Asymmetric Cryptography: A (Medium) Deep Dive

Eli Holderness — @eli.holderness.dev — they/them/theirs

Eli (pronounced /ˈiːlaɪ̯/) is a is a research software advocate, recovering mathematician, and audience participator.

They like people, the web, and learning weird facts about computers.

Agenda

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1. Brief history

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2. How RSA works

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3. How ECC works

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4. QC & Shor's Algorithms

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1. Brief history
2. How RSA works
3. How ECC works
4. QC & Shor’s Algorithms
5. What next?

1

A brief history of cryptography

Øredev is great!

A	B	C	D	E	F	G	H	I	J	K
G	H	I	J	K	L	M	N	O	P	Q

Uxkjkb oy mxkgz!

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Øredev is great!

	1	2	3	4	5	6	7	8	9	10	11
+6	7	8	9	10	11	12	13	14	15	16	17

Uxkjkb oy mxkgz!

ØREDEV

15 18 5 4 5 22

ØREDEV	15 18 5 4 5 22
+	+
SECRET	19 5 3 18 5 20

ØREDEV

+

SECRET

15 18 5 4 5 22

+

19 5 3 18 5 20

=

34 23 8 22 10 42

ØREDEV

+

SECRET

15 18 5 4 5 22

+

19 5 3 18 5 20

=

8 23 8 22 10 18

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ØREDEV	15 18 5 4 5 22
+	+
SECRET	19 5 3 18 5 20
=	=
HWHVJR	8 23 8 22 10 18

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symmetric cryptography requires both parties to know a specific secret asymmetric cryptography relies on mathematical solutions that are very expensive to compute asymmetric cryptography is largely an implementation detail which enables symmetric encryption

2

RSA & group theory

RSA cryptosystem

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security based on the difficulty of factoring large numbers N = pq where p, q prime

worked example with $N = 323 = 17 * 19$

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We need to know $\lambda(N)$, the smallest number where $a^{\lambda(N)} \equiv 1 \bmod N$ for every a coprime to N

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$\lambda(N) = 144$

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$$\lambda(N) = \mathrm{lcm}(\lambda(p), \lambda(q)) = \mathrm{lcm}(p-1, q-1) = \mathrm{lcm}(16, 18) = 144$$

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Choose e between 2 and N coprime to N; let's pick 5

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$\lambda(N) = 144$ $\quad e = 5;\ d = 29$

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Choose e between 2 and N coprime to N; let's pick 5

Find d such that $d * e \equiv 1 \bmod \lambda(N)$; this is 29

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$\lambda(N) = 144$ $e = 5; d = 29$

Our public key is $(N, e) = (323, 5)$ and our private key is $d = 29$

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To encrypt a number, they raise it to the power of $e = 5$:

$14^5, 4^5, 3^5 = 537824, 1024, 243$

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Then take the modulus of N:

$14^5, 4^5, 3^5 \equiv 29, 55, 243 \pmod{N}$

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We received the message (29, 55, 243)

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Decode by raising each number to the power of $d = 29$, then taking the modulus of N

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We received the message (29, 55, 243)

Decode by raising each number to the power of $d = 29$,
then taking the modulus of N

$29^{29}, 55^{29}, 243^{29} \equiv 14, 4, 3 \bmod N$

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$\lambda(\mathrm{N}) = 144$ $\quad e = 5;\ d = 29$ $\quad m = (29, 55, 243)$

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$$a^{\lambda(N)+1} = a^{145} = a^{5 \times 29} = (a^5)^{29}$$

So $(a^5)^{29} \equiv a \bmod N$ and we can recover the original message from the encrypted intermediate

limitations & considerations

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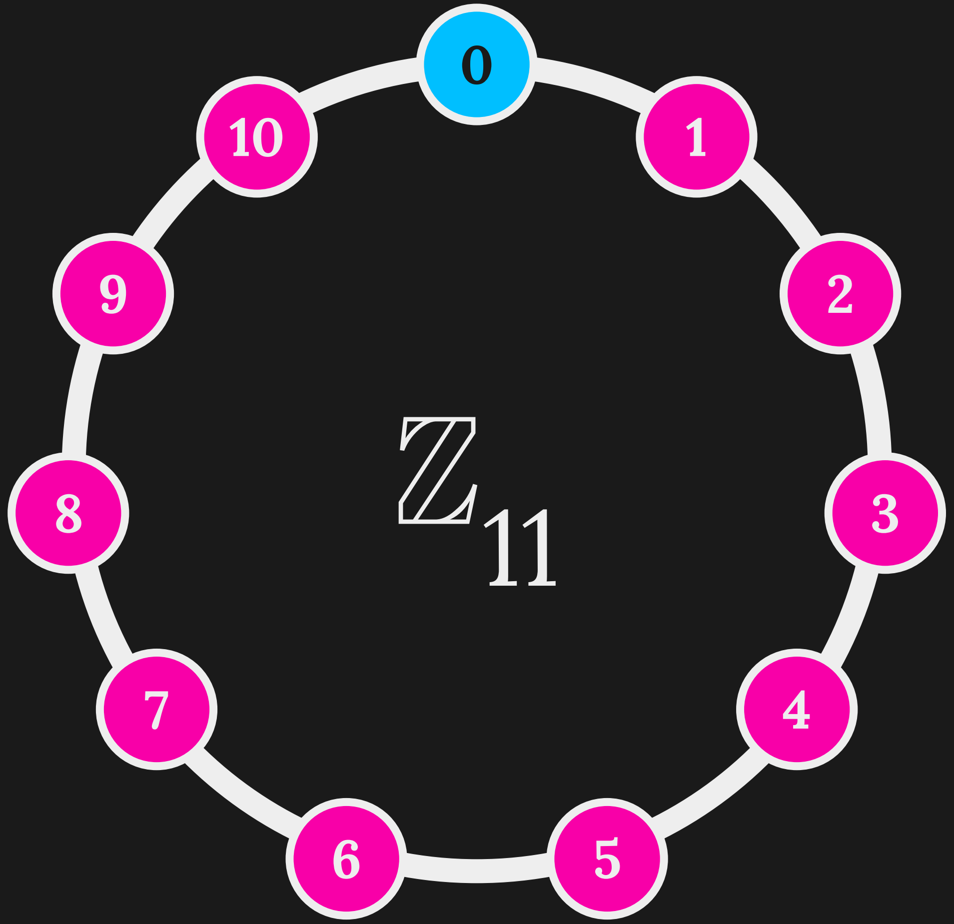
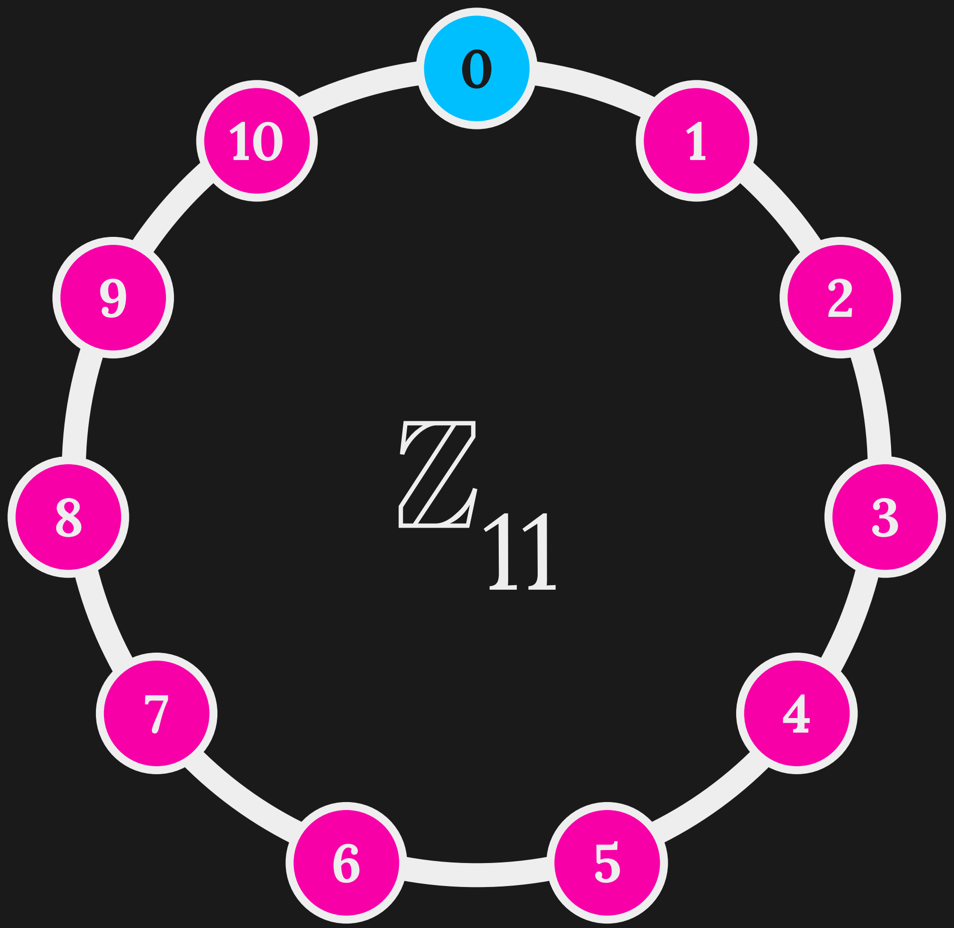
without padding, messages can be vulnerable to chosen plaintext attacks



0
1
2
3
4
5
6
7
8
9
10
$\mathbb{Z}_{11}$



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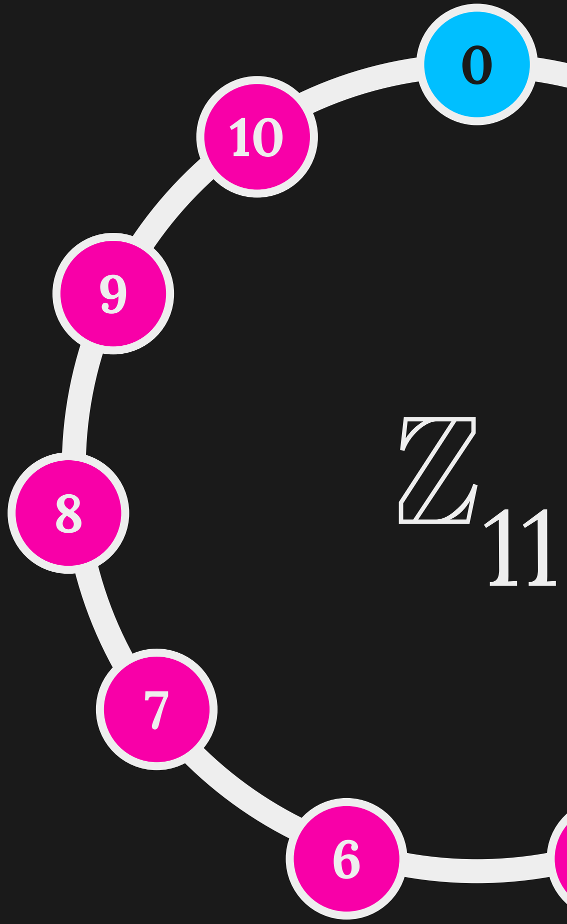

identity element

adding 0 doesn’t change an element

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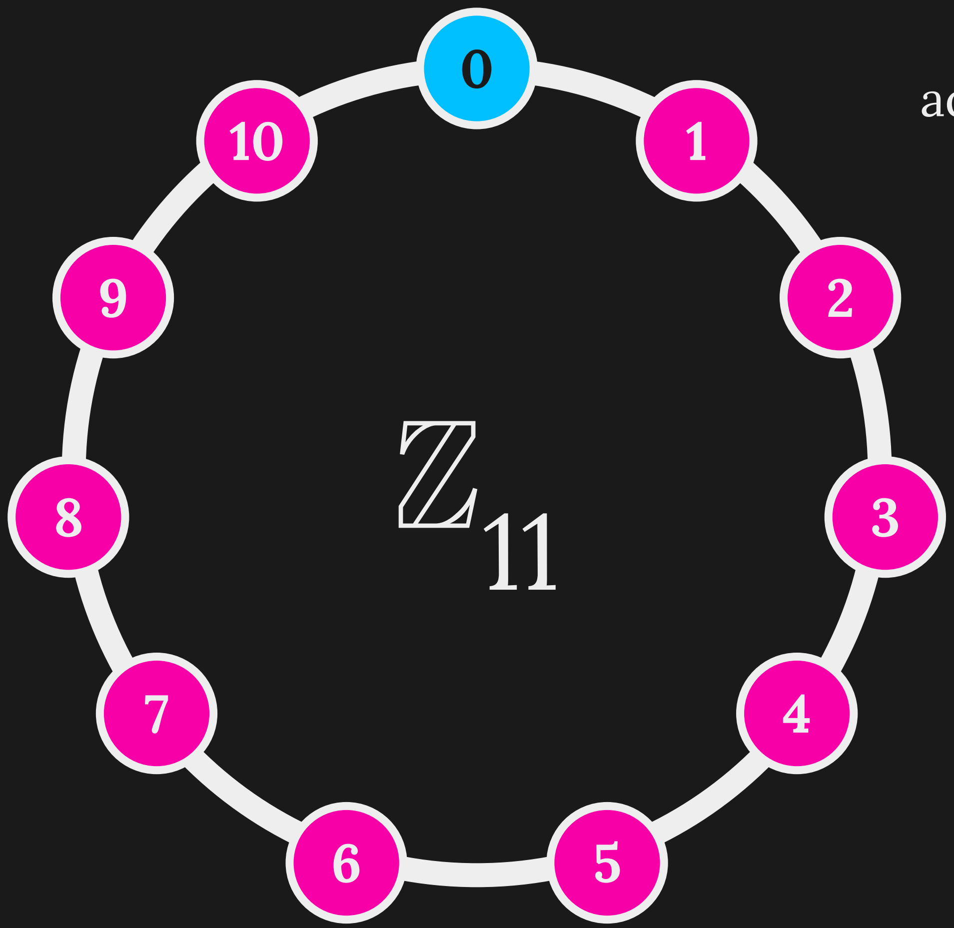

identity element

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inverses

for every a in the group, there's a b that makes $a + b = 0$ true

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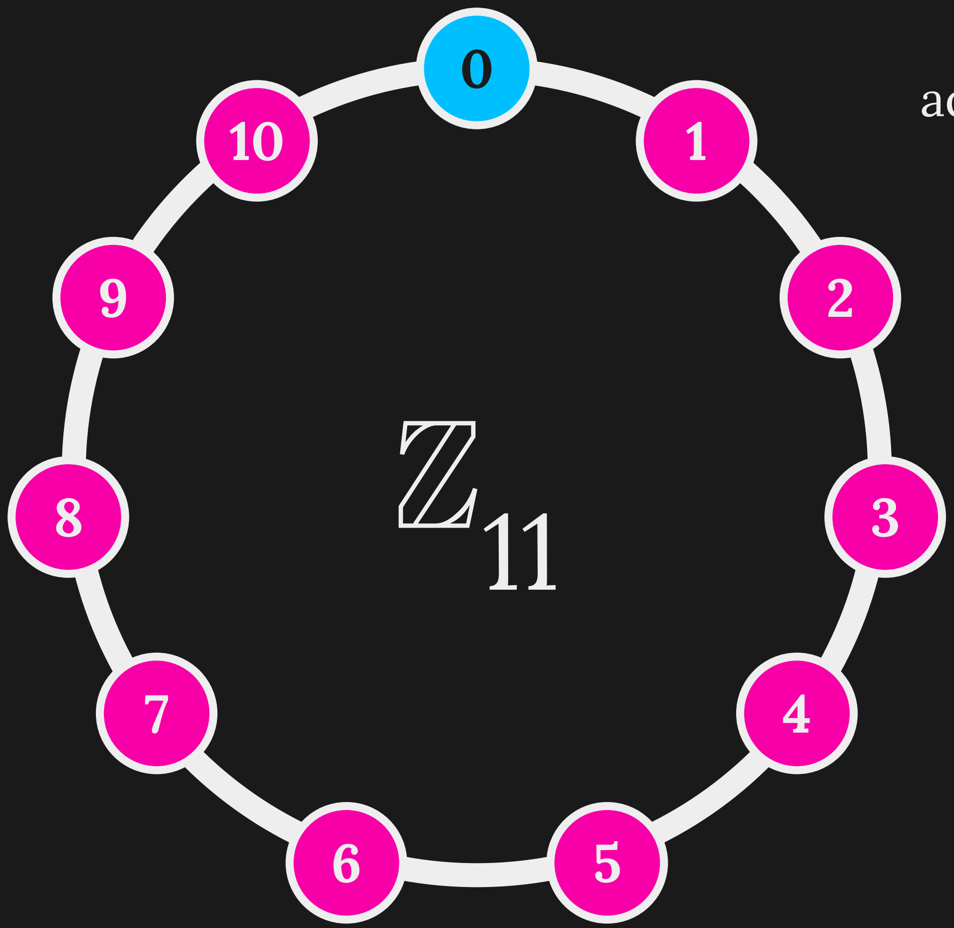

