

**Organization ←→ Communication
Emerging Perspectives V
The Renaissance in Systems Thinking**

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*Social Science Applications
of Nonequilibrium
Thermodynamics: Science or
Poetry? Procedures for the
Precise Measurement of
Energy in Social Systems*

*Joseph Woelfel**
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**NONEQUILIBRIUM THERMODYNAMICS AS
METAPHOR**

Illya Prigogine's pioneering analysis of the thermodynamics of nonequilibrium

* The basic research for this work was performed while the author was a Senior Fellow at the

physical processes not only had important implications for physical scientists, but promised revolutionary impact on the social sciences as well. Fundamentally, the applicability of the model to social settings can be understood in the light of the Benard Instability (Prigogine & Nicolis, 1977). The Benard instability can result from a vertical temperature gradient in a horizontal liquid layer. The lower face of the layer is maintained at a fixed level higher than the temperature of the upper level by an external energy source. This sets up a permanent heat flux from bottom to top. This far-from-equilibrium system is highly nonlinear, and can move abruptly to one of a multiplicity of stable states. Whether the stable heat flux does move to another state, and which state that might be, can be determined by extremely small random fluctuations in the system and its environment. In contrast, an equilibrium or near-equilibrium system has, in general, only one stable state. When perturbed slightly from equilibrium, the system will inevitably return to the single stable state.

Both physical and social scientists were quick to recognize the rough similarity between the situation described by the Benard Instability and everyday social and cultural phenomena. Loye and Eisler (1987), for example, considered "systems breakdowns" of two types to be candidates for description by nonequilibrium thermodynamic theory: microcosmic crises and discontinuities such as financial crises, food crises, political and military crises and the like, and macrocosmic crises, or "the great, overriding churning of history" of the type Polyani (1994) called "the great transformation." Smith and Gemmill (1991) rethought Lewin's (1947) classic formulation in terms of nonlinear thermodynamics to account for any kind of social restructuring in small groups. Contractor and Seibold (1993) described group decision support systems as examples of nonlinear thermodynamic systems. Woelfel and Fink (1980) considered human cognitive processes as thermodynamic systems, and Fink and Chen (1995) discussed organizational climate following these authors. Jang and Barnett (1994) considered the network of interconnected corporations a nonequilibrium thermodynamic system.

On closer examination, however, only a select few of the hundreds of discussions of nonequilibrium thermodynamics actually apply the theory to the phenomena investigated. By far, the largest majority of such applications use the theory only metaphorically, treating the phenomena modeled "as if" they were nonequilibrium thermodynamic processes like the Benard Instability. The gross characteristics of a non-equilibrium thermodynamic system are taken as analogs to similar gross characteristics of social phenomena. Thus, we can model macro-

cosmic social and cultural systems to be nonequilibrium thermodynamic systems; if they were such, they would indeed be expected to "churn," and we should expect "great transformations." Similarly, if group decision support systems are nonequilibrium thermodynamic systems, we should expect abrupt transitions and shifts, even system failures. These predictions, of course, are easily confirmed, lending credence to the hypothesis that social phenomena are indeed nonequilibrium thermodynamic processes. However, recognizing that social phenomena are instances of nonequilibrium thermodynamic processes is a far cry from successfully modeling them as such in a useful way. Even in cases where serious mathematics and equations are utilized, the equations themselves generally model only the gross characteristics of the phenomena, and seldom make precise predictions about outcomes. Even when they do, the absence of measurement procedures for the phenomena in question turns the equations into metaphor.

Qualitative vs. quantitative models

The essence of any thermodynamic model is energy. The fundamental laws of thermodynamics are written as functions of energy, and the differences among various existing thermodynamic systems is entirely describable in the way in which energy flows through the system. It may be well argued, then, that no successful model of any phenomenon at all can be written in the absence of a satisfactory definition of energy within that system and its surroundings. To be successful, such a definition must, at a minimum, specify how energy is to be measured. In the absence of a clear and consensual definition of energy, and in the absence of any agreement over procedures for the measurement of energy, any model based on thermodynamic principles will always be simply metaphorical, and, as such, poetry rather than science¹.

What is needed is a definition of energy within social systems that lends itself to precise measurement. In what follows, I try to produce a complete system that allows for the measurement of energy within the domain of cognitive processes.

A MECHANICAL MODEL

The development of mechanics itself rests on certain formal logical advancements that communication theorists for the most part have not yet adopted. In this chapter, specifically, it is my intention to attempt to model certain com-

¹ Those few papers which do produce successful models of social phenomena as thermodynamic processes of any type all come from a theoretical perspective—the Galileo Model—which enjoys a clear and measurable definition of energy. See Barnett & Woelfel, 1988; Woelfel, Barnett, Pruzek & Zimmelman, 1989; Kincaid, et. al., 1983; Kincaid, 1988.

munication phenomena in terms of some of the most powerful of mechanical forms, following what are most frequently called *variational principles*. At the same time, I hope to illustrate the logical developments required for such a step and to show how communication theory has, as yet, failed to pass through these developmental stages. The development of a variational mechanical model of any phenomena depends specifically on four logical operations. The first of these is what I have called elsewhere the Cartesian measurement model (Woelfel, 1977). The second is the development of functional representations of variables in terms of the ratios given by measurements (Barnett, 1982). The third operation requires the recognition and analysis of residual terms in the functions, and the fourth requires the stipulation of minimization of stationary principles. Once these steps have been accomplished, the equations of motion or change for any configuration of the system of variables under consideration can always be written down.

The Cartesian Measurement Model

The first step in the development of the mechanics of any phenomena is the adoption of the Cartesian measurement model. This model consists entirely in arbitrary agreements among participating scholars to measure phenomena as ratios to an arbitrary standard unit².

