MATH240 – Lecture 5

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1 Predicate Logic

p = "All men are mortal"

is an atomic proposition from the point of view of propositional logic (cannot be decomposed into simpler propositions). It still has some structure:

- "men" and "mortal" are predicates
- "All" is a quantifier
- "are" is a copula

A predicate is something that is true, or false, about a subject (which may vary)
ex: "mortal"

- Socratos is mortal: True
- Zeus is mortal: False

Mathematically, a predicate is a function

$$P : x \in U \to Bool : \{T, F\} \quad U : \text{Universe of discourse}$$
$$\text{Subject} \ \to \ \text{Truth value}$$

In predicate logic, instead of dealing with propositional variables, we deal with predicates, P(x) and our variables x range over any universe U

$$P(x) : "x \geq 0" \to \text{ When you give a value to x, you find a proposition}$$
$$P(2) : "2 \geq 0" : True$$
$$P(-1) : " -1 \geq 0" : False$$

Quantifiers:

- $\forall$: "for all": Universal quantifier
- $\exists$: "there exists": Universal quantifier

We can bind a variable (in a proposition) to a quantifier, so that it can no longer be freely set