Eigenvalue Decomposition:

can be real or complex numbers

eigenvectors of A

$$A = Q \Lambda Q^{-1}, \quad \Lambda = \mathrm{diag}(\lambda_1, \dots, \lambda_p) \text{ and } Q = [v_1 \ \dots \ v_p]$$

Not necessarily orthogonal

if $A^T = A$, then $Q^{-1} = Q^T$

Source:

https://math.uchicago.edu/~may/REU2013/REUPapers/Marsden.pdf

Singular-Value Decomposition:

columns in U & V are orthogonal (within U and V)

→ represent rotation linear transformations

$$A = U \Sigma V$$

identity matrix

UV is not necessarily $= I_p$

All entries in Σ are real & non-negative

Can be performed even if A is non-square

Multiplicity

Calculating the eigenvectors & eigenvalues of a matrix A requires determining its characteristic polynomial: $\det(\lambda I - A)$

The multiplicity of some eigenvalue λ^* refers to the number of times λ^* appears as the root of $\det(\lambda I - A) = 0$

E.g. $\det(\lambda I - A) = \lambda^2(\lambda - 3)^1 \Rightarrow \underbrace{\lambda_1 = 0}_{\text{multiplicity} = 2}, \ \underbrace{\lambda_2 = 3}_{\text{multiplicity} = 1}$

$\det(\lambda I - A) = (\lambda - 4)^4 (\lambda + 1)^3 \Rightarrow \underbrace{\lambda_1 = 4}_{\text{multiplicity} = 4}, \ \underbrace{\lambda_2 = -1}_{\text{multiplicity} = 3}$

Reading List: Rayleigh Principle