

Order Statistics Simulations

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Order Statistics Simulations

```
# Simulating function
sim_orders <- function(N, iter, sample_fn, ...){
  order_stats <- function(N, ...){
    x <- sample_fn(N, ...)
    data.frame(value = sort(x), order = (1:length(x))) # Get order statistics
  }
  map_dfr(rep(N, iter), order_stats, ...)
}

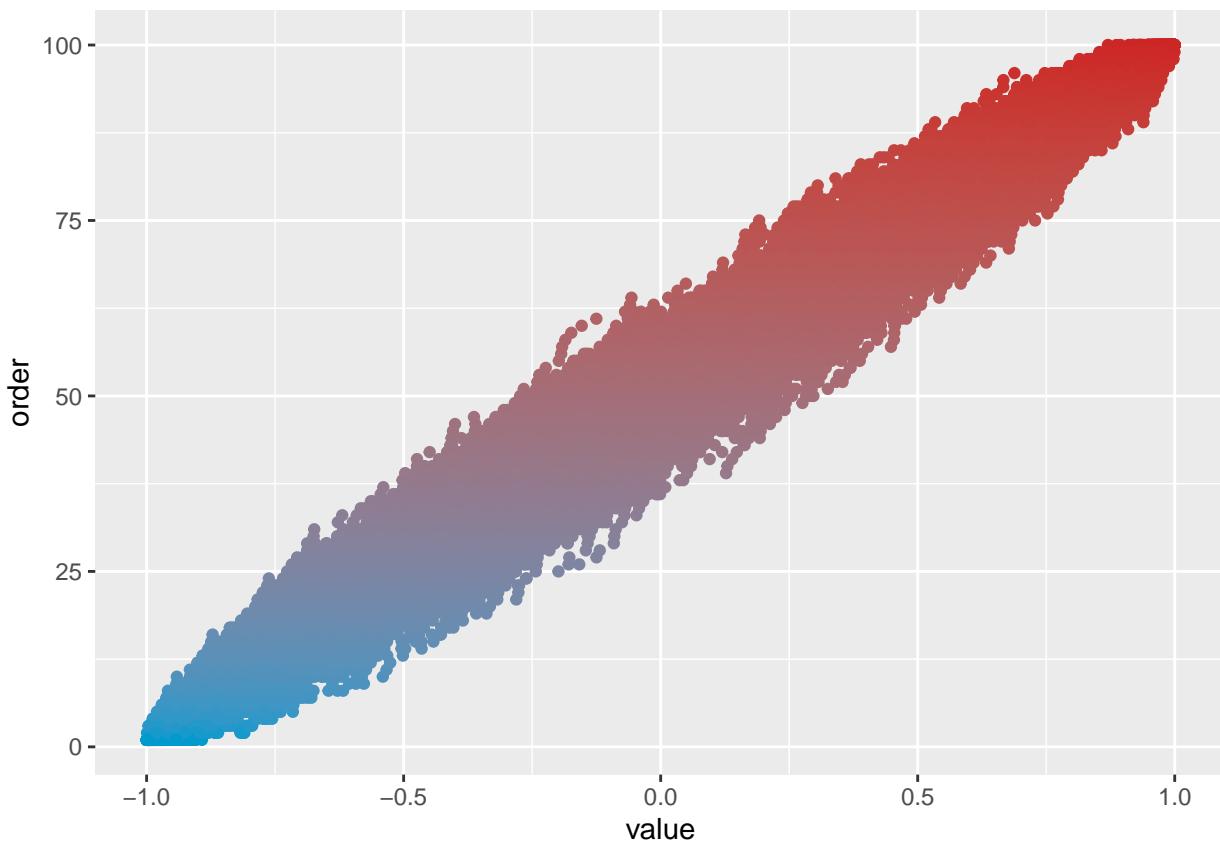
# Simulating function for complex versions
sim_orders_cplx <- function(N, iter, sample_fn, ...){
  order_stats <- function(N, ...){
    x <- abs(sample_fn(N, ...)) + 1i * sample_fn(N, ...) # Norm of complex version
    data.frame(value = sort(x), order = (1:length(x))) # Get order statistics
  }
  map_dfr(rep(N, iter), order_stats, ...)
}

# Plot a data frame of order data statistics
orders_plot <- function(orders, N){
  # Plotting parameters
  #order_cols <- scales::seq_gradient_pal("blue", "red", "Lab")(seq(0, 1, length.out = N))
  # Plot
  orders %>%
    ggplot(aes(value, order, color = order)) +
    geom_point(show.legend = FALSE) +
    #scale_y_continuous(breaks = 1:N) +
    scale_color_gradient(low = "deepskyblue3", high = "firebrick3")
}

# Simulation parameters
iter <- 500
N <- 100
```