We have described the Cartesian or ratio method of measurement elsewhere in greater detail (Woelfel, 1977; Woelfel & Fink, 1980), but it is useful here to point out the fundamental difference between this model and the categorical logic. The categorical logic is a logic of inclusion and exclusion based on category membership. Reasoning or deduction is possible only on the basis of certain types of overlaps among categories. Thus, it is evident that if Socrates is a man, then he is mortal because the entire category *man* is included in the category *mortal*. However, it is not possible to determine whether or not Shiela is a man on the basis of knowing she is featherless, because only part of the category *featherless* overlaps with the category *man*. In category logic, only three outcomes are possible: yes, no, and doubtful; that is, no answer. An equation or syllogism in category logic is either correct, wrong, or inconclusive.

In Cartesian or *comparative* logic, however, relationships are not only expressed in terms of inclusion and exclusion, but as infinitely variable numerical proportions or ratios. Thus, we may say that B is 2.6 times as long (or bright, or friendly) as A. Furthermore, if C is 5.2 times as long (or bright, or friendly) as

² Descartes, of course, did not invent this method, but we refer to it here as the Cartesian model due to his explicit formal recognition of this model as a complete alternative logic in contradistinction to the categorical logic of Plato, Aristotle, and Aquinas who dominated the intellectual arena prior to the Renaissance.

A, then C is twice as long as B.

There are two advantages to this system that are necessary for competent mechanics. First, this system can carry more information in a quantitative sense than the categorical logic; it is *much more informative*. Secondly, because it is quantitative rather than categorical, when deductions based on this system are wrong (that is, when they yield outcomes that do not correspond to measurements), they are not only wrong, as would be an erroneous deduction within a categorical syllogism, but they are *wrong by a measurable quantity*. Deductions within a categorical logic are either wrong or right, but each deduction within a comparative logic leaves a *residual term*. If the deduction is perfectly correct, the residual term is zero; as errors increase, the residual term grows larger. The residual term is (important as we will see) because it not only provides a basis for knowing how accurate the logical system may be in any particular instance, but because it also provides, at once the basis for correcting errors and developing general principles for dealing with the phenomena in the future.

As we will see, every principle of mechanical physics depends on comparative measurement and comparative logic, and no principle of physics is stated in any other terms than comparative terms. The measurement model of communication is still categorical, however, it cannot be described as "mechanical."

Functional representation and time

Just as the logic of categorical systems is based on the syllogism, the logic of comparative measurement is based on the *function*, a word introduced in 1694 by Leibnitz. The most elementary function within comparative logic is the function by which the position of an element is determined. As we suggested earlier, the length of an object (or its friendliness or brightness, or any attribute) is expressed as a ratio to some arbitrary length (or unit of length, unit of brightness, unit of friendliness, etc.). This arbitrary unit may be thought of with no loss in generality as a coordinate axis, and the position of the element to be measured is then given as a (ratio) function of the original arbitrary element. Thus, if the element of length is the meter, and we imagine a coordinate axis laid in the direction of the meter, the position of the endpoint of the element to be measured is given by:

$$y = f(m)$$

where, y is the length of the unit to be measured, f is the function, and m is the length of the unit (one meter). If the element to be measured is twice as long as the unit, we could write the function more explicitly as:

$$y = 2m$$

Less well-known is the fact that the Cartesian measurement model is com-

pletely abstract and rests on no empirical foundation; hence, it may be applied without modification to the measurement of any experience, even an experience as abstract and as personal, for example, as affection³ (Hamblin, 1973; Woelfel, 1977; Woelfel & Fink, 1980). We may choose as an arbitrary standard unit, for example, the amount of affection one feels for the average stranger, and represent the amount of affection one feels for any arbitrary person or object as a function of this standard. We might like a friend twice as much, for example, and thus, write:

$$y = f(x) \quad (1)$$

where y is the amount of affection we feel for our friend, f is the function equal to 2, and x is the amount of affection we feel for the average stranger. So, we write again:

$$y = 2x \quad (2)$$

This, of course, is the same functional form we wrote earlier to describe the length of an arbitrary unit or the position of an arbitrary object. The logical form is independent of the experience it represents. In order to illustrate the development of mechanics of communication rather than physical experience, we will continue to refer to affection toward a friend as the dependent variable, but it should be recalled that we might just as well refer to a physical point.

So far, by adopting this system of measurement over the categorical system, we have gained some advantage in that it is more informative to say how much we care for the friend than to simply say that we care. Yet, we gain a second advantage as important as the first—the ability to define new variables by applying functional reasoning. Suppose, for example, that we measure the attitude again at a later time and obtain a new value, y_1 . (We will now designate the old value as y_0 to avoid confusion). This allows us to create the new variable:

$$\Delta y = y_1 - y_0 \quad (3)$$

We create the variable *time* by the same ratio method,⁴ and designate the time of

³ Some communication researchers balk at the idea that ratio level measurement can be used as effectively for human data as other, cruder measures, such as Likert-type, semantic differential-type scales, or even simple ordinal measures. We have discussed these matters in detail elsewhere (Woelfel & Fink, 1980) as have others (Barnett, Hamblin & Danowski, 1981; Hamblin, 1973; Shinn, 1974; Stevens, 1951) and will not say more here, except to suggest that among those who are familiar with the empirical evidence about ratio measurement or "magnitude estimation" as it is often called, there remains little enthusiasm.

