

# Many-Objective Evolutionary Influence Maximization: Balancing Spread, Budget, Fairness, and Time

## Supplementary Material

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**Algorithm 1** Propagation model  $\sigma(S)$ .  $S$  is the seed set,  $G$  is the graph,  $\tau$  is the maximum number of timesteps,  $p$  (needed only for IC) is the probability that an edge will be activated.

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**Input:**  $S$

**Require:**  $\mathcal{G}, \tau, [p]$

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1:  $A \leftarrow S$ 
2:  $B \leftarrow S$ 
3:  $t = 0$ 
4: while  $B$  not empty and  $t < \tau$  do
5:    $C \leftarrow \emptyset$ 
6:   for  $n \in B$  do
7:     for  $m \in \text{neighbours}(n) \setminus A$  do
8:        $C \leftarrow C \cup \{m\}$  with probability  $p$ 
9:    $B \leftarrow C$ 
10:   $A \leftarrow A \cup B$ 
11:   $t = t + 1$ 
12: return  $|A|$ 

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- The set of activated nodes
- Nodes activated in the previous timestep
- Timestep counter
- Nodes activated in the current timestep
- Activation attempt ( $p$  differs for IC/WC/LT)
- Total number of activated nodes

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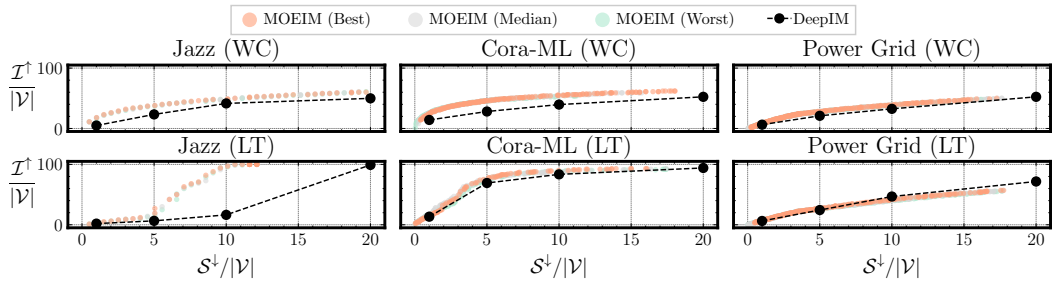


Fig. 1. Non-dominated solutions found MOEIM and DeepIM [4]. For MOEIM, we provide the worst, median and best sets of non-dominated solutions found across 10 runs. To allow for a direct comparison with the results reported in [4], the x-axis and y-axis show, respectively, the seed set size  $S^{\downarrow}$  and the final influence  $I^{\uparrow}$ , both normalized w.r.t. the size of each network,  $|\mathcal{V}|$ . For DeepIM, we show the results available in the original paper, where they are reported separately for each value of  $S^{\downarrow}/|\mathcal{V}|$ .

Table 1. Avg.  $\pm$  std. dev. hypervolume ( $HV$ ) achieved by the algorithms under comparison and our proposed MOEIM, across 10 runs for MOEIM, MOEA, and GDD, and (because of computational limits, as discussed in [1]) 3 runs for CELF ( $\mathcal{I}^\uparrow$ : Influence;  $\mathcal{S}^\downarrow$ : Seed set size;  $\mathcal{C}^\uparrow$ : Communities;  $\mathcal{F}^\uparrow$ : Fairness;  $\mathcal{B}^\downarrow$ : Budget;  $\mathcal{T}^\downarrow$ : Time). The column  $HV_{\mathcal{I}^\uparrow, \mathcal{S}^\downarrow}$  indicates the  $HV$  in the 2D  $\mathcal{I}^\uparrow, \mathcal{S}^\downarrow$  space, while the next four columns indicate the  $HV$  calculated in the corresponding 3D space, and  $HV_{\text{all}}$  indicates the  $HV$  calculated in a 6D space in which all objectives ( $\mathcal{I}^\uparrow, \mathcal{S}^\downarrow, \mathcal{C}^\uparrow, \mathcal{F}^\uparrow, \mathcal{B}^\downarrow, \mathcal{T}^\downarrow$ ) are considered. For MOEIM, we show the case where 2 objectives ( $\mathcal{I}^\uparrow, \mathcal{S}^\downarrow$ ), the combinations of 3 objectives ( $\mathcal{I}^\uparrow, \mathcal{S}^\downarrow$  plus either  $\mathcal{C}^\uparrow$ ,  $\mathcal{F}^\uparrow$ ,  $\mathcal{B}^\downarrow$  or  $\mathcal{S}^\downarrow$ ), and all objectives ( $\mathcal{I}^\uparrow, \mathcal{S}^\downarrow, \mathcal{C}^\uparrow, \mathcal{F}^\uparrow, \mathcal{B}^\downarrow, \mathcal{T}^\downarrow$ ) are considered during the evolutionary process. The colored (underlined) cells indicate the highest (second highest) value per column, separately per each dataset, propagation model, and combination of objectives considered to calculate the  $HV$ .

Algorithm	Independent Cascade (IC)						Weighted Cascade (WC)						
	$HV_{\mathcal{I}^\uparrow, \mathcal{S}^\downarrow}$	$HV_{\mathcal{I}^\uparrow, \mathcal{S}^\downarrow, \mathcal{C}^\uparrow}$	$HV_{\mathcal{I}^\uparrow, \mathcal{S}^\downarrow, \mathcal{F}^\uparrow}$	$HV_{\mathcal{I}^\uparrow, \mathcal{S}^\downarrow, \mathcal{B}^\downarrow}$	$HV_{\mathcal{I}^\uparrow, \mathcal{S}^\downarrow, \mathcal{T}^\downarrow}$	$HV_{\text{all}}$	$HV_{\mathcal{I}^\uparrow, \mathcal{S}^\downarrow}$	$HV_{\mathcal{I}^\uparrow, \mathcal{S}^\downarrow, \mathcal{C}^\uparrow}$	$HV_{\mathcal{I}^\uparrow, \mathcal{S}^\downarrow, \mathcal{F}^\uparrow}$	$HV_{\mathcal{I}^\uparrow, \mathcal{S}^\downarrow, \mathcal{B}^\downarrow}$	$HV_{\mathcal{I}^\uparrow, \mathcal{S}^\downarrow, \mathcal{T}^\downarrow}$	$HV_{\text{all}}$	
cellular-core	GDD [6]	4.99e-1 $\pm$ 3.72e-5	4.62e-1 $\pm$ 2.29e-4	4.19e-1 $\pm$ 1.83e-6	4.57e-1 $\pm$ 8.45e-5	6.10e-2 $\pm$ 3.62e-3	3.21e-2 $\pm$ 2.94e-3	4.11e-1 $\pm$ 2.26e-4	3.78e-1 $\pm$ 7.52e-5	3.01e-1 $\pm$ 1.21e-4	<u>3.10e-1 <math>\pm</math> 4.47e-4</u>	7.45e-2 $\pm$ 5.62e-3	2.68e-2 $\pm$ 4.93e-3
	CELF [3]	4.87e-1 $\pm$ 8.97e-4	4.58e-1 $\pm$ 7.80e-4	4.07e-1 $\pm$ 4.35e-3	4.60e-1 $\pm$ 1.12e-3	2.52e-2 $\pm$ 4.43e-2	1.43e-2 $\pm$ 1.05e-2	3.80e-1 $\pm$ 1.40e-3	3.47e-1 $\pm$ 1.49e-3	3.17e-1 $\pm$ 6.99e-3	2.53e-1 $\pm$ 1.84e-3	7.01e-2 $\pm$ 1.62e-3	3.59e-2 $\pm$ 3.62e-3
	MOEA [1]	4.96e-1 $\pm$ 1.84e-3	4.68e-1 $\pm$ 1.80e-3	4.17e-1 $\pm$ 1.19e-2	4.53e-1 $\pm$ 2.14e-3	1.97e-2 $\pm$ 2.79e-2	3.00e-3 $\pm$ 9.49e-3	3.59e-1 $\pm$ 6.40e-3	3.26e-1 $\pm$ 6.57e-3	3.15e-1 $\pm$ 7.68e-3	4.97e-2 $\pm$ 5.52e-3	2.11e-2 $\pm$ 5.25e-3	
	MOEIM ( $\mathcal{I}^\uparrow, \mathcal{S}^\downarrow$ )	<u>4.99e-1 <math>\pm</math> 3.59e-4</u>	<u>4.70e-1 <math>\pm</math> 5.52e-4</u>	4.18e-1 $\pm$ 4.33e-3	4.54e-1 $\pm$ 1.38e-3	0.00e0 $\pm$ 0.00e0	3.43e-1 $\pm$ 1.08e-3	<u>4.13e-1 <math>\pm</math> 1.37e-3</u>	<u>3.82e-1 <math>\pm</math> 1.36e-3</u>	3.52e-1 $\pm$ 1.63e-2	2.50e-1 $\pm$ 3.49e-4	7.06e-2 $\pm$ 5.46e-3	1.67e-2 $\pm$ 6.13e-3
	MOEIM ( $\mathcal{I}^\uparrow, \mathcal{S}^\downarrow, \mathcal{C}^\uparrow$ )	4.95e-1 $\pm$ 1.64e-3	<u>4.72e-1 <math>\pm</math> 3.53e-3</u>	4.17e-1 $\pm$ 5.05e-3	4.54e-1 $\pm$ 1.17e-3	1.92e-2 $\pm$ 2.49e-2	3.00e-3 $\pm$ 4.51e-3	4.09e-1 $\pm$ 1.29e-3	<u>3.80e-1 <math>\pm</math> 1.29e-3</u>	3.52e-1 $\pm$ 1.69e-3	2.57e-1 $\pm$ 8.29e-4	7.90e-2 $\pm$ 4.57e-3	2.76e-2 $\pm$ 4.99e-3
	MOEIM ( $\mathcal{I}^\uparrow, \mathcal{S}^\downarrow, \mathcal{F}^\uparrow$ )	4.95e-1 $\pm$ 6.50e-4	<u>4.67e-1 <math>\pm</math> 8.70e-4</u>	<u>4.64e-1 <math>\pm</math> 1.44e-3</u>	4.55e-1 $\pm$ 2.05e-3	5.67e-3 $\pm$ 4.11e-3	2.31e-3 $\pm$ 5.26e-3	4.06e-1 $\pm$ 1.39e-3	2.11e-1 $\pm$ 1.98e-3	<u>3.89e-1 <math>\pm</math> 1.28e-3</u>	2.60e-1 $\pm$ 1.79e-3	7.61e-2 $\pm$ 2.28e-3	3.40e-2 $\pm$ 4.03e-3
	MOEIM ( $\mathcal{I}^\uparrow, \mathcal{S}^\downarrow, \mathcal{B}^\downarrow$ )	4.91e-1 $\pm$ 1.82e-3	4.63e-1 $\pm$ 1.83e-3	4.41e-1 $\pm$ 1.83e-3	4.41e-1 $\pm$ 4.35e-3	<u>4.74e-1 <math>\pm</math> 1.79e-3</u>	1.38e-1 $\pm$ 2.81e-3	3.94e-1 $\pm$ 9.16e-4	3.63e-1 $\pm$ 6.83e-4	3.61e-1 $\pm$ 8.82e-3	2.66e-1 $\pm$ 1.16e-3	8.83e-2 $\pm$ 6.41e-4	3.07e-2 $\pm$ 1.45e-3
	MOEIM ( $\mathcal{I}^\uparrow, \mathcal{S}^\downarrow, \mathcal{T}^\downarrow$ )	<u>4.98e-1 <math>\pm</math> 6.38e-4</u>	4.69e-1 $\pm$ 5.90e-4	4.22e-1 $\pm$ 8.15e-3	<u>4.64e-1 <math>\pm</math> 2.32e-4</u>	<u>1.60e-1 <math>\pm</math> 3.66e-3</u>	4.57e-2 $\pm$ 3.57e-3	4.10e-1 $\pm$ 7.90e-4	3.79e-1 $\pm$ 7.93e-4	3.61e-1 $\pm$ 5.31e-3	2.59e-1 $\pm$ 1.23e-3	<u>9.84e-2 <math>\pm</math> 9.77e-4</u>	3.56e-2 $\pm$ 4.83e-4
	MOEIM (all)	4.86e-1 $\pm$ 5.43e-4	4.61e-1 $\pm$ 6.62e-4	<u>4.66e-1 <math>\pm</math> 2.55e-3</u>	4.66e-1 $\pm$ 8.19e-4	<u>1.60e-1 <math>\pm</math> 1.30e-3</u>	<u>7.72e-2 <math>\pm</math> 3.77e-3</u>	3.81e-1 $\pm$ 2.42e-3	3.50e-1 $\pm$ 2.35e-3	3.57e-1 $\pm$ 1.36e-3	2.55e-1 $\pm$ 8.23e-4	<u>8.92e-2 <math>\pm</math> 4.20e-4</u>	<u>8.25e-2 <math>\pm</math> 7.43e-4</u>
	feedback-combined	GDD [6]	5.15e-1 $\pm$ 4.42e-5	3.65e-1 $\pm$ 4.30e-3	2.95e-1 $\pm$ 2.38e-5	4.62e-1 $\pm$ 2.92e-4	0.00e0 $\pm$ 0.00e0	0.00e0 $\pm$ 0.00e0	2.07e-1 $\pm$ 9.42e-5	1.84e-1 $\pm$ 4.89e-5	8.76e-2 $\pm$ 3.38e-5	1.70e-1 $\pm$ 5.91e-5	4.66e-3 $\pm$ 3.30e-3
CELF [3]		5.18e-1 $\pm$ 4.99e-4	3.65e-1 $\pm$ 1.00e-3	2.84e-1 $\pm$ 9.19e-3	4.73e-1 $\pm$ 1.66e-3	0.00e0 $\pm$ 0.00e0	0.00e0 $\pm$ 0.00e0	2.46e-1 $\pm$ 1.10e-3	1.84e-1 $\pm$ 7.23e-4	1.17e-1 $\pm$ 6.10e-3	2.04e-1 $\pm$ 6.49e-4	2.33e-3 $\pm$ 3.27e-3	1.18e-4 $\pm$ 2.05e-4
MOEA [1]		5.02e-1 $\pm$ 4.56e-3	3.47e-1 $\pm$ 5.72e-3	2.08e-1 $\pm$ 5.97e-3	4.74e-1 $\pm$ 1.52e-3	0.00e0 $\pm$ 0.00e0	0.00e0 $\pm$ 0.00e0	1.06e-1 $\pm$ 5.52e-3	1.52e-1 $\pm$ 7.79e-3	1.21e-1 $\pm$ 4.25e-3	1.71e-1 $\pm$ 3.28e-3	0.00e0 $\pm$ 0.00e0	2.76e-4 $\pm$ 1.40e-4
MOEIM ( $\mathcal{I}^\uparrow, \mathcal{S}^\downarrow$ )		<u>5.22e-1 <math>\pm</math> 8.35e-4</u>	<u>3.78e-1 <math>\pm</math> 2.52e-3</u>	3.18e-1 $\pm$ 5.60e-3	4.80e-1 $\pm$ 4.46e-3	0.00e0 $\pm$ 0.00e0	0.00e0 $\pm$ 0.00e0	<u>2.50e-1 <math>\pm</math> 2.73e-4</u>	<u>1.89e-1 <math>\pm</math> 9.27e-4</u>	1.49e-1 $\pm$ 9.40e-4	2.06e-1 $\pm$ 1.25e-3	8.34e-3 $\pm$ 3.15e-3	2.56e-4 $\pm$ 2.42e-4
MOEIM ( $\mathcal{I}^\uparrow, \mathcal{S}^\downarrow, \mathcal{C}^\uparrow$ )		5.12e-1 $\pm$ 1.07e-3	<u>3.93e-1 <math>\pm</math> 1.88e-3</u>	3.18e-1 $\pm$ 9.67e-3	4.78e-1 $\pm$ 1.31e-3	0.00e0 $\pm$ 0.00e0	1.58e-4 $\pm$ 5.01e-4	2.46e-1 $\pm$ 1.13e-3	<u>1.95e-1 <math>\pm</math> 8.76e-4</u>	1.65e-1 $\pm$ 3.67e-4	2.04e-1 $\pm$ 1.37e-3	4.09e-3 $\pm$ 2.98e-3	1.97e-4 $\pm$ 2.49e-4
MOEIM ( $\mathcal{I}^\uparrow, \mathcal{S}^\downarrow, \mathcal{F}^\uparrow$ )		5.14e-1 $\pm$ 1.13e-3	3.71e-1 $\pm$ 4.36e-4	<u>3.73e-1 <math>\pm</math> 2.21e-3</u>	<u>4.86e-1 <math>\pm</math> 3.54e-3</u>	3.18e-3 $\pm$ 4.49e-3	1.13e-4 $\pm$ 4.39e-4	2.41e-1 $\pm$ 1.75e-3	1.87e-1 $\pm$ 2.13e-3	<u>1.79e-1 <math>\pm</math> 1.40e-3</u>	2.02e-1 $\pm$ 1.22e-3	8.46e-3 $\pm$ 2.61e-3	6.99e-4 $\pm$ 6.83e-4
MOEIM ( $\mathcal{I}^\uparrow, \mathcal{S}^\downarrow, \mathcal{B}^\downarrow$ )		5.05e-1 $\pm$ 1.40e-3	3.54e-1 $\pm$ 2.80e-3	3.28e-1 $\pm$ 6.31e-4	<u>4.91e-1 <math>\pm</math> 1.28e-3</u>	<u>7.66e-2 <math>\pm</math> 4.09e-3</u>	<u>6.22e-2 <math>\pm</math> 1.52e-3</u>	2.41e-1 $\pm$ 2.13e-3	1.80e-1 $\pm$ 2.15e-3	1.48e-1 $\pm$ 2.27e-3	<u>2.07e-1 <math>\pm</math> 1.47e-3</u>	<u>1.70e-2 <math>\pm</math> 1.35e-3</u>	2.12e-3 $\pm$ 6.82e-4
MOEIM ( $\mathcal{I}^\uparrow, \mathcal{S}^\downarrow, \mathcal{T}^\downarrow$ )		<u>5.21e-1 <math>\pm</math> 3.96e-4</u>	3.70e-1 $\pm$ 3.58e-3	3.11e-1 $\pm$ 1.25e-2	4.82e-1 $\pm$ 5.86e-4	<u>7.52e-2 <math>\pm</math> 2.84e-3</u>	<u>1.14e-2 <math>\pm</math> 1.94e-4</u>	<u>2.48e-1 <math>\pm</math> 7.09e-4</u>	1.88e-1 $\pm$ 1.06e-3	1.51e-1 $\pm$ 2.52e-3	<u>2.06e-1 <math>\pm</math> 9.59e-4</u>	<u>1.83e-2 <math>\pm</math> 1.35e-3</u>	<u>2.00e-3 <math>\pm</math> 6.38e-4</u>
MOEIM (all)		4.91e-1 $\pm$ 1.66e-3	3.53e-1 $\pm$ 3.71e-3	<u>3.50e-1 <math>\pm</math> 1.43e-3</u>	4.78e-1 $\pm$ 1.31e-3	6.40e-2 $\pm$ 2.02e-3	<u>9.22e-3 <math>\pm</math> 1.01e-3</u>	2.22e-1 $\pm$ 4.23e-3	1.66e-1 $\pm$ 3.35e-3	1.60e-1 $\pm$ 2.19e-3	1.83e-1 $\pm$ 3.30e-3	1.61e-2 $\pm$ 3.21e-4	<u>3.89e-3 <math>\pm</math> 3.53e-4</u>
majority		GDD [6]	<u>1.55e-2 <math>\pm</math> 7.05e-6</u>	<u>1.01e-2 <math>\pm</math> 9.04e-6</u>	9.90e-3 $\pm$ 3.79e-6	9.06e-3 $\pm$ 7.76e-6	9.75e-3 $\pm$ 1.46e-4	1.84e-3 $\pm$ 1.63e-5	1.64e-1 $\pm$ 1.64e-4	1.43e-1 $\pm$ 1.30e-4	1.04e-1 $\pm$ 1.07e-4	1.09e-1 $\pm$ 1.51e-4	6.95e-3 $\pm$ 2.25e-3
	CELF [3]	1.53e-2 $\pm$ 1.11e-4	8.34e-3 $\pm$ 1.71e-4	7.01e-3 $\pm$ 4.94e-4	9.48e-3 $\pm$ 3.19e-5	8.50e-3 $\pm$ 3.65e-4	1.33e-3 $\pm$ 1.56e-4	<u>2.95e-2 <math>\pm</math> 2.29e-3</u>	<u>2.60e-2 <math>\pm</math> 2.60e-3</u>	1.95e-1 $\pm$ 4.30e-3	<u>2.38e-1 <math>\pm</math> 1.60e-3</u>	3.63e-3 $\pm$ 3.36e-5	3.67e-4 $\pm$ 1.43e-4
	MOEA [1]	1.34e-2 $\pm$ 1.15e-4	7.84e-3 $\pm$ 2.36e-4	9.33e-3 $\pm$ 9.90e-5	8.58e-3 $\pm$ 6.39e-5	8.51e-3 $\pm$ 8.23e-5	1.70e-3 $\pm$ 7.55e-5	1.86e-1 $\pm$ 5.41e-3	1.63e-1 $\pm$ 5.68e-3	1.22e-1 $\pm$ 1.75e-3	1.93e-1 $\pm$ 1.19e-3	1.92e-4 $\pm$ 2.69e-4	
	MOEIM ( $\mathcal{I}^\uparrow, \mathcal{S}^\downarrow$ )	<u>1.55e-2 <math>\pm</math> 1.22e-4</u>	9.54e-3 $\pm$ 1.43e-4	1.06e-2 $\pm$ 2.45e-4	8.34e-3 $\pm$ 3.02e-5	9.06e-3 $\pm$ 1.13e-4	1.61e-3 $\pm$ 7.57e-5	<u>2.90e-1 <math>\pm</math> 1.20e-3</u>	<u>2.66e-1 <math>\pm</math> 9.10e-4</u>	2.08e-1 $\pm$ 1.24e-3	2.06e-1 $\pm$ 2.21e-3	2.83e-3 $\pm$ 6.74e-4	2.22e-4 $\pm$ 1.38e-4
	MOEIM ( $\mathcal{I}^\uparrow, \mathcal{S}^\downarrow, \mathcal{C}^\uparrow$ )	1.52e-2 $\pm$ 9.41e-5	<u>9.73e-3 <math>\pm</math> 1.11e-4</u>	<u>1.10e-2 <math>\pm</math> 2.22e-4</u>	8.28e-3 $\pm$ 6.27e-5	9.50e-3 $\pm$ 2.05e-4	2.02e-3 $\pm$ 1.0e-4	2.80e-1 $\pm$ 3.58e-3	2.61e-1 $\pm$ 3.37e-3	2.07e-1 $\pm$ 6.51e-3	2.00e-1 $\pm$ 3.09e-3	0.00e0 $\pm$ 0.00e0	3.46e-4 $\pm$ 2.95e-4
	MOEIM ( $\mathcal{I}^\uparrow, \mathcal{S}^\downarrow, \mathcal{F}^\uparrow$ )	1.42e-2 $\pm$ 2.73e-5	8.75e-3 $\pm$ 8.07e-5	<u>9.28e-3 <math>\pm</math> 1.55e-4</u>	8.57e-3 $\pm$ 3.50e-4	9.37e-3 $\pm$ 9.72e-5	2.47e-3 $\pm$ 1.16e-4	2.71e-1 $\pm$ 1.50e-3	2.49e-1 $\pm$ 1.53e-3	<u>2.33e-1 <math>\pm</math> 1.83e-3</u>	1.90e-1 $\pm$ 1.83e-3	6.40e-3 $\pm$ 2.60e-3	8.41e-4 $\pm$ 3.24e-4
	MOEIM ( $\mathcal{I}^\uparrow, \mathcal{S}^\downarrow, \mathcal{B}^\downarrow$ )	1.43e-2 $\pm$ 8.04e-5	8.52e-3 $\pm$ 1.50e-4	1.11e-2 $\pm$ 2.64e-4	<u>1.05e-2 <math>\pm</math> 6.82e-5</u>	1.04e-2 $\pm$ 2.95e-4	2.75e-3 $\pm$ 1.08e-4	2.82e-1 $\pm$ 1.56e-3	2.59e-1 $\pm$ 8.24e-4	<u>2.12e-1 <math>\pm</math> 1.37e-3</u>	2.19e-1 $\pm$ 2.68e-3	3.98e-3 $\pm$ 7.49e-4	4.47e-4 $\pm$ 1.46e-4
	MOEIM ( $\mathcal{I}^\uparrow, \mathcal{S}^\downarrow, \mathcal{T}^\downarrow$ )	1.52e-2 $\pm$ 2.64e-4	9.27e-3 $\pm$ 3.50e-4	1.08e-2 $\pm$ 4.58e-4	9.27e-3 $\pm$ 9.40e-5	<u>1.14e-2 <math>\pm</math> 1.94e-4</u>	2.26e-3 $\pm$ 1.19e-4	2.81e-1 $\pm$ 2.54e-3	2.57e-1 $\pm$ 2.44e-3	2.01e-1 $\pm$ 2.72e-3	<u>1.98e-1 <math>\pm</math> 2.72e-3</u>	<u>1.74e-2 <math>\pm</math> 1.26e-3</u>	4.28e-3 $\pm$ 8.34e-4
	MOEIM (all)	1.36e-2 $\pm$ 4.98e-5	7.86e-3 $\pm$ 8.30e-5	1.17e-2 $\pm$ 1.81e-4	<u>1.01e-2 <math>\pm</math> 2.77e-5</u>	<u>1.05e-2 <math>\pm</math> 1.65e-4</u>	<u>2.87e-3 <math>\pm</math> 9.96e-5</u>	2.46e-1 $\pm$ 1.92e-3	2.26e-1 $\pm$ 2.33e-3	2.05e-1 $\pm$ 7.72e-4	1.71e-1 $\pm$ 1.25e-3	<u>1.57e-2 <math>\pm</math> 1.40e-3</u>	<u>8.93e-3 <math>\pm</math> 9.00e-4</u>
	with noise	GDD [6]	1.79e-1 $\pm$ 3.33e-5	1.17e-1 $\pm$ 4.82e-5	1.63e-1 $\pm$ 3.54e-5	1.59e-1 $\pm$ 7.96e-5	2.13e-2 $\pm$ 2.28e-3	7.07e-3 $\pm$ 1.62e-3	6.94e-2 $\pm$ 1.15e-3	6.60e-2 $\pm$ 2.64e-3	6.43e-2 $\pm$ 9.02e-6	5.10e-2 $\pm$ 2.64e-5	1.47e-2 $\pm$ 6.13e-4
CELF [3]		1.76e-1 $\pm$ 1.01e-3	1.67e-1 $\pm$ 9.63e-5	1.35e-1 $\pm$ 9.44e-3	1.63e-1 $\pm$ 4.87e-4	5.96e-3 $\pm$ 8.43e-3	8.40e-4 $\pm$ 4.47e-3	8.06e-2 $\pm$ 8.86e-4	7.48e-2 $\pm$ 9.77e-4	6.88e-2 $\pm$ 2.10e-3	<u>5.96e-2 <math>\pm</math> 4.37e-4</u>	1.66e-2 $\pm$ 7.10e-4	9.01e-3 $\pm$ 2.32e-4
MOEA [1]		1.67e-1 $\pm$ 1.62e-3	1.51e-1 $\pm$ 1.00e-3	1.55e-1 $\pm$ 1.95e-3	1.57e-1 $\pm$ 1.34e-3	0.00e0 $\pm$ 0.00e0	0.00e0 $\pm$ 0.00e0	5.20e-2 $\pm$ 1.49e-3	4.69e-2 $\pm$ 1.75e-3	4.59e-2 $\pm$ 2.13e-3	1.13e-2 $\pm$ 9.34e-4	6.96e-3 $\pm$ 4.38e-4	
MOEIM ( $\mathcal{I}^\uparrow, \mathcal{S}^\downarrow$ )		<u>1.81e-1 <math>\pm</math> 1.24e-4</u>	<u>1.75e-1 <math>\pm</math> 1.16e-4</u>	1.45e-1 $\pm$ 2.03e-4	1.68e-1 $\pm$ 6.91e-4	1.66e-2 $\pm$ 4.13e-4	6.06e-3 $\pm$ 2.62e-3	<u>3.25e-2 <math>\pm</math> 9.97e-4</u>	<u>2.82e-2 <math>\pm</math> 1.02e-4</u>	7.82e-2 $\pm$ 1.61e-3	6.80e-2 $\pm$ 7.76e-5	1.80e-2 $\pm$ 5.14e-4	9.25e-3 $\pm$ 3.51e-4
MOEIM ( $\mathcal{I}^\uparrow, \mathcal{S}^\downarrow, \mathcal{C}^\uparrow$ )		1.81e-1 $\pm$ 4.71e-4	<u>1.72e-1 <math>\pm</math> 1.62e-4</u>	1.58e-1 $\pm$ 2.92e-4	1.67e-1 $\pm$ 6.20e-4	1.70e-2 $\pm$ 8.58e-5	3.10e-3 $\pm$ 1.16e-3	8.25e-2 $\pm$ 1.97e-4	7.52e-2 $\pm$ 2.10e-4	<u>6.80e-2 <math>\pm</math> 2.28e-4</u>	5.57e-2 $\pm$ 2.99e-4	1.77e-2 $\pm$ 1.25e-4	1.02e-2 $\pm$ 2.30e-4
MOEIM ( $\mathcal{I}^\uparrow, \mathcal{S}^\downarrow, \mathcal{F}^\uparrow$ )		1.81e-1 $\pm$ 4.94e-4	1.72e-1 $\pm$ 4.26e-4	<u>1.78e-1 <math>\pm</math> 1.62e-4</u>	1.67e-1 $\pm$ 6.52e-4	1.12e-2 $\pm$ 1.43e-4	4.77e-3 $\pm$ 1.96e-4	8.27e-2 $\pm$ 2.00e-4	7.86e-2 $\pm$ 2.71e-4	<u>7.20e-2 <math>\pm</math> 1.93e-4</u>	6.45e-2 $\pm$ 2.81e-4	1.80e-2 $\pm$ 1.25e-4	1.06e-2 $\pm$ 1.25e-4
MOEIM ( $\mathcal$													