Uniform Distribution Order Statistics

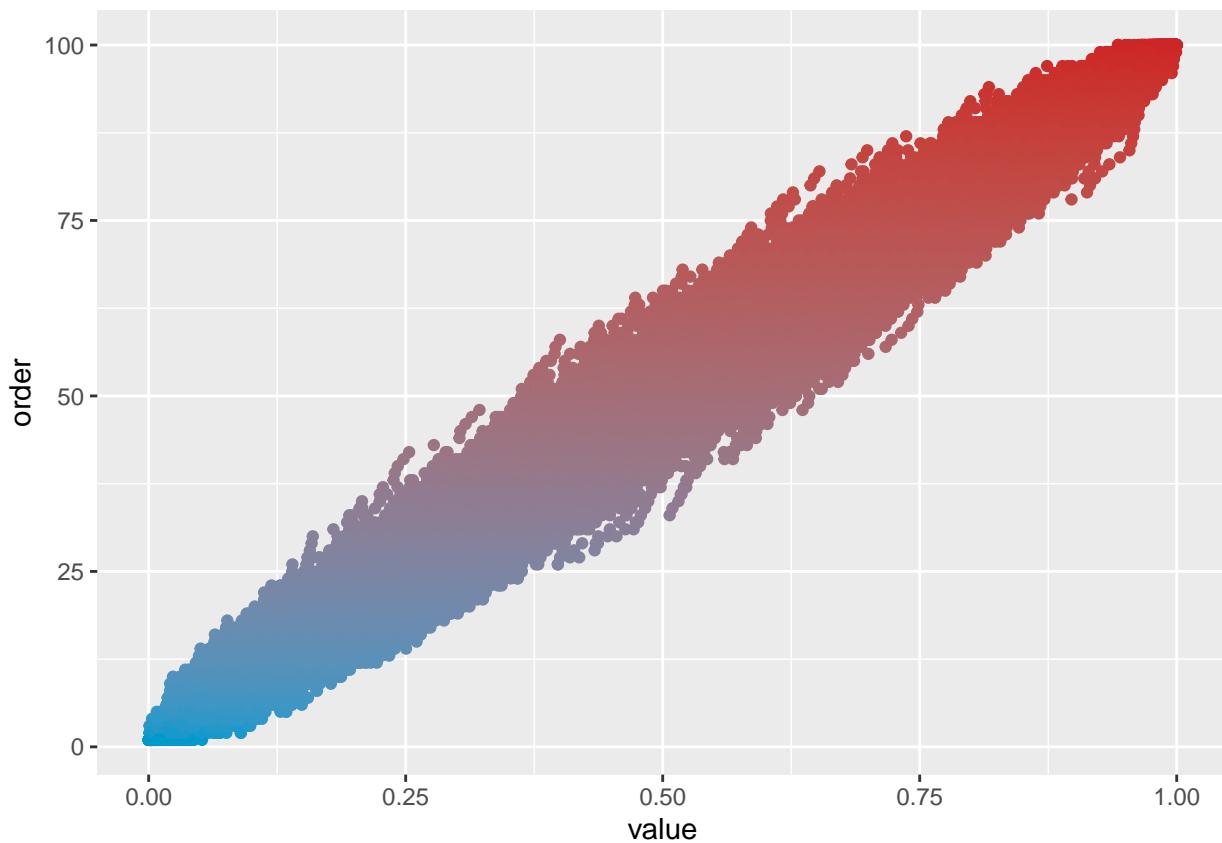
```
# Simulate uniformly distributed order statistics
sim_orders(N, iter, sample_fn = runif, min = -1, max = 1) %>% orders_plot(N)
```