⁴ It is worth noting that time is measured as ratios to a standard interval of time (usually the second), and time, of course, is at least as abstract as affection, yet time is the most precisely measured variable in science.

the first measurement as t_1 and the second as t_0 so that:

$$\Delta t = t_1 - t_0 \quad (4)$$

Combining these two functions, we may create yet another new variable:

$$v = \Delta y / \Delta t \quad (5)$$

where v is the *ratio* of the change in attitude to the change in time, or thus the *rate* of change of the attitude, or velocity of attitude change. In fact, this variable represents the average rate of change of the attitude over the interval (t , and leaves us ignorant of the rate of change or velocity at any precise moment, but we can solve this difficulty by imagining the time interval (t growing smaller without limit and, as it does so, the average velocity across the (even smaller) interval can be made as accurate an approximation as we like to the instantaneous velocity. This instantaneous velocity is referred to as the *time derivative* of the position (or attitude) as is symbolized by

$$\lim_{\Delta t \rightarrow 0} \frac{\Delta y}{\Delta t} = \frac{dy}{dt} \quad (6)$$

so that

$$v_t = \frac{dy}{dt} \quad (7)$$

Thus, v_t represents the instantaneous velocity or rate of change of the attitude at time t^5

Inertial principles and residual terms

It should be clear that Equation (7) defines the velocity of the attitude or its rate of change at a specific instant of time, and we are free to measure the velocity at

⁵ This notation was first published by Leibnitz in 1677, and the same procedure was independently developed about the same time by Newton precisely to deal with experiences of a *processual* nature, because they both believed that ordinary language was inadequate for the description of processes. To this day, the calculus remains the most powerful language known for describing processes. Although many communication scholars lay heavy emphasis on the processual nature of communication phenomena, and criticize contemporary communication theory and method for dealing with processes clumsily, the calculus is virtually non-existent in Communication journals and books, and very few human communication theorists have any knowledge of it at all. Thus, while we may agree that contemporary human communication theory deals with processes in a clumsy fashion, we cannot agree that it is because communication theory is mechanical, since a mechanics without ratio level measurement and the calculus could not rise much beyond the Greeks on logical grounds alone.

any number of such instants. This in turn gives rise to the possibility that the velocity itself might change over time, and hence, becomes, itself, a function of time, or:

$$v = f(t). \quad (8)$$

It is at this point that arbitrary but important stipulations or principles are introduced into a theory if it is to be a *mechanical* theory. These stipulations are rules in the sense that Cushman and Tompkins (1980) speak of them, but they are rules that scientists agree to honor solely for the purpose of developing a common framework for understanding and communicating their experiences. When they were originally postulated, their inventors and followers usually thought they were discoveries about the nature of reality, but later theorists realized they are, in fact, definitions of a frame of reference against which we gauge our experience. The first of these (usually called Newton's First Law) says that for any system that is left alone, (that is, which is not in communication with its near surroundings) there will be no changes in velocity. We might write:

$$v = f(t) = \text{constant} \quad (9)$$

to indicate that velocity is a constant of time, or using the notation of the calculus:

$$a_t = \frac{dv}{dt} = 0, \quad (10)$$

where a_t equals the *acceleration* of the attitude at time t , which means that the change in velocity with regard to the change in time (acceleration) is zero. This principle is not meant, of course, to say that changes in velocity are impossible, but merely that any such change must be accounted for by means of some communication of the moving element with its environment. If we evaluate the derivative of the velocity, with regard to time, at some point in time (i.e., enter numerical measurements into Expression (10)) and find that it does not equal zero, but rather equals:

$$a_t = \frac{dv}{dt} = 0 + k, \quad (11)$$

then the variable k expresses the magnitude of the communication from the environment. The actual value of this variable will depend on the arbitrary units in which v and t are themselves measured. If, perhaps at some other time or for some other sample of data, we evaluate the same expression and find:

$$a_{t1} = \frac{dv^1}{dt^1} = 0 + k_1, \quad (12)$$

then the *ratio* k/k_1 , will *not* depend on the units of measure, and will represent the

ratio of forces impressed on the system from the environment at the two times or for the two samples.

We have written Expressions (11) and (12) in what may seem a strange form to illustrate that these forces are measured as residual or leftover terms from what our stipulations would have led us to expect had the system been left alone; we will see again and again that what we call "explanatory variables" are always residual terms. *The process of scientific explanations always requires us to set up our reference frames in such a way that the residual terms are minimized, and to seek processes involving communication between the changing element and its environment that are correlated with the residual terms.*

One of the most important residual terms in mechanics is *force*, and force may be determined up to arbitrary constant by the method just described as

$$\frac{F_1}{F_2} = \frac{K}{K_1} = \frac{a_t}{a_{t1}} \quad (13)$$

This means that the ratio of forces is defined as the ratio of the observed accelerations. In this sense, force as a residual term is defined as "that which produces acceleration." As real as the concept of force has come to be to us after centuries of usage, we should nonetheless realize that there is no such entity in nature as "force," but rather force is a logical device which is wholly a product of the arbitrary conceptual system we have created to describe our experiences. It has as its counterpart one of the other most important residual variables in mechanics, inertial mass, which is defined up to an arbitrary constant as the inverse ratio of the accelerations of any two elements under a constant force, or

$$\frac{m_1}{m_2} = \frac{A_2}{A_1} \quad (14)$$

where

m_1 = the mass of the 1st element

m_2 = the mass of the 2nd element

a_1 = the acceleration of the 1st element

a_2 = the acceleration of the 2nd element

It is easy enough to deduce from (14) that mass is the reciprocal of force, or "that which resists acceleration." Mass does not mean "quantity of matter" as Newton thought it did, but is instead a residual term which accounts for the differential acceleration of different elements under constant force. There is no way to measure the mass of any body short of pushing it with a known (i.e., previously measured) force and measuring its acceleration relative to the acceleration of other elements (or attitudes) subject to the same force. Mass is not a quality of material objects, and it is every bit as sensible to say that some attitudes are harder to accelerate (have more mass) than others as it is to say that some material objects are harder to accelerate than others.

As long as we deal with only one attitude, there is no need for inertial mass to

enter the equations, since inertial mass only refers to the resistance an attitude offers to acceleration relative to other attitudes. When we consider more than one attitude, however, we must modify equation (11) to include the mass of the attitude. If the mass does not vary with the time, i.e., if

$$m = f(t) = \text{constant},$$

so that

$$\frac{dm}{dt} = 0, \quad (15)$$

we may write (11) as

$$m \frac{dv}{dt} = F \quad (16)$$

or

$$ma = F$$

which is Newton's Second Law. We should not consider this a discovery about nature, however, since it is a consequence of our stipulation given in (10).