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associativity

$1 + (4 + 2) = (1 + 4) + 2$

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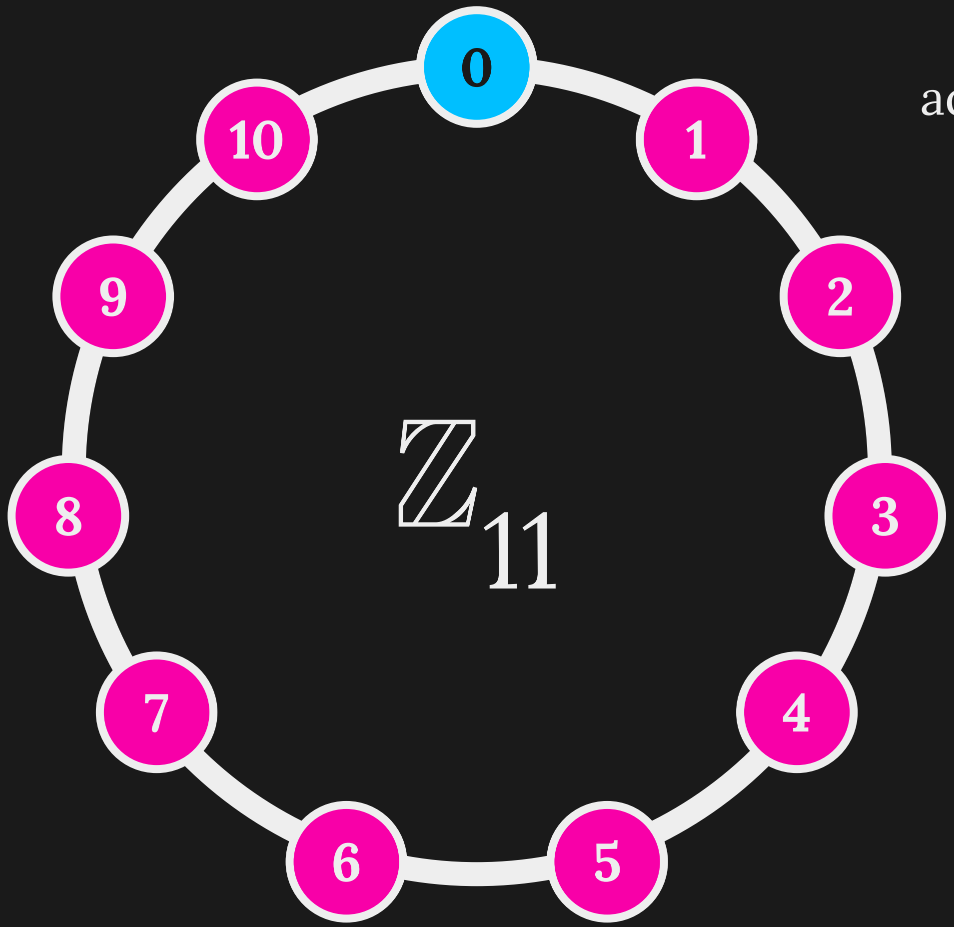

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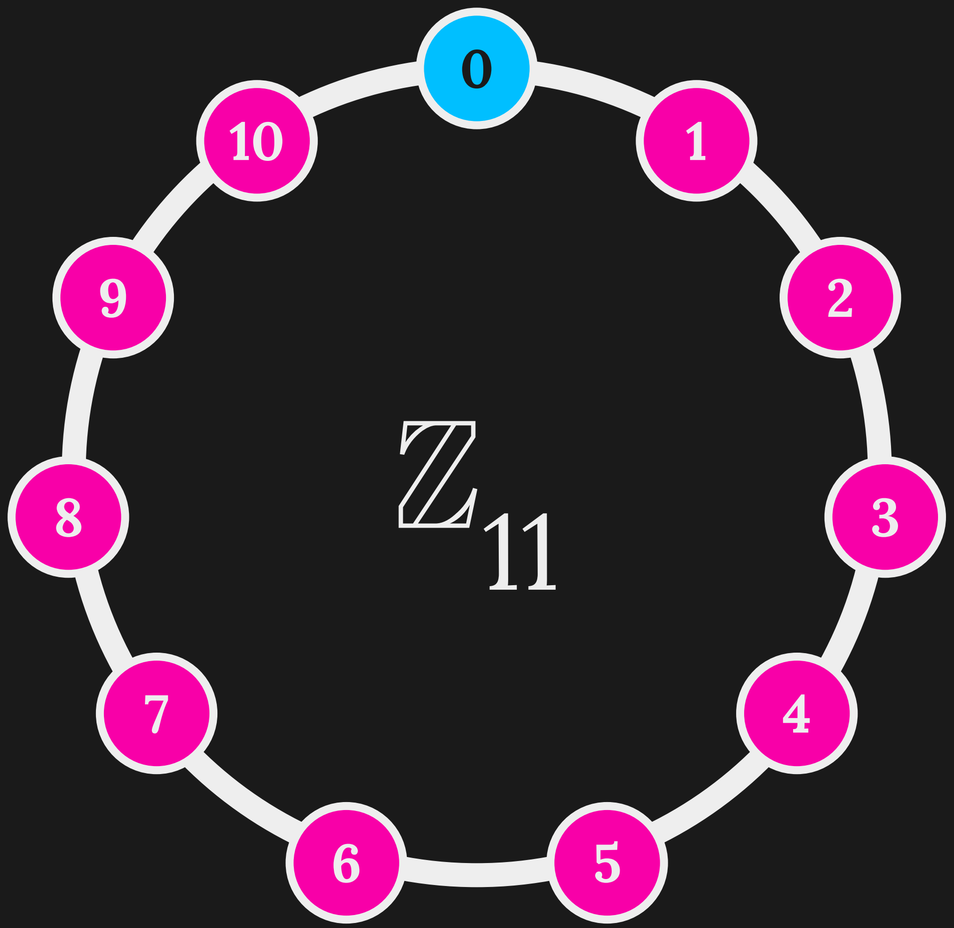
closure

If a and b are in the group and $a + b = c$, then c is in the group



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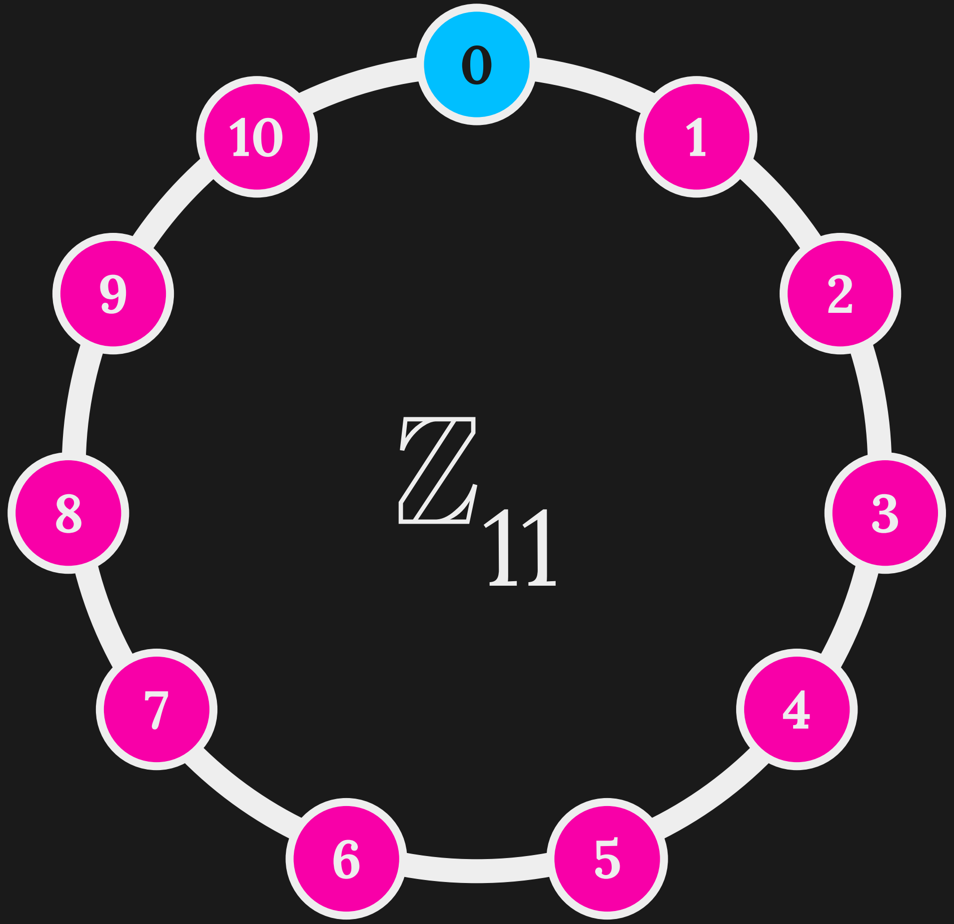
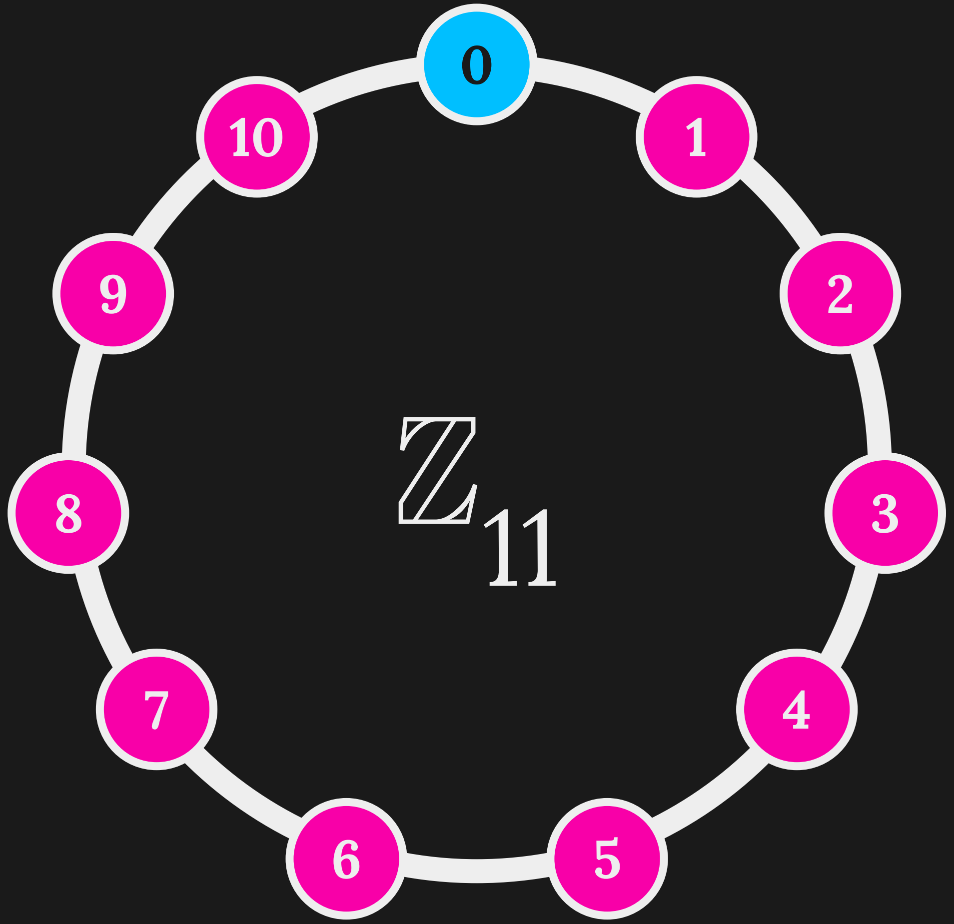


$4 \times 13 = 52$



$$4 \times 13 = 52$$

$$= (4 \times 11) + 8$$

$$= 8$$

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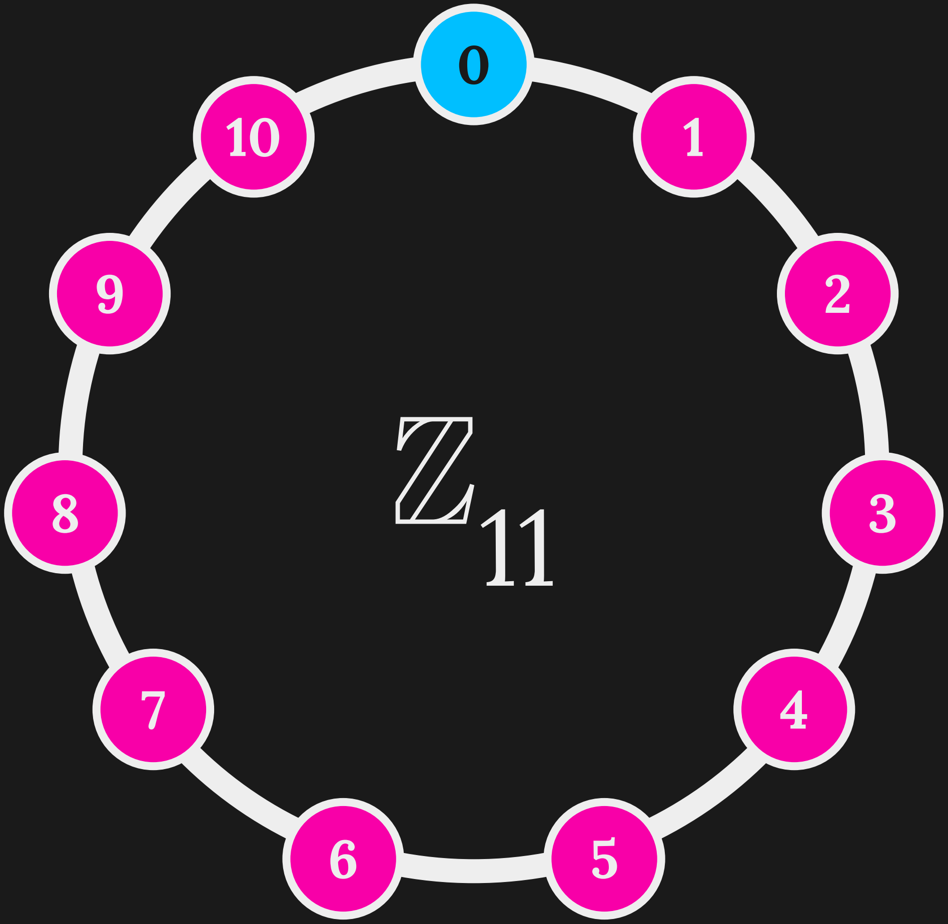

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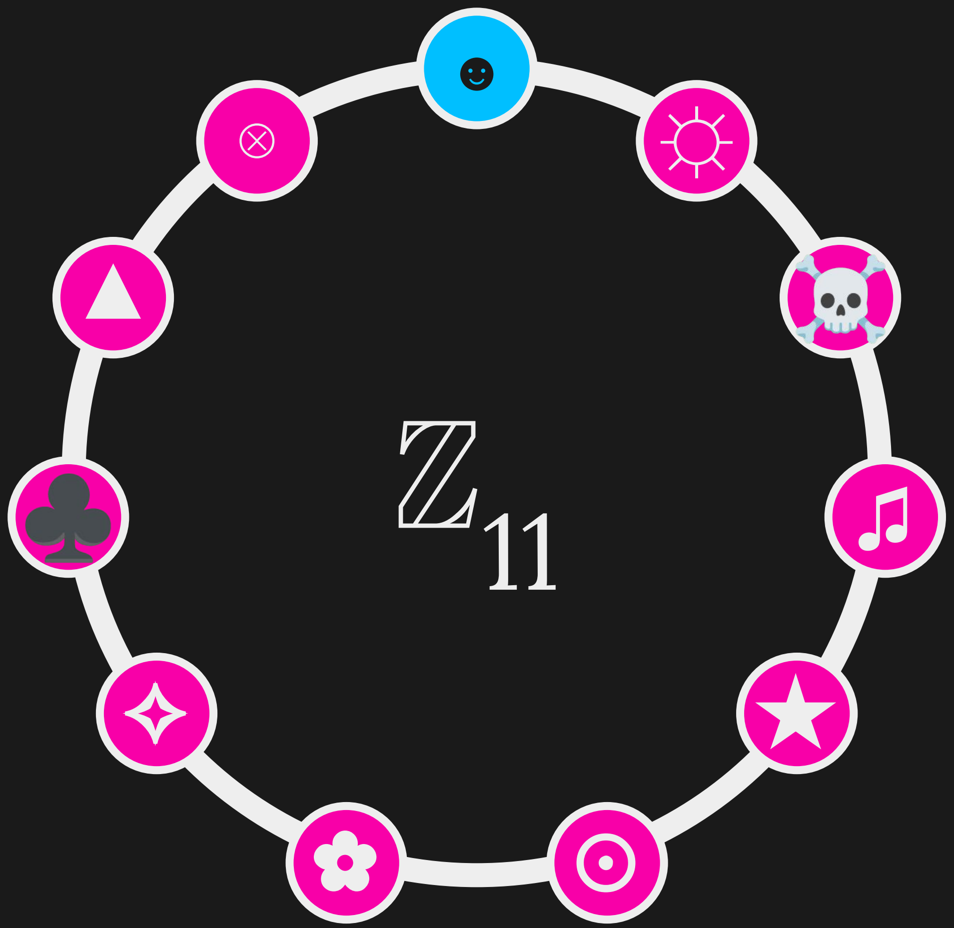
you can multiply an element of the group by something that is NOT in the group



 x 13 = 52

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= 

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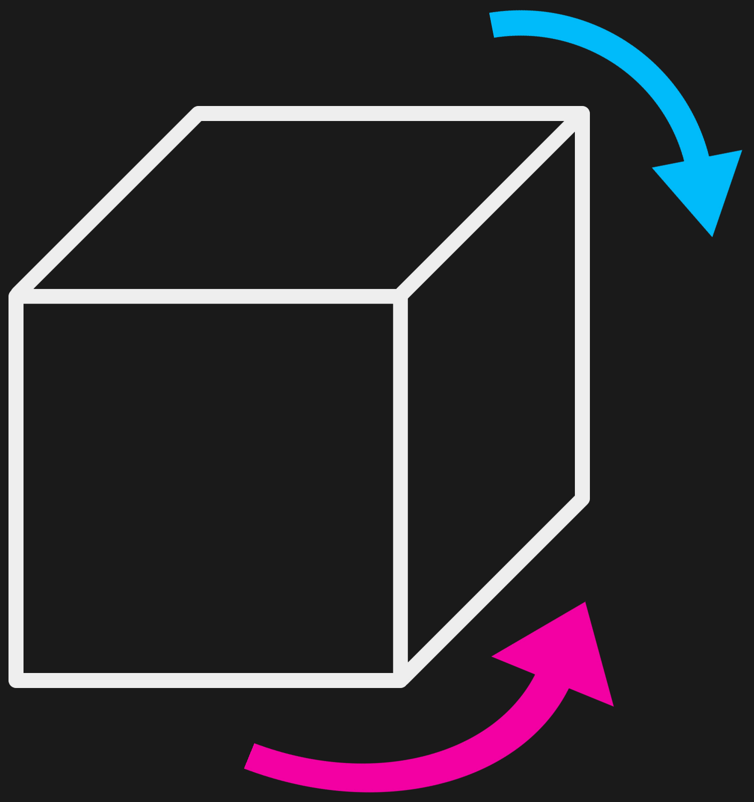

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$\{a\ ,b,\ c,\ \ldots\ldots\}$ & '+'

identity element

there is an element 0 such that
$0 + n = n$ for every n in the group

inverses

for every a in the group, there's
a b that makes $a + b = 0$ true

associativity

$a + (b + c) = (a + b) + c$

closure

If a and b are in the group and
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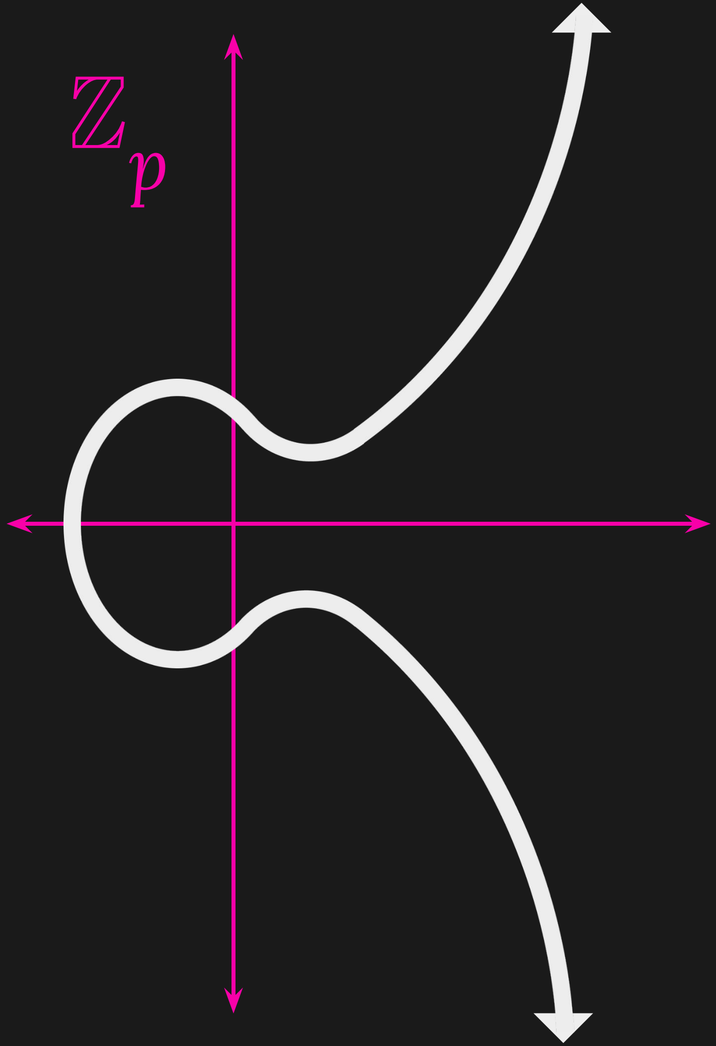
3

Elliptic Curve Cryptography



$$y^2 \equiv x^3 + \mathrm{a}x + \mathrm{b}$$

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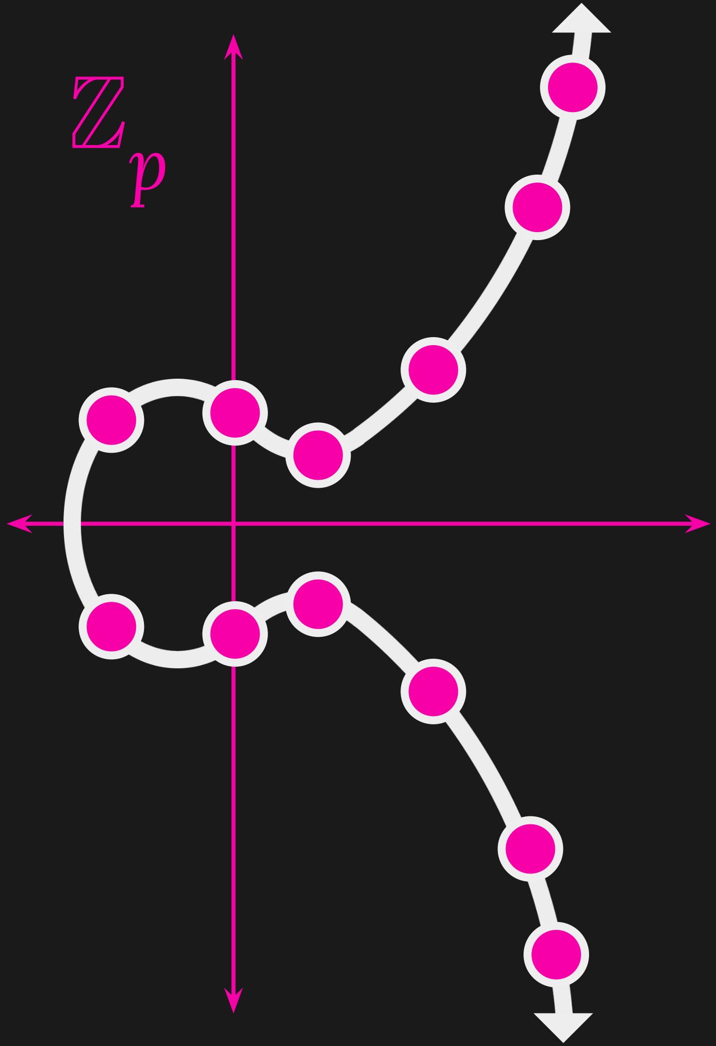

$$y^2 \equiv x^3 + ax + b$$

where x and y are in $\mathbb{Z}_p$

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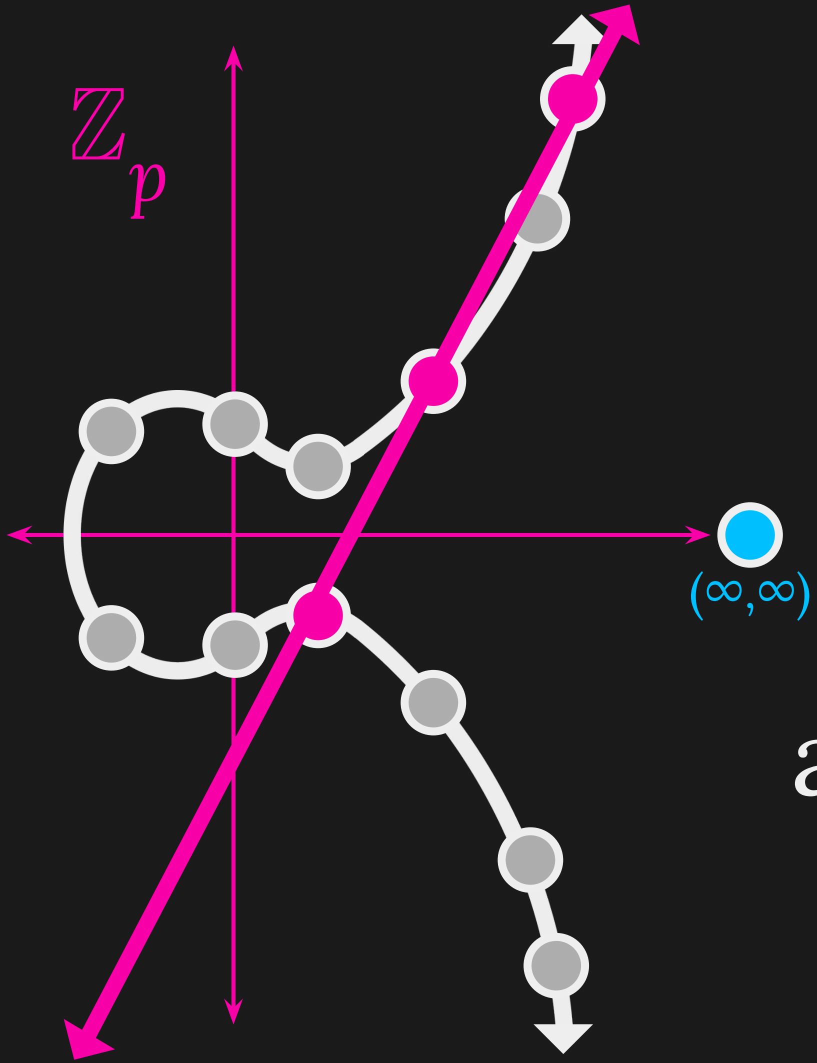
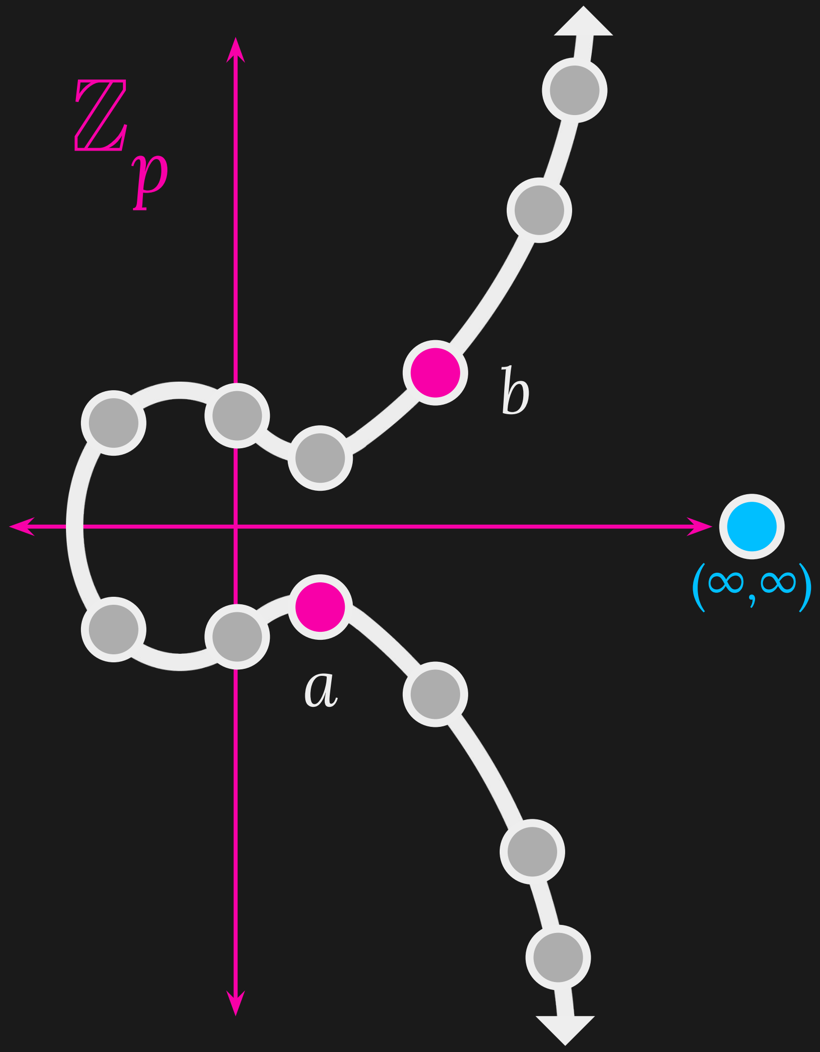


$$y^2 \equiv x^3 + ax + b$$

where x and y
are in $\mathbb{Z}_p$

and three collinear
points 'sum' to O



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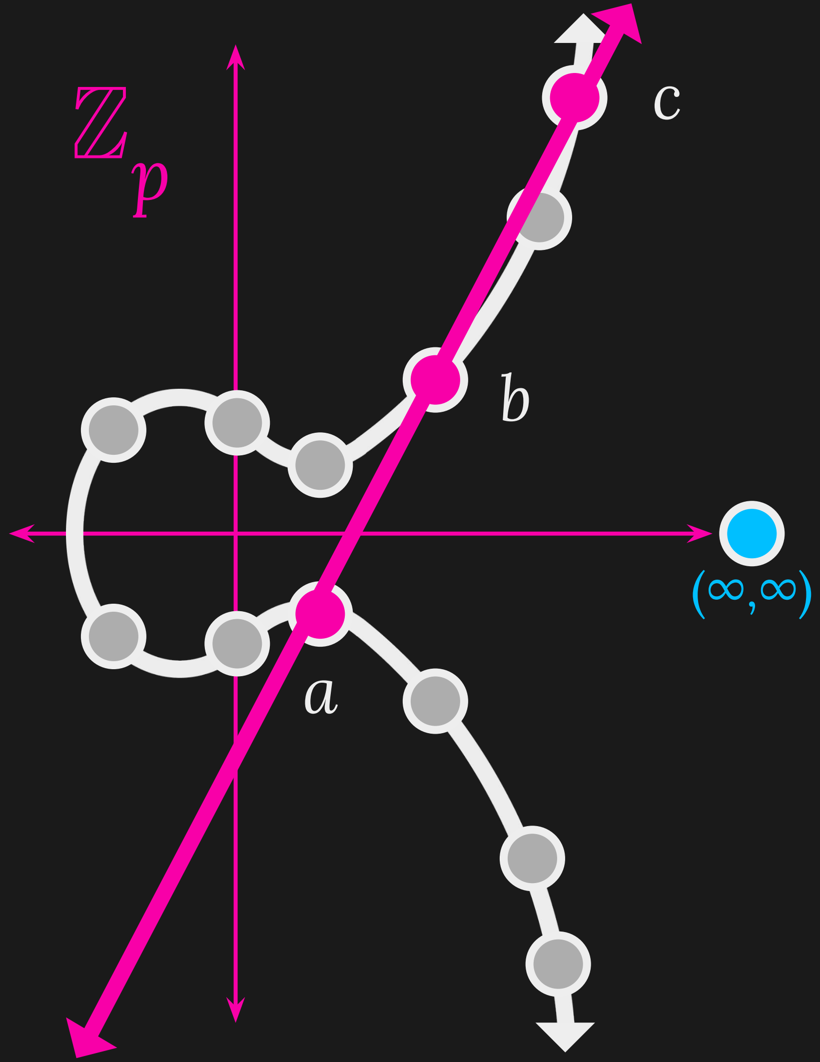

