MATH240 – Lecture 4

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1 Negating a sentence

Ex:

p = you can access the internet on campus only if you are a student or an employee

I = you can access the internet on campus

S = you are a student

E = you are an employee

$$p = I \Rightarrow (S \vee E)$$

Let's find a sentence for $\overline{p}$

$$\begin{aligned} \neg p &= \neg(I \Rightarrow (S \vee E)) \\ &= \neg(\neg I \vee (S \vee E)) \text{ (Def of } \Rightarrow) \\ &= \neg\neg I \wedge \neg(S \vee E) \text{ (De Morgan)} \\ &= I \wedge (\neg S \wedge \neg E) \text{ (De Morgan + Double negation)} \end{aligned}$$

$\neg p$ = You can access the internet on camput and you are not a student and you are not an employee

2 Four types of logical formulas

A logical formula is:

- satisfiable if its truth table contains at least one T
- a contradiction if not satisfiable (always false)
- falsifiable if its truth table contains at least one F
- a tautology if not falsifiable (always true)

ex1: $p \wedge \neg p$: contradiction

contradictions are always falsifiable

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

ex2: $p \vee \neg p$: tautology

tautology are always satisfiable

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

ex3: $p \iff q$: satisfiable and falsifiable

p	q	$p \iff q$
T	T	T
T	F	F
F	T	T
F	F	T