DCIT 212
Index No: 1084 1867

Session 1 - Assignment

$$\begin{bmatrix} 1 & 1 & 3 \\ 5 & 3 & 1 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$$

1. Cramer's Rule

~~let detem~~ let the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 5 & 3 & 1 \\ 2 & 3 & 1 \end{bmatrix} = A$

~~det~~

$$\det(A) = 1\begin{vmatrix} 3 & 1 \\ 3 & 1 \end{vmatrix} - 1\begin{vmatrix} 5 & 1 \\ 2 & 1 \end{vmatrix} + 3\begin{vmatrix} 5 & 3 \\ 2 & 3 \end{vmatrix}$$

$$= (3-3) - (5-2) + 3(15-6)$$

$$= 0 - 3 + 27$$

$$= 24$$

$$A_x : \begin{bmatrix} 2 & 1 & 3 \\ 3 & 3 & 1 \\ -1 & 3 & 1 \end{bmatrix}$$

$$\det(A_x) = 2\begin{vmatrix} 3 & 1 \\ 3 & 1 \end{vmatrix} - 1\begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix} + 3\begin{vmatrix} 3 & 3 \\ -1 & 3 \end{vmatrix}$$

$$= 2(3-3) - (3+1) + 3(9+3)$$

$$= 0 - 4 + 36$$

$$= 32$$

$$A_y : \begin{bmatrix} 1 & 2 & 3 \\ 5 & 3 & 1 \\ 2 & -1 & 1 \end{bmatrix}$$

$$\det(A_y) = 1\begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix} - 2\begin{vmatrix} 5 & 1 \\ 2 & 1 \end{vmatrix} + 3\begin{vmatrix} 5 & 3 \\ 2 & -1 \end{vmatrix}$$

$$= (3+1) - 2(5-2) + 3(-5-6)$$

$$\det(A_y) = 4 - 6 - 33$$
$$= -35$$

$$A_z : \begin{bmatrix} 1 & 1 & 2 \\ 5 & 3 & 3 \\ 2 & 3 & -1 \end{bmatrix}$$

$$\det(A_z) = 1\begin{vmatrix} 3 & 3 \\ 3 & -1 \end{vmatrix} - 1\begin{vmatrix} 5 & 3 \\ 2 & -1 \end{vmatrix} + 2\begin{vmatrix} 5 & 3 \\ 2 & 3 \end{vmatrix}$$
$$= (-3-9) - (-5-6) + 2(15-6)$$
$$= -12 + 11 + 18$$
$$= 17$$

$$x = \frac{A_x}{A} = \frac{32}{24}$$
$$= 1\frac{1}{3} \approx 1.333$$

$$y = \frac{A_y}{A} = -\frac{35}{24}$$
$$\approx -1.458333$$

$$z = \frac{17}{24} \approx 0.708333$$

Solution : $\left(\frac{32}{24}, -\frac{35}{24}, \frac{17}{24}\right)$
$\simeq (1.33, -1.46, 0.71)$

2. Gauss elimination without pivoting

$$\begin{bmatrix} 1 & 1 & 3 \\ 5 & 3 & 1 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$$

$R_2 = R_2 - 5R_1$ $\quad$ $R_3 = R_3 - 2R_1$

$= 5 - 5(1)$ $\quad$ $= 2 - 2(1)$

$= 0$ $\quad$ $= 0$

~~and $= 3 - 5(3) \Rightarrow$ Fo.~~

$\Rightarrow 3 - 5(1)$ $\quad$ $\Rightarrow 3 - 2(1)$

$= -12 - 2$ $\quad$ $= -3$ 1

$\Rightarrow 1 - 5(3)$ $\quad$ $\Rightarrow 1 - 2(3)$

$= -14$ $\quad$ $= -5$

$\Rightarrow 3 - 5(2)$ $\quad$ $\Rightarrow -1 - 2(2)$

$= -7$ $\quad$ $= -5$

Hence $\begin{bmatrix} 1 & 1 & 3 \\ 0 & -2 & -14 \\ 0 & 1 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \\ -5 \end{bmatrix}$

$R_2 = R_2 / -2$

$\Rightarrow \frac{-2}{-2} = 1$

$\Rightarrow \frac{-14}{-2} = 7$

$\Rightarrow \frac{-7}{-2} = \frac{7}{2} = 3.5$

Hence $\begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 7 \\ 0 & 1 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3.5 \\ -5 \end{bmatrix}$

$R_3 = R_3 - R_2$

$\Rightarrow 1 - 1$

$= 0$

$\Rightarrow -5 - 7$

$= -12$

$\Rightarrow -5 - 3.5$

$= -8.5$

Hence $\begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 7 \\ 0 & 0 & -12 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3.5 \\ -8.5 \end{bmatrix}$

