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Cognitive Processes as Motions in a Multidimensional Space: A General Linear Model*

THE EFFECTS of communication on the formation and change of attitudes has consistently attracted the attention of psychologists and communication researchers over the years, and is very likely one of the most carefully studied topics in the social science literature. While potentially powerful theories asserting curvilinear[1] relationships among key information (message) variables and attitude change have found some measure of empirical support (Hovland, *et. al.*, 1957; Sherif & Hovland, 1961; Sherif, *et. al.*, 1965; Sherif & Sherif, 1967; McLaughlin & Sharman, 1972) it is clear that most theories (and the analytic procedures used in researching them) assume a linear model, and treat departures from linearity as special cases requiring further explanation. Such a model assumes that attitudes (or other cognitive components) are some linear aggregate of some finite set of variables. Mathematically the general linear model takes the form of the familiar linear regression polynomial:

$$(1)\quad A = a + b_1x_1 + b_2x_2 + \ldots + b_nx_n = a + \sum_{i=i}^{n} b_ix_i$$

Where A = the dependent attitude

- a = the y intercept for the vector of the polynomial
- b_i = coefficients or weights indicating the relative net effectiveness of each of the variables (messages) x_i in effecting changes in the attitude A
- x_i = the variables (usually information-flow variables) assumed to exert causal influence over attitude formation and change.

In addition to its widespread use, there are several substantial reasons for a close analysis of the general linear model. First, although expressly curvilinear models show theoretical promise, none has shown impressively better empirical results overall than simple linear models (Bochner & Insko, 1966).

In general, empirical results show that statistically significant curvilinear effects are not frequently noted (Bochner & Insko, 1966; Aronson, *et. al.,* 1963; Bergin, 1962; Fisher & Lubin, 1958; Freedman, 1964; Goldberg, 1954; Helsen, Blake & Mouten, 1958; Hoffland & Pritzker, 1957; Rosenbaum & Franc, 1960; Tuddenham, 1958; Zimbardo, 1960). When found, curvilinear relationships between change advocated and change effected are found usually for messages sent by low or medium credibility sources (Bergin, 1962; Bochner & Insko, 1966; Aronson, *et. al.*, 1963; Freedman, 1964; Insko, Murashima & Sayadaian, 1966). Under special circumstances, however, clearcut curvilinear and even non-monotone relations of some substance ("boomerang effect") are noted (Cohen, 1962; Ablesen & Miller, 1967; Berscheid, 1966; Kelley & Volkhard, 1952; Mann, 1965). In spite of their infrequent appearance, these negative effects remain troublesome, and most investigators would probably agree that fully satisfactory explanations have not yet been made (Insko, 1967).

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A second reason for closer scrutiny of the general linear model is the fact that linear aggregation models, even in their simplest forms, are frequently very successful empirically, particularly in real life (non-experimental) settings. Using a simple linear model, Woelfel & Haller (1970) account for 64% of the variance in high school students' educational aspirations, principally on the strength of the *average educational expectations* of their "significant others;" Mettlin (1970) replicated these results on a second sample with equal success. Woelfel & Hernandez (1970) account for 86% of the variance in marijuana are using a linear model, and nearly equivalent levels of success are recorded for attitudes toward French Canadian Separatism (Woelfel, *et. al.,* 1974), cigarette smoking (Mettlin, 1973), and the extent to which children view television as "real" as opposed to fantasy (Reeves, 1974). Even though there may be situations in which the linear model fails, nonetheless its general utility in everyday life is clear from these findings.

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Still a third reason for examination of the linear model is the fact that it implies a theoretical model which is very parsimonious in its basic form, yet which can be expanded easily to encompass very complex empirical phenomena. With this in mind, this article will present one theory (sometimes called "Force Aggregation Theory," or FAT) based on the linear model, and assess its status in the light of available evidence.

