Problem 3

Let L be lower triangular, U be upper triangular, and D be diagonal.

1. Suppose L and U are both unit triangular and is diagonal.
2. In the case that D is singular, no. If for example, then L and U can be any lower and upper triangular matrices.
3. In the case that D is invertible, yes. Not that U must be lower triangular as:

And we have proven that inverse of lower triangular matrices are lower triangular. Similarly, L is upper triangular:

Therefore must be diagonal.

1. Suppose E = LDU is nonsingular and diagonal. Then L and U are diagonal.

* Note that L, D, U must be invertible as:
* So we have again,

Implying by the same reasoning both are diagonal.

1. Suppose L and U are both unit triangular and if LDU is diagonal, does it follow that L = U = I?

* In the case that D is singular, no, as D = 0, leaving L and U unconstrained unit triangular matrices
* In the case D in nonsingular, we know L and U are diagonal, and since they are unit triangular, that diagonal must be of 1’s. So

Problem 5

Code for Cholesky:

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Output:

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Problem 6

1. Code:

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1. Does Gauss-Siedel converge on this systems for all values of c?

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Clearly when , the matrix is no longer diagonally dominant, and no longer theoretically guaranteed to converge for any initial vector . But it does converge for smaller values of .

Based on these data points, it seems like it is necessary for this particular matrix to be diagonally dominant for Gauss-Siedel to converge, perhaps because if it is not diagonally dominant, it is not positive definite.

* Note: Varying n did not change the values of c for which this matrix converged

When the Gauss Siedel method does converge (c = 3), increasing doe not seem to affect number of iterations need to converge dramatically:

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1. Performing the computation we get that

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* Therefore the iterative process (Gauss-Siedel) converges as the spectral radius of B is less than 1.

On the other hand when c=2, the spectral radius is computed to be 12, and so the Gauss-Siedel method does not converge.

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