Norm Version

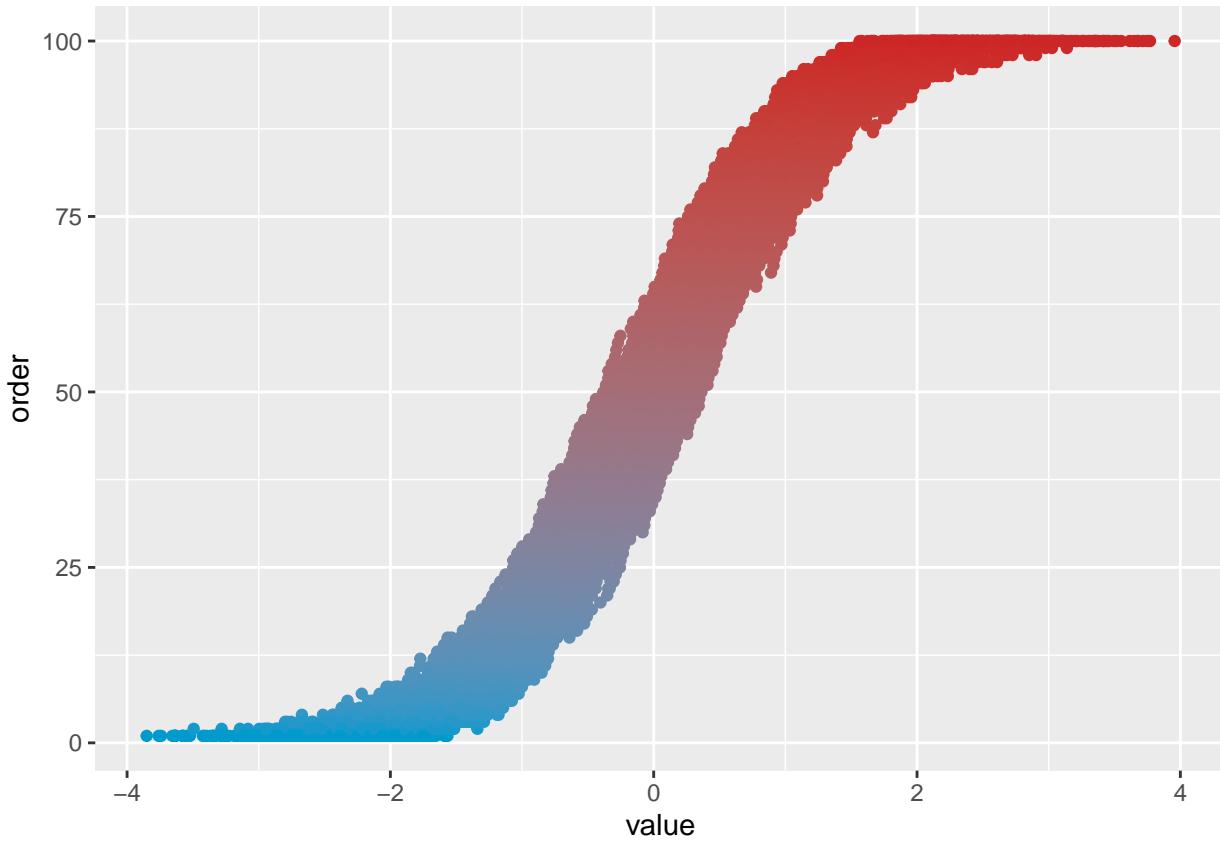
```
# Define norm of random uniform distribution function
NORM_runif <- function(n, min, max){abs(runif(n, min, max))}

# Simulate uniformly distributed order statistics
sim_orders(N, iter, sample_fn = NORM_runif, min = -1, max = 1) %>% orders_plot(N)
```