I. *The Theoretical Model*

The simplest linear theory that could be posited to explain the joint effects of a set of messages $x_1, x_2, \ldots x_n$ on an attitude a would be one which assumed each message had an effect equal to each other message and that no other variables had substantial effects. This formulation[2], as shown in equation (2) below, stipulates that the resulting attitude a should equal the arithmetic mean of all messages:

$$(2) \quad a = \frac{1}{n}x_1 + \frac{1}{n}x_2 + \ldots + \frac{1}{n}x_n = \sum_{i=i}^{n} \frac{x_i}{n}$$

It follows immediately from (2) that this simple theory is a "balance" theory, since, for any mean $\overline{x}$,

$$(3) \quad \sum_{i=i}^{n} (x_i - \overline{x}) = 0; \text{ and hence } \sum_{i=i}^{n} (x_i - a) = 0$$

If each message x_i is construed as a "force" which "pulls" the attitude one way or another, expression (3) shows that the mean constitutes that point at which such forces sum to zero or "balance."[3] Simple though it is, this theory suggests a continuously-scaled least-squares balance point, which is a considerably more powerful mathematical model than the discrete graph-theoretic representations of many balance formulations (Newcomb, Heider, Osgood, Tannebaum & Suci, *et. al.*).

Given equation (2), it is possible to derive the expression for the value of the new attitude a_2 given the receipt of new information by the individual as

$$(4) \quad a_2 = \frac{n_1 a_1 + n_m m}{n_1 + n_m}$$

Where a_2 = the new attitude
a_1 = the old attitude
n_1 = the number of messages out of which the old attitude was formed
m = the average value of the messages received about the attitude over the time interval $t_1 - t_2$
n_m = the number of messages about the attitude received during the interval $t_1 - t_2$

More informative is the same equation solved for the amount of change in the attitude ($a_2 - a_1$) given the receipt of the new messages:

$$(5) \quad a_2 - a_1 = \frac{n_m (m - a_1)}{n_m + n_1}$$

Equation (5) is graphic, in that it shows clearly that three factors are causally related to attitude change according to this theory: (1) n_m, or the number of new messages, (2) n_1, or the number of old messages out of which the original attitude is composed, and (3) (m-a_1), or the amount of discrepancy between the old attitude and the mean position advocated by the new messages. More precisely, the amount of attitude change is directly related to the product of the average discrepancy between incoming information and the old attitude (average change advocated) and the number of such messages, and inversely related to the sum of the number of messages out of which the change message and the original attitude is composed.[4]

The role of n_m, or the number of new messages with which the old attitude is impacted, seems clear and has been a fundamental principle of contemporary advertizing, political campaigning and other large-scale persuasive activities, and its role in attitude change is generally confirmed in the laboratory (Sherif, 1935; Asch, 1951). The role of n_1, that is, the number of messages out of which the original attitude was formed, has been explored systematically in at least one study (Saltiel & Woelfel, 1974). Briefly, using a two-stage least squares path analytic procedure on panel data from 126 high school students, these researchers found changes in overall value positions of the students over a six-month interval were inversely related to a reliable measure of the amount of communication the students had had about those values prior to the onset of the research. They found further (as the theory predicts) that changes over the interval were not related to the stress the students reported experiencing, the students' own reports of the strength and certainty with which they held their attitudes about the topics, or the strength and certainty the "significant others" of the students reported holding about the information they had originally transmitted to the students about the attitudes in question (Saltiel & Woelfel, 1974).

The effects of the third of these variables, $(m-a_1)$ or the average change advocated, on $a_2 - a_1$ (the amount of change observed) has been very carefully studied by many investigators. Experimental work has shown consistently that these two variables are related, that they appear to be linearly related when the change message is delivered by high credibility sources (although the definition of credibility is not precise), but non-linear and even sometimes non-monotone in the case of low credibility sources (see Roloff, 1974). Plausible hypothetical explanations for this latter ("boomerang") finding have been offered by the non-linear social judgment theory (Hovland, *et. al.*, 1957; Sherif & Hovland, 1961, Sherif, *et. al.,* 1965; Sherif & Sherif, 1967; McLaughlin & Sharmon, 1972), and by the frequently but not universally found tendency to derrogate the source of the message (Bergin, 1962) and the message itself (Bochner & Insko, 1966). Data do not support any of these explanations unambiguously, however, and boomerang effect remains troublesome to the attitude change researcher (Ablesen & Miller, 1967).

II. Scaling Models

Even in this relatively simple form, the linear model has a fair record of predictive accuracy. Before elaborating the model further, however, it seems wise to consider the scaling procedures available for use with the theory, particularly since the scaling requirements of this mathematical model are severe, calling in the ideal case for continuous ratio scaling as a prerequisite.

Early formulations of this theory (Woelfel & Haller, 1970; Mettlin, 1970) attempted to circumvent this problem by utilizing rate-like dependent variables such as *educational aspirations* (measured in years of school a student expects to attain). Later studies explicitly utilize *rates of behavior* as dependent variables, like frequency of attendance at French Canadian Separatist rallies and demonstrations (Woelfel, *et. al.*, 1974) and rate of marijuana use (Woelfel & Hernandez, 1970).