If the mass does depend on the time, then we must modify expression (14) to make it somewhat more general, and we obtain

$$\frac{d}{dt} (mv) = F. \quad (17)$$

Once again, this generalized equation should not be taken as a discovery about the nature of the world, but rather as a statement of principle. In words, equation 17 says that neither the mass nor the velocity nor both of them together ought to be expected to change spontaneously, but rather any change must be considered the result of the application of some force. The quantity in brackets in Equation (17) is called the linear momentum of the attitude, and (17) asserts that any changes in linear momentum must be the result of some communication with the environment; in fact, it must be equal to the magnitude of that communication.

D'Alembert made this point explicitly by suggesting we write (17) as:

$$F - \frac{d}{dt} (mv) = 0, \quad (18)$$

and create the variable:

$$I = - \frac{d}{dt} (mv), \quad (19)$$

which allows us to write:

$$F + I = 0 . \quad (20)$$

We are able to interpret this new variable I as the force due to the motion of the attitude, or what we may call the inertial force, although we may refer to the force resulting from communication with the environment as the impressed force. Thus, Equation (20) may be interpreted as a principle that says the impressed forces plus the inertial forces (forces due to the motion) will sum to zero. As Lanczos says (1962):

A given system of impressed forces will generally not be in equilibrium. This requires the fulfilling of special conditions. The total virtual work of the impressed forces will usually be different from zero. In that case, the *motion* of the system makes up for the deficiency. The body moves in such a way that the additional forces, produced by the motion, bring the balance up to zero. In this way, d'Alembert's principle gives *the equations of motion of an arbitrary mechanical system* (Emphasis in original). (p. 90)

So far, by adopting the Cartesian measurement rule, we were able to construct two quantitative primitive variables—position (or length) and time. From these and the functional method of comparative logic, we have derived the additional descriptive variables—velocity and acceleration—the additional explanatory variables—force and mass—and the mixed variable—momentum. None of these variables have any epistemological status other than as creations of the logical system, but nonetheless, they are of great utility in constructing consensual and informative statements about our experiences. By saying that these variables have no epistemological status, I mean that their form does not depend exclusively on any inherent characteristics of the experiences out of which they are modeled, and so they may be applied to any experiences, physical or otherwise. In fact, it may be that what we call physical those experiences that have already been cast successfully into the form of these variables. Two other variables will be of great interest to us. The first of these can be obtained again through the logic of functional analysis by multiplying Equation (16) by the velocity, dx/dt to obtain:

$$vF = vma = \frac{m}{2} \cdot \frac{d}{dt} (vv) = \frac{d}{dt} \left(\frac{mv^2}{2} \right), \quad (21)$$

and, multiply through by dt :

$$Fdx = d \left(\frac{mv^2}{2} \right). \quad (22)$$

The quantity on the left side of Equation (22) is the product of the force and an infinitesimal distance, and represents the work done by the force through the interval of distances dx , although the quantity on the right hand side represents the differential element of the kinetic energy, so that:

$$T = \frac{mv^2}{2} \quad (23)$$

defines a quantity called the kinetic energy, or the energy that an element (or attitude) has by virtue of its motion. With this in mind, we can understand (22) as a definition that requires the differential element of the work to be equal to the differential element of the kinetic energy. The work done, of course, is to change the magnitude of the kinetic energy. If the attitude moves through no force, there is no change in the kinetic energy.

Although the mechanical system is not yet complete, it has already become useful as a vehicle for recording and ordering experiences. We may find, for example, after we deflect the attitude from its original position x_0 to another position x_1 that, even though subsequently left alone (that is, even though no communication between the attitude and its environment takes place), nevertheless the attitude tends to return to its original position. Unlike the relationships we have discussed so far, this outcome is not required by the internal logic of the mechanical system, and we might just as well find that the attitude is "content" to remain wherever we put it. Therefore, the extent to which an attitude tends to return to its starting point represents an empirical finding. In fact, some research has observed this effect (Woelfel, Holmes, Cody, & Fink, 1977), although it is outside the scope of this chapter to discuss the empirical character of the attitude domain. If this should be the case, however, we can obviously suggest that the work done moving the attitude from x_1 to x_2 should be equal and opposite to the work required to move it back again from x_2 to x_1 . Given this fact, along with the relationship between work and kinetic energy established in (22) and (23), we may see that the work required to move the attitude from x_1 to x_2 is given by

$$\int_{x_1}^{x_2} Fdx = T_{x_2} - T_{x_1} = - \int_{x_2}^{x_1} Fdx = T_{x_1} - T_{x_2} \quad (24)$$

where T_{x_2} is the kinetic energy of the attitude at x_2 , T_{x_1} is the kinetic energy of the attitude at x_1 , and

$\int_{x_1}^{x_2} Fdx$ is the total work done through the interval x_1-x_2 .

Equation (24) means that the total work exerted through the distance from x_1 to x_2 equals the change in the kinetic energy. Once again, if the force is zero (or perpendicular to the direction from x_1 to x_2), no work is done and the kinetic energy remains constant.

From our definition (16) we know that the attitude will not accelerate back towards its starting point unless exposed to some force, but, because there is some point at which the attitude is stable, we know that the restoring force is itself a function of the position of the attitude, or:

$$F = F(x),$$

then we can see that the sum of these forces acting through differential elements of distance between any two points will behave similarly to (24). We introduce the new quantity, $V(x)$, which is given by:

$$V(x) = \int_x^{x_s} F(x) dx = - \int_{x_s}^x F(x) dx = , \quad (25)$$

where $V(x)$ is called the *potential energy*, and s is selected as a standard reference point. Potential energy is that energy an element (or attitude) possesses by virtue of its position, and as long as the forces acting on the attitude depend only on its position, we may write, following (24) and (25):

$$\begin{aligned} T_{x_2} - T_{x_1} &= \int_{x_1}^{x_s} F(x) dx + \int_{x_s}^{x_2} F(x) dx \\ &= \int_{x_s}^{x_1} F(x) dx + \int_{x_s}^{x_2} F(x) dx \\ &= V_{x_1} - V_{x_2}. \end{aligned} \quad (26)$$

From this, we can see that:

$$T_{x_1} + V_{x_1} = T_{x_2} + V_{x_2} = T + V = E$$

where E is a constant called the *total mechanical energy*. As long as no forces from the environment impinge on the attitude system, this quantity, the total energy, remains a constant, or is conserved.

