Sample Questions and Problems for the Final Exam (Intro to Numerical Mathematics Math 3620/5620)

Topics to be covered by the Exam (this may still change slightly). The final exam will be comprehensive.

1. The exam will cover the following main topics (included in Chapters 1-8, 11 of the textbook):
2. Floating-point arithmetic (significant digits, absolute and relative error, roundoff error, normalized floating-point representation, single and double precision, underflow and overflow, relative machine precision, loss of significance, subtractive cancellation)
3. Exercises: p.15 (2,3,4), p.32 (4,5,25), p.52 (3), p.53 (15, 18), p.63 (1), p.65 (26)
4. Solving nonlinear equations (bisection method, Newton's method)
5. Exercises: p.123 (8), p.136 (1,3)
6. Polynomial interpolation and numerical differentiation (polynomials, evaluation of polynomials by Horner's scheme – nested multiplication, truncated Taylor series, truncation error, Lagrange and Newton forms of interpolating polynomials, divided differences, divided difference table, error formula for polynomial interpolation, basic numerical differentiation formulas, the conflict between subtractive cancellation and the truncation error, Richardson extrapolation)
7. Exercises: p.174 (1–7), p.186 (6,10,16), p.198 (1-4,6), p.199 (13,18), p.200 (24(a))
8. Numerical integration (trapezoid rule, Simpson's rule, and the idea behind Gauss quadrature)
9. Exercises: p.214 (1,2,8), p.237 (1,4), p.248 (1,2(a),5,6), p.249 (10)
10. Linear systems of equations (selected topics from Chapters 2 and 8): Naive Gauss elimination (§2.1), Scaled partial pivoting (§2.2), Banded matrices, tridiagonal matrices (§2.3), Matrix factorizations (§8.1: LU, LDL, Cholesky, and basic definition of Singular value decomposition), Iterative methods (§8.4: Jacobi, Gauss-Seidel, definition of SOR, analysis of convergence using the notion of spectral radius).
11. Exercises: 2.1 (3a,3b,5,6,), 2.2 (1,2,3,6,8), 2.3 (4,5,6), 8.1 (1a,2a,6,9,10), 8.4 (3,4,5,7,8)
12. Splines (§6.1 and §6.2): Definition of splines in one variable. Natural splines (optimality property and algorithm for computing them).
13. Exercises: 6.1 (1,10,12,15,16,21), 6.2 (1,2,4,5,6,7a,12,13,15,19,32,33,39)
14. Initial Value problems (§7.1, §7.2, §7.5): Taylor series methods, Euler method, local truncation error and global error, Runge-Kutta methods, idea (only) of Adams-Bashforth-Moulton methods.
15. Exercises: 7.1 (9,10,12), 7.2 (3,7a,8,9,18,)
16. (Assuming we have time to cover this) Boundary value problems (§11.2): Finite-difference methods
17. Exercises: 11.2 (1,2,7,8)