MATH240 – Lecture 1

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1 Set Theory

$A = \{0, 1, 5\}$ finite set with 3 elements

1.1 $\in$ (is element of)

Notation: $x \in A \to x$ is element of A

$$A = \{0, 1, 2, 3\}$$
$$0 \in A \qquad 42 \notin A$$

1.2 Set by extension

$$\emptyset = \{\}$$
$$\mathbb{N} = \{0, 1, 2, \ldots\}$$
$$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$$
$$\mathbb{Q} = \text{ Rational numbers (fractions) } = \{0, \frac{1}{2}, \frac{2}{3}, 5, \frac{-42}{11}, \ldots\}$$

1.3 Set by comprehension

Notation: $A = \{x \in U \mid P(x) \text{ is true}\}$

$$\begin{aligned}
\text{even numbers: } E &= \{\ldots, -4, -2, 0, 2, 4, \ldots\} \\
&= \{x \in \mathbb{Z} \mid x = 2n \text{ for some } n \in \mathbb{Z}\} \\
\text{odd numbers: } O &= \{\ldots, -3, -1, 3, \ldots\} \\
&= \{x \in \mathbb{Z} \mid x = 2n + 1 \text{ for some } n \in \mathbb{Z}\} \\
\text{also: } &= \{n + 1 \mid n \in E\} \\
&= \{x \in \mathbb{Z} \mid x = n + 1 \text{ for some } n \in E\} \\
\text{rational numbers: } \mathbb{Q} &= \{\frac{a}{b} \mid a \in \mathbb{Z}, b \in \mathbb{Z}, b \neq 0, b > 0, GCD(a, b) = 1\}
\end{aligned}$$

1.4 Subsets

If every element of set $A$is also element of set B, then A is subset B
Notation: $A \subseteq B$

$$A = \{0, 1, 5\} \subseteq \mathbb{N}$$
$$\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q}$$

2 sets are equal if $A \subseteq B$ and $B \subseteq A$: $A = B$

1.5 Cardinality

Number of elements in a set
Notation: $|A|$

$$A = \{1, 3, 5\} \qquad |A| = 3$$

1.6 Russel set paradox

Russel set: $R = \{x \text{ is a set} \mid x \notin x\}$
ex:

$$\mathbb{N} \notin \mathbb{N}, \text{ so } \mathbb{N} \in \mathbb{R}$$
$$x = \{1, 2, x\} \text{ satisfies } x \in x \Rightarrow x \notin \mathbb{R}$$

Question: is $R \in R$?

- If $R \in R$ then $R \notin R$
- If $R \notin R$ then $R \in R$