While fairly successful in explaining the phenomena they address, these studies reveal difficulties which suggest even more elaborate scaling procedures, as well as some mathematical elaborations in the linear model developed so far.

First, while a surprising number of theoretically and substantively important phenomena may be expressed as rates, not all variables may be so conceived, and this limits the generality of the model. Secondly, while the linear model anticipates that identical messages from different sources may be differentially forceful,[5] both the separatism study (Woelfel, *et. al.*) and the marijuana use study (Woelfel & Hernandez) find that, controlling for the content of the message, messages sent interpersonally have very substantial effects over attitudes, while messages sent via mass media generally have very tiny effects, even though sample sizes are quite large in some cases (over 400 in the separatism study). Not only is such an outcome unanticipated by the theory, but it clearly contradicts equation (5), unless one is willing to assume media messages are nearly totally massless, which is unlikely.

Woelfel & Hernandez present an alternative explanation. They speculate rather that the implicit assumption of the typical unidimensionally scaled studies that all forces are expressed completely along the vector of the dependent variable is unrealistic. They theorize instead that the attitude change process may be construed as a multidimensional space in which every message x_1 exerts a force given by $u_i\ (a_1 - x_i)$ is the discrepancy between the position advocated by the message and the position held by the receiver. They further hypothesize, however, that this force is exerted at an angle α to the dependent attitude vector, which they interpret as the "relevance" of the message. α Greek alpha

Taking advantage of the fact that the relative net effectiveness of any two messages is given in the multiple regression equation by the ratios of their respective slopes, and further that the angle α

between the vector of the message and the vector of the dependent variable is given by the arc cosine of the correlation coefficient (Woelfel, 1973), these authors estimate the inertial masses of messages from the several media.[6] These preliminary estimates show the masses of the media messages are roughly equivalent to those of interpersonal messages, but that the angles between media message vectors and the dependent variable are significantly close to 90°. (While the mass of messages from movies was estimated as between three and four times as great as that of interpersonal messages, for example, the angle included between the movie message vector and the dependent attitude vector was greater than 89°. Clearly whatever effects these messages might have cannot be exerted along the dependent variable vector, since they are nearly orthogonal to it.)

While the Woelfel and Hernandez paper provides some evidence for a multi-dimensional representation of the attitude change process, they do not provide a procedure for obtaining such a space, and use correlational techniques to estimate the appropriate angles between vectors taken two at a time. Procedures for multi-dimensional attitude scaling are well known in the psychometric literature, however, and are rapidly gaining currency in the communication literature. (Torgersen, 1958; Shephard, Romney & Nerlove, 1972; Woelfel, 1973, 1974a, 1974b; Woelfel & Barnett, 1974; Serota, 1974; Barnett, 1974; McLaughlin & Sharmon, 1972).

These techniques begin by assuming that the process of perceiving and identifying any "object" is basically a process of differentiation wherein the individual learns to discriminate the stimuli which are the mechanism of the perception of the object from other stimuli representing other objects on the basis of their dissimilarities with regard to certain underlying attributes (Torgersen, 1958). Thus, for example, one identifies a yellow ball as different from a red ball because she or he recognizes them to be dissimilar by a certain amount in terms of the attribute *color*. Although in the example given the two objects presumably differ only in color, it is most frequently the case that objects differ with regard to *many* attributes at once. Two *persons*, for example, may differ in regard to the attributes *sex, age, height,* and so on through many attributes. The *aggregate* of all these dissimilarities can be taken as a measure of the *overall difference* or dissimilarity of these two persons.

Dissimilarities among cognitive objects may be represented by a continuous numbering system such that two objects considered to be completely identical are assigned a paired dissimilarity score or distance score of zero (0), and objects of increasing dissimilarity are represented by numbers of increasing value. Assuming that the definition of an object or concept is constituted by the pattern of its relationship to other objects, the definition of any object may be represented by a 1 x n vector where s_{11} represents the distance or

$$s_{11}, s_{12}, s_{13}, \ldots s_{1n}$$

dissimilarity of object 1 from itself (thus $s_{11} = 0$ by definition), s_{12} represents the distance or dissimilarity between objects 1 and 2, and s_{1n} represents the distance between the 1st and the nth objects. Similarly, the second object may be represented by a second vector

$$s_{21}, s_{22}, s_{23}, \ldots s_{2n}$$

and the definition of any set of concepts or objects may therefore be represented in terms of the matrix

$$\begin{matrix} s_{11}, & s_{12}, & \ldots & s_{1n} \\ s_{21}, & s_{22}, & \ldots & s_{2n} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ s_{n1}, & s_{n2}, & \ldots & s_{nn} \end{matrix}$$

where any entry s_{ij} represents the dissimilarity or distance between i and j.