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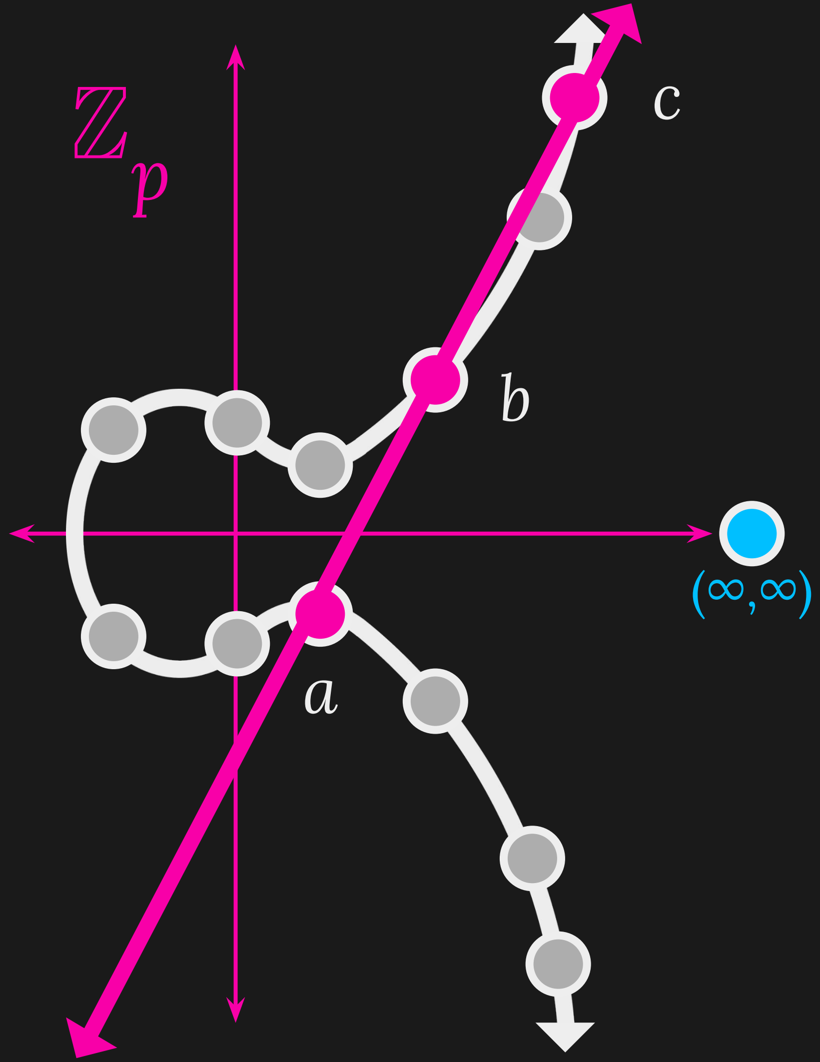

$$a + b + c = \mathbf{O}$$

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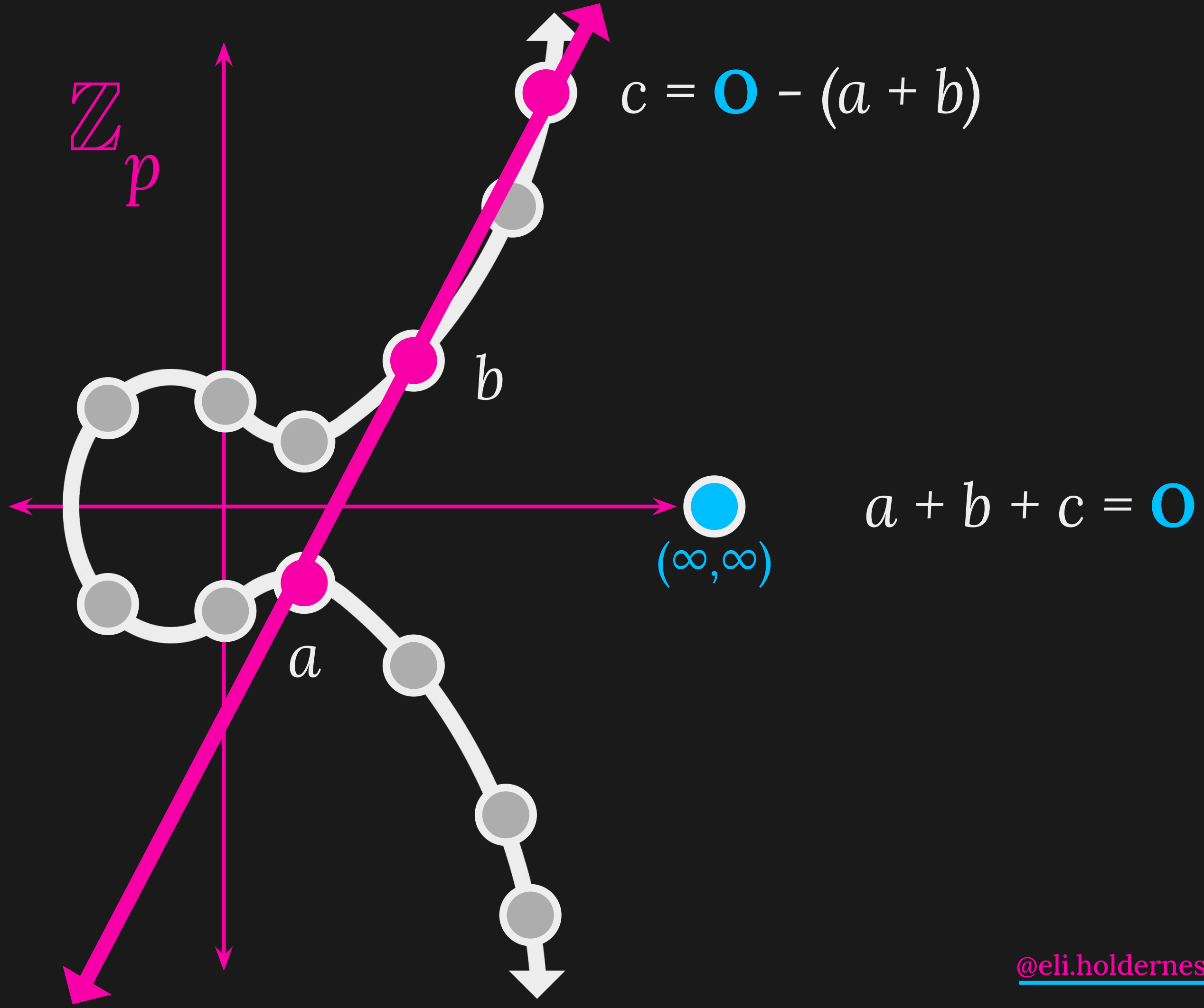

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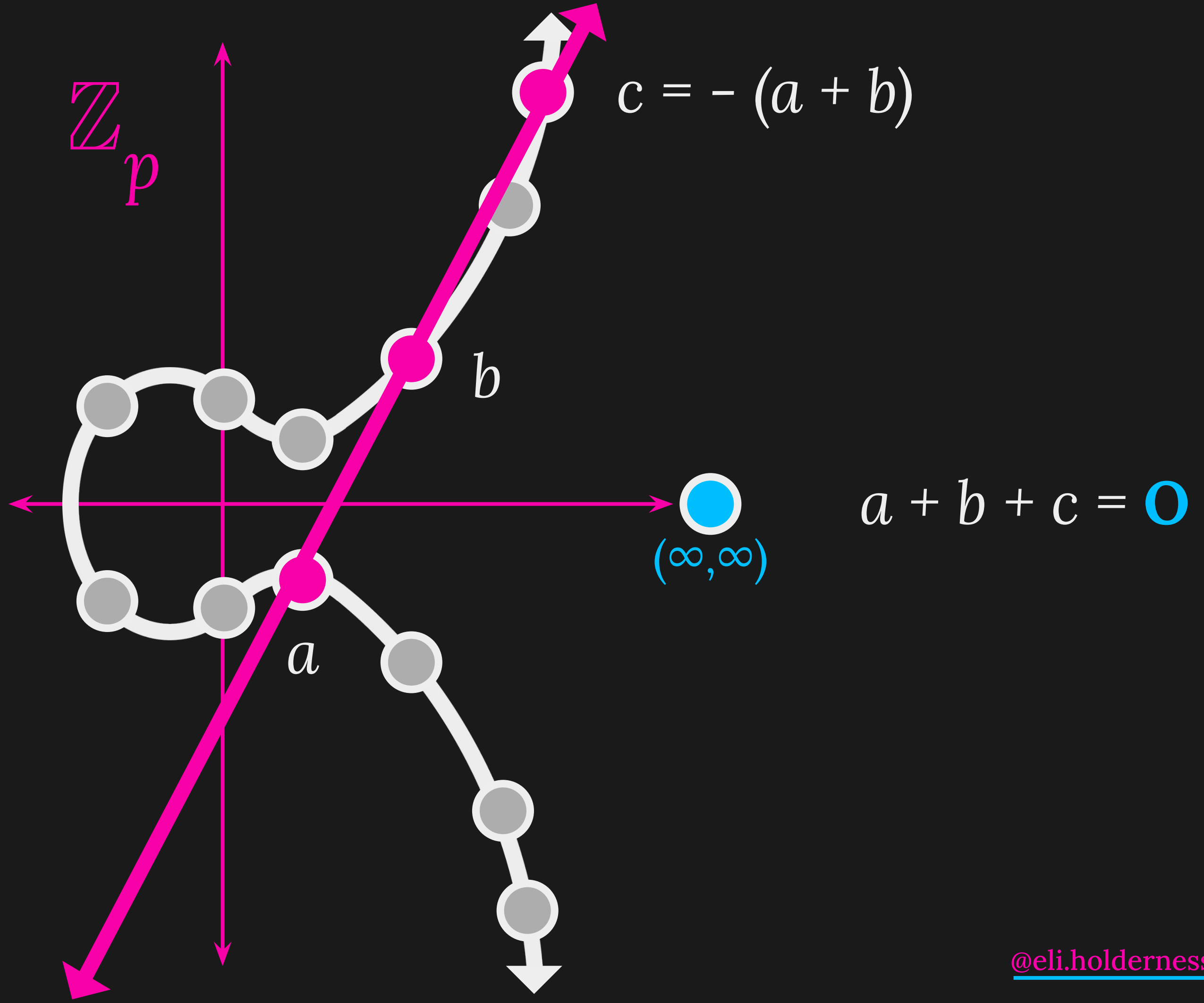
$\mathbb{Z}_p$
$c = -(a + b)$
b
a
(∞, ∞)
$a + b + c = \mathbf{O}$
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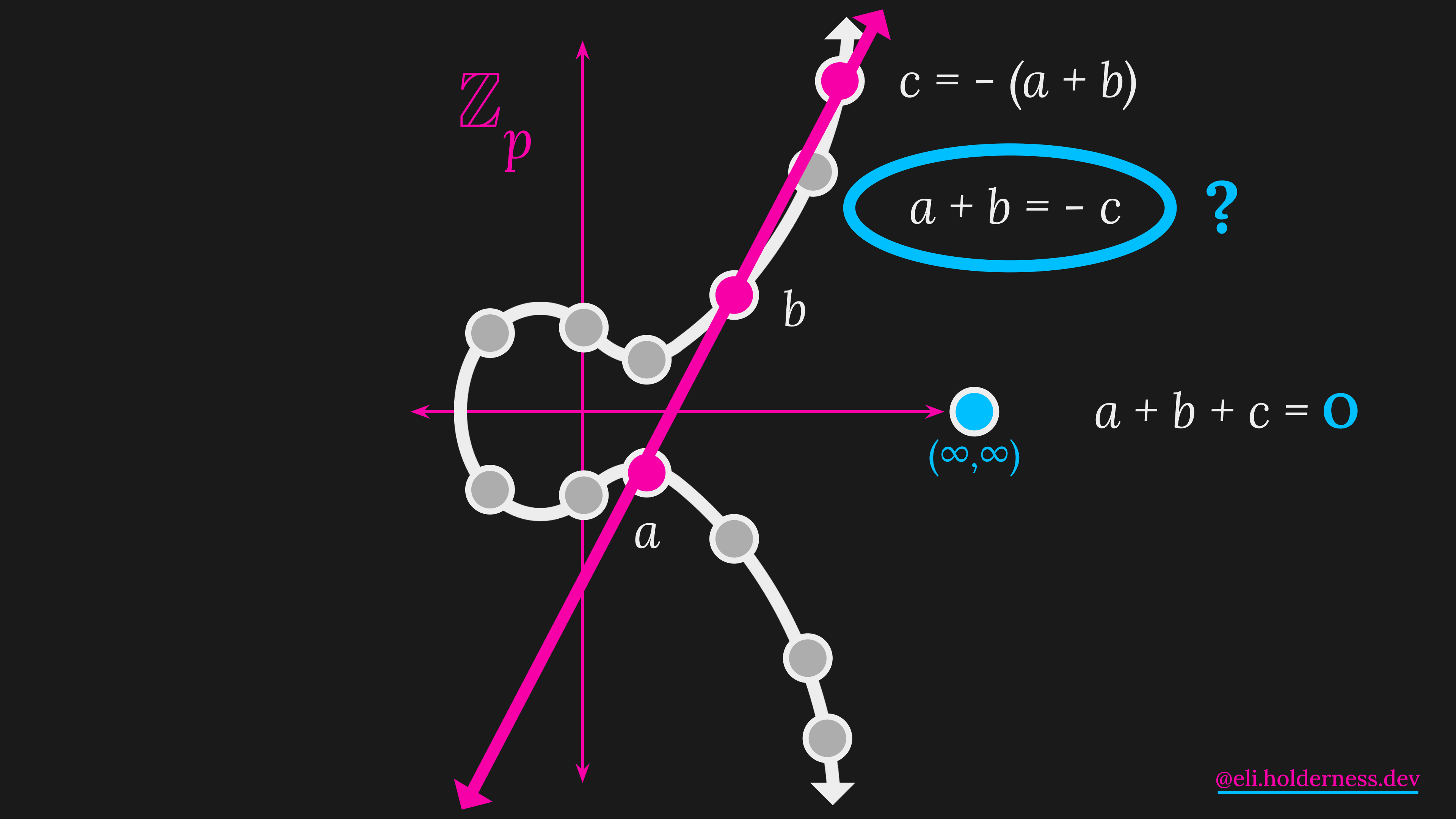

$\mathbb{Z}_p$
$c = -(a + b)$
$a + b = -c$
?
b
$a + b + c = \mathbf{O}$
(∞,∞)
a
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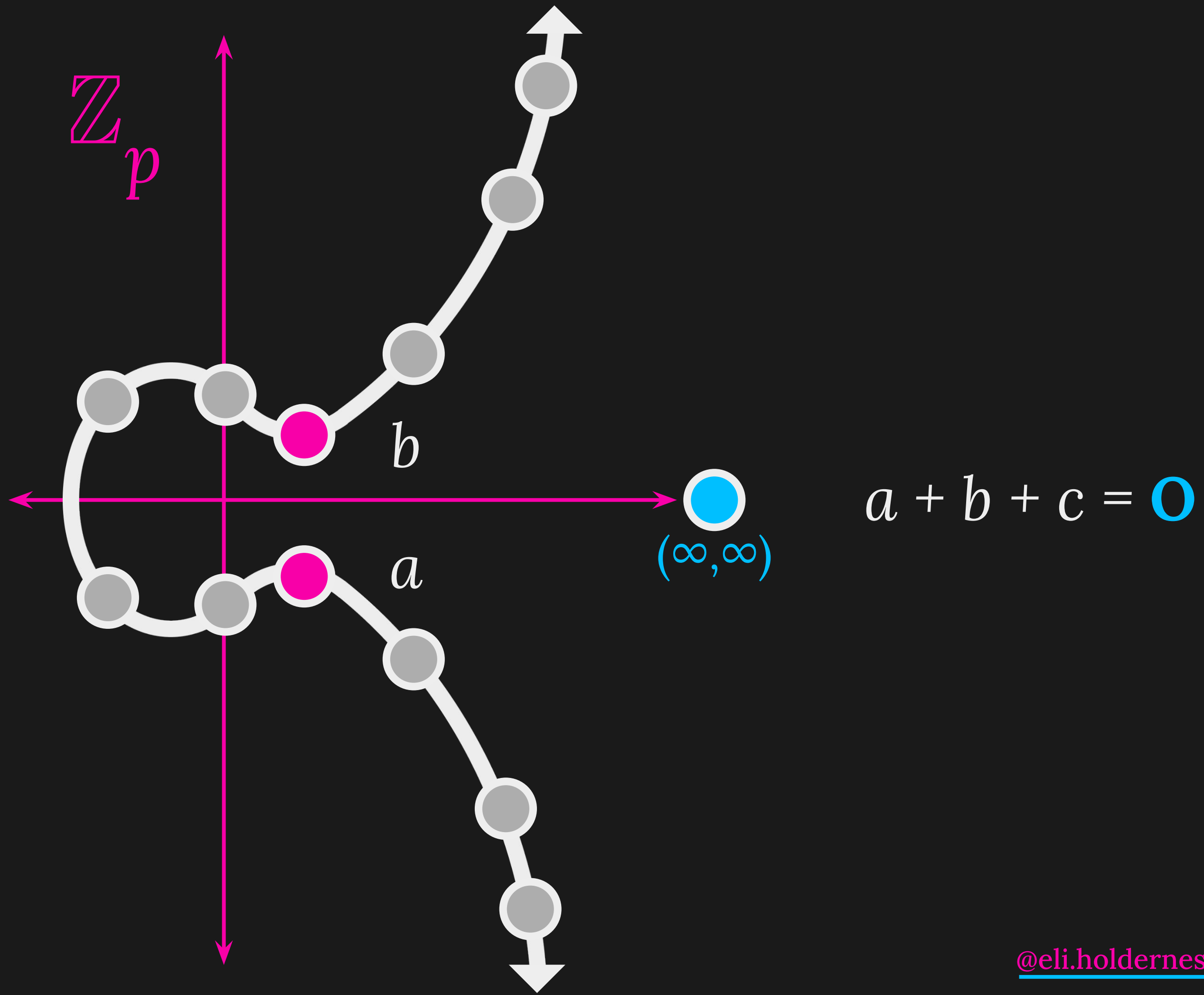

