[CS 235] Assignment 6

Wednesday, November 13, 2024 8:52 PM

Assignment 8

Computer Science 235

Reading. Sections 5.1 and 5.3

1) Show that EQ_{CFG} is undecidable. (*Hint*: Use the fact that ALL_{CFG} is undecidable as indicated in Theorem 5.13 on page 225.)

Suppose EQ_{CFG} is decidable.

Let $C_1 = ALL_{CFG}$ and C_2 be another CFG that recognizes the same language.

Then, we have $L(C_1) = L(C_2)$ such that the pair $C_1, C_2 \in EQ_{CFG}$.

However, since we know that ALL_{CFG} is undecidable, C_2 must also be undecidable.

Since both C_1 and C_2 are undecidable, we cannot determine if the equivalence of the pair.

2) Show that EQ_{CFG} is co-Turing-recognizable. Reduces to emptiness co-TM recognizable problem.

Problem 1: $\exists$ TM to recognize every $(C_1, C_2) \in EQ_{CFG}$ s.t. $L(C_1) = L(C_2)$.

M = "On input (C_1, C_2)...

1. Find the difference between sets:

   $diff = \{A_{C_1}\} - \{A_{C_2}\}$

   where A_{C_1} and A_{C_2} are the set of all accepted inputs/strings of the CFGs C_1 and C_2, respectively.
2. Check if $diff \in E_{CFG}$ or $diff = \emptyset$.
3. If $diff \in E_{CFG}$ or $diff = \emptyset$, accept. Otherwise, reject.

Problem 2: $\exists$ TM to recognize every $C_1, C_2 \in \overline{EQ_{CFG}}$ s.t. $L(C_1) \neq L(C_2)$.

M = "On input (C_1, C_2)...

1. Find the difference between sets:

   $diff = \{A_{C_1}\} - \{A_{C_2}\}$

   where A_{C_1} and A_{C_2} are the set of all accepted inputs/strings of the CFGs C_1 and C_2, respectively.
2. Check if $diff \notin E_{CFG}$ or $|diff| > 0$.
3. If $diff \notin E_{CFG}$ or $|diff| > 0$, accept. Otherwise, reject.

3) Let T = { <M> | M is a TM that accepts w^R whenever it accepts w}. Show that T is undecidable.

Reduces to A_{TM} undecidable problem.

pf: Assume T is decidable.

Suppose we construct 2 sub-problems/machines to decide A_M (the accepted inputs of machine M as defined).

Let M_1 be a machine that accepts w and M_2 be a machine that accepts w^R if w is accepted by M_1.

However, $\langle M_1, w\rangle \in A_{TM}$, which we know is undecidable.

Clearly, $\langle M_2, w\rangle$ cannot be decidable either since M_2 is only decidable if M_1 is decidable.

Hence, T is undecidable, a contradiction.

4) Consider the problem of determining whether a single-tape Turing machine ever writes a blank symbol over a nonblank symbol during the course of its computation on any input string. Formulate this problem as a language and show that it is undecidable.

$B_{TM} = \{ \langle M \rangle \mid M$ writes $\sqcup$ over a non-blank symbol on some symbol on an input $x\}$.

Assume $\exists$ D to decide B_{TM}.

For any TM M and input w, construct

M' = "on input w:

1) Run M on w

2) If M accepts w, write $\sqcup$ over non-blank.

D is decidable iff M is decidable.

Since A_{TM} is undecidable, D is also undecidable.

Thus B_{TM} is undecidable (cannot be decided by D).

5) Consider the problem of determining whether a Turing machine M on an input w ever attempts to move its head left when its head is on the left-most tape cell. Formulate this problem as a language and show that it is undecidable.

Let $L(M) = \{\langle M, w \rangle \mid M$ attempts to move its head left when at leftmost position on tape on input $w\}$.

Let $H_{TM} = \{\langle M, w \rangle \mid M$ halts on input $w\}$.

Construct M' that runs M on w (simulates M).

If M halts on w, M' moves tape head to leftmost position and tries to move left.

If M is not halted, M' is stuck.

Thus M' is only possible iff M halts on w.

Since H_{TM} is undecidable, L_{TM} is also undecidable.