Once these dissimilarities have been estimated,[7] the matrix will represent the pattern of differences among the stimuli across whatever attributes the respondent (or sample members in the aggregate case) perceive the stimuli to differ. While this matrix S contains an immense amount of information about the interrelationships among the stimuli scaled therein, much of this information (like the attributes along which the respondent(s) see the stimuli differing) is in latent form, and, furthermore, the matrix is usually large and unwieldy. Fortunately, techniques for the recovery of this latent information which also generally reduce the size and complexity of the matrix have been well developed, particularly since the early work of Torgersen. (See especially Young & Householder, 1938; Torgersen, 1967, 1968; Shephard, Romney & Nerlove, 1972; Woelfel, 1974; Barnett, 1974; Woelfel & Barnett, 1974; Serota, 1974). While many variants of these multidimensional scaling techniques (MDS) have been developed (Torgersen, 1958; Shephard, 1962, 1966; Kruskal, 1964; Young & Torgersen, 1967; Guttman, 1968; Lingoes, 1972; Carroll & Chang, 1970; Woelfel, 1972), all existing variants may be classified into two types depending on the rigor of the scaling assumptions required. When the matrix of dissimilarities can be trusted to be equi-interval and reliable (as is the case given the direct quantitative pair-comparison estimates suggested in Woelfel, 1974a, b), then "metric" techniques are appropriate and techniques defined by Young & Householder (1938) and elaborated by Torgersen (1958) are utilized. Specifically, the dissimilarities matrix D is converted to a scaler product matrix[8] B^*, which is then factored by any standard factor analytic procedure like principal components analysis or jacobi. The result is a spatial coordinate system represented in an m x r matrix R, where m = the number of objects or stimuli represented in S; $r \leq m-1$, orthogonal dimensions of the space and where any entry of R_{jk} represents the coordinate value of the jth concept or stimulus on the kth dimension. This matrix has the strong mathematical property that

$$s_{ij} = \sum_{k=1}^{r} (R_{ik} - R_{jk})^2$$

That is, the matrix R and the matrix R are exactly equivalent; R does not in any way distort the relationships defined in S. (See Woelfel, 1974).

When the data in the matrix S are poorly measured, the "nonmetric" procedures are typically applied. These differ from metric procedures only in that the matrix R is estimated by an iterative procedure which produces a configuration constrained to fit only the *ordering* of the dissimilarities in R; R is then related monotonically to the matrix R, which is a weaker mathematical assumption than that of metric procedure. (Shephard, 1962, 1966; Kruskal, 1964; Young, 1972).

Regardless of whether metric or non-metric procedures are utilized (although metric procedures seem clearly preferable for time-series or process models like the one implied in the present case) the result may be seen as an r-dimensional space in which a set of m objects are arrayed such that the distances between any two objects corresponds (exactly or monotonically, as the procedures are respectively metric or non-metric) to their distances in the data. When these data consist of perceptions of dissimilarities among objects, the definition of each object or stimulus within the domain of objects or stimuli scaled is given wholly by its location in the space; i.e., vis-a-vis all other objects arrayed in the space. Changes in the definition of any object may be represented as movement of the object in the space relative to the other objects. This is exactly the kind of multidimensional representation called for by the Woelfel-Hernandez findings.

How this may be made relevant to the attitude change process is illustrated in Figure One.

Figure One represents a hypothetical two-dimensional plot of the cognitive structure[9] of an individual (or set of individuals) including: 1) a definition (given in the space, of course, wholly by its location relative to the other objects in the space) of the source of a message, like a medium or a person, 2) a definition of "own position," i.e., the position of the receiver vis-a-vis the source, the position advocated and other concepts, like good-bad, etc., 3) a definition of the *position advocated*, and 4) a definition, vis-a-vis the other concepts, of the concepts "good," "bad," "credible" and "not credible."

