frac{L}{\sigma}}\left(\frac{\cancel{4}}{d^2\pi}\right) > \cancel{\frac{L}{\sigma}}\left(\frac{\cancel{4}}{3d^2\pi}\right)$$

$$\frac{1}{d^2\pi} > \frac{1}{3d^2\pi}$$

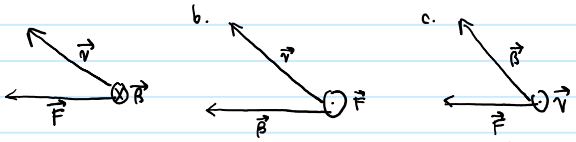
$$\frac{}{\cancel{\sigma}}\left(\frac{}{d^2\pi}\right) > \frac{}{\cancel{\sigma}}\left(\frac{}{3d^2\pi}\right)$$

$$\frac{1}{d^2\pi} > \frac{1}{3d^2\pi}$$

$$R_{w_1} > R_{w_2}$$

Conductor 1 has the higher resistance of $\frac{4L}{\sigma\pi d^2}$.

3) a.



Middle $\vec{B}$	$\otimes$	$\leftarrow$	$\uparrow$
Index $\vec{v}$	$\uparrow$	$\uparrow$	$\odot$
Thumb $\vec{F}$	$\leftarrow$	$\odot$	$\leftarrow$
Reasonable?	Yes	Yes	Yes

All three configurations are physically reasonable as they follow the RHR; the directions of each axes align with our orientations defined by right-hand direction conventions.

4) a. $\vec{A}\times\vec{B} = AB\sin\theta\,(\hat{C})$

$\|\vec{C}\| = 3$

$\|\vec{D}\| = 5$

$\theta = 30°$ above x-axis

$\vec{C} = 3\hat{x}$

$\vec{D} = 5\cos30\,\hat{x} + 5\sin30\,\hat{y}$

$$\begin{aligned}\vec{C}\times\vec{D} &= (3)(5)\sin30\,\hat{z}\\ &= 15\sin30\,\hat{z}\\ &= 15\left(\tfrac{1}{2}\right)\hat{z}\\ &= 7.5\,\hat{z}\end{aligned}$$

b. $\vec{A} = 2\hat{x} - 5\hat{z}$

$\vec{B} = -5\hat{x} + 3\hat{y} + 5\hat{z}$

$$\vec{C} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 2 & 0 & -5 \\ -5 & 3 & 5 \end{vmatrix}$$

$$= \hat{x}\begin{vmatrix} 0 & -5 \\ 3 & 5 \end{vmatrix} - \hat{y}\begin{vmatrix} 2 & -5 \\ -5 & 5 \end{vmatrix} + \hat{z}\begin{vmatrix} 2 & 0 \\ -5 & 3 \end{vmatrix}$$

$$= \hat{x}(0-(-15)) - \hat{y}(10-25) + \hat{z}(6-0)$$

$$= \hat{x}(15) - \hat{y}(-15) + \hat{z}(6)$$

$$= 15\hat{x} + 15\hat{y} + 6\hat{z}$$

c. $\vec{A} = d\hat{y} + 2d\hat{z}$

$\vec{B} = -r\hat{x} - d\hat{y}$

$$\vec{C} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & d & 2d \\ -r & -d & 0 \end{vmatrix}$$

$$= \hat{x}\begin{vmatrix} d & 2d \\ & \end{vmatrix} - \hat{y}\begin{vmatrix} 0 & 2d \\ & \end{vmatrix} + \hat{z}\begin{vmatrix} 0 & d \\ & \end{vmatrix}$$