Neither kinetic nor potential energy exist in nature other than as a consequence of the comparative logic used and the stipulations or principles laid down, but they provide extremely useful conceptual structures against which we may gauge our experiences, particularly when defined in such a way as to remain invariant under conditions of no communication with the environment. Both kinetic and potential energy are built up of several orders of ratios from the original ratios on which measured values are established, and so, are complete consequences of the character of the comparative logic.

Another such variable, *linear momentum*, which was defined in (17) is also conserved during conditions of isolation from its surroundings. If we multiply (16) by dt , we obtain:

$$F dt = m \frac{dv}{dt} dt = d(mv) = dp, \quad (26)$$

where p is the linear momentum.

Integrating between two times, t_1 and t_2 yields:

$$\int_{t_1}^{t_2} F dt = P_{t_2} - P_{t_1}, \quad (27)$$

where the quantity on the left hand side of (27) is called the impulse delivered by the force F during the interval from t_1 to t_2 , Equation (27) defines the change in linear momentum over the interval of time to be equal to the impulse delivered by the force, and simultaneously asserts that the linear momentum will remain invariant if no force from the environment is present.

This system of variables, as well as the interrelationships they obey, constitutes a pattern against which we may express our experiences. Therefore the definitions we have presented of conserved quantities within the system should not be taken as arguments that such conservative systems exist because no system that actually exists is absolutely conservative. This is equivalent to saying that no empirical system is absolutely isolated from its environment, and, furthermore, is an acknowledgment that the relationships any system has with its near surroundings are likely to be too complicated to be expressed in minute detail. To the extent that quantities like energy and momentum are not found to be conserved in an empirical system, nonconservative forces must be postulated. These departures from the conservative ideal are therefore attributed to "nonconservative forces," that may in turn be characterized as forces that cannot be ascribed to some specific action of the environment. As Triffet (1968) suggests, "...nonconservative forces can usually be recognized by the fact that they are not gradients of time-independent scalar potential functions." Lanczos (1962) calls them "polygenic" for the same reason. In any event, insofar as they are residuals or "leftovers," they are nonetheless subject to calculation. In general, they will be found to be functions of time or velocity, and so, we may modify Equation (16) to yield:

$$ma - F^c = 0 \quad (28)$$

where F^c represents conservative forces, showing that the inertial forces (ma) minus the conservative forces equal zero when the system is not under the influence of any nonconservative forces. This makes it easy to take account of time-dependent nonconservative forces by means of the expression:

$$ma - F^c = F(t), \quad (29)$$

and similarly, we may take account of velocity-dependent nonconservative forces by writing:

$$ma - f^c = F(v). \quad (30)$$

There is, of course, no limit to the complexity of the situation that may be dealt

with in this way, and we may consider nonconservative forces that depend on both the velocities and the time by setting:

$$ma - F^c = F(v, t), \quad (31)$$

where the nonconservative function F can take on any form.

Equation (29) does not constitute so much a discovery as a means of searching, for it indicates that some anomaly has been detected in the system's performance. What is actually accomplished is simply the naming of the anomaly (force) and its expression as a function of time. In this situation, time serves as a surrogate variable against which other processes may be calibrated so that they might be compared to each other.

Time-dependent forces are commonplace in communication phenomena, particularly when they are cyclical, as they often are (for example, in daily or seasonal fluctuations); they can easily be controlled, even if unexplained (See Barnett, Chang, Fink, & Richards, 1991).

Velocity-dependent nonconservative forces are also common, and frequently depend on the velocity in a fairly simple way. Forces of friction or the resistance due to a viscous medium are examples of velocity-dependent forces that are relatively simple functions of the velocity for physical systems, as is random error in message transmission within human communication, or random forgetting in cultural processes.

As a simple example of nonconservative forces within a communication system, consider a single attitude that has been dislodged from its equilibrium position by some message. The notion that an equilibrium position exists means that there is a potential function; that is, the force acting on the attitude depends on its position. We further assume that random forces proportional to the velocity of attitude change must be considered (although it is useful for purposes of the example to consider them completely unknown); and so, we may write:

$$ma + Cv + kv = 0, \quad (32)$$

which means that the system will be in equilibrium under the forces due to inertia (ma), the nonconservative velocity-dependent forces (Cv) and the restoring forces due to position (kv), of course, assuming there is no other communication with the environment. The constant C represents the magnitude of the resistive force per unit of velocity, and the constant K represents the force per unit of deflection from the equilibrium point (assuming such forces are linear).

Equation (32) clearly shows that the acceleration of the attitude will be zero at equilibrium, but the further the attitude is deflected from equilibrium, the stronger the forces that attempt to restore it to equilibrium. The more massive the attitude, the less quickly it will respond to those restoring forces. The higher the velocity of the attitude change, the greater the resistive forces slowing the change.

There is good reason to believe this equation (or one much like it) describes (at least to an order of approximation) the process of attitude change in humans and cultures. The restoring force is necessitated by the commonplace assumption in communication theory that there exists an equilibrium position for beliefs, attitudes, or, indeed whole cultures. This requires position-dependent potentials. The mass term recognizes the empirical fact that some attitudes, beliefs, and cultural elements are harder to change than others, and we simply specify some scalar value, based on measured resistance to change, which quantifies that differential resistance. The velocity-dependent force is required because, as a little thought will show, otherwise, the system, once deflected from the equilibrium position, would oscillate endlessly around the equilibrium point.

This Equation (32) models the system when it is isolated from its environment—when no communications are being received from the surround. It is easy, using the comparative logic, to extend (32) to the case in which communication from the surroundings is present, so that:

$$ma + Cv + Kx = F \quad (33)$$

where F represents the force of the external communication. If the external communication varies with the time, we may write:

$$ma + Cv + Kx = F(t). \quad (34)$$

Equation (34), therefore, expresses the state of motion of the attitude at any time as a function of the communication received from the surroundings.

