# Eigenvectors of Symmetric Matrices

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## Computational Evidence: Real Symmetric Matrices have Real Eigenvectors

In this document, we hope to show that given any arbitrary element of the set of $M \times M$ symmetric matrices, denote it $S \in \mathcal{SM}_{\mathbb{R}}[M \times M]$ has a set of real eigenvectors $[\lambda_i] \in \mathbb{R}^M$.

To simulate a generic element $S$, we use the following method:

(1) First, pick some $f \in [0, 1]$, letting it denote the fraction of positive entries of $S$. That is;

$$\text{Want: } f \approx \frac{|\{s_{ij} > 0\}|}{M^2}$$

We hope to show that our condition is invariant to the value of $f$, since there is the possibility that the sign proportions of our matrix $S$ influences the $\det(S)$.

(2) To simulate a symmetric matrix $S$ with a fraction of positive entries $f$, we will sample from the distribution:

$$s_{ij} \sim \text{Unif}(f - 1, f)$$

(3) To not constrict the sizes of $|s_{ij}|$, we will add an $\epsilon$ term and scale our endpoints to preserve the fraction $f$.

$$s_{ij} \sim \text{Unif}(\epsilon(f - 1), \epsilon f)$$

(4) Having our uniform distribution, we will generate $M^2$ entries and insert them in the matrix $S$. Then, we delete the lower triangular matrix, then duplicate the entries from the upper triangle to the lower triangle.

(5) Now, if we let $f \sim \text{Unif}(0, 1)$ and let $\epsilon \to \infty$, we can well approximate $S \in \mathcal{SM}_{\mathbb{R}}[M \times M]$.

```r
RM_symm(5,0.5,10)
```

```
##            [,1]      [,2]       [,3]       [,4]       [,5]
## [1,]  4.2573266 -1.226771  3.0619321  1.3922001  1.7420834
## [2,] -1.2267710 -2.443325 -0.4383707 -2.5688747 -1.9790974
## [3,]  3.0619321 -2.568875  3.1935870  1.9974003  2.2200583
## [4,] -0.4383707  1.997400 -1.9790974  0.3239135 -3.1167550
## [5,]  1.3922001  1.742083  2.2200583 -3.1167550  0.1564236
```

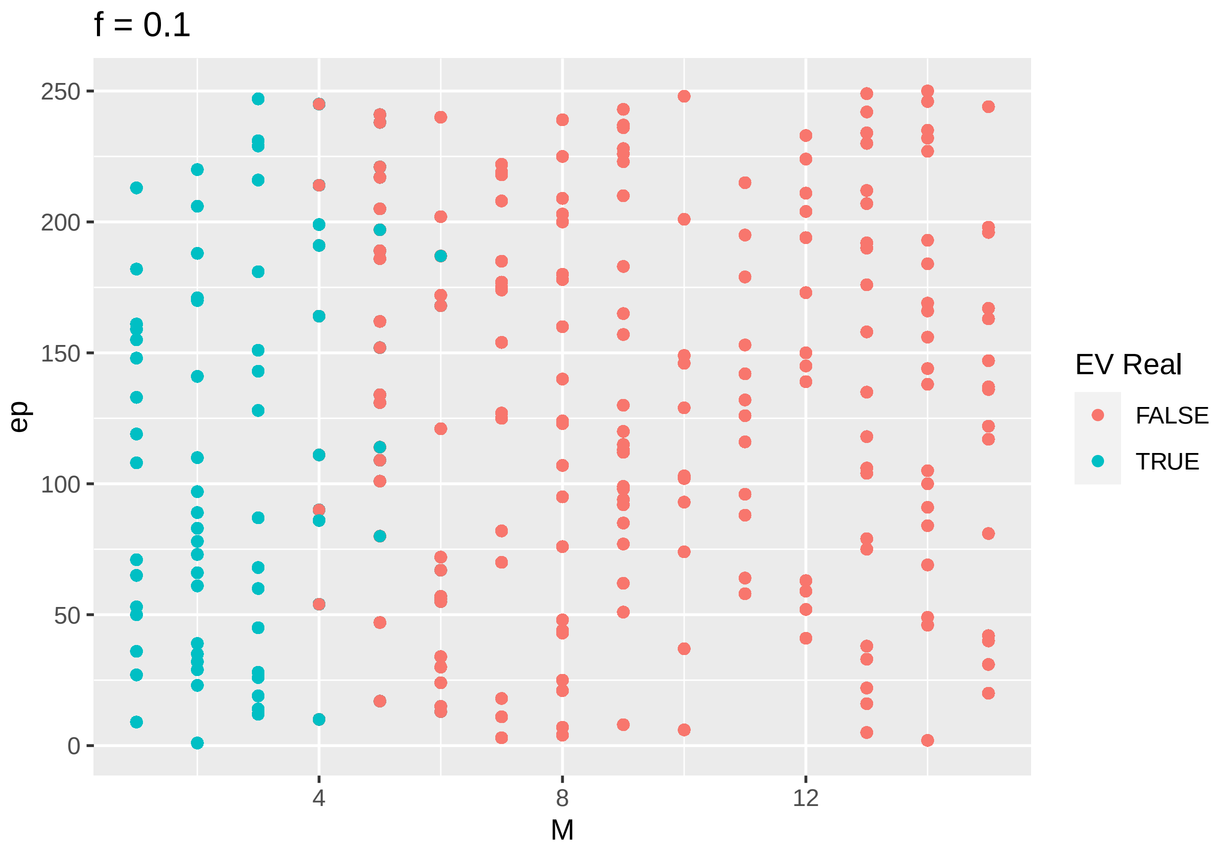
# Simulation

```r
simulate_by_f <- function(f,M_max,ep_max,draws){
  M_vec <- sample(1:M_max, draws, replace = T)
  ep_vec <- sample(1:ep_max, draws, replace = F)
  table <- data.frame(M = M_vec, ep = rep(ep_vec,length(M_vec)))

  bool_vec <- rep(NA, length(table$M))

  for(i in 1:length(table$M)){
    S_curr <- RM_symm(table$M[i],f,table$ep[i])
    bool_vec[i] <- check_real_eigenvectors(eigen_frame(S_curr))
  }
  cbind(table,bool_vec)
}

plot_f_table <- function(table, f){
  ggplot() +
    geom_point(data = table, aes(x=M, y=ep, color = factor(bool_vec))) +
    labs(color = "EV Real", title = paste("f = ",f,sep=""))
}
```

```r
table <- simulate_by_f(f = 0.1, M_max = 15, ep_max = 250, draws = 250)
head(table)
```

```
##    M  ep bool_vec
## 1  1  27     TRUE
## 2 14  49    FALSE
## 3 14 232    FALSE
## 4  9 183    FALSE
## 5  6  55    FALSE
## 6  8  44    FALSE
```

```r
plot_f_table(table, f = 0.1)
```

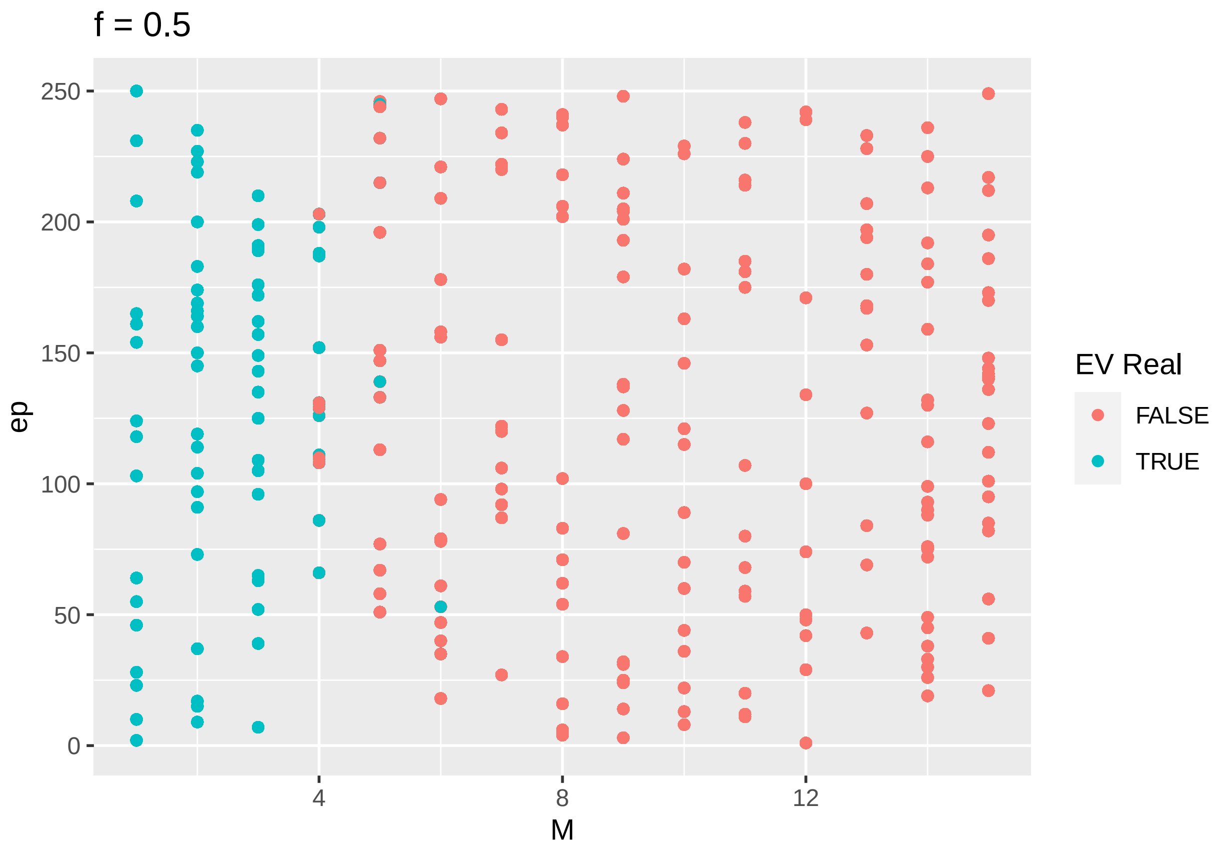



}

```
table <- simulate_by_f(f = 0.5, M_max = 15, ep_max = 250, draws = 250)
head(table)
```

```
##    M  ep bool_vec
## 1 11  11    FALSE
## 2  3 189     TRUE
## 3  8 241    FALSE
## 4  5 151     TRUE
## 5 12   1    FALSE
## 6 10  44    FALSE
```

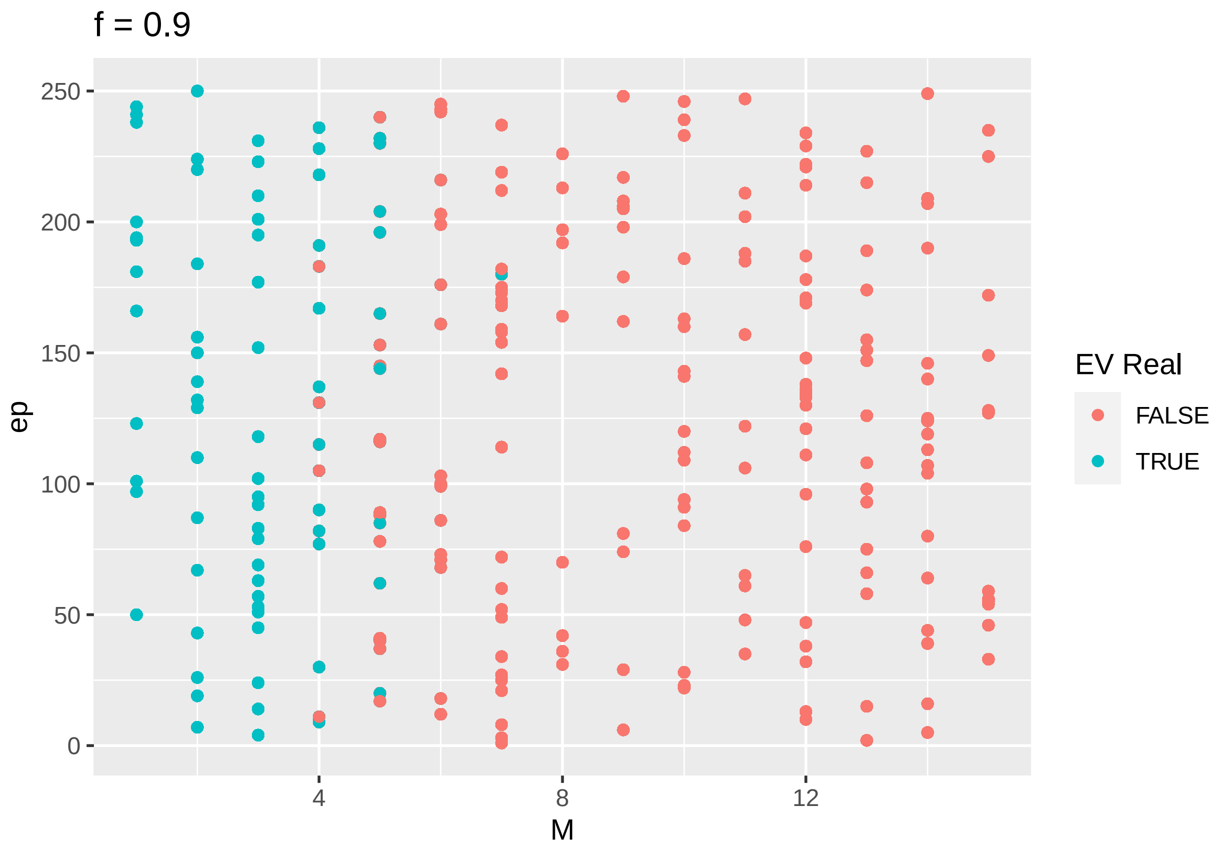
```
plot_f_table(table, f = 0.5)
```




```
table <- simulate_by_f(f = 0.9, M_max = 15, ep_max = 250, draws = 250)
head(table)
```

```
##    M  ep bool_vec
## 1 14 209    FALSE
## 2 11 202    FALSE
## 3 13 155    FALSE
## 4 12 135    FALSE
## 5  7  27    FALSE
## 6  5  20    FALSE
```

```
plot_f_table(table, f = 0.9)
```



f = 0.9
250
200
150
100
50
0
ep
4
8
12
M
EV Real
FALSE
TRUE


“‘