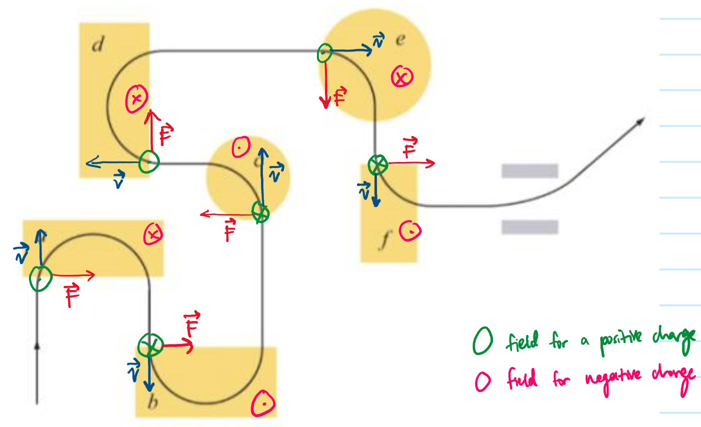
$$= \hat{x}\begin{vmatrix} d & 2d \\ -d & 0\end{vmatrix} - \hat{y}\begin{vmatrix} 0 & 2d \\ -r & 0\end{vmatrix} + \hat{z}\begin{vmatrix} 0 & d \\ -r & -d\end{vmatrix}$$

$$= \hat{x}\left(0-(-2d^2)\right) - \hat{y}\left(0-(-2dr)\right) + \hat{z}\left(0-(-dr)\right)$$

$$= \hat{x}(2d^2) - \hat{y}(2dr) + \hat{z}(dr)$$

$$= 2d^2\hat{x} - 2dr\hat{y} + dr\hat{z}$$

5)



field for a positive charge

field for negative charge

Magnetic Fields of Each Region:

a: $\otimes$ (out of page)

b: $\odot$ (into page)

c: $\odot$

d: $\otimes$

e: $\otimes$

f: $\odot$

Charge is negative b/c particle deflects towards higher potential plate. Negative charged particles still want to have high → low ΔU, since $\Delta U = q\Delta V$, ΔV is positive (expected behavior w/o any external work/energy) when q is negative since $\Delta V = \Delta V_{low} - \Delta V_{high}$ = negative.

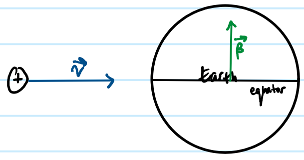
6) a. $\vec{v} = 2.8\times10^7$ m/s

$\vec{B} = 50\,\mu T = 50\times10^{-6}$ T

$m = 1.67\times10^{-27}$ kg

$g = 9.8$ m/s²

$q = +e = 1.6\times10^{-19}$ C



b. $F_B = qvB\sin\theta$

$= (1.6\times10^{-19}\text{ C})(2.8\times10^7\text{ m/s})(50\times10^{-6}\text{ T})\sin 90$

$= 2.24\times10^{-16}$ N

c. $F_g = mg$

$= (1.67\times10^{-27}\text{ kg})(9.8\text{ m/s}^2)$

$= 1.637\times10^{-26}$ N

d. $\frac{F_B}{F_g} = 1.369\times10^{10}$ $\quad F_B \gg F_g$

7) a. $q = 2e = 2(1.6\times10^{-19}\ C) = 3.2\times10^{-19}\ C$

$m = 4.0u = 4(1.67\times10^{-27}\ kg) = 6.68\times10^{-27}\ kg$

$r = 4.5\ cm = 0.045\ m$

$B = 1.2\ T$

$$v = \frac{qBr}{m} = \frac{(3.2\times10^{-19}\ C)(1.2\ T)(0.045\ m)}{6.68\times10^{-27}\ kg}$$

$$= \frac{1.728\times10^{-20}}{6.68\times10^{-27}}\ \frac{C\cdot T\cdot m}{kg}$$

$$= 2.5868\times10^{6}\ m/s$$

b. $$\omega = \frac{qB}{m} = \frac{(3.2\times10^{-19}C)(1.2\ T)}{(6.68\times10^{-27}\ kg)}$$

$$= 5.756\times10^{7}\ rad/s$$

c. $KE = \frac{1}{2}mv^2$ ans from pt. a

$$= \frac{1}{2}(6.68\times10^{-27}\ kg)(2.5868\times10^{6}\ m/s)^2$$

$$= 2.235\times10^{-14}\ J$$

d. $$V = \frac{KE}{q} = \frac{(2.235\times10^{-14}\ J)}{(3.2\times10^{-19}\ C)}$$

$$= 69842.88\ V$$

$$= 69.84\ kV$$