Normal Distribution Order Statistics

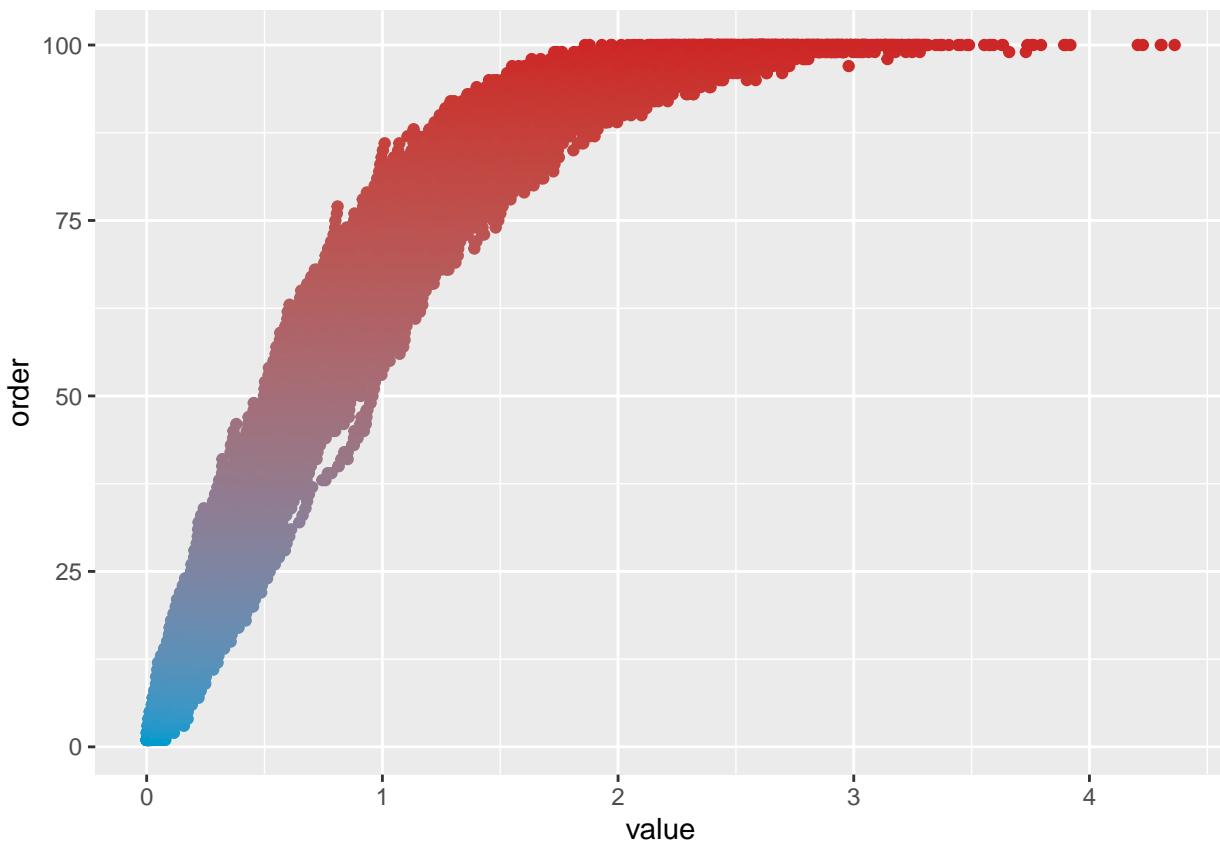
```
# Simulate normally distributed order statistics
sim_orders(N, iter, sample_fn = rnorm) %>% orders_plot(N)
```