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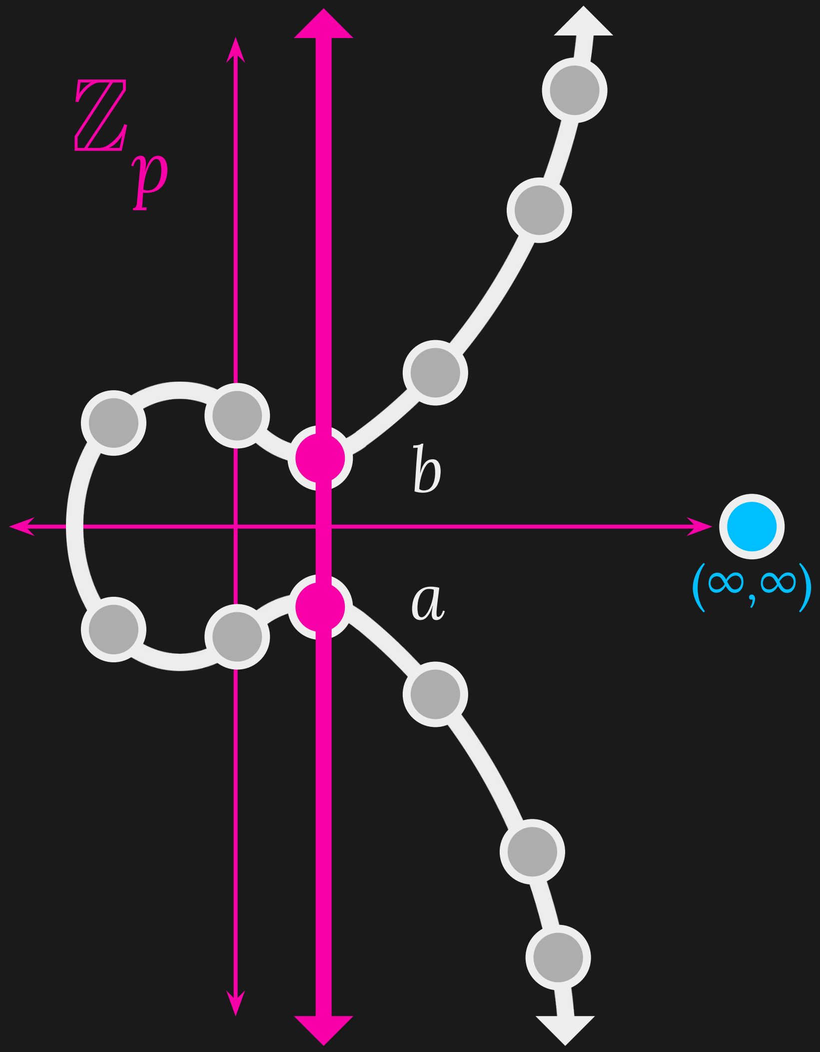

?????

$$a + b + c = \mathbf{O}$$

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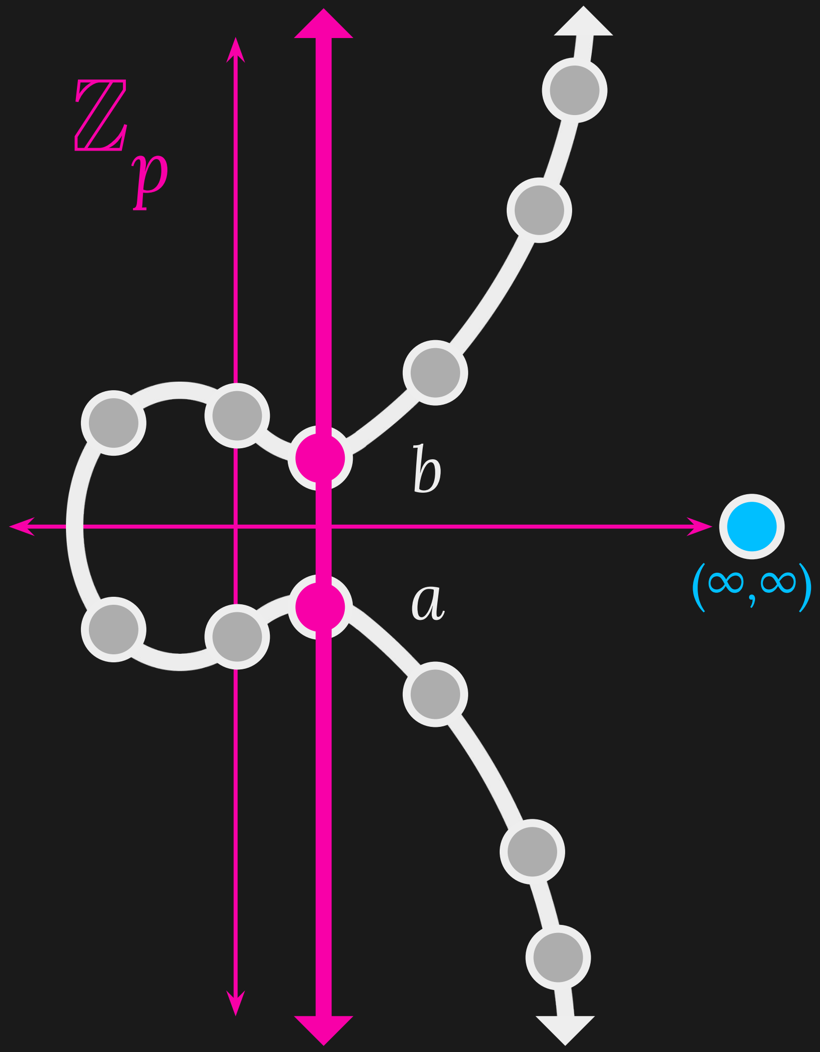

$a + b + \mathbf{O} = \mathbf{O}$

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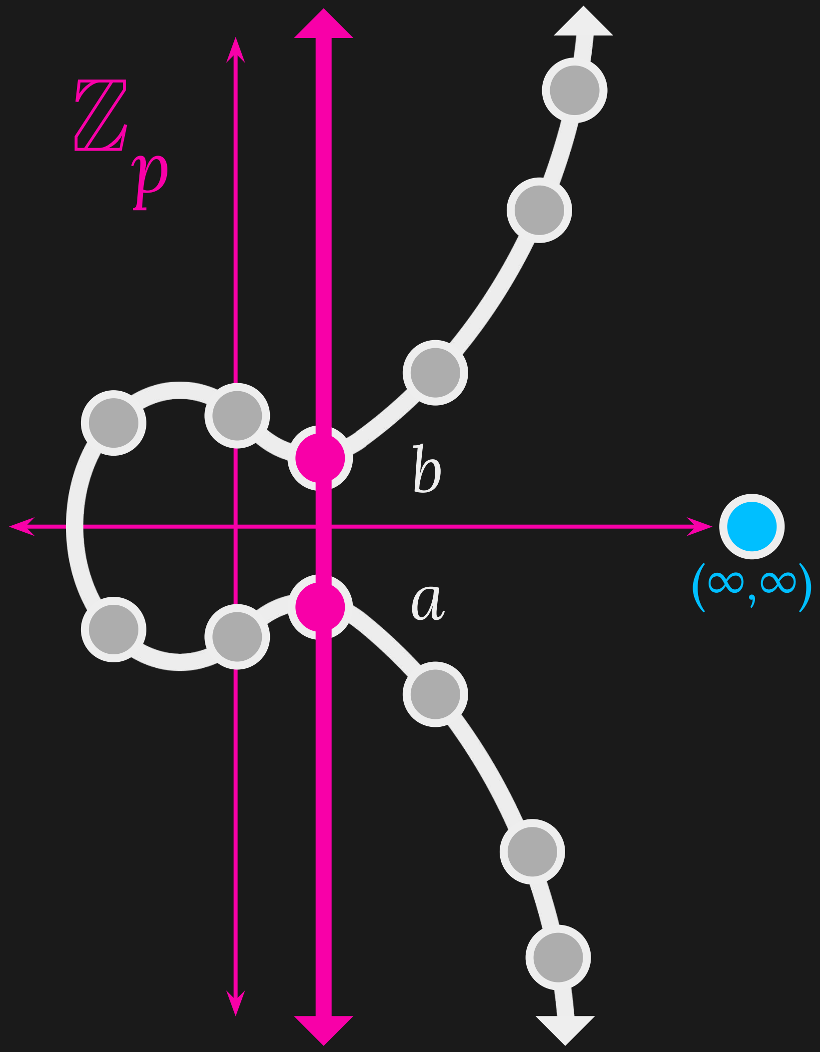

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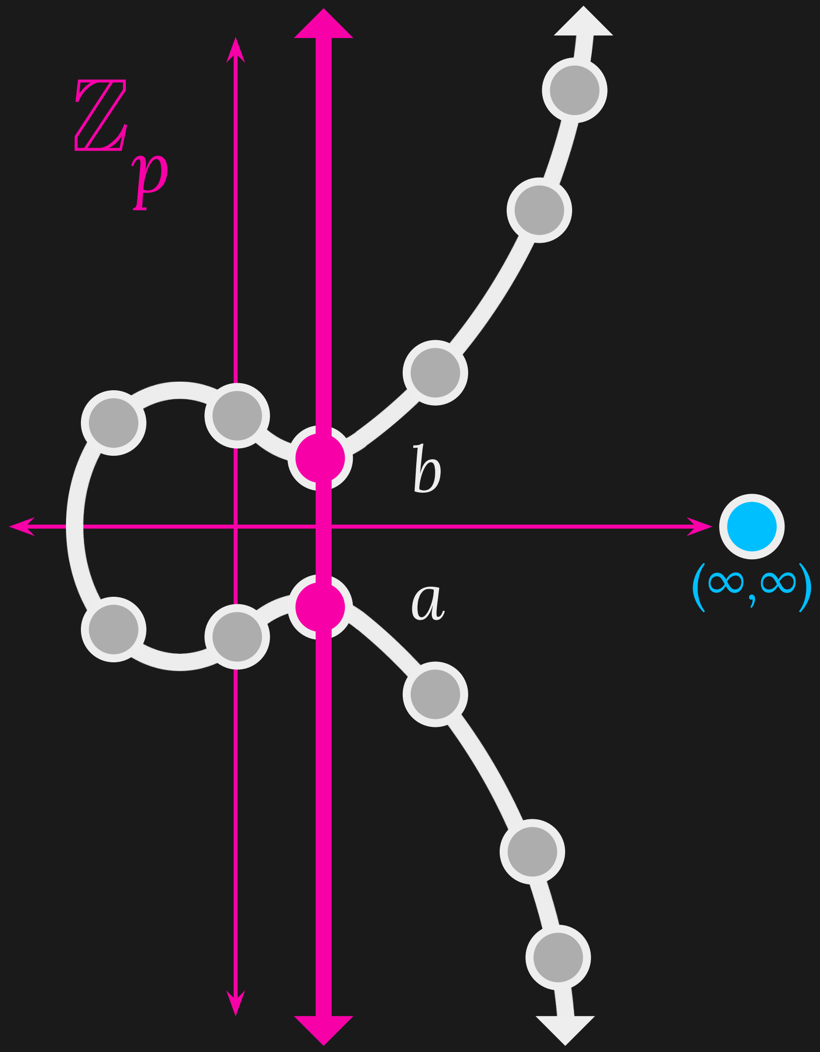

$$a + b + \mathbf{O} = \mathbf{O}$$

$$\Downarrow$$

$$a + b = \mathbf{O}$$

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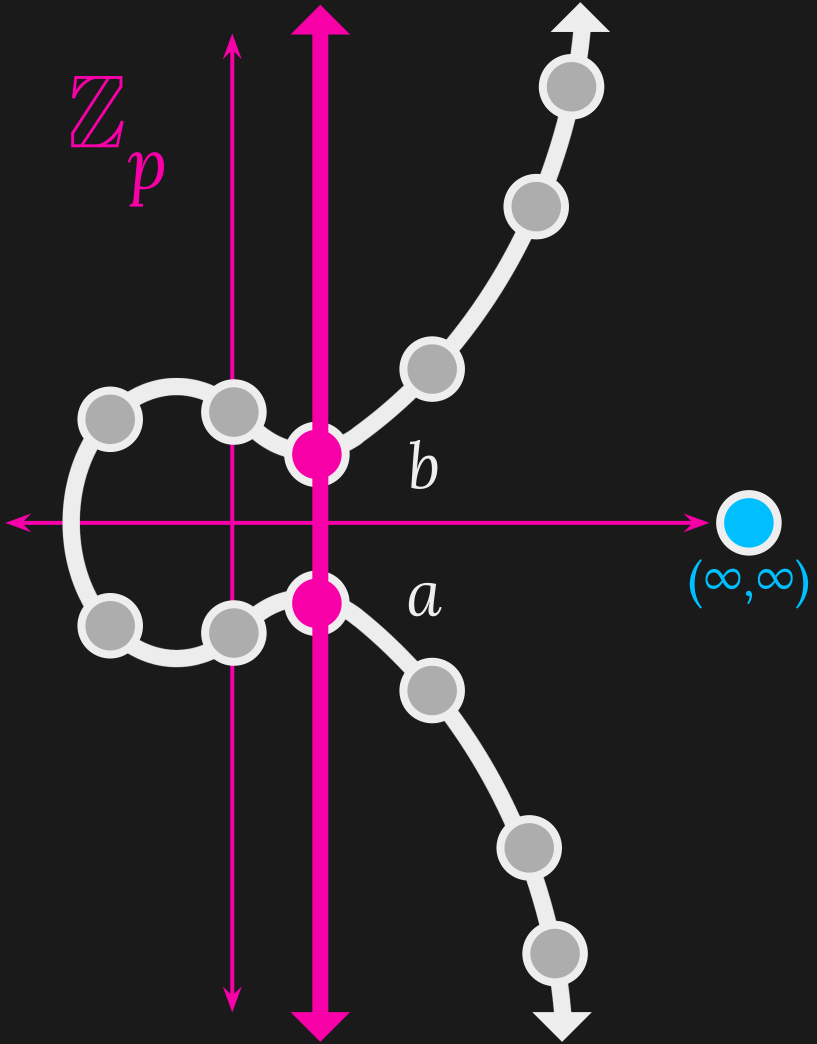


