MATH240 – Lecture 2

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1 Set algebra

When are sets equal? For instance:

$$A = \{x \in \mathbb{Z} \mid x = 2k - 1 \text{ for some } k \in \mathbb{Z}\}$$
$$B = \{x \in \mathbb{Z} \mid x = 2n + 1 \text{ for some } n \in \mathbb{Z}\}$$

We need to prove:

1. $A \in B$
2. $B \in A$

1. NTS (need to show): if $x \in A$ then $x \in B$

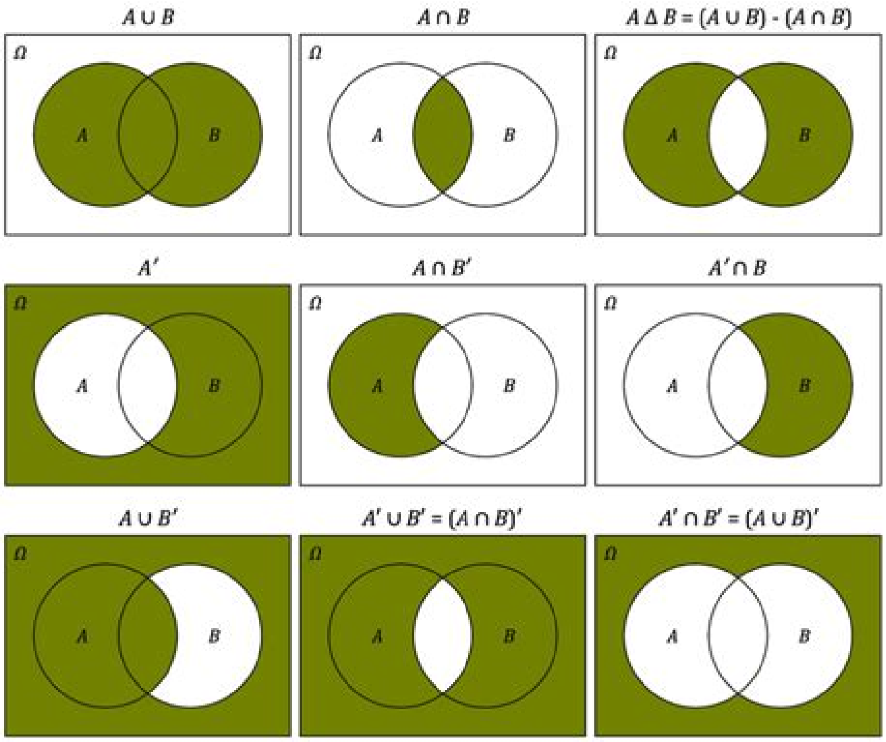
$$\text{Assume } x \in A \text{ so } x = 2k - 1 \text{ for some } k \in \mathbb{Z} = 2k - 2 + 2 - 1 = 2(k-1) + 1$$
$$\text{With } n = k - 1 \text{ we have } x = 2n + 1$$
$$\text{therefore } A = B$$

2. $x \in B \Rightarrow x \in A$ Let $x = 2n + 1$ where $n \in \mathbb{Z}$, then

$$\begin{aligned} x &= 2n + 2 - 2 + 1 \\ &= 2(n+1) - 1 \end{aligned}$$

If $k = n + 1$ then $x = 2k - 1 \Rightarrow x \in A$

2 Set operations



$U =$ The universe of objects

- Union

$$A \cup B = \{x \in U \mid x \in A \text{ or } x \in B\}$$

- Intersection

$$A \cap B = \{x \in U \mid x \in A \text{ and } x \in B\}$$

- Difference

$$A \backslash B = A - B = \{x \in A \mid x \notin B\}$$

- Complement

$$\overline{A} = A' = \{x \in U \mid x \notin A\} = U \backslash A$$

ex:

$$\begin{aligned} A &= \{1, 2, 3\} \\ U &= \mathbb{N} \\ \overline{A} &= \{0, 4, 5, \dots\} \end{aligned}$$

- Symmetric Difference

$$\begin{aligned} A \oplus B &= \{x \in U \mid x \in A \text{ or } x \in B \text{ , but not both } \} \\ &= (A \backslash B) \cup (B \backslash A) \\ &= (A \cup B) \backslash (A \cap B) \\ &= (A \cup B) \cap (A \cup B) \end{aligned}$$

  ex:

$$\begin{aligned} A &= \{1, 2, 3\} \\ B &= \{3, 4, 5\} \\ A \oplus B &= \{1, 2, 4, 5\} \end{aligned}$$

3 Set identities

Theorem: If two sets A and B have the same Venn diagram, then $A = B$
Proof: We need to show $A \subseteq B$ and $B \subseteq A$ (They have similar proof)

1. $A \subseteq B$
   Let $x \in A$, then x is in the shaded region of A's Venn diagram. So it's also in shaded version of B's Venn diagram, so $x \in B$

2. $B \subseteq A$
   Let $x \in B$, then x is in A's region. So it's also in B's region, so $x \in A$

$\square$

3.1 Laws of boolean algebra

- Identify law:

$$\begin{aligned} A \cup \emptyset &= A \\ A \cap U &= A \end{aligned}$$

- Idempotent laws:

$$\begin{aligned} A \cup A &= A \\ A \cap A &= A \end{aligned}$$

- Domination laws:

$$\begin{aligned} A \cup U &= U \\ A \cap \emptyset &= \emptyset \end{aligned}$$

- Double complement laws:

$$\overline{\overline{A}} = A$$

- Commutative laws:

$$A \cup B = B \cup A$$
$$A \cap B = B \cap A$$

- Complement laws:

$$A \cup \overline{A} = U$$
$$A \cap \overline{A} = \emptyset$$