Enlai Li - 2023/01/04

Set Theory

A set is a collection of objects of the same nature

Examples:

- $A = \{0, 1, 5\}$
- $\emptyset = \{\}$
- $\mathbb{N} = \{1, 2, 3, 4, ...\}$
- $\mathbb{Z} = \{..., -3, -2, -1, 0, 1, 2, 3, ...\}$

Notation: $x \in A$ means "x is an element of A"

Ex:

- $3 \in \mathbb{N}$
- $-4 \notin \mathbb{N}$
- $x \notin \emptyset$ (no matter what x is)

> All set above were defined by extension, which means, by listing their individual elements. But this is not always a good method...

Ex: $\mathbb{Q} = \{0, \frac{1}{2}, \frac{2}{3}, \frac{-42}{11}, ...\}$ *What's the pattern?*

We can also define sets by comprehension , i.e. by restricting a larger set to its elements with a certain property:

$$S = \{x \in A | p(x)\}$$

ex:

- even numbers: $E = \{n \in \mathbb{Z} \mid n = 2k \; for \; some \; k \in \mathbb{Z}\}$
- odd numbers: $O = \{n \in \mathbb{Z} \mid n = 2k + 1 \; for \; some \; k \in \mathbb{Z}\}$
- rational numbers: $\mathbb{Q} = \{\frac{a}{b} \mid a \in \mathbb{Z}, \; b \in \mathbb{N} \; and \; GCD(a, b) = 1\}$

Subsets: if every element of A is also an element of B, we say that A is a <u>subset</u> of B, and we write $A \subseteq B$

ex: $\{1, 2, 3\} \in \mathbb{N} \in \mathbb{Z}$

> We say that $\mathbb{Z} \subseteq \mathbb{Q}$, since each $n \in \mathbb{Z}$ corresponds to the rational number $\frac{n}{1}$ $\emptyset \subseteq X$ for any set X

Equality: 2 sets A and B are equal (written A = B) if $A \subseteq B$ and $B \subseteq A$

ex: $\{1, 2, 3\} = \{3, 1, 2\}$

Cardinality: