Stoyanoff
(3)

Metric Measurement of Cultural Processes

Joseph Woelfel

Michigan State University

December, 1974

A paper presented to the Speech Communication Association
Annual Meetings, Chicago, Illinois, December, 1974

Rah Press

It is axiomatic not only in thesocial sciences in general but in communication in particular that the linguistic system or encoding symbols utilized by an observer play a fundamental role in the perceptions of that observer and in the cognitive processing of data yielded by such perceptions. The development of language, from this standpoint, may be seen to provide a set of shared symbols by which observers can apply common perceptual frameworks to yield increasingly consensual statements about their perceptions.

Science too may be encompassed within this view, and the past century has seen the emergence of a consistent and appealing view of science as an extension of this fundamental human process. Largely through the work of Lagrange (1788), Helmholtz (1869), Mach (1883), Hertz (1894), Poincare (1902), Einstein (1952) and many others, many physicists have come to see the principle work of science as the development of shared symbol systems and relations defined on those sysmols which define the domain of possible perceptions, which are set into correspondence with perceptions of observers. Such a set of symbols, along with a stipulation of the relations possible among the elements of the set and rules for setting the symbols into correspondence with perceptions of observers constitute a scientific theory. Scientific theories can be said to be an advance over everyday language in three ways: first, within the domain of phenomena to which they apply, they are more complete, that is, no matter how finely observations may be discriminated, some symbol in the set may be set into correspondence with each percept. Second, the relations defined on the set -- that is, the relations among percepts considered possible -- are logically consistent according to some rule. Third, rules for establishing correspondence between symbols and percepts are more clearly and consensually specified. To the extent to which these criteria are met, scientific theories create increased predictive capability in those who understand them.

This paper presents the thesis that the notions of <u>perceived discrepancy</u> and <u>time</u>, when setinto correspondence with the set of complex numbers, along with the relations defined by multidimensional scaling and physical mechanics, provide a theory which is consistently superior to ordinary language systems for the perception and prediction of communication phenomena.

Contemporary scientists (Kramer, 1970) and philosophers of science (Reichenbach, 1951) generally agree that scientific theories have their ultimate roots in certain fundamental or primitive variables which cannot be defined in terms of yet more basic concepts, but rather are defined, following Kant, by some a priori call to the experience of observers. The fundamental variables of physics are usually considered to be distance, time, force or mass[1] and temperature. (King, 1962) Of these, two -- distance and time -- are special in that they alone refer directly to observations made by observers. These are usually called descriptive fundamental variables, whereas the latter are deriveable as ratios of the two descriptive variables and are usually called fundamental explanatory variables. From these variables all others in modern physical science may be derived.

This paper suggests that all variables required for a useful science of communication phenomena can similarly be derived from two fundamental variables, perceived discrepancy and time. Since time in communication is the same as time in physical science, perceived discrepancy will be treated here initially, but time and its relation to discrepency will be discussed shortly.

Perceived discrepancy as a fundamental descriptive variable:

In his now classic text, (Torgerson, 1958) Warren Torgerson established a classification scheme which, in spite of substantial technical advances, remains a viable and exhaustive system for the description of known psychological measurement procedures. Among other criteria, scaling methods may be described in terms of the information

---

1.) Since force and mass are reciprocals, that is,

$F = m/a$ or $a = F/m$, only one of the two need be considered. Some writers include angle as a fundmental variable, but angle can be derived from the ratio of the chord of a circle included in the angle to the radius of the circle, both of which are measures of distance.

they require from respondents. In the most demanding case (which Torgerson calls categorical subjective estimate methods) subjects are required to distinguish for a series of stimuli how much of some attribute each possesses or represents. (How tall is Mary?) Implicit in this task is a comparison among stimuli in which each is distinguished or "discriminated" from all others in terms of the absolute amount of some attribute or quality they possess or represent. Less demanding scaling methods, such as the comparative variability models, simply require that subjects discriminate stimuli from each other with regard to stated attributes, but do not require a quantitative estimate of the magnitude of the discrepancy. If we remove finally even the notion of attribute, the simplest judgment that could be made about the relation between two stimuli is whether they [illegible] be distinguished at all. At the basis of every perception lies the fundamental notion of discrepancy.

Within the limits of current knowledge, these discriminations of difference between objects cannot be defined or accounted for by yet more fundamental concepts, that is, there is no way that the experimenter can specify what is meant by "different" in the question "are A and B different?", or even the question "are a and b different in pitch?", or "are α and β different in length?" Such measurements require the assumption that each respondent can in an *a priori* way define "difference" and relate it to his or her perception. The definition of "different" is ultimately made not by the experimenter but rather by the respondent and in that sense is *a priori* or fundamental as that word is used here. To be sure, discrepancies of many different kinds may be identified; stimuli might differ in terms of color, intensity, pitch, hardness or in other ways, and even physical distance can be seen as the special case of discrepancy in location, but all these differences share ultimately the more fundamental notion of simple difference or discrepancy. This means that all perception, information and communication can be derived from the fundamental notion of discrepacny.

Assuming that "perceived discrepancy" may be considered a fundamental concept, the next step in the development of a measure of this concept is the specification of rules or procedures for the estimation of relative magnitudes of discrepancies.

Here Einstein's conception of the measurement of distqnce is instructive: (Einstein, 1961)

> Tor this purpose(the measurement of distance) we require a "distance" (Rod S) which is to be used once and for all, and which we employ as a standard measure. If, now, A and B are two points on a rigid body, we can construct the line joining them according to the rules of geometry; then, starting from A, we can mark off the distance S time after time until we reach B. The number of these operations required is the numerical measure of the distance AB This is the basis of all measurement of length.

Einstein's measurement procedure is two-staged: first, an arbitrary[2] distance (or discrepancy, in the general case) is stipulated by the scientist. It is vital to note that rules for the perception or measurement of this initial measurement distance or discrepancy are not stated; rather the scientist must assume the subject and himself/herself share a common referent for the ordianry language symbol "distance" or "difference", and that the subject can make this initial recognition unaided by further definition. Ultimately it is this <u>a priori</u> call to common experience as codified in ordinary language symbols that establishes a link between the everyday experience of the observer and the scientific theory.

Secondly, the scientist specifies a rule by whichother instances of distance or discrepancy are to be compared to this unity. In this case, the observer is asked to make ratio comparisons of all other distances or discrepancies to this arbitrary standard. Clearly, fundamental measurement represents a joint activity of scientist and observer.

2 .) While the choice of the unit of measure is arbitrary, choice of different standards will have consequences for the patterns of measurements made with the sysetm. Choosing as Rod S some ordinary language symbol who relation to other such symbols is stable over time might make results of the measurement more clearly interpretable in terms of the ordinary language system that would a Rod S defined by a symbol whose meaning fluctuates in the vernacular system. Good scaling practice, furthermore, would suggest a standard midway between the large and smallest discrepancy likely to be encountered, so that judgments of extremely large or extremenly small discrepancies are minimized.

Since this technique yields both a true zero (that is, no difference between two stimuli) and a standard unit or interval of measure (Rod S), it may be seen to constitute, by definition, a ratio scale whose validity rests on the conventional linguistic symbol system. This means that numbers yielded by these procedures represent discrepancies among stimuli as they appear to the respondent, rather than as defined by the scientist's fiat. Formally, these procedures performed for a single observer over the $(N(N-1))/2$ possible non-redundant pairs of N stimuli, yield the N X N symmetric matrix S (see figure one) where any cell s_{ij} represents the discrepancy or difference between the <u>ith</u> and <u>jth</u> stimuli as reported by the observer and expressed as a ratio to some arbitrary discrepancy s_{xy}.

Initially the recognition of the fundamental dependence of this type of measurement on philosophical assumptions may be disquieting, but the pattern of responses of subjects to the task yield evidence pertinent to the validity of the <u>a priori</u> assumption. Specifically, if the outcomes of these procedures yield "lawful patterns" among measures, so that the results of certain applications of the measure might be predicted by other measurements, a system with predictive validity can be seen to exist.

Several such outcomes are possible. First, respondents may fail to respond at all, indicating they are unable to make the observation required, and in such cases the assumption must be held suspect. Second, subjects may respond, but responses across individuals might vary greatly and unsystematically. In this case, the assumption that all persons make the same observations in accordance with the same principles is suspect. Third, we might find that responses made by an individual are randomly related to responses made by the same individual at subsequent times, in which case we could not assume the same subject makes the same <u>a priori</u> observations across time. Finally, we might find systematic agreement among the responses of individuals over time and/or across individuals, or even within an aggregate of individual responses across time. These outcomes support stronger or weaker variations of the hypothesis favoring the a priori capabilities of observers. The validity of the system of fundamental measurement of perceived discrepancies can be shown operationally, therefore, by the demonstration of extensive patterning among the variables so measured.

The Geometry of Discrepancy:

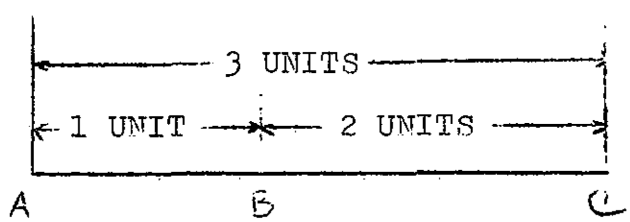
The concept of a geometry of discrepancy capitalizes on the recognition that physical distance is a special case of discrepancy in general, and therefore is isomorphic to discrepancy in formal structure. Since this is so, the seperation or difference between two discrepant stimuli may be depicted as a distance. Since the procedures for measuring and conceiving distance are so well articulated and widely shared, they represent a useful set of symbols or "psychological framework" for interpreting discrepancies. The discrepancy score for any cell s_{ij} of the discrepancy matrix S may be pictured as a distance between the <u>ith</u> and the <u>jth</u> stimuli. The interrelations of all these distances can be seen to result in a shape or geometric pattern among the stimuli. Thus consider the matrix:

$$S = \begin{array}{c|ccc} & A & B & C \\ \hline A & 0 & 0 & 0 \\ B & & 0 & 0 \\ C & & & 0 \end{array}$$

Here, since $s_{AB} = s_{ac} = s_{BC} = 0$, the three stimuli lie on a point in a 0 dimesnional space. In the Matrix

$$S = \begin{array}{c|ccc} & A & B & C \\ \hline A & 0 & 1 & 3 \\ B & & 0 & 2 \\ C & & & 0 \end{array}$$

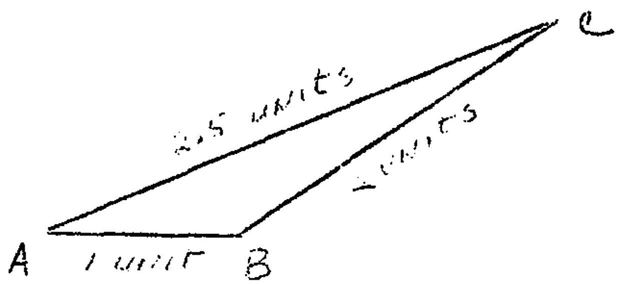
the discrepancies form a line segment in a one-dimensional space, that is,



and the matrix

$$S = \begin{array}{c|ccc} & A & B & C \\ \hline A & 0 & 1 & 2.5 \\ B & & 0 & 2 \\ C & & & 0 \end{array}$$

represents a triangle in a two-dimensional space



and finally the matrix

$$S = \begin{array}{c|ccc} & A & B & C \\ \hline A & 0 & 1 & 4 \\ B & & 0 & 2 \\ C & & & 0 \end{array}$$

represents a complex (non-Euclidian) space of 2 dimensions. In general, the configuration will always be a geometric shape which fits into a space of k n - 1 dimensions for *n* stimuli. Figure two represents the spatial pattern underlying the discrepancy matrix of city locations presented in figure one.

(figure two about here)

This geometric realization makes it simple to state the intuitive basis for the validity of the fundamental measure of discrepancy: similarity of patterns across individuals is given by the extent to which these configurations are congruent across individuals. Over time patterning is shown by stability and relative continuity of the shape over time. Noise or random components will be represented as rapid random motion or "jiggle" in the shape over time, rather like 'camera shake' in a movie. Should the configuration meet these criteria, evidence for the validity of the procedure should be considered substantial, since clearly such lawfully changing patterns can provide a basis for predictions of future states of the configuration, or for predicting the observations of one observer from the observations made by another.

Metric Multidimensional Scaling:

Techniques which map the structure of discrepancy or dissimilarity data onto a space where they may be interpreted as distances are well known in the multidimensional scaling literature and have been since Torgerson defined the procedure. (Torgerson, 1958) Computational equations for Torgerson's method, called metric or classical or Torgerson multidimensional scaling, have been detailed in several places (Torgerson, 1958; Woelfel, 1974; Serota, 1974) but certain salient aspects deserve mention here.

First, metric multidimensional scalling (MMDS) yields a coordinate system of $k \leq N$ orthogonal dimensions for N stimuli. Second, the mapping of discrepancies into this space is one-to-one, that is, no information is lost by MMDS. Third, the function which maps discrepencies (s_{ij}) reported by the respondents onto distances in the space (s'_{ij}) is the simple

$$s_{ij} = s'_{ij}$$

that is, distances in the space conform exactly to discrepancies reported by the respondent. Such a system of measure can be seen to conform in essential respects to the spatial coordinate system of classical (and modern) mechanics. Should the system of measure which provides the input discrepancies to the MMDS process prove to have predictive validity, the exceptional conceptual apparatus developed by mechanics would therefore apply directly to the description of whatever processes might be found in the data.

As we suggested earlier, the structure of interrelations among discrepant stimuli will be conveyed in the MMDS space as a geometric pattern, and processes among the stimuli will be represented as change in this pattern over time. Any single stimulus in the pattern will be described as the curve of a point through the space over time. (See figure three)

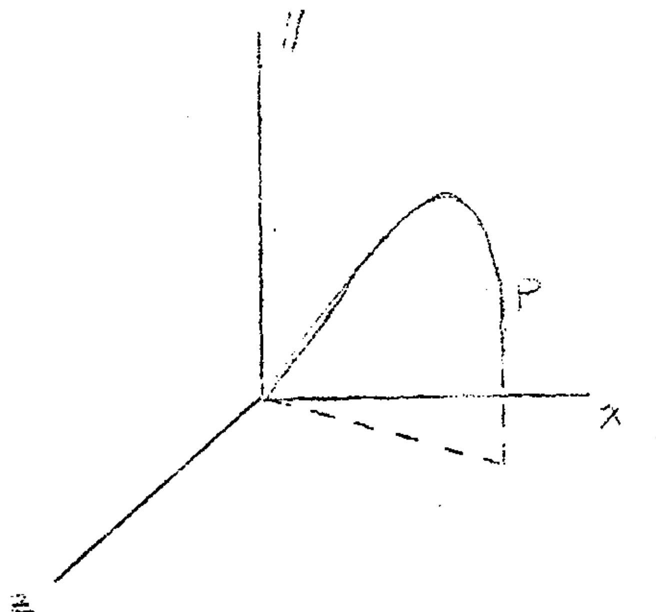


Figure 3. Three-dimensional representation of a stimulus moving over time in a MMDS space.

Following Lagrange (1788) this motion may be decomposed into its components along the orthogonal dimensions, and velocities and accelerations may be computed as derivitives of the resulting curves against tiem, as (for a 3 space)

$$Vt = \frac{ds}{dt} = \frac{ds_i}{dt} + \frac{dt_j}{dt} + \frac{ds_k}{dt}$$

where V_t = the velocity of the point p at time t.

or, for k dimensions,

$$V_t = \frac{ds}{dt} = \sum_{i=1}^{k} \frac{ds_i}{dt}$$

Similarly, accelerations in the space are given by the second derivitive:

$$a_t = \frac{d^2s}{dt^2} = \sum_{i=1}^{k} \frac{d^2s_i}{dt^2}$$

where a_t = the acceleration of the point p at some time t.

While these are, of course, only descriptive variables completely deriveable from the fundamental descriptive variables perceived discrepancy and time, nonetheless extrapolatin g the curves into the future and integrating can be seen to yield predictions of future states of the pattern. Furthermore, the precision with which the state of the system can be measured from moment to moment enhances greatly the likelihood of identifying the sources of perturbations in the pattern.

These are very real advantages, and a discipline like communication which is sensitive to the effects of symbols on cognitive processes should be especially cognizant of the advantages of gaining the use of the elegant and elaborate symbolic apparatus of mechanics for the description ofsocial and communication processes. This, of course, would be a highly desireable outcome, and we should therefore be attentive to arguments for and against the possibility that such a symbol system might be fitted to our experiences of social phenomena.

Non metric scaling mod ls:

In spite of these potential advantages, psychometricians have spent relatively little energy on the development and use of the metric model since Torgerson's work, but have turned rather to the "non-metric" or ordinal models.

Proponents of the non metric models generally reject the MMDS model on the basis of the following assumptions: First, respondents are generally assumed to be unable to make reliable ratio judgments of discrepancies among stimuli. (Shephard, 1966, 1972; Kruskal, 1966) Second, many psychometricians, for philosophical or heuristic reasons, resist the notion that <u>k</u>, the dimensionality of the space, should be left a free parameter to be discovered inductively as a consequence of the rule for measuring distances, but rather feel <u>k</u> should be set at some arbitrarily small value and distances (discrepancies) reported by observers adjusted accordingly. This last assumption is similar in intention to the practice of relativity theorists, who generally assume the 4-dimensional character of the space-time continuum as an axiom and adjust distance observations to fit this constraint, but differs importantly in procedure. (Weyl, 1952; Riemann, 1953; Reichenbach, 1958)

As a consequence of accepting these assumptions, nonmetric advocates 1.) select some small and arbitrary value of <u>k</u>; 2.) determine some algorithm (generally an iterative procedure) which transforms the configuration and its discrepancy relations in any way which does not violate the order relations among the reported discrepancies, and which maps the modified configuration into the <u>k</u> space, and 3.) measure the discrepancy between the modified configuration and the original configuration. To this discrepancy some "goodness of fit" criterion like Kruskal's <u>Stress</u> (kruskal, 1968) is then applied, and an arbitrary judgment of the adequacy of fit is made. Several routines for non metric MDS are currently available, the best known being Kruskal's MDSCAL (Kruskal, 1964); Young-Torgerson's Torsca (Young and Torgerson, 1967); and Guttman-Lingoes series of non metric programs (Lingoes, 1972).

In spite of the promise of the non metric techniques, all share a potentially serious shortcoming: as Reichenbach notes, in general a space of <u>r</u> dimensions cannot be mapped one-to-one into a space of <u>k</u> dimensions where $k < r$. This means that, in general, no function relates the resultant configuration to the original discrepancies reported by the respondent. Although operations like those specified above for the derivation of descriptive variables like velocity and acceleration can be defined for the non metric spaces, no one-to-one corespondence can be established between such symbolic operations and the perceptions of observers. Thus non metric procedures leave an inherent indeterminacy between the concepts and operations of theory and the perceptions of observers that is not present in metric models. Clearly this is an undesireable state of affairs, and in fact one that would not be tolerated at all except for the assumption of imprecision of the original discrepancy scores. Several arguments may be levelled against this assumption.

Whatever error might occur in the discrepancy judgments of individuals must be, first of all, error due to simple unreliability rather than systematic bias. This is clearly true, since the phenomena under study are perceptions of observers, and criteria for systematic misestimates of perceived discrepancies must assume the _a priori_ existence of "true perceived discrepancies" whose values are independent of the perceptions of human observers. In the same _a priori_ sense that the definition of discrepancy is provided by the respondent rather than given by the scientist, one cannot be systematically in error in reporting one's perception. This would require not misperceiving 'reality', but misperceiving one's own perceptions. In this important sense, no systematic error of estimate could ever be detected, and its existence or non existence becomes a metaphysical matter.

One might argue, however, that, at least in the case of psychophysical measures (such as loudness, distance, color, etc.) such a criterion exists. (Torgerson, 1958) While initially appealing, this view is also flawed.

The development by scientists of a systematic symbolic procedure through which the observations of observers may be encoded enhances the observational capacity of the observer, so that he/she may experience phenomena more systematically and more precisely than when observations are experienced and encoded by vernacular coding systems. Furthermore, by precisely specifying the operational rule by which observations are to be made, the scientist standardizes the observations of observers. We should expect that, prior to the development of a standardized measuring rule, different observers would differ widely among themselves, even, for example, about physical distances, and that establishment of the standard measure would create inter-observer correspondence. The importance of this standardization has seldom been fully recognized, and there is reason to believe that it may be the sole contribution of science to human understanding.

What we today call physical measures, such as distance, loudness, light intensity, and so on, are measures themselves developed by science, and quite recently. It is a main thesis of this paper that developing or learning such a system of measure "improves" the perceptual abilities of those who know the system, and, since it standardizes the operations of observation across observers, creates a consensual or "true" view, although clearly the result is not a "true" view in a metaphysical sense.

Rather than serving as a "true measure" against which psychological discriminations may be validated, physical measures of stimuli ought more reasonably be considered simply an alternative set of procedures for observation which, although more complete and more consistent, is not metaphysically more "correct." It would be unlikely that observations of the subject using more primitive vernacular methods would yield results identical to these scientific procedures, but nonetheless these vernacular observations should not be viewed as in error in any absolute sense.

We might then expect that the development and dissemination of the measurement methods described in this paper would not show their full effects immediately. The measurement methods described here, as is true of others, are not simply passive scales upon which the perceptions of respondents are projected, but rather serve themselves as symbolic mechanisms of perception which alter the perceptual abilities of the respondents. They are constructed by the joint act of scientist, who specifies the unit of measure and defines rules for its application to perceptions, and the observer, who provides the referent (definition) for the standard unit. Practice in their use and communication about their outcomes results in standardization of meaning and perception in the same way as vernacular linguistic symbols are developed. Thus a physical scientist should be able to make distance judgments that are more nearly equivalent to those resulting from scientific procedures than those made by non scientists, and surveyors should do better still. Similarly, audiologists familiar with the decibel scale should be able to convert perceived loudness into sound intensity (a "physical" measure) insofar as they are familiar with the logistic relation between the two. Continued practice, both for the individual observer and the collective culture should yield increased correspondence among observers and over time.

We should not be surprized, therefore, if the immediate application of the procedures for measuring discrepancy described here would result initially in unreliable measures, just as we would not be surprized if members of a mathematically naive culture made unreliable measures of distance after a few minutes of instruction in surveying. We should expect, however, that the development of a cadre of scientists trained in the method, along with subsequent diffusion of the system through the educational system should result in increasingly precise judgments of discrepancies by the population in general.

Finally, even though precise and reliable measurements of perceived discrepancies by individuals may be a long way off, nonetheless discrepancy estimates averaged over a large sample of individuals can yield extremely precise and reliable estimates of discrepancies as perceived by that aggregate of people even now. (Woefel, 1974; Barnett, 1972) These measures might well be seen as descriptions of discrepancies as seen by the culture from whom the sample of individuals is drawn. While we might hope that this theory and its associated measurement system can eventually serve as a precise tool for the analysis of individual cognitive processes, it would seem to be fully able to serve now as a precise system for the analysis of cultural or macro-communication processes. In fact, recent applications of thesetechniques to political processes (Barnett, Serota and Taylor, 1974) mass media effects (Barnett, 1972), Cultural definitions of sex roles (Saltiel, 1974), and even individual cognitive processes (Marlier, 1974) have shown great promise even at this early stage.

Should this line of reasoning prove correct, communication scientists might well consider less willingness to settle for the weaker and less satisfactory non metric approach and concentrate greater effort on the more difficult but potentially more rewarding task ofestablishing measures sufficiently precise to fit the metric model. Perhaps more importantly, the development of a general theory of discrepancy may well lave important consequences for physical science by providing a more comprehensive symbolic representation of the relationship of human observers to what are now considered physical phenomena.

	Atlanta	Boston	Chicago	Cleveland	Dallas	Denver	Detroit	Los Angeles	Miami	New Orleans	New York	Phoenix	Pittsburgh	San Francisco	Seattle	Washington
Atlanta	0	931	587	554	721	1212	596	1936	604	424	748	1592	521	2139	2182	543
Boston	931	0	851	551	1551	1769	613	2596	1255	1359	188	2300	483	2699	2473	393
Chicago	587	851	0	308	803	920	238	1745	1188	833	713	1453	410	1858	1737	597
Cleveland	554	551	308	0	1035	1227	90	2049	1087	924	405	1749	115	2166	2026	306
Dallas	721	1551	803	1035	0	663	999	1240	1111	443	1374	887	1070	1483	1681	1135
Denver	1212	1769	920	1227	663	0	1156	831	1726	1082	1631	586	1320	949	1021	1494
Detroit	596	613	238	90	999	1156	0	1983	1152	939	482	1690	205	2091	1938	396
Los Angeles	1936	2596	1745	2044	1246	831	1983	0	2339	1673	2451	357	2136	347	959	2300
Miami	604	1255	1188	1087	1111	1726	1152	2339	0	669	1092	1982	1010	2594	2734	923
New Orleans	424	1359	833	934	443	1082	939	1673	669	0	1171	1316	919	1926	2101	966
New York	748	188	713	405	1374	1631	482	2451	1092	1171	0	2145	317	2571	2408	205
Phoenix	1592	2300	1453	1749	887	586	1690	357	1982	1316	2145	0	1828	653	1114	1983
Pittsburgh	521	483	410	115	1070	1320	205	2136	1010	919	317	1828	0	2264	2138	192
San Francisco	2139	2699	1858	2166	1483	949	2091	347	2594	1926	2571	653	2264	0	678	2442
Seattle	2182	2473	1737	2026	1681	1021	1938	959	2734	2101	2408	1114	2138	678	0	2392
Washington	543	393	597	306	1185	1494	396	2300	923	966	205	1983	192	2442	2392	0

TABLE: 1 U.S. Inter-City Air Distances

(One unit equals one statute mile)

BIBLIOGRAPHY

Barnett, George A., Reliability and Metric Multidimensional Scaling, (unpublished).
1972

Barnett, George; Serota, Kim B.; and Taylor, James A.; "A Method for Political
1974 Communication Research," Association for Education in Journalism, Aug. 18-20, 1974, San Diego.

Bohr, Niels, Atomic Physics and Human Knowledge, Science Editions, New York.
1961

Danes, J., and Woelfel, J. "An Alternative to the 'Traditional' Scaling Para-
1975 digm in Mass Communication Research: Multidimensional Reduction of Ratio Judgments of Separation," a paper presented to the Mass Communication Division of the International Communication Association, Chicago, Illinois, April, 1975.

Durkheim, Emile. Rules of the Sociological Method, Free Press.
1951

Einstein, Albert, "On the Electrodynamics of Moving Bodies," in The Principle
1952 of Relativity, a collection of original memoirs, New York, Dover Publications, pp. 37-65.

__________, Relativity: The Special and General Theory, Crown, New York.
1961

Helmholtz, Principles of Physics.
1869

Hertz, H., The Principles of Mechanics, New York, Dover Publications.
1956

King, A., Thermophysics, San Francisco, W. H. Freeman and Company.
1962

Kramer, Edna, The History and Development of Modern Mathematics.
1970 Greenwich, Connecticut: Fawcett Publications, Inc.

Kruskal, J. B., "Multidimensional Scaling by Optimizing Goodness of Fit to a
1964 Nonmetric Hypothesis," Psychometrika, 29, pp. 1-27(a).

__________, "Nonmetric Multidimensional Scaling: A Numerical Method." Psycho-
1964 metrika, 29, pp. 115-129(b).

Lagrange, Mechanique Analytique. Paris V^{e} Courcier.
1788

Lingoes, J. C., "A General Survey of the Guttman-Lingoes Nonmetric Program Ser-
1972 ies," in R. Shepard, A. K. Rommey and S. B. Merlove (eds.), Multidimensional Scaling: Theory and Applications in the Behavioral Sciences, New York, Seminar Press.

Mach, Ernst, <u>The Science of Mechanics</u>, The Open Court Publishing Co., Chicago
1919 and London.

Marlier, John, <u>Procedures for the Precise Test of Social Judgment Predictions of Assimilation and Contrast</u>, paper presented to Speech Communication
1974 Association annual meeting, Chicago, December, 1974.

Michels, Walter, <u>et al</u>., <u>Foundations of Physics</u>, Princeton, Van Nostrand.
1968

Mistretta, Martin J., "Criminal Law and the Collective Conscience: Multidimen-
1975 sional Scaling of Collective Cultural Perceptions of Crime," unpublished manuscript, University of Illinois, Urbana.

Pfanzagl, J. <u>Theory of Measurement</u>, Wurzburg, Germany, Physica-verlag.
1968

Poincare, Henri, <u>The Foundations of Science</u>, Lancaster, Pa., Science Press.
1946

Reichenbach, H., <u>The Rise of Scientific Philosophy</u>, Berkeley, University of
1951 California Press.

__________, <u>The Philosophy of Space and Time</u>, New York, Dover Publication.
1958

Riemann, B. "On the Hypotheses Which Lie at the Foundations of Geometry," in
1959 <u>A Source Book in Mathematics</u>, vol. 11, David E. Smith (ed.), New York, Dover Publications.

Saltiel, John, "Multidimensional Scaling Analysis of Changes in Sex Role Ste-
1974 reotypes," paper presented at Speech Communication Association annual meeting, Chicago, December, 1974.

Serota, Kim B., <u>Metric Multidimensional Scaling and Communication: Theory and</u>
1974 <u>Implementation</u>, master's thesis, Michigan State University, East Lansing, Michigan.

Serota, Kim B., Edward L. Fink, Jay Noel, and Joseph Woelfel, Communication, Ideology, and Political Behavior: A Multidimensional Analysis. A paper presented to ICA annual meeting. Chicago, April, 1975.

Shepard, R., "Metric Structures in Ordinal Data," <u>Journal of Mathematical Psy-</u>
1966 <u>chology</u>, 3, pp. 287-315.

__________, "Introduction to Volume I," in R. Shepard, A. K. Romney, and S. B.
1972 Norlove (eds.), <u>Multidimensional Scaling: Theory and Applications in the Behavioral Sciences</u>, New York, Seminar Press.

Torgerson, Warren S., <u>Theory and Methods of Scaling</u>, New York, Wiley.
1958

Weyl, H., "Gravitation and Electricity," in The Principle of Relativity, a collec-
1952 tion of original memoirs, New York, Dover Publications, pp. 199-216.

Woelfel, Joseph, Procedures for the Precise Measurement of Cultural Processes,
1974 paper presented at Southwestern Social Science Association Annual Meetings, Dallas.

Young, Forest W., and Warren S. Torgerson, "Torsca: A FORTRAN IV Program for Shephard-Kruskal Multidimensional Scaling Analysis," Behavioral Science, 12, p. 498.