An LMI-based approach for the control of semi-active magnetorheological suspensions

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Abstract: We address modeling and control of semi-active suspensions based on magnetorheological (MR) fluids. We first introduce a saturation-based model of the underlying nonlinear phenomena, and combine the MR suspension with a quarter car model whose parameters resemble a SUV-like vehicle. Then we present simulation results to test the passenger comfort arising from the passive configuration (namely holding the input constant over the whole simulation) and the use of the Skyhook controller (which is adapted from its typical use in electrohydraulic suspensions). Finally, we propose LMI-based control designs exploiting an approximated model and typical generalized sector conditions for saturation nonlinearities. The simulation results show the advantages of the proposed design.

Keywords: Semi-active suspensions, vehicle dynamics, magnetic suspension, ride comfort, passive suspensions

1. INTRODUCTION

Semi-active suspensions correspond to shock absorbers whose dynamical nature can be manipulated by way of some suitable control inputs with the constraint that no energy can be injected into the system (as opposed to active suspensions that can inject energy, see Savaresi et al. (2010)). These suspensions have become popular in the automotive field because they are cost-effective solutions that can be employed to considerably increase the driving comfort.

The first semi-active suspensions technology consisted in the so-called linear adaptive configuration, wherein the viscous damping coefficient c of an electro-hydraulic (EH) shock absorber can be adjusted in real-time between a bounded (and positive) set of allowable ranges $[c_{\min}, c_{\max}]$, by manipulating the size of the orifices within the suspension. While the passive EH configuration is fully linear (because viscous damping is a linear phenomenon), the input c of the linear adaptive solution enters nonlinearly in the vehicle vertical dynamics, thereby posing interesting control design challenges for improved passenger comfort and/or road handling. Different approaches have been proposed for the control of such devices; among them, switching strategies (Savaresi and Spelta, 2007), optimal controls (Savaresi et al., 2004; Poussot-Vassal et al., 2006), MPC (Canale et al., 2006) and LPV approaches (Poussot-Vassal et al., 2008) can be mentioned.

A different semi-active suspensions technology is based on the magnetorheological (MR) effects of certain fluids (Wereley, 2013), usually oil. When subjected to a magnetic field, the oil exhibits an increased viscosity, to the point of becoming a viscoelastic solid: such an effect is employed to change the equivalent damping force exerted by the suspension, without the need of changing its geometry. The control input in this case is once again nonlinear and corresponds to the yield stress of the fluid, accurately adjusted by varying the magnetic field intensity with a simple electromagnet. The peculiarity of the magnetorheological technology lies in the highly non-linear characteristic of its damping force. Typically, it appears that the non-linear characteristic resembles the overall effect of a constant viscous term, plus a saturation-like term, whose amplitude is proportional to the control input.

Even in the passive configuration (namely when the control input is held constant), the magnetorheological suspension provides a nonlinear behaviour: as a consequence, controlling these devices is an even more complicated task. There are two main streams in the scientific literature: on the one side state-of-the-art solutions developed for the EH technology have been reused in the MR case, see *e.g.* Yao et al. (2002); Corno et al. (2019). On the other side, researchers also developed custom control solutions explicitly addressing the nonlinearity of the MR technology, see *e.g.* Du et al. (2005); Turnip et al. (2008); Zapateiro et al. (2011).

In this paper we set in the latter mainstream, by proposing a model-based controller design for a semi-active MR suspension, to improve the passenger comfort. With respect to the available literature, the MR suspension is here

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modelled by exploiting the saturation nonlinear elements. As a result, the control problem is addressed using the Linear Matrix Inequalities (LMIs) framework stemming from regional sector properties of the saturation function, as summarized in Tarbouriech et al. (2011). The proposed control strategy is preliminarily validated against a passive configuration and a Skyhook strategy, a consolidated switching approach for the control of EH semi-active suspensions.

The reminder of the paper is as follows: in Section 2 the vehicle and the MR suspension model are introduced, along with the road generation and the performance cost function. The benchmark strategies are presented in Section 3 and the proposed LMI-based solution is given in Section 4. The simulations in Section 5 show the effectiveness of the proposed controller and its characteristics. Concluding remarks are finally given in Section 6.

2. SYSTEM MODELLING AND PERFORMANCE METRIC

2.1 Quarter-car model

In order to test the proposed control strategy and compare it with known state-of-the-art solutions, a quarter car model is used; this approach is a consolidated procedure in the scientific community (see Savaresi et al. (2010)), when dealing with vehicle vertical dynamics as it can yield, through a relatively simple model, a proper overview of the expected performances, especially when a comparative study of different control strategies is assessed.

The quarter car model is schematically depicted in Figure 1: it features a sprung mass (m_s) representing a quarter of the chassis mass that is supported by one suspension, an unsprung mass m_u representing the wheel, and finally the suspension element which connects them. The differential equations that describe it follow:

$$\begin{cases} m_s \ddot{z}_s = -k\tilde{z} - f_d(\tilde{z}) \\ m_u \ddot{z}_u = -k_t(z_u - z_r) + k\tilde{z} + f_d(\dot{\tilde{z}}), \end{cases} \tag{1}$$

where $\tilde{z} := z_s - z_u$. In equations (1), the suspension is modelled as a linear spring, coupled with the damper whose force depends only on the suspension stroke speed $\dot{\tilde{z}}$, as further elaborated in the following.

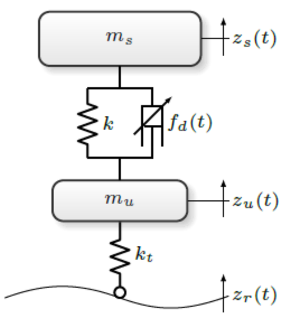

Fig. 1. Schematic representation of the quarter car model.

The quarter car model parameters, listed in Table 1, are tuned to match those of a SUV-like vehicle. Indeed such vehicles benefit from the use of MR suspensions for the vertical dynamics control: on the one side their ground clearance is typically increased to allow for an off-road usage, which significantly excites the vertical dynamics. On the other side, their high sprung mass value, naturally coupled with a decrease of the natural frequency of the chassis oscillations, requires significant damping forces at low stroke speeds, which is one of the distinguishing features of MR dampers.

Parameter	Symbol	Value
Unsprung Mass	m_u	$70\,Kg$
Sprung Mass	m_s	$450\,Kg$
Suspension stiffness	k	$27000\,\frac{N}{m}$
Tire stiffness	k_t	$300000\,\frac{N}{m}$
Minimum damping	$c_{\min}$	$800\,\frac{Ns}{m}$
Saturation slope	k_0	$38000\,\frac{Ns}{m}$
Maximum saturation level	$\tilde{f}_{\max}$	$3000\,N$

Table 1. Quarter car and MR suspension model parameters.

2.2 MR semi-active suspension model

Modelling a semi-active magnetorheological suspension is a known topic in the scientific literature. There are two distinguishing features of such devices:

(1) A highly nonlinear force-speed relationship, made up by a pre-yield and a post-yield region: in the former the force increases drastically with the stroke speed whereas in the second one the force exerted by the device bends. Thanks to the semi-active capability, the pre-yield region can be increased or decreased: for this reason semi-active MR dampers can produce a significant force even at very low stroke speeds.
(2) A hysteretic behaviour highly dependent on the fluid properties, that can be more or less pronounced and which is usually neglected when dealing with control oriented models, see for example Du et al. (2005); Do et al. (2012); Turnip et al. (2008); Zapateiro et al. (2011); Pepe et al. (2019)

In the present work the MR damper is modelled focusing on the nonlinear force-speed relationship, neglecting the possible hysteretic behaviour of the device. This choice is consistent with the classical scientific literature approach, as discussed, and with the objective of this paper, namely highlighting the benefits of designing a force-speed shape aware control algorithm, as compared to the state-of-the-art solutions that are typically agnostic with respect to the device characteristics. Within this perspective, avoiding the inclusion of other nonlinear effects (the hysteretic behaviour) allows focusing on the possible benefits of the proposed approach.

The upper plot of Figure 2 shows an example of the typical force-speed curves of a magnetorheological suspension, for different values of applied current. It can be seen that when the pre-yield region is null, the suspension characteristic resembles a linear one (red dash-dotted line); for higher current values, the pre-yield region expands with a significant increase of the damping force.

The mathematical model used in this work to describe the force-speed nonlinearity $f_d(\dot{\tilde{z}})$ of the MR damper is

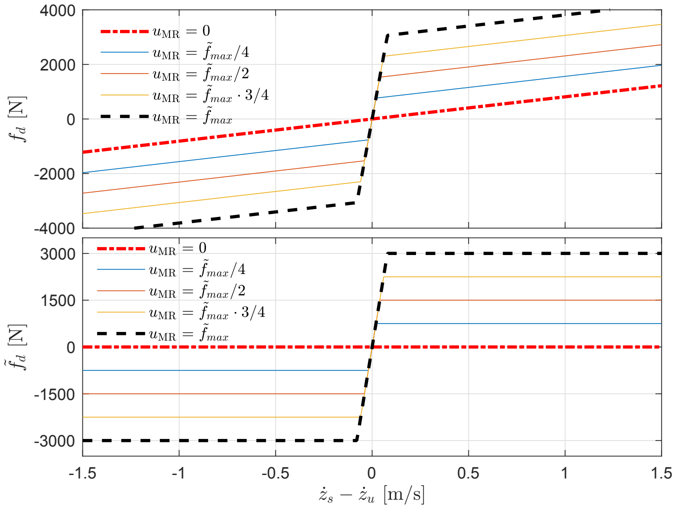

Fig. 2. Typical force-speed curve associated to a MR suspension (upper plot) and the corresponding nonlinearity (lower plot).

$$f_d = c_{\min}(\dot{z}_s - \dot{z}_u) + \tilde{f}_d(u_{\mathrm{MR}}, \dot{z}_s - \dot{z}_u). \tag{2}$$

which is composed by two elements: a linear one, which represents the minimum damping (the red dashed-dotted line in the upper plot of Figure 2) and the nonlinear term $\tilde{f}_d$ which accounts for the nonlinearity of the suspension. In particular, the nonlinear term can be described by the saturation-like function shown in the bottom plot of Figure 2, modelled as:

$$\tilde{f}_d(u_{\mathrm{MR}}, \dot{z}_s - \dot{z}_u) = \mathrm{sat}_{u_{\mathrm{MR}}}(k_0(\dot{z}_s - \dot{z}_u)), \quad u_{\mathrm{MR}} \in [0, \tilde{f}_{\max}]. \tag{3}$$

where $\mathrm{sat}_{\bar{u}}(s) := \max\{\min\{s, \bar{u}\}, -\bar{u}\}$ represents the symmetric scalar saturation function having limits $\pm\bar{u}$, and the constant k_0 corresponds to the slope of the saturation function near the origin. Notice that in equation (3) the control variable of the semi-active suspension has been explicitly defined, and corresponds to the saturation level u_{MR}.

The parameters that completely define equations (2) and (3) are listed in Table 1: they have been chosen according to the results presented in Gołdasz and Dzierżek (2016), to represent an average automotive MR damper.

2.3 Performance characterization and road definition

The performances of the passive suspension and the semi-active control strategies are compared in terms of passenger perceived comfort. As widely accepted in the scientific literature, a means to quantitatively evaluate this aspect is the index J corresponding to the *rms* of the output acceleration $y = \ddot{z}_s$:

$$J(\ddot{z}_s) = \left(\frac{1}{T} \int_0^T |\ddot{z}_s(\tau)|^2 d\tau \right)^{\frac{1}{2}}, \tag{4}$$

where T is the duration of the simulation (or experiment).

The driving comfort is evaluated by simulating the quarter car model (1) for a road profile z_r generated according to the ISO-8608 standard, as presented in Agostinacchio et al. (2014). An equivalent implementation of the proposed approach consists in defining the road profile by filtering a Gaussian noise $n(t)$ through the transfer function $G_r(s)$ (see Figure 3). The transfer function $G_r(s)$ is defined as:

$$G_r(s) = \frac{k_r s}{s^2 + 2\xi_r \omega_r s + \omega_r^2}, \tag{5}$$

where the parameter ξ_r is a constant ($\xi_r = 0.7$) and ω_r depends on the velocity as follows:

$$\omega_r = \frac{2\pi v}{l_c}. \tag{6}$$

The role of the transfer function G_r is to shape the frequency content of the Gaussian broad-band noise, in order to fit the typical power spectral density of a road. The parameter l_c is usually constant and represents the maximum spatial resolution considered: large values of l_c allow considering low frequency variations of the road profile (like hills or long slopes, which might be interesting when energy-management strategies are addressed), whereas smaller values of l_c result in almost flat roads with distributed micro-roughness. The parameter k_r accounts for the road damage severity. All the simulations and the results of the next sections are achieved by considering a road profile of class C-D (representing a light off-road condition, see Agostinacchio et al. (2014)), with a velocity of $v = 90\frac{km}{h}$, resulting in a value of $k_r = 0.025$. Figure 4 shows an example of the considered road profile, as a function of the longitudinal travelled distance.

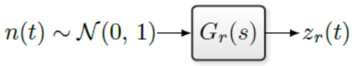

Fig. 3. Road surface generation scheme.

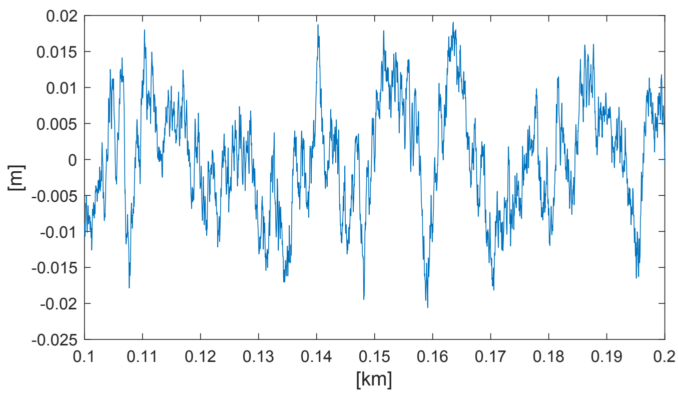

Fig. 4. Example of a generated road profile.

Since the road is generated as a realization of a random vector, our results might change from a simulation to another. In order to avoid this variability, instead of repeating the simulations and averaging the obtained results, the considered road has been selected to be sufficiently long (that is, $2.5km$ long) so that the functional J directly performs the required averaging.

2.4 Overall system model

Combining all the elements of the simulation framework discussed above, it is possible to write a unified expression for the considered system. By defining the state variable vector $x = [z_s \ \tilde{z} \ \dot{z}_s \ \dot{\tilde{z}}]^\top$, considering as external input the excitation input z_r coming from the road, and as the performance output the vertical acceleration $y = \ddot{z}_s$, the overall system can be described by the following state-space equations:

$$\Sigma_{\mathrm{MR}} : \begin{cases} \dot{x} = A(c_{\min})x - B\tilde{f}_d(u_{\mathrm{MR}}, \dot{\tilde{z}}) + Ez_r \\ y = \ddot{z}_s = C(c_{\min})x - D\tilde{f}_d(u_{\mathrm{MR}}, \dot{\tilde{z}}) \end{cases} \tag{7}$$

with

$$\left[\begin{array}{c|c|c} A(c_{\min}) & B & E \\ C(c_{\min}) & D & \end{array}\right] =$$

$$\left[\begin{array}{cccc|c|c} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -\dfrac{k}{m_s} & 0 & -\dfrac{c_{\min}}{m_s} & \dfrac{1}{m_s} & 0 \\ \dfrac{k_t}{m_u} & -\dfrac{k}{m_s}-\dfrac{k_t+k}{m_u} & 0 & -\dfrac{c_{\min}(m_s+m_u)}{m_u m_s} & \dfrac{m_s+m_u}{m_u m_s} & -\dfrac{k_t}{m_u} \\ \hline 0 & -\dfrac{k}{m_s} & 0 & -\dfrac{c_{\min}}{m_s} & \dfrac{1}{m_s} & \end{array}\right]$$

The proposed model is generically nonlinear, due to the saturation that defines $\tilde{f}_d(u_{\mathrm{MR}}, \dot{\tilde{z}})$; when $u_{\mathrm{MR}} = 0$ the model turns into a linear one, with minimum suspension damping, equal to c_{min}.

3. PASSIVE AND SEMI-ACTIVE BENCHMARKS

The control strategy proposed in Section 4 is tested against two classical benchmarks when dealing with semi-active suspensions: the fully passive and the Skyhook semi-active control strategy.

3.1 Passive benchmark

The basic comparison of any semi-active control strategy is the corresponding passive suspension one. It is well-known that the choice of the suspension damping, in the passive case, undergoes a performances trade-off and the best comfort performances are found for intermediate (neither too low, nor too high) damping values. Thus, a sensitivity analysis of the comfort index with respect to different, constant, values of the control variable u_{MR} has been performed: the results are shown in Figure 5. As a matter

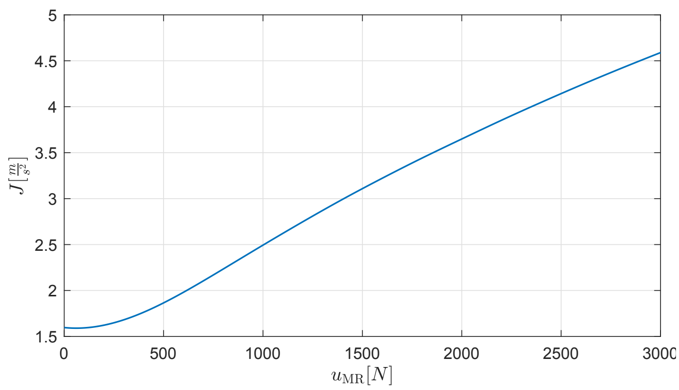

Fig. 5. Sensitivity analysis of the comfort index J with different passive MR damping curves, obtained for different constant selections of the control variable $u_{\mathrm{MR}} \in [0,\, \tilde{f}_{max}]$.

of fact, the best performances obtained for the passive case ($J \cong 1.6\frac{m}{s^2}$) correspond to setting $u_{\mathrm{MR}} = 0$. This result is consistent with the selected value of the linear damping c_{min} which is, as a matter of fact, very close to the optimal value that can be obtained for a linear suspension; within this perspective it is clear that introducing an additional MR damping force in the system (by using values of $u_{\mathrm{MR}} > 0$) worsens the comfort performances in the same way as it would happen for the linear case.

3.2 Skyhook benchmark

The Skyhook semi-active controller is a classical comfort-oriented control strategy developed for semi-active suspensions, Savaresi et al. (2010). In particular, the so called two-state Skyhook is a switching control strategy that switches the suspension damping between its minimum and maximum values, according to the following law:

$$c = \begin{cases} c_{\min}, & \text{if } \dot{z}_s\dot{\tilde{z}} \leq 0 \\ c_{\max}, & \text{if } \dot{z}_s\dot{\tilde{z}} > 0. \end{cases} \tag{8}$$

Despite being conceived and designed for linear semi-active suspensions (typically realised with the Electro-Hydraulic technology) this control strategy has been directly applied also to the MR suspensions, as recalled in the introduction. In the present work, we consider this strategy as another benchmark to be compared with the LMI-based solutions of the next sections. In the magnetorheological configuration, rather than switching between two damping factors $c_{\min}$ and $c_{\max}$, the control input u_{MR} in (3) switches directly between 0 and $\tilde{f}_{max}$. In this way, when $u_{\mathrm{MR}} = 0$ the total saturation contribution in (2) is zero and the only force injected into the system is the minimum linear one. Instead, when $u_{\mathrm{MR}} = \tilde{f}_{max}$, the term $\tilde{f}_d$ in (2) saturates at the maximum allowable level (i.e. $3000N$), according to the same logic used in the linear case. In summary, to adapt the two-states Skyhook algorithm to the MR system, the control switching logic becomes:

$$u_{\mathrm{MR}} = \begin{cases} 0, & \text{if } \dot{z}_s\dot{\tilde{z}} \leq 0 \\ \tilde{f}_{max}, & \text{if } \dot{z}_s\dot{\tilde{z}} > 0 \end{cases} \tag{9}$$

When simulating the quarter-car model with the control law (9) the resulting performance index is $J \cong 2.35\frac{m}{s^2}$, which is higher than the passive benchmark. On the one side this result is consistent with the Skyhook formulation. Indeed, the Skyhook is designed to limit the chassis velocity, rather than its accelerations: it is a known drawback of this solution that when the difference between the minimum and the maximum damping increases, some performance worsening due to the large accelerations induced by the damping switches should be expected. However, such a performance worsening is amplified in the MR case, mainly due to the nonlinearities of the force-speed maps of these dampers.

The obtained result motivates the development of the MR control strategy proposed in the next section, which explicitly includes the shape of the force-speed maps in the controller design.

4. LMI-BASED CONTROLLER

The results of the previous section show that the two-states skyhook controller merely applied to the MR suspension is not effective at improving the mere passive

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