Norm Version

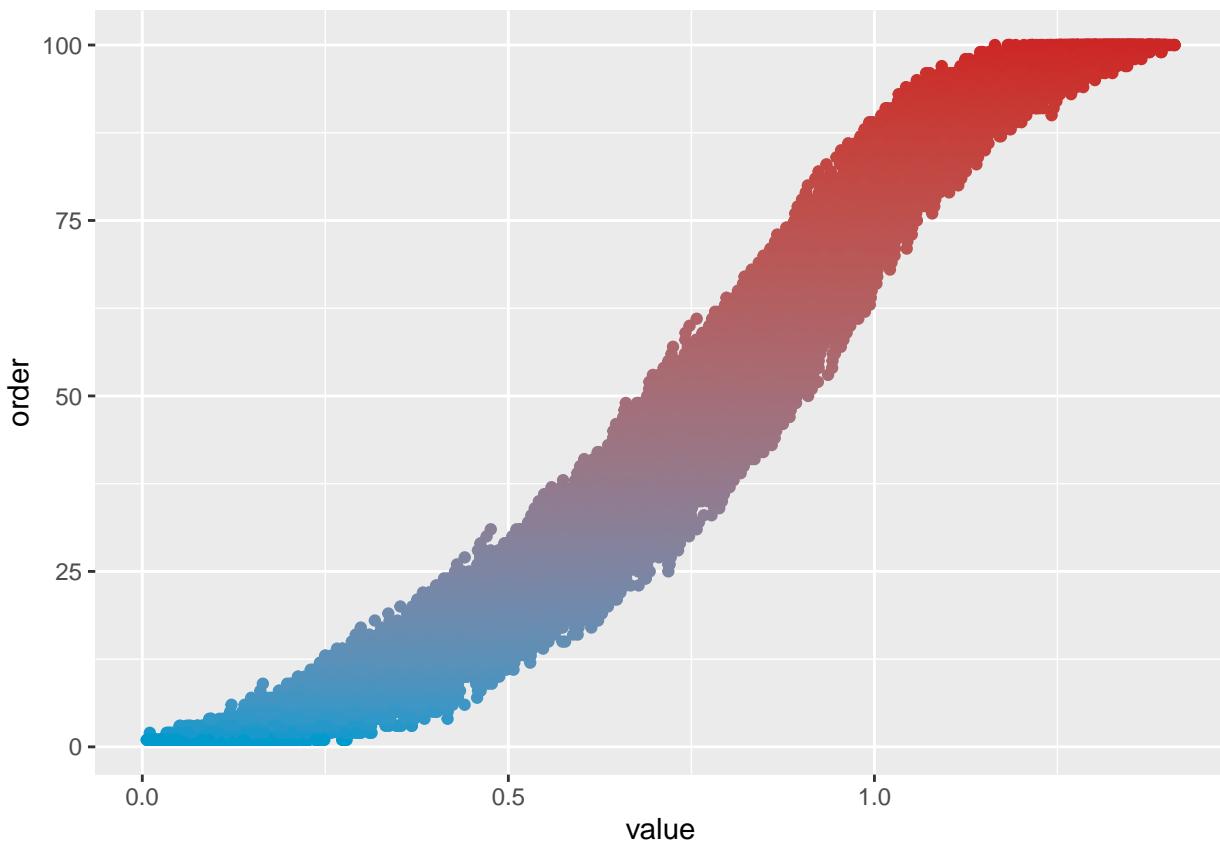
```
# Define norm of random uniform distribution function
NORM_rnorm <- function(n, ...){abs(rnorm(n, ...))}

# Simulate normally distributed order statistics
sim_orders(N, iter, sample_fn = NORM_rnorm) %>% orders_plot(N)
```



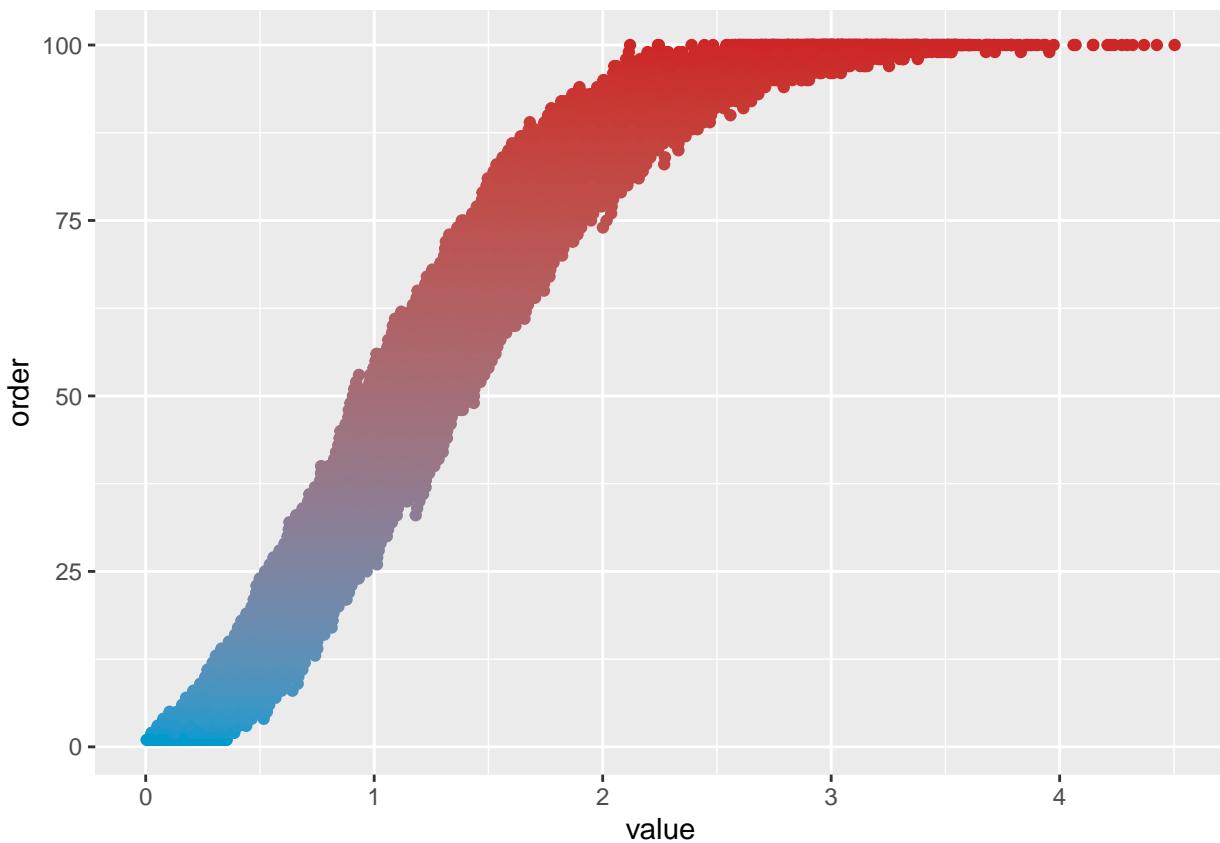
Norm of Uniformly Distributed Complex Number Order Statistics

```
# Simulate
sim_orders_cplx(N, iter, runif, min = -1, max = 1) %>% orders_plot(N)
```