SYSTEMS OF ATTITUDES AND BELIEFS

Up to this point, we have dealt only with a single attitude, which may be interpreted as a single "object" (a friend) whose position varies on a single dimension or attribute, (in this case, "affection"). The comparative logic, however, makes it possible to generalize this case to any degree of complexity. As a first step, we may consider the case of several objects varying independently along a single attribute. By "independently," we mean that the change in position of any one of the objects has no effect over the position of any of the other objects.

In this case, we may generalize (32) to the set of n independent equations:

$$m_i a_i + C_i v_i + K_i x_i = 0 \quad (35)$$

where i is $1, 2, \dots, n$, and n is the number of objects.

Similarly, Equations (33) and (34) can be generalized by adding the index i to the force term. If the motions of the set of n objects are not independent of each

other, however, that is, if the motion of each object influences the motion of each of the others, then the motion of any object j depends on the characteristics of the others, and we may write:

$$m_{ij}a_j + C_{ij}v_j + K_{ij}x_j = 0, \quad (36)^6$$

where ij is $1, 2, \dots, n$, or, for the case in which the system is in communication with its surroundings,

$$m_{ij}a_j + C_{ij}v_j + K_{ij}x_j = F_j, \quad (37)$$

or, when the communication with the surroundings is time-dependent,

$$\begin{aligned} m_{ij}a_j + C_{ij}v_j + K_{ij}x_j &= F_j(t), \\ i, j &= 1, 2, \dots, n. \end{aligned} \quad (38)$$

While equations represented in (38) are quite compact, they describe a very complex system of interrelationships. Briefly the equations in (38) say that the state of motion of any of the attitudes j is determined by the masses of the j^{th} attitude and all $n-1$ of the others, by the resistive forces due to the velocities of all n particles, by the $n(n-1)/2$ pairwise forces expressing the mutual interactions of their potential energies due to their positions, and by the forces from outside the system working directly on the j^{th} attitude.

As complicated as they are, equations (38) deal only with motions along a single dimension or attribute, as we mentioned earlier. In a realistic individual or cultural belief system, the definition of each object involves positions and motions with regard to many attributes, and so the equations in (38) remain incomplete. If each of the attributes is perpendicular (independent) to each of the others, then we may proceed simply by adding another index l to (38), but it is possible to show a more general procedure that will work regardless of whether the attributes are orthogonal or not, or even in the case of general curvilinear coordinates.

As we have seen, the state of motion of a system of beliefs may be described completely in terms of its potential energy, kinetic energy and outside (impressed) forces. This as we have seen is not a description of "reality" but rather a consequence of the logic of the descriptive system which is called "mechanical." When a belief changes within more than one dimension, the comparative logic system, based as it is on ratios, allows us to establish the *proportions* of the kinetic energy, potential energy and forces which are projected on each of these dimensions. For the case of kinetic energy these variables may be projected on the two generalized coordinates axes (dimensions) q_1 and q_2 . The vector V represents the (arbi-

⁶We adopt here the summation convention, so that repeated indicies are to be summed over, and the summation sign may be omitted.

trary) velocity of an arbitrary belief of mass m. The component V_1 of V projected on q_1 is given by the ratio V/V , or, by an elementary result of geometry, $V_1 = \cos \theta$. Working in the other direction, V itself may be given in terms of its components along the axes q_1 and q_2 by the generalized theorem of Pythagoras

Multiplying through by $1/2m$ gives the kinetic energy as a function of its projections on the generalized coordinates. When the generalized coordinates are linear and orthogonal (i.e., independent), $\theta = 0$ and the scalar product term $2V_1V_2 \cos\theta$ in (39) vanishes and the components of the kinetic energy become linearly additive. Even if the coordinates P_1 and P_2 are curved in an arbitrary way, (39) will still be true for arbitrarily small elements, and we may write:

$$dV^2 = dV_1^2 + dV_2^2 - 2dV_1V_2 \cos\theta. \quad (40)$$

where the d 's refer, as previously, to the differential element. The component of the velocity projected on the i^{th} axes is actually a ratio of infinitesimal elements, controlling for the projections on the other axes, which is represented by the partial derivative:

$$\frac{\partial}{\partial} \frac{V}{q_i}, \quad (41)$$

and similarly, the component of the kinetic energy with regard to each of the generalized coordinates will be given by the same formalism as:

$$\frac{\partial}{\partial} \frac{T}{q_i}, \quad (42)$$

as will the components of the potential energy:

$$\frac{\partial}{\partial} \frac{V}{q_i}. \quad (43)$$

These expressions (41), (42) and (43) are ratios of infinitesimal arcs of the curved coordinates, controlling for all the other such ratios, and are therefore called *partial differential slopes*. They are analogous to partial linear regression coefficients, except that they are generalized for the nonlinear case.

Now that we have established the way in which the kinetic and potential energies will project on any arbitrary set of generalized coordinates, it only remains to establish in a precise way how these same energies are related to the motion of a system on one dimension and a generalization to any number of arbitrary coordinates is possible.

Although the chain of reasoning is lengthy, it is not difficult. To simplify the notation somewhat, we will define the derivative with regard to time according

to Newton's notation, so that:

$$\frac{dq_i}{dt} = \dot{q}_i = vq_i, \quad (44)$$

where q_i is the position of the belief projected onto the i^{th} generalized coordinate and vq_i is the velocity of change of the belief projected onto the i^{th} generalized coordinate.

Similarly, the second derivative:

$$\frac{d}{dt} \frac{dq_i}{dt} = \frac{d\dot{q}_i}{dt} = \frac{d^2q_i}{dt^2} = \ddot{q}_i = a_{q_i}, \quad (45)$$

where a_{q_i} is the acceleration of the belief projected on the i^{th} generalized coordinate.