Table 2. Results of the statistical tests, performed by using the Holm-Bonferroni procedure [2], using “MOEIM (all)” as reference (Rank=8.82). Note that, for simplicity of notation, we omit “MOEIM” from the other settings of our algorithm. “Rank” refers to the ranking of the algorithm (the higher, the better). “z” is the statistic computed on the rank, “p” is the p-value, computed as the value of the cumulative normal distribution related to z.  $\alpha/j$  is the significance threshold adjusted for each row of the table. “Reject” indicates if the sequential null hypothesis (of equivalence between the reference and the algorithm on the row) can be rejected or not.

Method	Rank	z	p	$\alpha/j$	Rejected
$\mathcal{I}^\uparrow\text{-}\mathcal{S}^\downarrow\text{-}\mathcal{B}^\downarrow$	6.56e+00	-4.71e+00	1.21e-06	5.00e-02	True
$\mathcal{I}^\uparrow\text{-}\mathcal{S}^\downarrow\text{-}\mathcal{T}^\downarrow$	6.52e+00	-4.83e+00	6.76e-07	2.50e-02	True
$\mathcal{I}^\uparrow\text{-}\mathcal{S}^\downarrow\text{-}\mathcal{F}^\uparrow$	4.97e+00	-9.22e+00	1.54e-20	1.67e-02	True
$\mathcal{I}^\uparrow\text{-}\mathcal{S}^\downarrow\text{-}\mathcal{C}^\uparrow$	3.55e+00	-1.32e+01	3.24e-40	1.25e-02	True
GGD	3.46e+00	-1.35e+01	9.96e-42	1.00e-02	True
CELF	3.36e+00	-1.38e+01	2.31e-43	8.33e-03	True
MOEA	2.27e+00	-1.69e+01	5.01e-64	7.14e-03	True
$\mathcal{I}^\uparrow\text{-}\mathcal{S}^\downarrow$	2.15e+00	-1.72e+01	1.79e-66	6.25e-03	True