Norm of Normally Distributed Complex Number Order Statistics

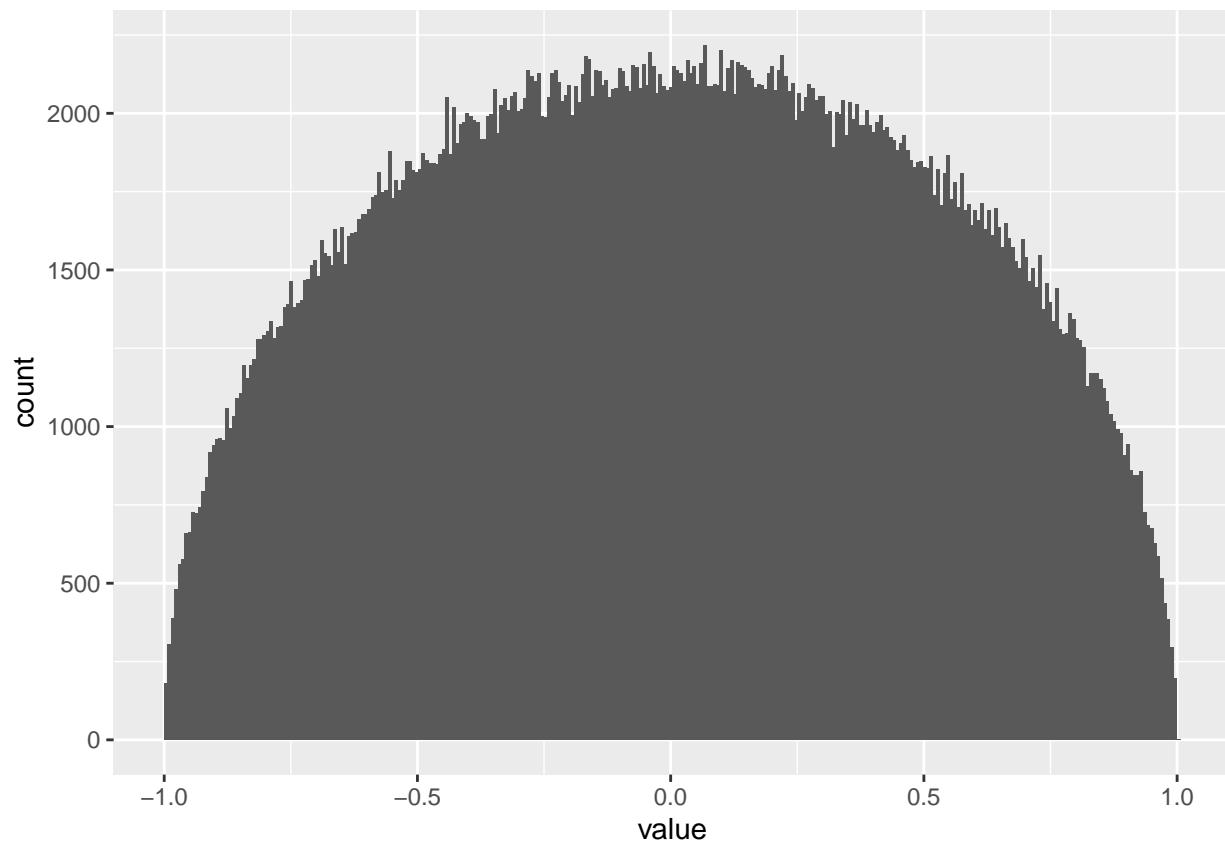
```
# Simulate
sim_orders_cplx(N, iter, rnorm, mean = 0, sd = 1) %>% orders_plot(N)
```



```
# Source code for simulating from the semicircle distribution
source("semicircle.R")
```