Using this notation, and considering the one dimensional case in which the belief or attitude is varying only along a single attribute x , we may write:

$$\dot{T} = \frac{d}{dt} \frac{m}{2} \dot{x}^2 = m\dot{x}\ddot{x}, \quad (46)$$

which says, in words, that the time-derivative of the kinetic energy equals the mass times the velocity times the acceleration. This suggests that we take the derivative of (46) to eliminate the velocity term (recalling that a derivative is a ratio), so that we obtain:

$$\frac{d}{dx} \dot{T} = m\ddot{x} = ma, \quad (47)$$

which says, in words, that the derivative of the time derivative of the kinetic energy, with respect to velocity, equals the mass times the acceleration. This means that due to the comparative method, we are able to express the mass and acceleration of a belief or attitude (or any object) in terms of its kinetic energy alone.

As it turns out, we can express the force acting on a belief (at least conservative forces in the sense that we discussed them earlier) in terms of its potential energy. As we say in Expression (25), the increment in potential energy equals the negative of the work done. Yet work is force through distance, so the derivative of the potential energy, with regard to the distance, is the force acting across an arbitrarily small distance. Formally, as we saw in (25), which we rewrite here without the subscript designating the x dimension, the potential energy V is equal to the integral of force over distance, or:

$$V = \int_1^2 F dx, \quad (48)$$

and so, the potential energy at a point will be:

$$dV = F dx. \quad (49)$$

Dividing both sides by an arbitrary small distance dx gives:

$$\frac{dV}{dx} = F. \quad (50)$$

This, of course, is the ratio of the change in potential energy to a change in position of the belief for very small changes in position.

Because we know that, for a conservative system, $F=ma$, we may write (using (47) and (50)):

$$\frac{d}{dx} \dot{T} = \frac{d}{dx} V, \quad (51)$$

or, equivalently,

$$\frac{d}{dx} \dot{T} - \frac{d}{dx} V = 0. \quad (52)$$

This is equivalent to:

$$\frac{d}{dx} \left(\frac{dT}{dx} \right) - \frac{dV}{dx} = 0. \quad (53)$$

When working in a curvilinear nonorthogonal system, we need to perform these operations relative to the components of the kinetic and potential energies along the generalized coordinates q_i , as we saw earlier. We learned how to do this in (42) and (43), and so, we may write:

$$\frac{d}{dt} \left(\frac{\partial}{\partial} \frac{T}{q_i} \right) - \frac{\partial}{\partial} \frac{V}{q_i} = 0, \quad (54)$$

or, what is the same thing,

$$\frac{d}{dt} \left(\frac{\partial}{\partial} \frac{(T-V)}{q_i} \right) - \frac{\partial}{\partial} \frac{(T-V)}{q_i} = 0, \quad (55)$$

Setting $L = T-V$, we may write (55) as:

$$\frac{d}{dt} \left(\frac{\partial}{\partial} \frac{L}{q_i} \right) - \frac{\partial}{\partial} \frac{L}{q_i} = 0, \quad (56)$$

Expression (56) is the usual formulation of the equations of Lagrange for a conservative system not in communication with its environment. If forces from the surroundings are impressed on the system, we may write:

$$\frac{d}{dt} \left(\frac{\partial}{\partial} \frac{L}{q_i} \right) - \frac{\partial}{\partial} \frac{L}{q_i} = Q_i, \quad (57)$$

where the Q_i refers to the generalized force components directed along the generalized coordinates.

Although very compact, the Lagrangian equations can encompass virtually infinite complexity, and the ramifications of these equations is far beyond the scope of this chapter. Nevertheless, we can hint at some of the range of communication systems to which these equations may refer by rewriting (57) as:

$$\frac{d}{dt} \left(\frac{\partial}{\partial} \frac{T}{q_i} - \frac{\partial}{\partial} \frac{V}{q_i} \right) - \left(\frac{\partial}{\partial} \frac{T}{q_i} - \frac{\partial}{\partial} \frac{V}{q_i} \right) = Q_i \quad (58)$$

and then:

$$\frac{d}{dt} \frac{\partial}{\partial} \frac{T}{q_i} - \frac{d}{dt} \frac{\partial}{\partial} \frac{V}{q_i} + \frac{\partial}{\partial} \frac{V}{q_i} - \frac{\partial}{\partial} \frac{T}{q_i} = Q_i. \quad (59)$$

As we have already noted, if the system is isolated and conservative, the right hand side Q_i vanishes. If the potential energy does not depend on the velocity (as it usually does not) the second term on the left vanishes; if the kinetic energy does not depend on position (as it usually does not), the fourth term on the left-hand side vanishes. In this case, the equations reduce to Newton's familiar Equation (14). This will also be true if there is no potential function, in which only the first term will remain, but, in the absence of a generalized force term Q_i on the right-hand side, this term will also vanish and there will be no acceleration.

If the kinetic energy depends on the velocities (as it usually does), and if a potential function can be defined as a function of position (which is a condition for a system having an equilibrium value, whether or not it is at or near such a value at any time) and if there are dissipative forces dependent on the velocities, then the second term on the left-hand side will not vanish and we will find a linearly resistive term like the second term in (34). Although this is only the briefest of sketches of the potential of the Lagrangian form, it should suffice to show that, although the Lagrangian equations are capable of expressing or describing virtually any communication system or system of beliefs, attitudes, and their changes, they do not impose themselves on the system modeled, but rather adapt their shape to the system observed. Moreover, it is a language and logic capable of a much fuller description of these complexities than is a verbal and categorical language.

Moreover, the treatment is greatly simplifying. This is by no means to suggest that the process of working through the many equations presented in this chapter, or the many more required to achieve actual solutions for interesting human belief systems is easy, but it is to say that, without such a system of thought, such has proven to be impossible. With some thought, it is possible to show, for example, that to define the equations for any system of beliefs or attitudes is only necessary for the theorist to specify the equilibrium state of the system so that a potential function may be defined. Once the potential energy function has been defined, the remainder of the equations may be written down at once.