$0x + 0y + -12z = -8.5$

$z = \frac{-8.5}{-12} = 0.71$

$0x + y + 7z = 3.5$

$y + 7(0.71) = 3.5$

$y = 3.5 - 7(0.71)$

$y = -1.47$

$x + y + 3z = 2$

$x + -1.47 + 3(0.71) = 2$

$x = 2 + 1.47 - 3(0.71)$

$x = 1.34 \therefore$ solution $(1.34, -1.47, 0.71)$

3- Gauss-Jordan elimination

$$\begin{bmatrix} 1 & 1 & 3 \\ 5 & 3 & 1 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$$

$R_2 \to -5r_1 + r_2 \Rightarrow$

$-5(1)+5=0$

$-5(1)+3=-2$

$-5(2)+3=-7$

$-5(3)+1=-14$

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & -2 & -14 & -7 \\ 0 & 1 & -5 & -5 \end{array}\right]$$

$R_3 \to r_2 + 2r_3 \Rightarrow$

$\Rightarrow -2+2(1)=0$

$-14+2(-5)=-24$

$-7+2(-5)=-17$

$R_2 = r_2/-12$

$R_3 = r_3/-24 \to$

$R_3 \to -2r_1 + r_3$

$\Rightarrow -2(1)+2=0$

$-2(1)+3=1$

$-2(3)+1=-5$

$-2(2)-1=-5$

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & -2 & -14 & -7 \\ 0 & 0 & -24 & -17 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & 1 & 7 & 7/2 \\ 0 & 0 & 1 & 17/24 \end{array}\right]$$

$0x_1 + 0x_2 + x_3 = 17/24$

$x_3 = 17/24 \simeq 0.71$

$x_2 + 7x_3 = 7/2$

$x_2 = \frac{7}{2} - 7(17/24)$

$x_2 = -1.46 = -\frac{35}{24}$

$$x_1 + x_2 + 3x_3 = 2$$

$$x_1 - \frac{35}{24} + 3\left(\frac{17}{24}\right) = 2$$

$$x_1 = 2 + \frac{35}{24} - \frac{3 \times 17}{24}$$

$$x_1 = \frac{4}{3} \simeq 1.33$$

Solution: $(1.33, -1.46, 0.71)$

4. $\begin{bmatrix} 1 & 1 & 3 \\ 5 & 3 & 1 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$

$$Ax = B$$

let $A = LU$

$\therefore LUx = B$

let $Ux = Y$

$\therefore LY = B$

~~SP~~ $R_2 \to -5r_1 + r_2$

$-5(1) + 5 = 0$

$-5(1) + 3 = -2$

$-5(3) + 1 = -14$

$-5(2) + 3 = -7$

$R_3 \to -2r_1 + r_3$

$-2(1) + 2 = 0$

$-2(1) + 3 = 1$

$-2(3) + 1 = -5$

$R_3 \to \frac{1}{2}$

$$\to \begin{bmatrix} 1 & 1 & 3 \\ 0 & -2 & -14 \\ 2 & 3 & 1 \end{bmatrix} \begin{matrix} \\ -7 \\ -1 \end{matrix}$$

$$\to \begin{bmatrix} 1 & 1 & 3 \\ 0 & -2 & -14 \\ 0 & 1 & 5 \end{bmatrix}$$

$R_3 \rightarrow \frac{1}{2}r_2 + r_3$

$\frac{1}{2}(-2) + 1 = 0.$

$\frac{1}{2}(-14) + (-5) = -12$

$$\rightarrow \begin{bmatrix} 1 & 1 & 3 \\ 0 & -2 & -14 \\ 0 & 0 & -12 \end{bmatrix} = U$$

Using the opposite of the multiples

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ 2 & -\frac{1}{2} & 1 \end{bmatrix}$$

$LY = B$

$$\begin{bmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ 2 & -0.5 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$$

$y_1 = 2$

$5y_1 + y_2 = 3$

$5(2) + y_2 = 3$

$y_2 = 3 - 10$

$y_2 = -7$

$2y_1 - 0.5y_2 + y_3 = -1$

$2(2) - 0.5(-7) + y_3 = -1$

$y_3 = -1 - 2(2) + 0.5(-7)$

$y_3 = -8.5$

$$\rightarrow Y = \begin{bmatrix} 2 \\ -7 \\ -8.5 \end{bmatrix}$$

$UX = Y$

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & -2 & -14 \\ 0 & 0 & -12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \\ -8.5 \end{bmatrix}$$