(B) Assume these are arrayed as in Fig. 1, and assume the source (S) has sent to the receiver (R) a message suggesting (R) adopt S's own position, (P). Following FAT theory (and entirely consistent with dissonance theory and other balance formulations), this message generates a set of forces which may be described as follows: By saying, "you should adopt position P," the source sets up forces toward the convergence of R and P (remember P's location in the space represents R's definition of P prior to the message) which are represented by the dotted vectors a and b. Similarly, by suggesting that P's position is his or her own, S is arguing that R has erroneously defined P and S as non-coincident, and sets up further forces for the convergence of P and S on each other represented by the dotted vectors c and d. Furthermore, since the message clearly implies that R should move closer to S, forces for convergence of these concepts on each other are set up, and are represented by the dotted vectors e and f. The resulting vectors (solid lines 1, 2, and 3) represent the resolution of these forces according to the mathematical model explicit in the linear model.

While this illustration represents the main variables in the theoretical model quite graphically, several questions still require clarification. First, this representation introduces into the discussion the concept of "force" as an explicit variable. Specifically it hypothesizes three main forces, F_{ab} (the force for the convergence of [R] and [P] on each other), F_{cd} (the force for the convergence of [P] and [S] on each other), and F_{eg} (the force for the convergence of [S] and [R] on each other). Since nothing is implied in the message given in this example other than that each pair of concepts should converge on each other (and following standard theoretical practice in physics), each of these forces has been divided such that $F_{ab} = F_a + F_b$ where $/F_a/ = /F_b/$; $F_{cd} = F_c + F_d$ where $/F_c/ = /F_d/$; and $F_{eg} = F_e + F_g$, $/F_e/ = /F_g/$. A fundamental question still open, however, is how the forces F_{ab}, F_{cd} and F_{eg} are to be estimated. Perhaps the simplest hypothesis would suggest that the force for the convergence of any two objects on each other is proportional to the distance between them, since this seems initially to be consonant with the notion that the amount of change effected (CE) is proportional to the amount of change advocated (CA). Following from this assumption, and since (in this hypothetical case) the distances between all pairs of concepts are roughly equal (i.e., $d_{pr} \cong d_{ps} \cong d_{rs}$) then FAB≅FCD≅FEG; further, $F_a = F_b = F_c = F_d = F_e = F_g$, and the resulting force vectors are given by (1), (2), and (3) in Figure One.

Even should this hypothesis prove correct, we should still not expect the *changes* in the cognitive structure pictured to conform exactly to these force vectors, since the theory (see particularly equation (5) above) expects change to follow not only from message factors (i.e., amount of change advocated) but from characteristics of the concepts as well. Specifically, equation (5) requires that attitudes resist forces toward change by a function of *N* or the *number of messages* out of which the old conception was composed. Following the example of physicists, this quality of the attitude which resists acceleration is defined here formally as the *inertial mass* of the concept. Inertial mass, therefore, is operationally defined in equation (5) as proportional to N. Such a formulation, in fact, is in the spirit of less precise equivalent concepts like "embeddedness," (Sherif, 1961). At this early stage of theorizing, it would be apt to suggest, perhaps, that the *acceleration* of the concept through the space will be roughly proportional to the force to which it is exposed and inversely proportional to its inertial mass.[10]

If we assume for convenience that all inertial masses in Figure One are equal[11] (i.e., $m_s = m_p = m_r$), then the distances moved after one unit of time will be proportional to the force vectors (1), (2), and (3). Sensibly, these vectors predict a convergence of all three concepts, S, P, and R, if the interaction continues without limit.

This seems intuitively sensible (and corresponds with most –– but not all –– empirical evidence), yet note some interesting anomalies: Assuming that the concepts good-bad, credible-not credible are arrayed in the space as shown, then the result of all these motions would be a *decrease* in source credibility, but the source would move somewhat closer to *both* "*good*" and "*bad.*" Clearly, attempts to measure *this* phenomenon on unidimensional scales would result in unpredictable outcomes, probably substantial increases in unreliability of measure. This would explain the mixed results in source derogation very parsimoniously, without requiring additional theoretical premises.

Of course, *where* in the space all these concepts are located is an empirical matter; if, for example, "credible" and "not credible" were *reversed*, then the result of the situation would be an *increase* in the credibility of the source. This possibility may have been overlooked because of the implicit assumption that all changes advocated would be viewed by receivers as undesirable changes,[12] and this in turn probably follows from the empirically unsupported but widely held notion that people themselves take the position they think "best" in some sense.