Although it is true that the Lagrangian form does not impose itself on the system under scrutiny, it is also the case that the Lagrangian formulation allows the

theorist to constrain the system with particular ease. This is due entirely to the fact that the Lagrangian equations may be expressed in terms of any generalized coordinates, rather than exclusively rectilinear Cartesian coordinates. To give an example of such constraints for an interesting communication situation, assume two cultural belief systems, *A* and *B*, that might represent the pattern of beliefs of two countries, groups, persons, or any social unit at all, about *n* concepts. Assume further that these groups are placed in communication with each other. If on theoretical grounds (or prior measurement experience) we may stipulate that, within each group, beliefs and opinions will not change relative to other beliefs and opinions, but solely with regard to those in the other group, then we have specified each system as a rigid body, i.e., a body in which distances among any pair of points (beliefs or attitudes) remains invariant over time. Under these constraints, each structure *A* and *B* may move closer to or further from the other, and/or either *A* or *B* may rotate relative to the other. Assuming the internal structure of each culture has been measured with a Galileo-type procedure (Woelfel & Fink, 1980), the cultural systems may be represented by $(n_a + n_b)$ concepts in an *r*-dimensional space, where *r* is the number of dimensions in the larger space. Ordinarily, $r(n_a + n_b)$ equations would result from the Lagrangian formulation (or would be required in the Newtonian mode), but the constraints established by requiring each structure *A* and *B* to be rigid bodies reduces the number of parameters that need to be estimated substantially. Specifically, only the position of the center of one of the structures relative to the other is needed to specify the distance of *A* from *B* overall; *r* angles are needed to specify the orientation of each body relative to the other, because we need only specify the orientation of the axes of the first body to the axes of the second. (These angles are usually referred to as the Euler angles, after their discoverer L. Euler). It requires *r* variables (which may be interpreted as the *r* rectilinear coordinates of the center of one of the cultural rigid bodies, x_1, x_2, \dots, x_r) to describe the location of the center of one of the structures in the space, and *r* additional variables that may be interpreted as the *r* position or Euler angles a_1, a_2, \dots, a_r to specify the state of orientation between the two bodies, so only $2r$ parameters are required. Because these themselves may depend on the time, we may write both the kinetic energies and the potential energies of the system as functions of $2r$ generalized coordinates:

$$T = T(\dot{q}_1, \dot{q}_2, \dots, \dot{q}_{2r}) \quad (60)$$

and

$$V = V(\dot{q}_1, \dot{q}_2, \dots, \dot{q}_{2r}) \quad (61)$$

In general, the Lagrangian form will yield $nr - k$ equations where *n* is the number of concepts, *r* is the number of dimensions within which the process takes place, and *k* is the number of constraints that may be established on the system a

priori. Although it is too early in the development of communication as a mechanical science to establish many solid constraints over the behaviors of beliefs and attitudes under general conditions, the constraints just discussed may be seen as a useful first approximation to the likely conditions that may prevail during the first stages of initial encounters between recently introduced cultures, because they will describe the rotations by means of which purely artifactual differences of opinion (due solely to differences in orientation or viewpoint) are transformed away. This problem has been studied from a more technical point of view by Woelfel, Holmes, and Kincaid (1979), in which the potential energy function has been described as a least-squares minimum problem.

CONCLUSION

This chapter has by no means established a complete mechanical science of communication, and indeed the implication that such would be possible in a single short monograph runs contrary to the spirit of this work. Rather, we have tried to recognize the great empirical complexity of human communication systems, and on this basis, have argued that the complexity is so great that the task of understanding such systems by means of a premechanical categorical model is hopeless. What we have tried to establish is not that a mechanical representation of human communication systems can be complete because the essence of scientific method consists in ignoring those aspects of any empirical situation that can be ignored without important loss. Any mechanical representation of any empirical system will always be, to some extent, an idealization. On the other hand, it does not follow that because a mechanical representation is incomplete, it should be rejected; prior to such a decision, we must consider the available alternative models. As I have tried to show, the available alternative model—the categorical verbal model—is dramatically less complete. To a large extent, communication scholars have been prevented from realizing the advantages of a mechanical treatment because they have been misled as to what a mechanical model implies. I have tried to show that a mechanical model does not imply machines, or clockwork or wires and springs, but rather implies simply a wholly abstract logical form of argument. If it aids the imagination of any scholar to conceive of interacting beliefs as a set of points connected to each other by elastic strings moving through a volume of fluid, then he or she ought to, by all means, make use of such an analogy, and the Lagrangian equations will model the system effectively. However, the strings and fluids are only a picture in the mind of the investigator, and the equations have no need of them they simply describe observed changes in a wholly abstract way.

The essence of the mechanical model is the notion of proportionality. Mechanical theory makes statements about ratios of abstract quantities that have important counterparts in experience, only insofar as measurements of observations of experience have been made with a proportional measurement system.

Attempts to cast categorical experiences into the logic of mechanical systems are bound to fail by formal logical reasons alone. Overwhelmingly, relatively casual efforts to fit categorical data (such as Likert-type, semantic differential-type or even ordinal-type measures) into a mechanical format, and of other logically indefensible practices such as substituting partial linear regression coefficients for partial differential regression coefficients into arbitrary functional relations have made it impossible to develop meaningful definitions of force, mass, velocity, acceleration, work, momentum, and potential and kinetic energy within the human communication disciplines. As a consequence, these terms are used imprecisely and analogously if at all. One may not simply use the words referring to concepts from mechanics and assume, therefore, that one makes use of the concepts themselves.

Of course, it may turn out that mechanical models of communication fail to produce the clarity and power that is implied in this chapter. No scientist would ever foreclose any possibility on the basis of reasoning alone. It should be clear from this chapter, however, that there is no present basis in evidence for saying that mechanical models do not fit human communication processes to useful tolerances. Such evidence, one way or the other, can only be provided by trying. Hopefully, insofar as it describes in some detail how a truly mechanical model of communication might be developed, this chapter might provide some impetus toward a rational answer to the question.

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