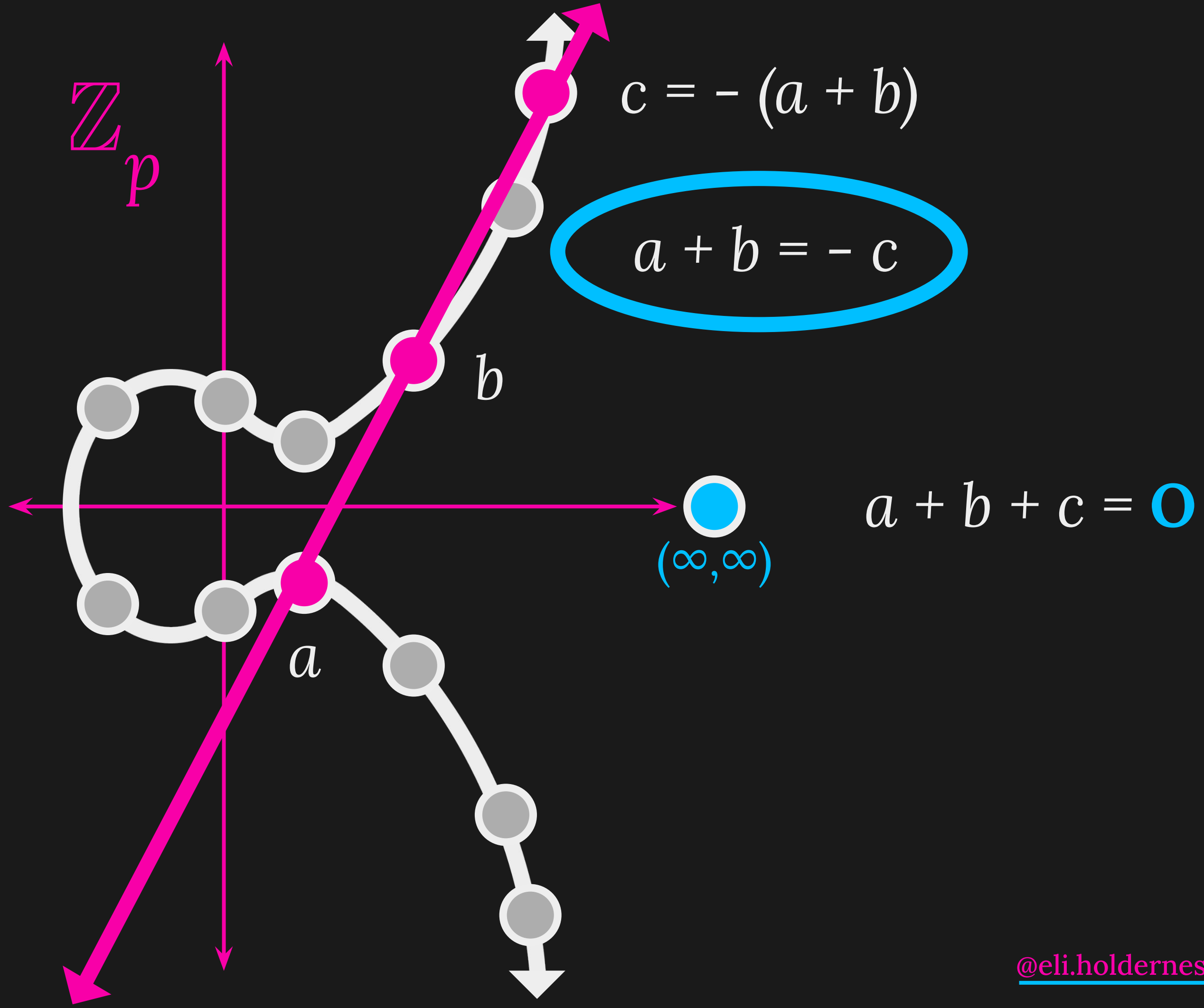
$$a + b + O = O$$

$$\Downarrow$$

$$a + b = O$$

$$\Downarrow$$

$$a = -b$$



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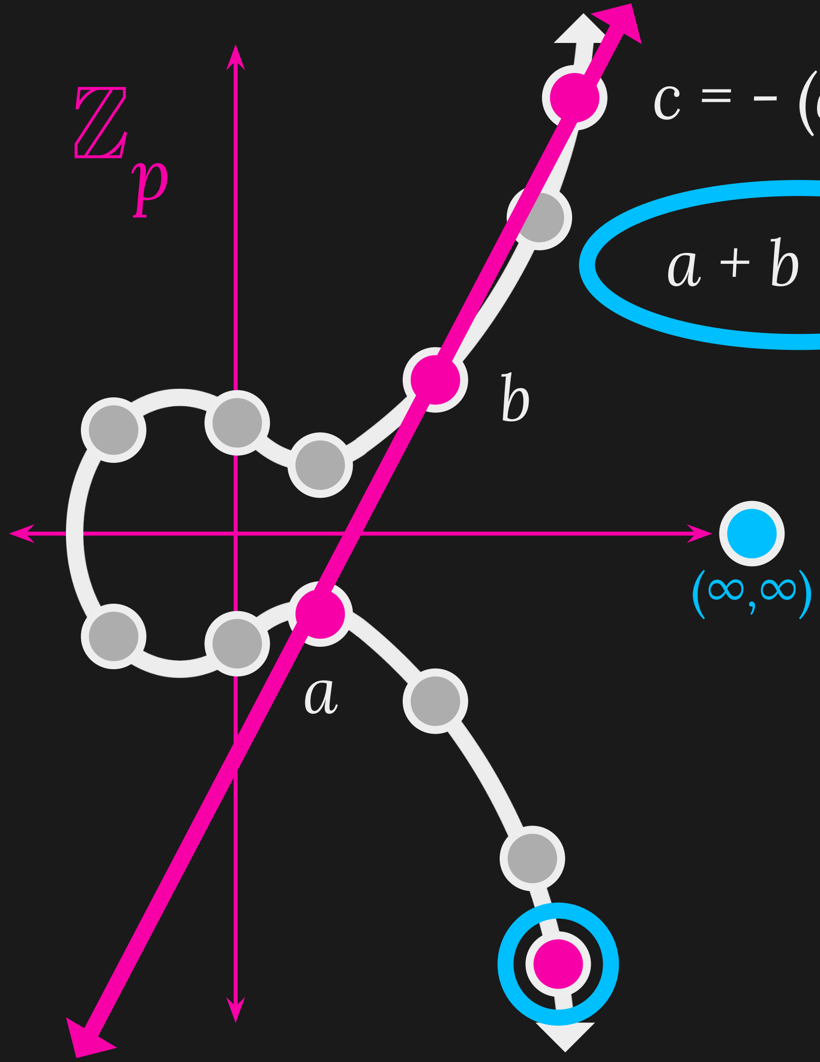

$c = -(a+b)$

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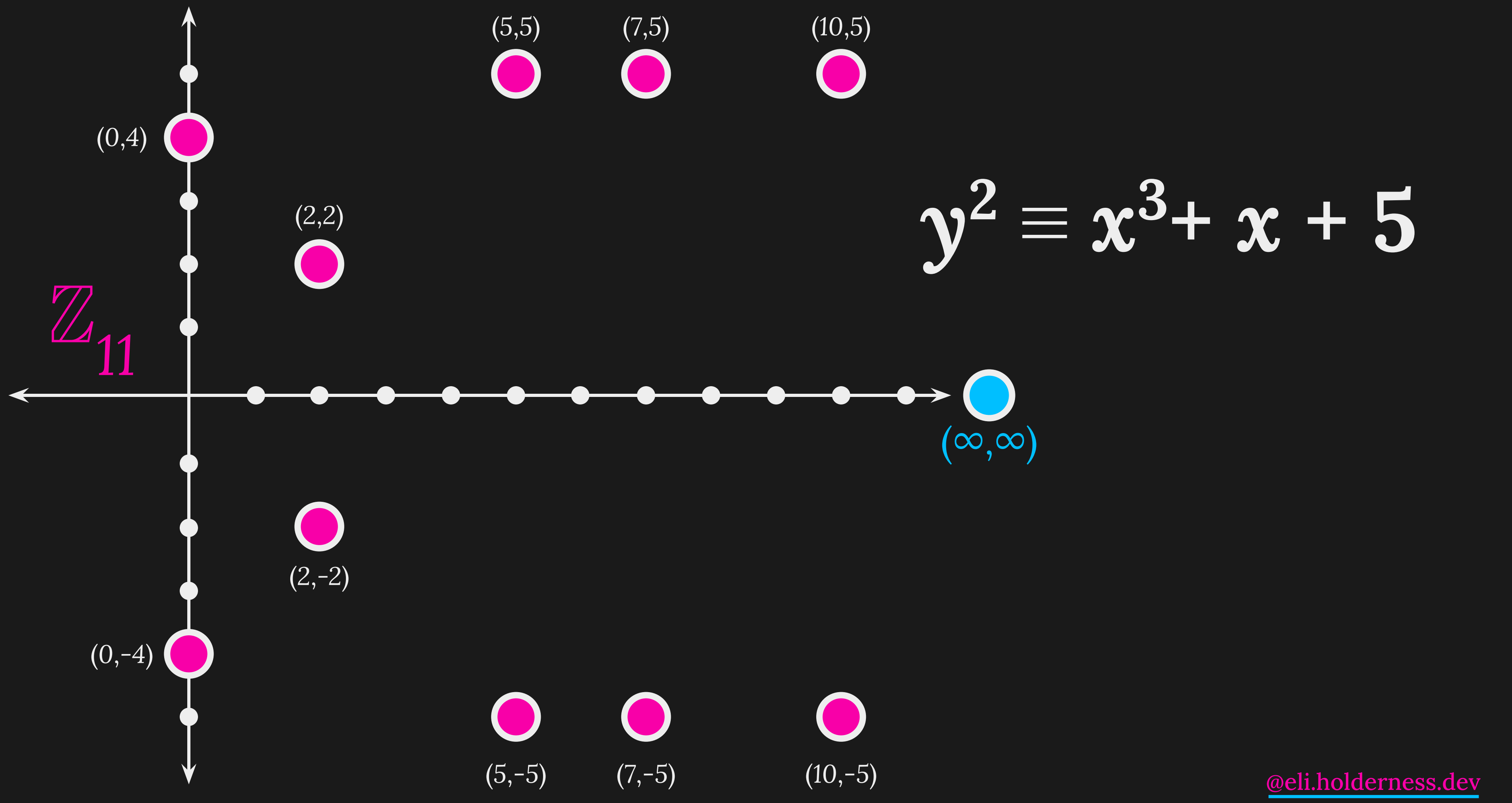
$a + b + c = O$



(5,5)
(7,5)
(10,5)
(0,4)
(2,2)
$y^2 \equiv x^3 + x + 5$
$\mathbb{Z}_{11}$
(∞,∞)
(2,-2)
(0,-4)
(5,-5)
(7,-5)
(10,-5)
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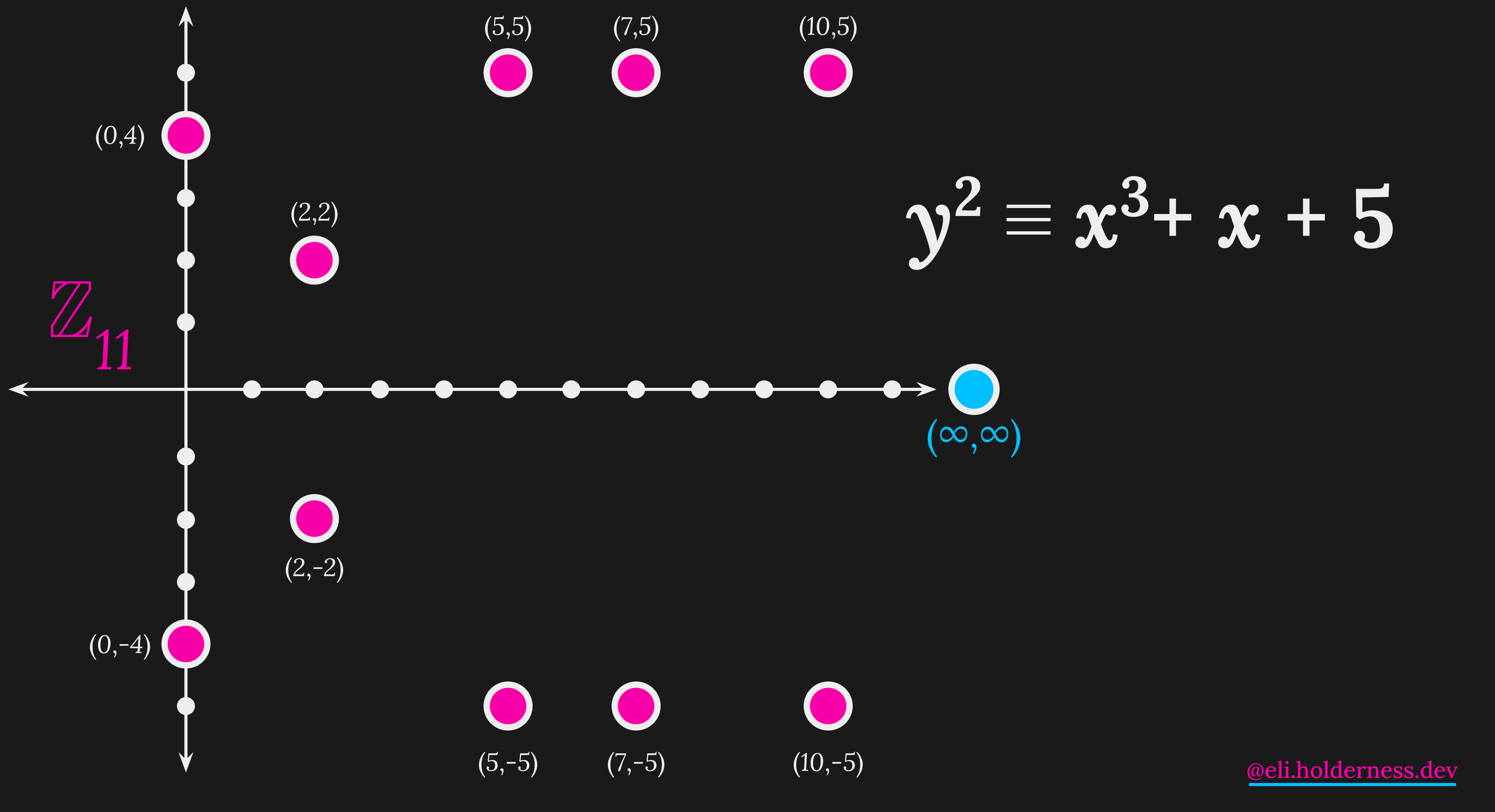

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elliptic curve domain parameters over $\boldsymbol{F}_p$

$$T = (p, a, b, G, n, h)$$

elliptic curve domain parameters over F_p

$$T = (p, a, b, \mathrm{G}, n, h)$$

an integer defining
the field F_p

Certicom Research: http://www.secg.org/sec2-v2.pdf

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elliptic curve domain parameters over F_p

$$T = (p, a, b, G, n, h)$$

two elements of F_p defining

$$E: y^2 \equiv x^3 + ax + b$$

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elliptic curve domain parameters over F_p

$$T = (p, a, b, G, n, h)$$

a point on $E(F_p)$ written as

$$G = (x_G, y_G)$$

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elliptic curve domain parameters over $\boldsymbol{F}_p$

$$T = (p, a, b, \mathrm{G}, n, h)$$

the *order* of G in $E(F_p)$ - i.e.,

$$n \times \mathrm{G} = \mathbf{O}$$

Certicom Research: http://www.secg.org/sec2-v2.pdf

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elliptic curve domain parameters over $\boldsymbol{F}_p$

$$T = (p, a, b, \mathrm{G}, n, h)$$

the *cofactor* of G in $E(F_p)$, which is $|E(F_p)| \,/\, n$

elliptic curve domain parameters over $\boldsymbol{F}_p$

$$T = (p, a, b, G, n, h)$$

or more properly, $orb(G)$

the *cofactor* of G in $E(F_p)$, which is

$$|E(F_p)| \,/\, n$$

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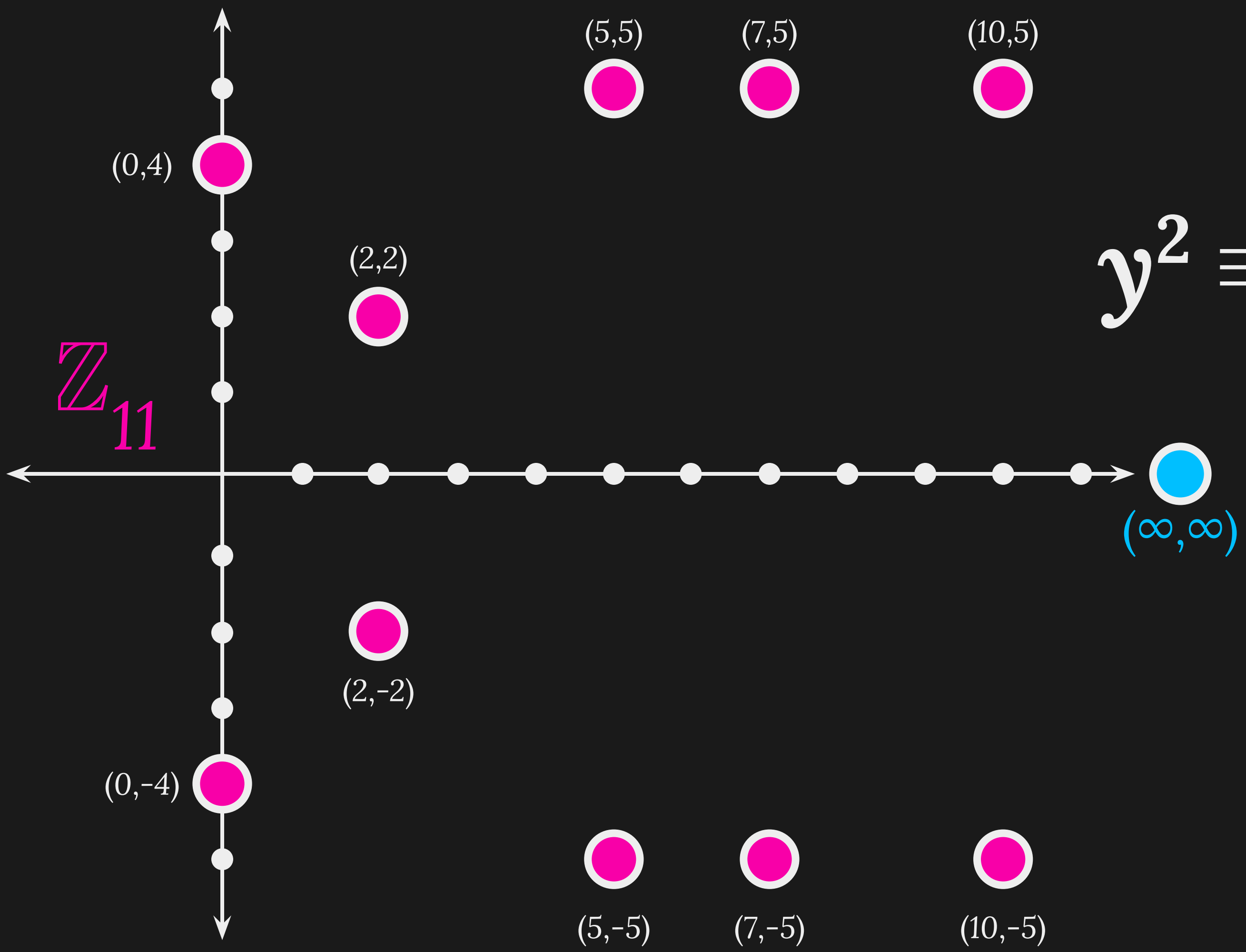