Even in this crude form, the multidimensional representation of the general linear model can be seen to account for a great deal of what is known about attitude change. Still, it should be made clear that the combination of inertial masses, forces and coordinate values of concepts in the space necessary to predict negative attitude change, that is change in a direction opposite that advocated ("boomerang effect"), does not seem to correspond to the conditions under which "boomerang effect" is usually observed under the assumption that force for change is directly proportional to change advocated. Figure Two depicts the two general cases in which the general linear model and the assumption that force is proportional to amount of change advocated predicts boomerang effect.

Assuming that the force generated toward attitude changes is proportional to the amount of change advocated, clearly the forces F_{rs} in (1) and $F_{s^1p^1}$ in (2) are larger than the respective forces F_{pr} in (1) and $F_{r^1p^1}$ in (2). Given the pattern of masses indicated, resultant motions will be proportional to the vectors a, b, c, and $a^1b^1c^1$ in Figure Two. The result of these motions will be an

Figure 2. Conditions under which the direct linear model predicts negative attitude change.

$$
\begin{array}{llll}
1 & P_h \xrightarrow{u} R_1 \xrightarrow{b} & & \xleftarrow{c} S_h \\
2 & {R^1}_h \xrightarrow{a^1} P_1 \xrightarrow{b^1} & & \xleftarrow{c^1} {S^1}_h
\end{array}
$$

P = position advocated in message $\quad$ a,b,c,a^1,b^1,c^1, = distances travelled in one unit of time
R = receiver of message
S = source of message
subscripting = mass of conception: h = high; l = low

absolute *increase* in the distances s_{PR} and $s_{P^1R^1}$, i.e., a net movement *away* from the position advocated, or boomerang effect.

As suggested earlier, these conditions do not generally conform to those in which boomerang effect is observed, particularly since they imply greatest boomerang effect when the smallest changes are advocated, and negative effects are most frequently notes when large changes are advocated. While these speculations are hardly conclusive, still the failure of the linear model under these specifications to predict a known outcome is sufficient reason to explore alternative formulations.

The beginnings of such a formulation may be found in the fact that the pattern of movement predicted in Figure Two is a consequence not of the linear form of the model, but of the substantive hypothesis that the *amount of force* toward change is directly proportional to the amount of change advocated. This assumption, coupled with the pattern of masses indicated in Figure Two, leads to the pattern of motion implied in the vectors a, b, c, a^1, b^1, and c^1. This assumption seemed plausible since the *amount of change effected* is generally found to be proportional to the *amount of change advocated*, and it seems a very reasonable assumption that the amount of change effected should be proportional to the force for change created by the message. While this may be plausible, it need not be the case, however, that the amount of change effected should be proportional to the *instantaneous* force generated by the message. To understand why this is the case, it is necessary to introduce *time* as an explicit variable in the attitude change process. Figure Three represents two parallel attitude-change situations.

Figure Three. Change Effected by Change Advocated for Small (Case 1) and Large (Case 2) Advocated Changes

1 $R \longrightarrow P$

2 $R' \longrightarrow P'$

In the first of these cases, only a relatively small attitude change is implied. In the second, considerably more change is advocated. Rather than assuming that the process of attitude change takes place instantaneously, we assume that delivery of the change message (R should adopt position P) sets up a process of change which takes place over time (t), and which continues until the equilibrium point predicted by equation (5) is reached. Given that the inertial masses of the concepts involved are in the order $m_P = m_p1$; $m_r = m_r1$, equation five predicts the total change ΔR^1P^1 will be larger than DRP. What equation (5) *does not predict*, however, is the length of time it will take for the changes $\Delta R^1 P^1$ and ΔRP to take place. A plausible assumption might be that the *rate of change*, rather than the *amount of change*, is proportional to the force generated by the change message. Under this assumption, Figure Four shows that the total changes of attitude might well be in the order $F_{PR} > F_{P^1R^1}$, giving $\frac{\Delta PR}{\Delta t} > \frac{\Delta P^1R^1}{\Delta t}$, and still result in $\Delta P^1R^1 > \Delta PR$ as long as the interval Δt is made larger without limit. Thus, as Figure Four shows, if $\Delta t < 9$, $\Delta PR > \Delta P^1R^1$, but where $\Delta t > 9$, $\Delta PR < \Delta P^1R^1$, since ΔPR approaches its maximum (6 units in Fig. 4) as Δt approaches 4, and ΔP^1R^1 approaches 6 only as Δt approaches 9. After 9 units of time have passed, however, ΔP^1R^1 continues to increase until it reaches its maximum of 11 at $\Delta t = 15$.

The point of this discussion is to show that the assumption (given in equation 5) that amount of change is directly proportional to amount of change advocated can be maintained even though it is also assumed that the amount of *force* for attitude change is *inversely* proportioned to the amount of change advocated. Not only is such a view possible, but it fits available evidence closely enough to warrant further careful investigation.

First of all, this inverse linear model predicts negative influence or boomerang effect under conditions like those in which it is usually observed, as Figure Five illustrates:

Figure Five. Conditions of negative influence (boomerang effect) as predicted by the inverse linear model.

1. $R_h \xrightarrow{a}$ $\qquad P_1 \xrightarrow{b} \quad \xleftarrow{c} S_h$

2 $P^1{}_h \xrightarrow{a^1}$ $\qquad R^1{}_1 \xrightarrow{b^1} \quad \xleftarrow{c^1} S^1{}_h$

R = receiver
P = position advocated
S = source

Subscripting:
h = high mass
1 = low mass

The inverse linear model quite clearly predicts that the forces toward convergence will be in the order
$F_{sp} > F_{pr} > F_{sr}$ and $F_{r^1s^1} > F_{r^1p^1} > F_{p^1s^1}$

Given that the patterns of masses are as shown in Figure 5, resultant movements are given by the vectors a, b, c, d, a^1, b^1, c^1, and d^1. When $b > a$ and $b^1 > a^1$, the result will be a net *increase* in the distances S_{rp} and $S_{p^1r^1}$, or boomerang. Of the two cases, the first seems most likely empirical, in that it represents a large change empirically,
advocated by a source whose position is also viewed as extreme, and further assumes the receiver's conception of his or her own position and the position of the source to be stable (high mass). The result (which is quite consistent with most dissonance formulations) predicted by the theory is a redefinition of the position advocated as even further from the receiver's own, even though the overall result of the change attempt is a change of the receiver's position in the direction advocated. Although not every relationship implicated in Figure Five has been examined empirically, it is the case that boomerang effect is noted when large changes are advocated

(Bochner & Insko; Ableson, *et. al.*, Aronson, *et. al.*) and when the receiver's own position is massive ("embedded").

Very little is known empirically about the second case, but boomerang effect does not seem implausible under these conditions, since it represents the case in which a receiver whose own position on an issue is tentative receives a message from a source whose position the receiver firmly perceives to be very discrepant from the position advocated by the same source. This situation would seem very likely to generate disbelief in the receiver, and should be expected to yield significant message derogation (Bochner & Insko). Under these circumstances, negative influence or boomerang effect seems plausible, as the inverse linear model predicts.

In the light of evidence currently available it is difficult to assess the relative fit of the direct and inverse linear models to the data. First, although the inverse linear model seems to fit the pattern of "boomerang" effects observed somewhat better than the direct linear model (and about as well as non-linear models hypothesized), the data are not unambiguous in their support. Secondly, it follows from Figures 3 through 5 and the attendant discussion that both models predict the same outcome (i.e., that amount of change effected will be linearly proportional to the amount of change advocated) when the interval of time (Δt) elapsed between delivery of the change message and measurement of its effects is greater than the interval of time required for the smallest of the changes advocated to take place. Data from an unpublished study by Woelfel & Walker tend to indicate that the interval of time needed for changes generated in a laboratory induced low-mass attitude was less than five minutes, but studies by Walster (1964) and Roloff (1974) show changes still occur respectively 90 minutes and several days after the change stimulus when the masses of attitudes are higher. Roloff's data further show that the correlations between changes advocated and changes effected vary about as predicted by the inverse linear model when the interval from stimulus to measurement is varied (Roloff, 1974). Several studies (McGuire, 1969) have indicated that attitudes move back toward their original positions after they have changed when measured later ("sleeper effect"), and these findings plausibly support the notion of an "equilibrium point" beyond which further change is inhibited, an occurrence anticipated by equation (5) and distinguished: 1) A direct model based on the assumption that force for change is proportional to amount of change advocated, and 2) an inverse model, which assumes that force for change is inversely proportional to the amount of change advocated. Comparison of these models requires an explicit examination of the role of time in the attitude change process, and such a consideration makes useful the specification of concepts borrowed from dynamics like *velocity, acceleration, force,* and *mass.* Although tentative, currently available data will support a theory which suggests that these variables are related in a way at least roughly analogous to their relation in physics, particularly when these concepts are combined in a metric multidimensional distance model of cognitive structure and process. While it would be rash to suggest such a model adequately represents the process of attitude change, it is clear that currently available data are consistent with such a view, and further investigations of this dynamic model represent potentially fruitful grounds for future research.