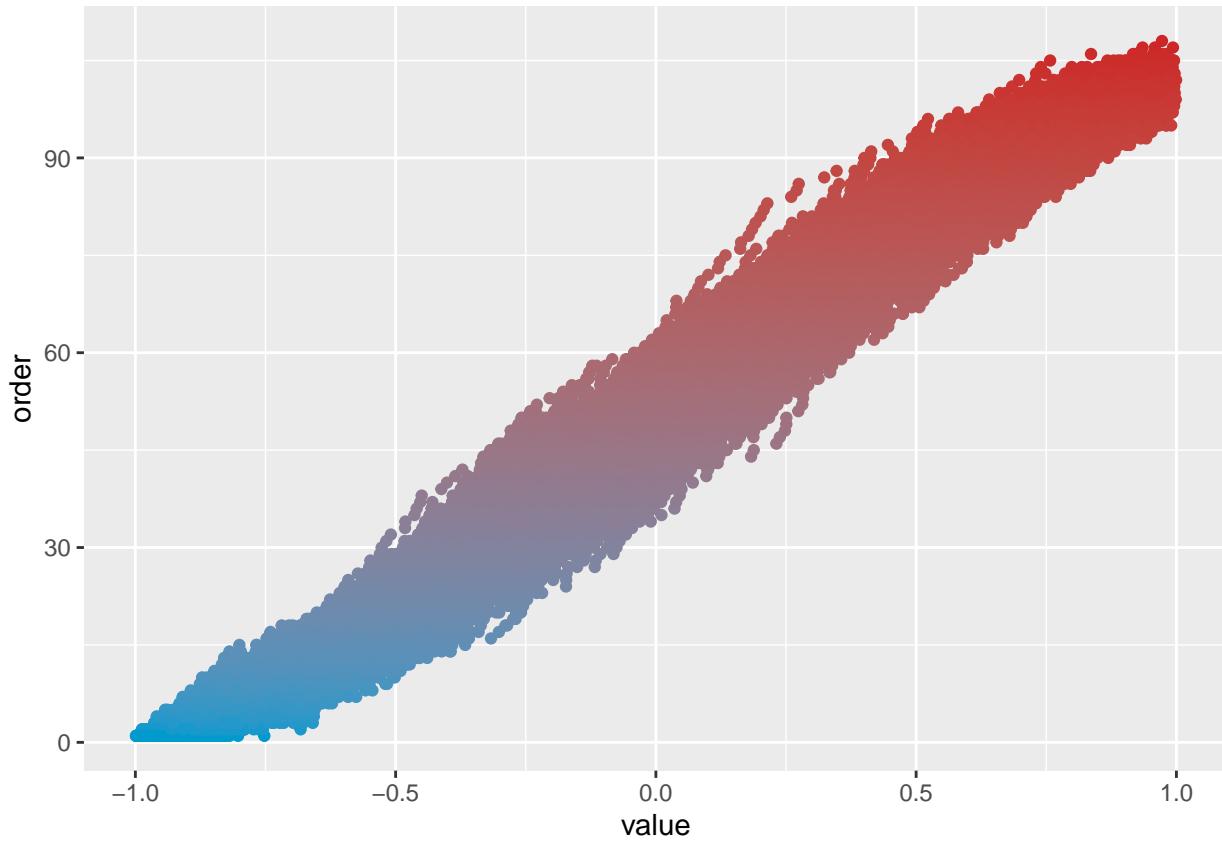
The Semicircle Distribution

```
sample <- data.frame(value = sample_semicircle(n = 500000, R = 1))
sample %>%
  ggplot() +
  geom_histogram(aes(x = value), bins = 300)
```



Semicircle Distribution Order Statistics

```
# Simulate
sim_orders(N, iter, sample_semicircle, R = 1) %>% orders_plot(N)
```



Norm of Semi-Circle Distributed Complex Number Order Statistics

```
# Simulate
sim_orders_cplx(N, iter, sample_semicircle, R = 1) %>% orders_plot(N)
```

