

# MATH240 – Lecture 1

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## 1 Set Theory

$A = \{0, 1, 5\}$  finite set with 3 elements

### 1.1 $\in$ (is element of)

Notation:  $x \in A \rightarrow x$  is element of  $A$

$$\begin{aligned} A &= \{0, 1, 2, 3\} \\ 0 &\in A \quad 42 \notin A \end{aligned}$$

### 1.2 Set by extension

$$\begin{aligned} \emptyset &= \{\} \\ \mathbb{N} &= \{0, 1, 2, \dots\} \\ \mathbb{Z} &= \{\dots, -2, -1, 0, 1, 2, \dots\} \\ \mathbb{Q} &= \text{Rational numbers (fractions)} = \{0, \frac{1}{2}, \frac{2}{3}, 5, \frac{-42}{11}, \dots\} \end{aligned}$$

### 1.3 Set by comprehension

Notation:  $A = \{x \in U \mid P(x) \text{ is true}\}$

$$\begin{aligned} \text{even numbers: } E &= \{\dots, -4, -2, 0, 2, 4, \dots\} \\ &= \{x \in \mathbb{Z} \mid x = 2n \text{ for some } n \in \mathbb{Z}\} \\ \text{odd numbers: } O &= \{\dots, -3, -1, 1, 3, \dots\} \\ &= \{x \in \mathbb{Z} \mid x = 2n + 1 \text{ for some } n \in \mathbb{Z}\} \\ \text{also: } &= \{n + 1 \mid n \in E\} \\ &= \{x \in \mathbb{Z} \mid x = n + 1 \text{ for some } n \in E\} \\ \text{rational numbers: } \mathbb{Q} &= \{\frac{a}{b} \mid a \in \mathbb{Z}, b \in \mathbb{Z}, b \neq 0, b > 0, GCD(a, b) = 1\} \end{aligned}$$

## 1.4 Subsets

If every element of set  $A$  is also element of set  $B$ , then  $A$  is subset  $B$

Notation:  $A \subseteq B$

$$A = \{0, 1, 5\} \subseteq \mathbb{N}$$

$$\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q}$$

2 sets are equal if  $A \subseteq B$  and  $B \subseteq A$ :  $A = B$

## 1.5 Cardinality

Number of elements in a set

Notation:  $|A|$

$$A = \{1, 3, 5\} \quad |A| = 3$$

## 1.6 Russel set paradox

Russel set:  $R = \{x \text{ is a set} \mid x \notin x\}$

ex:

$$\mathbb{N} \notin \mathbb{N}, \text{ so } \mathbb{N} \in R$$

$$x = \{1, 2, x\} \text{ satisfies } x \in x \Rightarrow x \notin R$$

Question: is  $R \in R$ ?

- If  $R \in R$  then  $R \notin R$
- If  $R \notin R$  then  $R \in R$