FOOTNOTES

[1] While the question of linear vs. curvilinear relationships among variables is a real question, the characterization of *theories* as linear or non-linear is often ill-considered. We call this theory linear because it considers the *new attitude* to be a linear combination of the old attitude plus message effects, but many curvilinear relationships among other variables are also predicted by this formulation, as the following pages will show.

[2] This model assumes a continuous ratio scale metric as a precondition. Techniques for providing such variables and their measures are outlined in Section II of this chapter.

[3] It can easily be shown that expression (3), as well as the results that follow, apply equally to those *more complex models* which assume each message to be of *differential effectiveness.* To do so it is sufficient to show that

$$\sum_{i=1}^{n} (b_i x_i = a) = 0 \quad \text{when } a = \sum_{i=1}^{n} \frac{b_i x_i}{n}$$

This follows directly from the property of closure under multiplication, which implies that $b_i x_i = x_i 1$, where $x_i 1$ is a scaler;

thus $\sum_{i=1}^{n} \frac{x_i 1}{n} = a$, and therefore $\sum_{i=1}^{n} (x_{i^1} - a) = 0$, which is

equivalent to (3).

[4] Interestingly, even though derived from the general linear model, equation (5) argues for a 3-way interaction among n_1, n_m and $m\text{-}a_1$. Empirically this equation can be estimated by an ordinary least squares regression equation of the form

$$\text{Log } /a_2 - a_1/ = B_1 \log/m - a_1/ + B_2 \log n_m +$$

$$B_3 \text{Log}(n_m + n^1) + B_4 \text{ Log } U$$

where U is the residual error term. This equation, of course, anticipates non-linear relations among the variables (cf. note 1).

[5] See note (3) above.

[6] The dependent variable in the Woelfel-Hernandez study is frequency of use of marijuana. Woelfel & Hernandez do not imply these results hold for any dependent variable, but rather are specific to this one.

[7] Numerous procedures for establishing numerical estimates of the distances among cognitive objects have been developed. See particularly Torgersen, 1958; Miller & Nicely, 1955; Shephard, 1962. Techniques which provide continuous ratio scale measurement implicit in the theory described here are described in Woelfel, 1973, 1974a, 1974b; Barnett, 1974; Serota, 1974.

[8] Traditionally this matrix is double centered to yield an origin on the centroid of the distribution of the objects or stimuli. The exact transformation for any cell b^*_{ij} is given by

$$b^*_{ij} = 1/2 \left(\frac{\sum_{i=1}^{n} d^2_{ij}}{n} + \frac{\sum_{j=1}^{n} d^2_{ij}}{n} - \frac{\sum_{i=1}^{n} \sum_{i=1}^{n} d_{ij}^2}{n^2} - d_{ij}^2 \right)$$

[9]This configuration can be obtained by metric procedures applied to an aggregate matrix of direct quantitative pair-comparison dissimilarities estimates averaged over a large number of people. See Woelfel, 1974a; Serota, 1974.

[10]While this formulation may sound unduly similar to Newtonian physical notions, it should be recalled that it fits the data about as well as any other.

[11]This assumption is solely for simplicity of illustration, and calculations involving unequal and non-constant masses and forces are straightforward.

[12]In fact, most experimental studies of attitude change have attempted to induce attitude changes unfavorable to the subjects. Ableson & Miller (1967), for example, induce "boomerang" effect by exposing naive subjects to personal insult; Cohen (1962) mobilized large scale apparatus to lower subjects' self-esteem; Aronson, Carlesmith & Turner (1963) advocate higher tuitions to student subjects; etc. Curiously, Fisher & Lubin (1958) who advocate essentially *judgmental* changes (e.g., the *number of paratroopers* in a photograph) do not observe source derogation.

[13]Many of these technical problems have been reduced by computer software specifically designed to deal with process models of metric multidimensional scaling (Serota, 1974). Copies of this program (Galileo I) are available on request from the author.

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