

# Set Theory

A set is a collection of objects of the same nature

Examples:

- $A = \{0, 1, 5\}$
- $\emptyset = \{\}$
- $\mathbb{N} = \{1, 2, 3, 4, \dots\}$
- $\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$

Notation:  $x \in A$  means "x is an element of A"

Ex:

- $3 \in \mathbb{N}$
- $-4 \notin \mathbb{N}$
- $x \notin \emptyset$  (no matter what x is)

All set above were defined by extension, which means, by listing their individual elements.  
But this is not always a good method...

Ex:  $\mathbb{Q} = \{0, \frac{1}{2}, \frac{2}{3}, \frac{-42}{11}, \dots\}$  *What's the pattern?*

We can also define sets by comprehension, i.e. by restricting a larger set to its elements with a certain property:

$$S = \{x \in A \mid p(x)\}$$

ex:

- even numbers:  $E = \{n \in \mathbb{Z} \mid n = 2k \text{ for some } k \in \mathbb{Z}\}$
- odd numbers:  $O = \{n \in \mathbb{Z} \mid n = 2k + 1 \text{ for some } k \in \mathbb{Z}\}$
- rational numbers:  $\mathbb{Q} = \{\frac{a}{b} \mid a \in \mathbb{Z}, b \in \mathbb{N} \text{ and } GCD(a, b) = 1\}$

**Subsets:** if every element of  $A$  is also an element of  $B$ , we say that  $A$  is a subset of  $B$ , and we write  $A \subseteq B$

ex:  $\{1, 2, 3\} \subseteq \mathbb{N} \subseteq \mathbb{Z}$

We say that  $\mathbb{Z} \subseteq \mathbb{Q}$ , since each  $n \in \mathbb{Z}$  corresponds to the rational number  $\frac{n}{1}$ .  $\emptyset \subseteq X$  for any set  $X$

**Equality:** 2 sets  $A$  and  $B$  are equal (written  $A = B$ ) if  $A \subseteq B$  and  $B \subseteq A$

ex:  $\{1, 2, 3\} = \{3, 1, 2\}$

**Cardinality:**