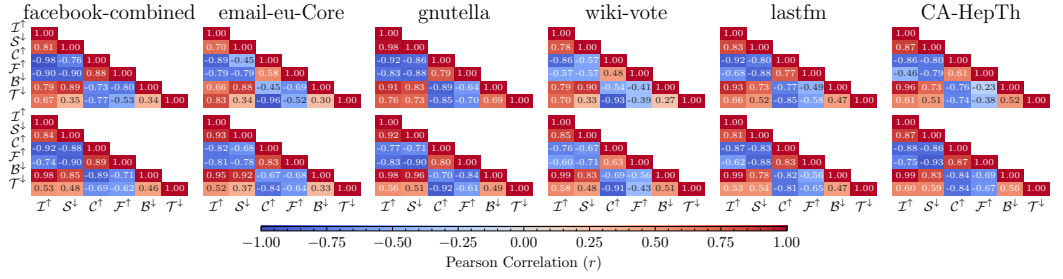


Fig. 2. Pearson correlation among the objectives ( $\mathcal{I}^\uparrow$ : Influence;  $\mathcal{S}^\downarrow$ : Seed set size;  $\mathcal{C}^\uparrow$ : Communities;  $\mathcal{F}^\uparrow$ : Fairness;  $\mathcal{B}^\downarrow$ : Budget;  $\mathcal{T}^\downarrow$ : Time). This correlation has been computed over the results of the 10 runs available for the “MOEIM (all)” setting (see Table 1 for details). Top row: IC model, bottom row: WC model.

Table 3. Results of the statistical tests using the Tukey HSD procedure [5] for pairwise comparisons. Note that, for simplicity of notation, we omit “MOEIM” from the settings of our algorithm. “Mean difference” refers to the mean of the difference on the hypervolume achieved by the methods in each pairwise comparison, “ $p$ -adj” refers to the adjusted  $p$ -value, “Lower CI” and “Upper CI” refer to the lower and upper bound for the 95% CI (Confidence Interval). “Reject” indicates if the null hypothesis (of statistical equivalence between Methods 1 and 2) can be rejected or not. We highlight the cases for which the hypothesis can be rejected.

