

MATH240 – Lecture 1

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1 Set Theory

$A = \{0, 1, 5\}$ finite set with 3 elements

1.1 \in (is element of)

Notation: $x \in A \rightarrow x$ is element of A

$$\begin{aligned} A &= \{0, 1, 2, 3\} \\ 0 &\in A \quad 42 \notin A \end{aligned}$$

1.2 Set by extension

$$\begin{aligned} \emptyset &= \{\} \\ \mathbb{N} &= \{0, 1, 2, \dots\} \\ \mathbb{Z} &= \{\dots, -2, -1, 0, 1, 2, \dots\} \\ \mathbb{Q} &= \text{Rational numbers (fractions)} = \{0, \frac{1}{2}, \frac{2}{3}, 5, \frac{-42}{11}, \dots\} \end{aligned}$$

1.3 Set by comprehension

Notation: $A = \{x \in U \mid P(x) \text{ is true}\}$

$$\begin{aligned} \text{even numbers: } E &= \{\dots, -4, -2, 0, 2, 4, \dots\} \\ &= \{x \in \mathbb{Z} \mid x = 2n \text{ for some } n \in \mathbb{Z}\} \\ \text{odd numbers: } O &= \{\dots, -3, -1, 3, \dots\} \\ &= \{x \in \mathbb{Z} \mid x = 2n + 1 \text{ for some } n \in \mathbb{Z}\} \\ \text{also: } &= \{n + 1 \mid n \in E\} \\ &= \{x \in \mathbb{Z} \mid x = n + 1 \text{ for some } n \in E\} \\ \text{rational numbers: } \mathbb{Q} &= \{\frac{a}{b} \mid a \in \mathbb{Z}, b \in \mathbb{Z}, b \neq 0, b > 0, GCD(a, b) = 1\} \end{aligned}$$

1.4 Subsets

If every element of set A is also element of set B , then A is subset B

Notation: $A \subseteq B$

$$A = \{0, 1, 5\} \subseteq \mathbb{N}$$

$$\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q}$$

2 sets are equal if $A \subseteq B$ and $B \subseteq A$: $A = B$

1.5 Cardinality

Number of elements in a set

Notation: $|A|$

$$A = \{1, 3, 5\} \quad |A| = 3$$