$-12x_3 = -8.5$

$x_3 = 0.708 = \frac{17}{24}$

$-2x_1 - 14x_2 + 3x_3 = 7$

$-2x_1 - 14(5/14) + 3x_3 = 7$

$-2x_1 - 5 + 3x_3 = 7$

$-2x_1 + 3x_3 = 7 + 5$

$-2x_1 + 3x_3 = 12$

$x_1 = \frac{-3 + 12}{-2}$

$x_1 = \frac{-3 + 12}{-2}$

$x_1 = -\frac{33}{24} \approx -1.46$

$x_1 + x_2 + 3x_3 = 2$

$x_1 = -1.46$

$x_1 = -\frac{35}{24} + 3(17/24) = 2$

$x_1 = \frac{33}{24} \approx 1.37$

Solution: $(1.33, -1.46, 0.71)$

$R_2 \to r_2/0.6$

$0.6/0.6 = 1$

$2/0.6 = 10/3$

$3.2/0.6 = 16/3$

$\begin{bmatrix} 1 & 2/3 & 0 & 0.6 & 11/3 \\ 0 & 1 & 1.2 & 0 & 5.6 \\ 0 & 0 & 1 & 1/2 & 14/3 \\ 0 & 0 & 2 & 3 & 7 \end{bmatrix}$

$R_4 \to -2r_3 + r_4$

$-2(1) + 2 = 0$

$-2(10/3) + 3 = -11/3$

$-2(14/3) + 7 = -1/3$

$\begin{bmatrix} 1 & 2/3 & 0.6 & 11/3 \\ 0 & 1 & 1.2 & 5.6 \\ 0 & 0 & 1 & 1/2 \\ 0 & 0 & 0 & -1/3 \end{bmatrix}$

From this

$-1/3 x_4 = -11/3$

$x_4 = 1$

Session 2 - Assignment

$$\begin{bmatrix} 3 & 2 & 0 & 0 \\ 2 & 3 & 2 & 0 \\ 0 & 2 & 3 & 2 \\ 0 & 0 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 12 \\ 17 \\ 14 \\ 7 \end{bmatrix}$$

1. Thomas algorithm

$R_1 \to r_1/3$

$3/3 = 1$

$2/3$

$12/3$

$$\to \left[\begin{array}{cccc|c} 1 & 2/3 & 0 & 0 & 12/3 \\ 2 & 3 & 2 & 0 & 17 \\ 0 & 2 & 3 & 2 & 14 \\ 0 & 0 & 2 & 3 & 7 \end{array}\right]$$

$R_2 \to -2r_1 + r_2$

$-2(1)+2 = 0$

$-2(2/3)+3 = 5/3$

$-2(12/3)+17 = 9$

$$\to \left[\begin{array}{cccc|c} 1 & 2/3 & 0 & 0 & 12/3 \\ 0 & 5/3 & 2 & 0 & 9 \\ 0 & 2 & 3 & 2 & 14 \\ 0 & 0 & 2 & 3 & 7 \end{array}\right]$$

$R_2 \to r_2 / 5/3$

$5/3 \div 5/3 = 1$

$2 \div 5/3 = 1.2$

$9 \div 5/3 = 5.4$

$$\to \left[\begin{array}{cccc|c} 1 & 2/3 & 0 & 0 & 12/3 \\ 0 & 1 & 1.2 & 0 & 5.4 \\ 0 & 2 & 3 & 2 & 14 \\ 0 & 0 & 2 & 3 & 7 \end{array}\right]$$

$R_3 \to -2r_2 + r_3$

$-2(1)+2 = 0$

$-2(1.2)+3 = 0.6$

$-2(5.4)+14 = 3.2$

$$\to \left[\begin{array}{cccc|c} 1 & 2/3 & 0 & 0 & 12/3 \\ 0 & 1 & 1.2 & 0 & 5.4 \\ 0 & 0 & 0.6 & 2 & 3.2 \\ 0 & 0 & 2 & 3 & 7 \end{array}\right]$$

$$x_3 + {}^{10}/_3 x_4 = {}^{16}/_3$$

$$x_3 + {}^{10}/_3(1) = {}^{16}/_3$$

$$x_3 = \frac{16}{3} - \frac{10}{3}$$

$$x_3 = 2$$

$$x_2 + 1.2x_3 = 5.4$$

$$x_2 + 1.2(2) = 5.4$$

$$x_2 = 5.4 - 1.2(2)$$

$$x_2 = 3$$

$$x_1 + {}^{2}/_3 x_2 = {}^{12}/_3$$

$$x_1 + {}^{2}/_3(3) = {}^{12}/_3$$

$$x_1 = \frac{12}{3} - \frac{2}{3}(3)$$

$$x_1 = 2$$

Solution (2, 3, 2, 1)