Method 1	Method 2	Mean difference	$p$ -adj	Lower CI	Upper CI	Rejected
$I^\uparrow-S^\downarrow$	$I^\uparrow-S^\downarrow-B^\downarrow$	6.90e-3	3.00e-4	2.10e-3	1.16e-2	True
$I^\uparrow-S^\downarrow$	$I^\uparrow-S^\downarrow-C^\uparrow$	1.40e-3	9.93e-1	-3.40e-3	6.20e-3	False
$I^\uparrow-S^\downarrow$	$I^\uparrow-S^\downarrow-F^\uparrow$	2.10e-3	9.06e-1	-2.70e-3	6.90e-3	False
$I^\uparrow-S^\downarrow$	$I^\uparrow-S^\downarrow-T^\downarrow$	7.70e-3	0.00e+0	3.00e-3	1.25e-2	True
$I^\uparrow-S^\downarrow$	all	1.20e-2	0.00e+0	7.30e-3	1.68e-2	True
$I^\uparrow-S^\downarrow$	CELF	2.40e-3	9.80e-1	-4.60e-3	9.40e-3	False
$I^\uparrow-S^\downarrow$	GGD	3.80e-3	2.62e-1	-1.00e-3	8.50e-3	False
$I^\uparrow-S^\downarrow$	MOEA	0.00e+0	1.00e+0	-4.70e-3	4.80e-3	False
$I^\uparrow-S^\downarrow-B^\downarrow$	$I^\uparrow-S^\downarrow-C^\uparrow$	-5.50e-3	1.09e-2	-1.03e-2	-7.00e-4	True
$I^\uparrow-S^\downarrow-B^\downarrow$	$I^\uparrow-S^\downarrow-F^\uparrow$	-4.80e-3	5.11e-2	-9.50e-3	0.00e+0	False
$I^\uparrow-S^\downarrow-B^\downarrow$	$I^\uparrow-S^\downarrow-T^\downarrow$	9.00e-4	1.00e+0	-3.90e-3	5.60e-3	False
$I^\uparrow-S^\downarrow-B^\downarrow$	all	5.20e-3	2.28e-2	4.00e-4	9.90e-3	True
$I^\uparrow-S^\downarrow-B^\downarrow$	CELF	-4.50e-3	5.53e-1	-1.15e-2	2.50e-3	False
$I^\uparrow-S^\downarrow-B^\downarrow$	GGD	-3.10e-3	5.19e-1	-7.90e-3	1.60e-3	False
$I^\uparrow-S^\downarrow-B^\downarrow$	MOEA	-6.90e-3	3.00e-4	-1.16e-2	-2.10e-3	True
$I^\uparrow-S^\downarrow-C^\uparrow$	$I^\uparrow-S^\downarrow-F^\uparrow$	7.00e-4	1.00e+0	-4.00e-3	5.50e-3	False
$I^\uparrow-S^\downarrow-C^\uparrow$	$I^\uparrow-S^\downarrow-T^\downarrow$	6.40e-3	1.30e-3	1.60e-3	1.11e-2	True
$I^\uparrow-S^\downarrow-C^\uparrow$	all	1.07e-2	0.00e+0	5.90e-3	1.54e-2	True
$I^\uparrow-S^\downarrow-C^\uparrow$	CELF	1.00e-3	1.00e+0	-6.00e-3	8.00e-3	False
$I^\uparrow-S^\downarrow-C^\uparrow$	GGD	2.40e-3	8.36e-1	-2.40e-3	7.10e-3	False
$I^\uparrow-S^\downarrow-C^\uparrow$	MOEA	-1.40e-3	9.94e-1	-6.10e-3	3.40e-3	False
$I^\uparrow-S^\downarrow-F^\uparrow$	$I^\uparrow-S^\downarrow-T^\downarrow$	5.60e-3	8.10e-3	8.00e-4	1.04e-2	True
$I^\uparrow-S^\downarrow-F^\uparrow$	all	9.90e-3	0.00e+0	5.10e-3	1.47e-2	True
$I^\uparrow-S^\downarrow-F^\uparrow$	CELF	3.00e-4	1.00e+0	-6.80e-3	7.30e-3	False
$I^\uparrow-S^\downarrow-F^\uparrow$	GGD	1.60e-3	9.79e-1	-3.10e-3	6.40e-3	False
$I^\uparrow-S^\downarrow-F^\uparrow$	MOEA	-2.10e-3	9.12e-1	-6.90e-3	2.70e-3	False
$I^\uparrow-S^\downarrow-T^\downarrow$	all	4.30e-3	1.17e-1	-5.00e-4	9.10e-3	False
$I^\uparrow-S^\downarrow-T^\downarrow$	CELF	-5.40e-3	3.03e-1	-1.24e-2	1.70e-3	False
$I^\uparrow-S^\downarrow-T^\downarrow$	GGD	-4.00e-3	1.89e-1	-8.80e-3	8.00e-4	False
$I^\uparrow-S^\downarrow-T^\downarrow$	MOEA	-7.70e-3	0.00e+0	-1.25e-2	-2.90e-3	True
all	CELF	-9.70e-3	7.00e-4	-1.67e-2	-2.60e-3	True
all	GGD	-8.30e-3	0.00e+0	-1.31e-2	-3.50e-3	True
all	MOEA	-1.20e-2	0.00e+0	-1.68e-2	-7.20e-3	True
CELF	GGD	1.40e-3	1.00e+0	-5.70e-3	8.40e-3	False
CELF	MOEA	-2.40e-3	9.81e-1	-9.40e-3	4.70e-3	False
GGD	MOEA	-3.70e-3	2.71e-1	-8.50e-3	1.00e-3	False

## REFERENCES

- [1] Doina Bucur, Giovanni Iacca, Andrea Marcelli, Giovanni Squillero, and Alberto Tonda. 2018. Improving multi-objective evolutionary influence maximization in social networks. In *Applications of Evolutionary Computation*. Springer, Cham, Switzerland, 117–124.
- [2] Holm, Sture. 1979. A simple sequentially rejective multiple test procedure. *Scandinavian journal of statistics* 6, 2 (1979), 65–70.
- [3] Jure Leskovec, Andreas Krause, Carlos Guestrin, Christos Faloutsos, Jeanne VanBriesen, and Natalie Glance. 2007. Cost-effective outbreak detection in networks. In *ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*. ACM, New York, NY, USA, 420–429.
- [4] Chen Ling, Junji Jiang, Junxiang Wang, My T Thai, Renhao Xue, James Song, Meikang Qiu, and Liang Zhao. 2023. Deep graph representation learning and optimization for influence maximization. In *International Conference on Machine Learning*. PMLR, Honolulu, HI, USA, 21350–21361.
- [5] Tukey, John W. 1949. Comparing individual means in the analysis of variance. *Biometrics* 5, 2 (1949), 99–114.
- [6] Xiaojie Wang, Xue Zhang, Chengli Zhao, and Dongyun Yi. 2016. Maximizing the spread of influence via generalized degree discount. *PLoS ONE* 11, 10 (2016), e0164393.