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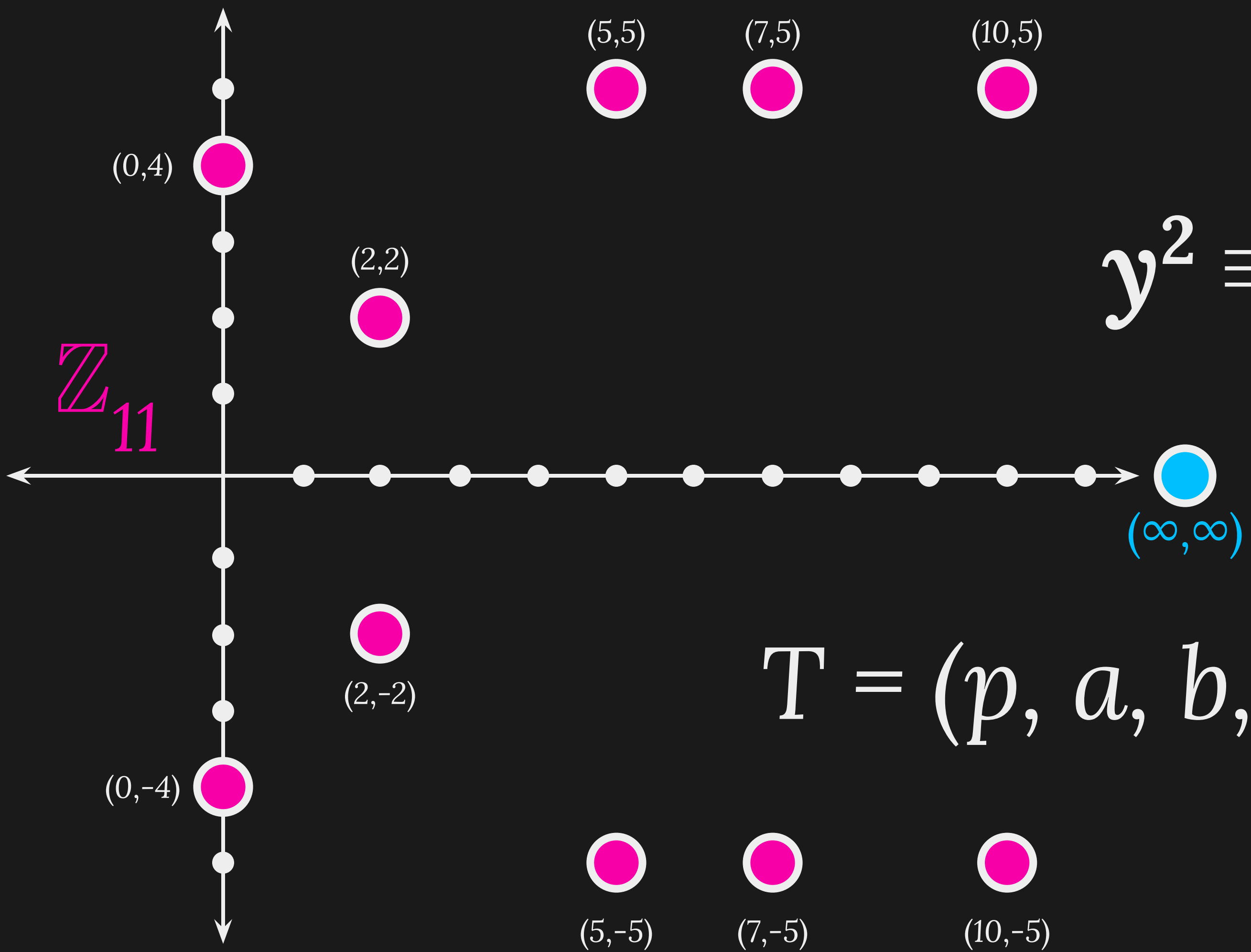

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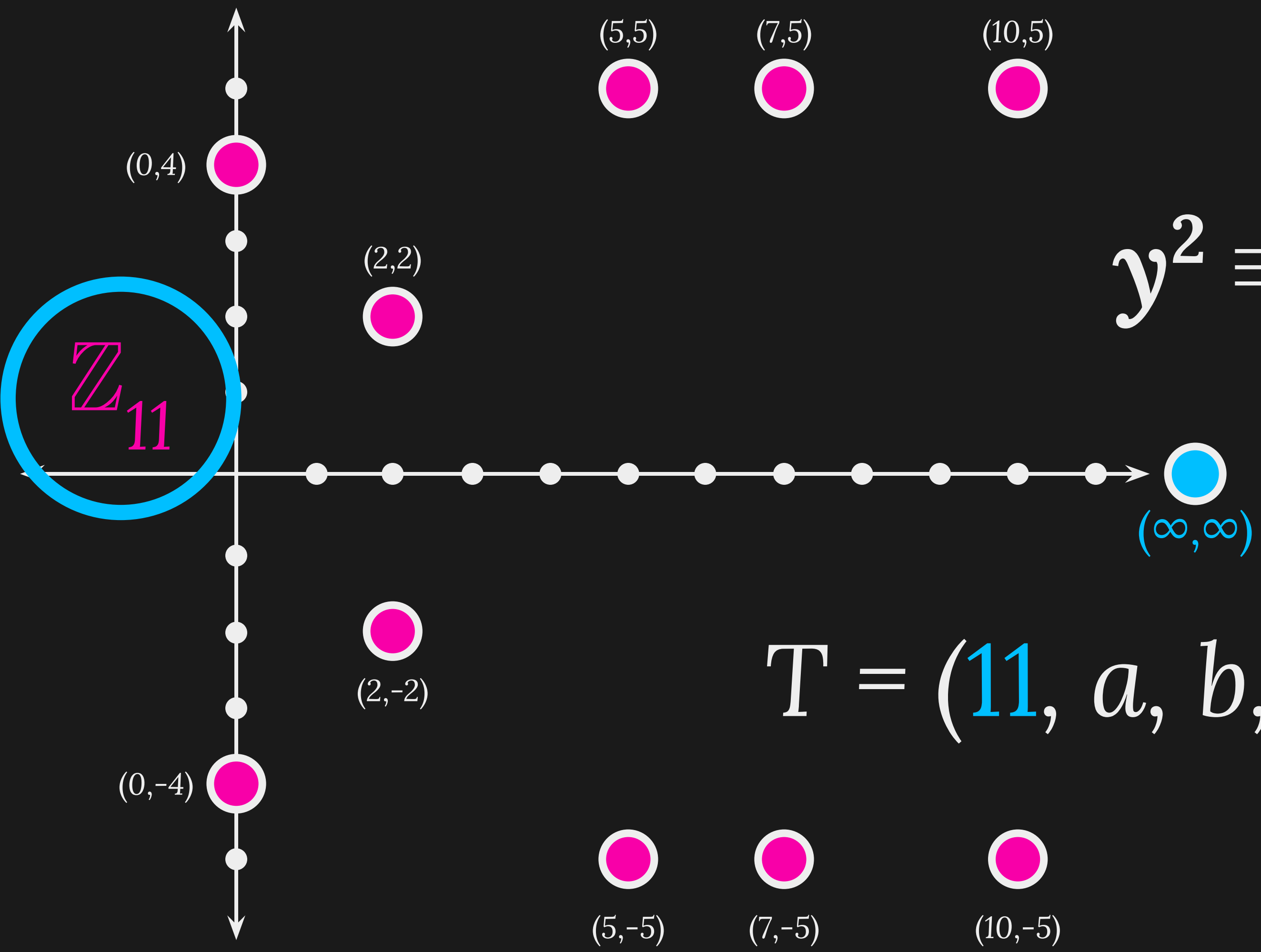

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$$T = (11, a, b, G, n, h)$$

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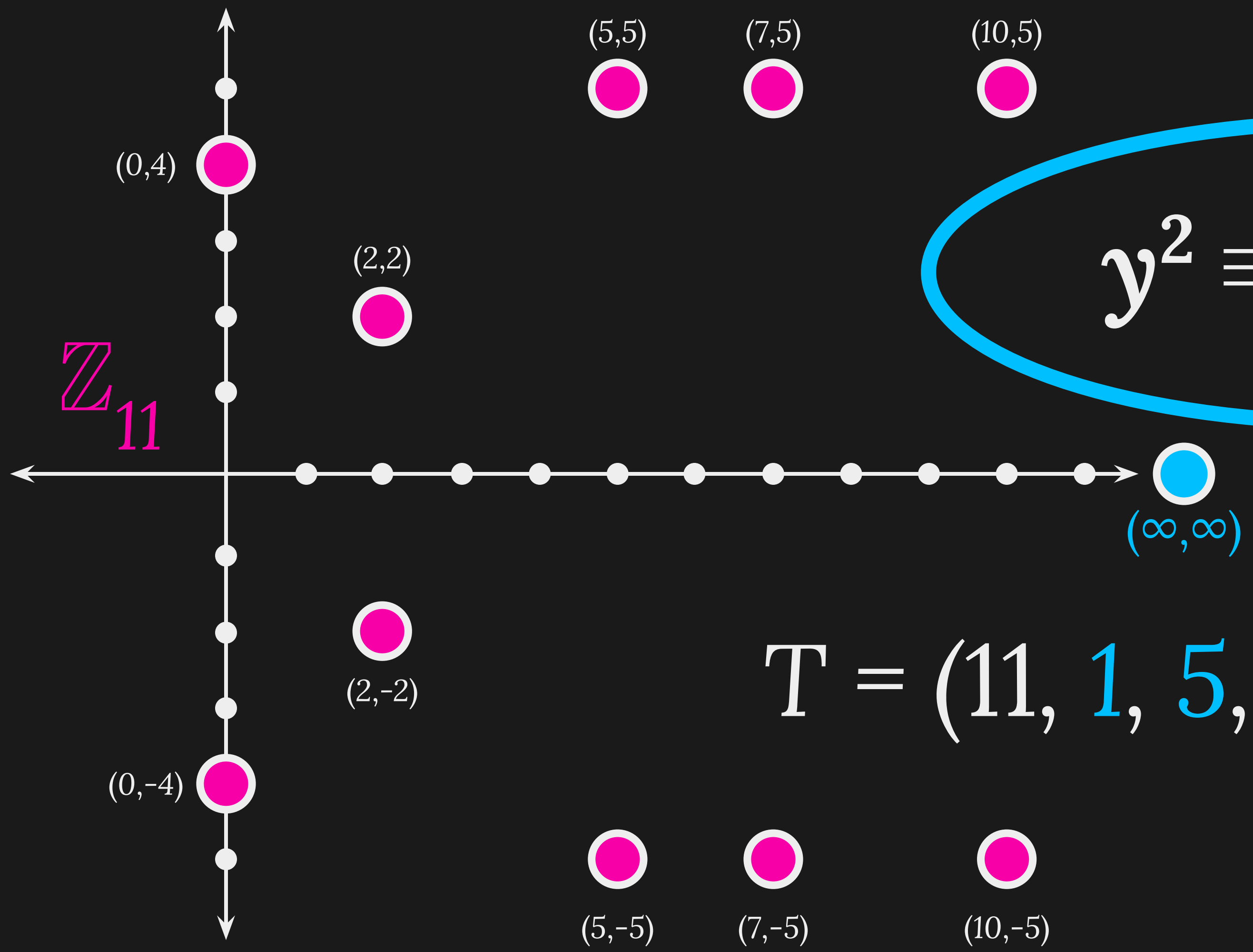

$$y^2 \equiv x^3 + x + 5$$

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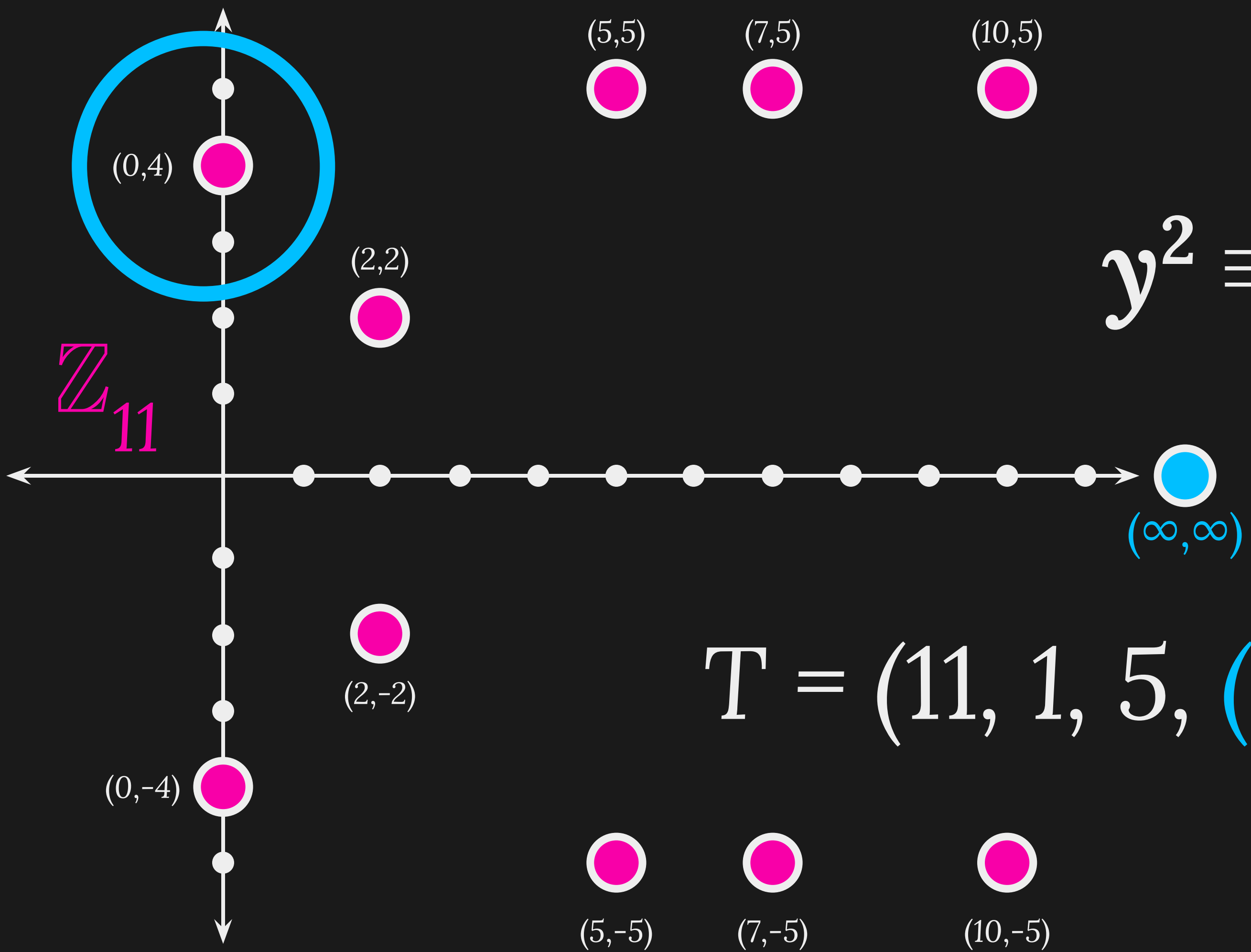

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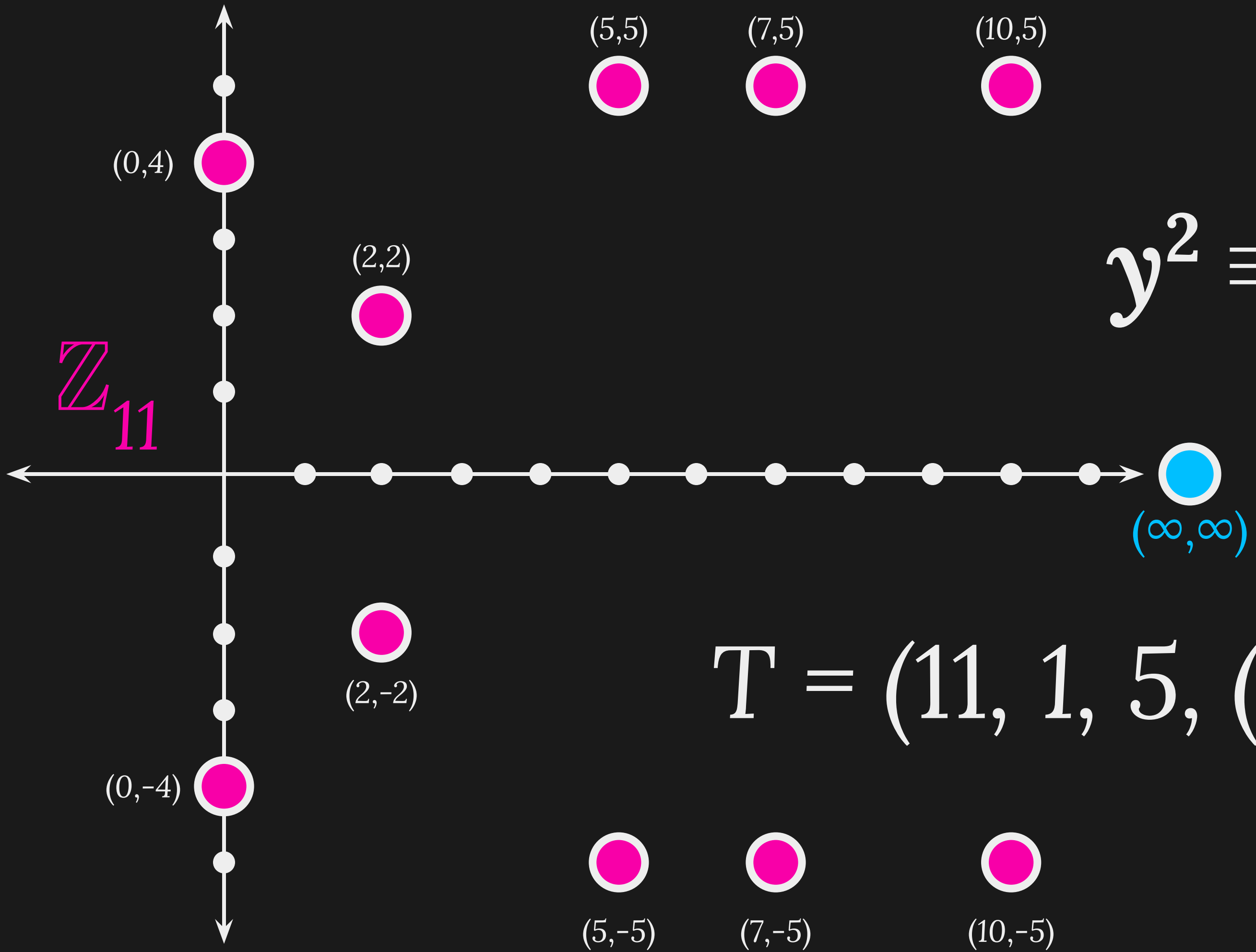

$$y^2 \equiv x^3 + x + 5$$

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Point addition [edit]

With 2 distinct points, *P* and *Q*, addition is defined as the negation of the point resulting from the intersection of the curve, *E*, and the straight line defined by the points *P* and *Q*, giving the point, *R*.[1]

$$\begin{aligned} P + Q &= R \\ (x_p, y_p) + (x_q, y_q) &= (x_r, y_r) \end{aligned}$$

Assuming the elliptic curve, *E*, is given by $y^2 = x^3 + ax + b$, this can be calculated as:

$$\begin{aligned} \lambda &= \frac{y_q - y_p}{x_q - x_p} \\ x_r &= \lambda^2 - x_p - x_q \\ y_r &= \lambda(x_p - x_r) - y_p \end{aligned}$$

These equations are correct when neither point is the point at infinity, $\mathcal{O}$, and if the points have different x coordinates (they're not mutual inverses). This is important for the ECDSA verification algorithm where the hash value could be zero.

Point doubling [edit]

Where the points *P* and *Q* are coincident (at the same coordinates), addition is similar, except that there is no well-defined straight line through *P*, so the operation is closed using a limiting case, the tangent to the curve, *E*, at *P*.

This is calculated as above, taking derivatives (dE/dx)/(dE/dy):[1]

$$\lambda = \frac{3x_p^2 + a}{2y_p}$$

where *a* is from the defining equation of the curve, *E*, above.

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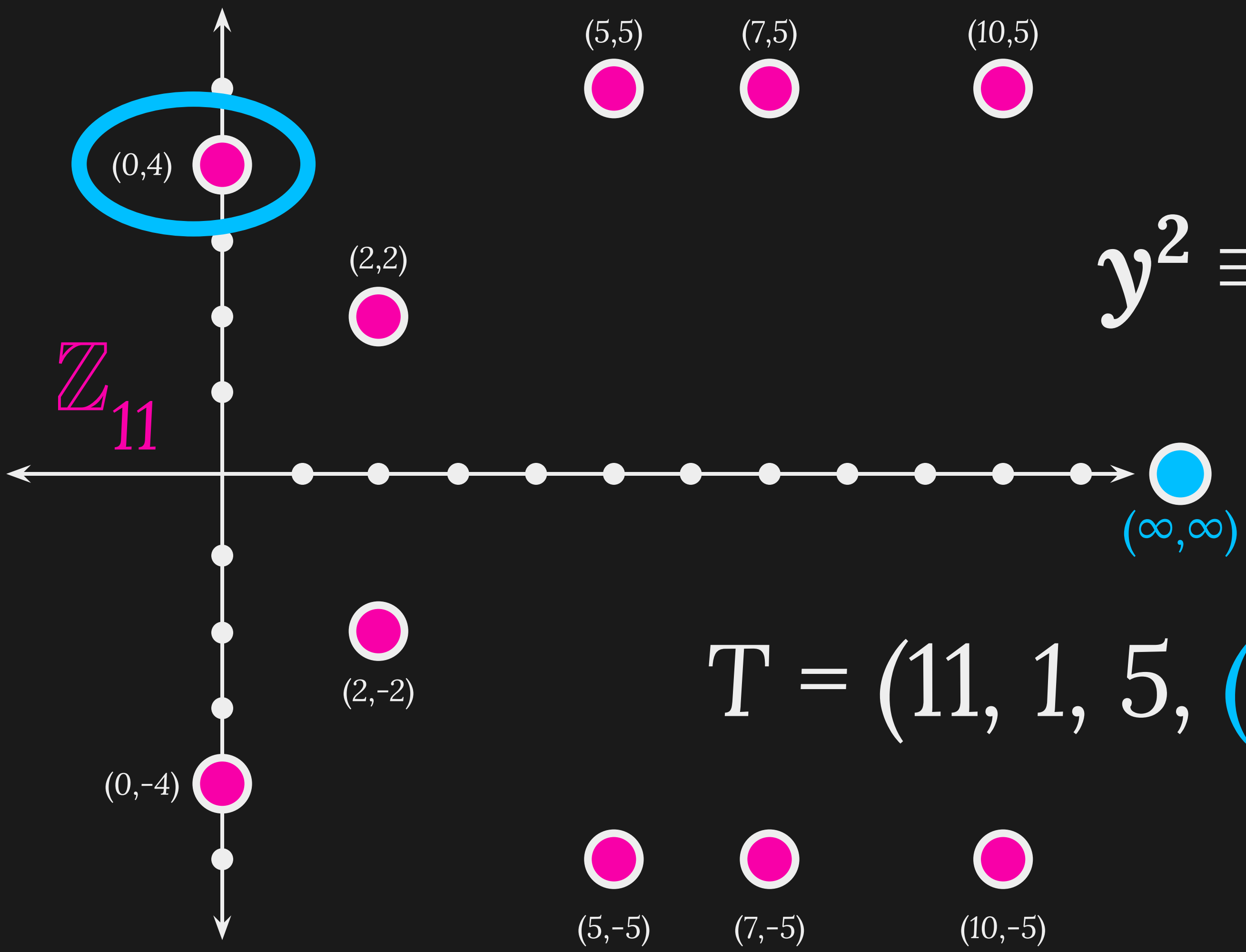

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$$3 \times G = (10, 5)$$
$$4 \times G = (2, -2)$$
$$5 \times G = (7, -5)$$
$$6 \times G = (7, 5)$$
$$7 \times G = (2, 2)$$
$$8 \times G = (10, -5)$$
$$9 \times G = (5, -5)$$
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comparison with RSA

comparison with RSA

smaller key size per security

comparison with RSA

smaller key size per security

smaller payload size

comparison with RSA

smaller key size per security

smaller payload size

faster computation

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Quantum Computing & Shor's Algorithms

the Integer Factorisation problem

if pq = N with p & q prime, find p and q given only N

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the Discrete Logarithm problem

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Shor's order-finding algorithm

for a given number N, and any number a between 1 and N, we can find the smallest r such that $a^r \equiv 1 \bmod N$, in polynomial time

Shor’s order-finding algorithm

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let N = 323. Choose $a = 11$.

Shor's algorithm gives us that $11^{48} \equiv 1 \bmod 323$

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this breaks RSA!

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Post-quantum Cryptography

the isogeny-finding problem

given two elliptic curves between which we know there exists an isogeny, find the mapping that describes it

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SIKE and SIDH, which are considered insecure

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CSIDH

Quantum Security Analysis of CSIDH and Ordinary Isogeny-based Schemes

Xavier Bonnetain[1,2] and André Schrottenloher[2]

[1] Sorbonne Université, Collège Doctoral, F-75005 Paris, France
[2] Inria, France

Abstract. CSIDH is a recent proposal by Castryck, Lange, Martindale, Panny and Renes for post-quantum non-interactive key-exchange. It is similar in design to a scheme by Couveignes, Rostovtsev and Stolbunov

https://who.rocq.inria.fr/Xavier.Bonnetain/pdfs/csidh-attack.pdf

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7 Conclusion

We presented a comprehensive quantum security assessment of CSIDH. In particular, when compared to the cost of a classical key-exchange, we showed that the parameters set in [6] actually seem to provide only around half of the expected security, as summarized in Table 7.

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the isogeny-finding problem

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SIKE and SIDH, which are considered insecure

CSIDH, which should also be considered insecure

the Learning With Errors problem

introducing noise to encodings and using probability to decode

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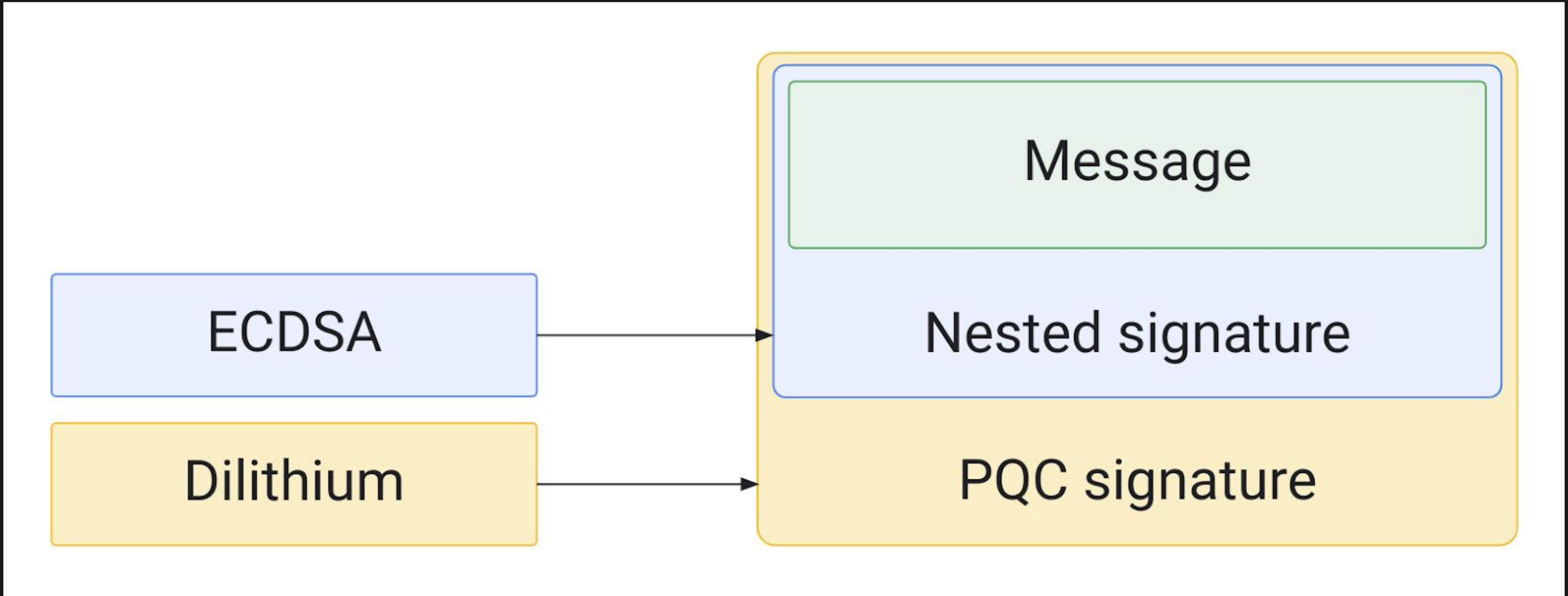
introducing noise to encodings and using probability to decode

CRYSTALS-Kyber (key encapsulation) and
CRYSTALS-Dilithium (signatures)



https://security.googleblog.com/2023/08/toward-quantum-resilient-security-keys.html

In Chrome, you can now enable ‘X25519Kyber768’ for key exchange during TLS

32 bits generated by X25519	32 bits generated by Kyber768

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https://openquantumsafe.org/

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Microsoft Brings Post-Quantum Cryptography To Windows And Linux In Early Access Rollout

Quantum Computing Business   Matt Swayne • May 21, 2025

https://thequantuminsider.com/2025/05/21/microsoft-brings-post-quantum-cryptography-to-windows-and-linux-in-early-access-rollout

what I hope to see

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more diverse quantum-resilient cryptosystems

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more diverse quantum-resilient cryptosystems

quantum-resilient hardware tokens

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quantum-resilient hardware tokens

wider accessibility & rollout

wrapping up

how we got here

how we got here

RSA & ECDSA

how we got here

RSA & ECDSA

...and how quantum breaks them

how we got here

RSA & ECDSA

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what's next

Asymmetric Cryptography: A Deep Dive

Eli Holderness
@eli.holderness.dev
they/them/theirs

sources: history

https://www.redhat.com/en/blog/brief-history-cryptography

sources: RSA + group theory

https://ee.stanford.edu/~hellman/publications/24.pdf

https://weakdh.org/imperfect-forward-secrecy-ccs15.pdf

https://en.wikipedia.org/wiki/Padding_(cryptography)

sources: ECC

https://scholar.rose-hulman.edu/cgi/viewcontent.cgi?article=1389&context=rhumj

http://koclab.cs.ucsb.edu/teaching/ecc/eccPapers/Washington-ch04.pdf

http://www.secg.org/sec2-v2.pdf

sources: QC & Shor

https://research.kudelskisecurity.com/2021/08/24/quantum-attack-resource-estimate-using-shors-algorithm-to-break-rsa-vs-dh-dsa-vs-ecc/

https://arxiv.org/pdf/quant-ph/9508027.pdf

https://www.omnicalculator.com/math/power-modulo

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https://thequantuminsider.com/2025/05/21/microsoft-brings-post-quantum-cryptography-to-windows-and-linux-in-early-access-rollout/