

# Many-Objective Evolutionary Influence Maximization: Balancing Spread, Budget, Fairness, and Time

## Supplementary Material

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**Algorithm 1** Propagation model  $\sigma(S)$ .  $S$  is the seed set,  $G$  is the graph,  $\tau$  is the maximum number of timesteps,  $p$  (needed only for IC) is the probability that an edge will be activated.

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**Input:**  $S$

**Require:**  $\mathcal{G}, \tau, [p]$

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1:  $A \leftarrow S$                                 ▶ The set of activated nodes
2:  $B \leftarrow S$                                 ▶ Nodes activated in the previous timestep
3:  $t = 0$                                         ▶ Timestep counter
4: while  $B$  not empty and  $t < \tau$  do
5:    $C \leftarrow \emptyset$                         ▶ Nodes activated in the current timestep
6:   for  $n \in B$  do
7:     for  $m \in \text{neighbours}(n) \setminus A$  do
8:        $C \leftarrow C \cup \{m\}$  with probability  $p$   ▶ Activation attempt ( $p$  differs for IC/WC/LT)
9:    $B \leftarrow C$ 
10:   $A \leftarrow A \cup B$ 
11:   $t = t + 1$ 
12: return  $|A|$                                 ▶ Total number of activated nodes
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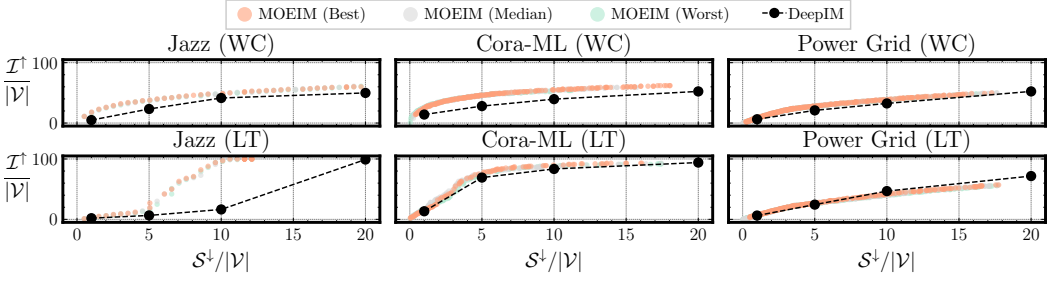


Fig. 1. Non-dominated solutions found MOEIM and DeepIM [4]. For MOEIM, we provide the worst, median and best sets of non-dominated solutions found across 10 runs. To allow for a direct comparison with the results reported in [4], the x-axis and y-axis show, respectively, the seed set size  $S^\downarrow$  and the final influence  $I^\uparrow$ , both normalized w.r.t. the size of each network,  $|V|$ . For DeepIM, we show the results available in the original paper, where they are reported separately for each value of  $S^\downarrow/|V|$ .

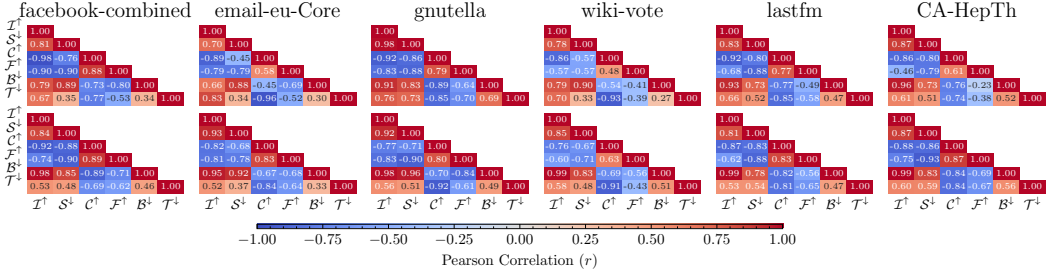


Fig. 2. Pearson correlation among the objectives ( $I^\uparrow$ : Influence;  $S^\downarrow$ : Seed set size;  $C^\uparrow$ : Communities;  $F^\uparrow$ : Fairness;  $B^\downarrow$ : Budget;  $T^\downarrow$ : Time). This correlation has been computed over the results of the 10 runs available for the “MOEIM (all)” setting (see Table 1 for details). Top row: IC model, bottom row: WC model.

Table 1. Avg.  $\pm$  std. dev. hypervolume ( $HV$ ) achieved by the algorithms under comparison and our proposed MOEIM, across 10 runs for MOEIM, MOEA, and GDD, and (because of computational limits, as discussed in [1]) 3 runs for CELF ( $\mathcal{I}^\uparrow$ : Influence;  $\mathcal{S}^\downarrow$ : Seed set size;  $\mathcal{C}^\uparrow$ : Communities;  $\mathcal{F}^\uparrow$ : Fairness;  $\mathcal{B}^\downarrow$ : Budget;  $\mathcal{T}^\downarrow$ : Time). The column  $HV_{\mathcal{I}^\uparrow, \mathcal{S}^\downarrow}$  indicates the  $HV$  in the 2D  $\mathcal{I}^\uparrow\text{-}\mathcal{S}^\downarrow$  space, while the next four columns indicate the  $HV$  calculated in the corresponding 3D space, and  $HV_{\text{all}}$  indicates the  $HV$  calculated in a 6D space in which all objectives ( $\mathcal{I}^\uparrow\text{-}\mathcal{S}^\downarrow\text{-}\mathcal{C}^\uparrow\text{-}\mathcal{F}^\uparrow\text{-}\mathcal{B}^\downarrow\text{-}\mathcal{T}^\downarrow$ ) are considered. For MOEIM, we show the case where 2 objectives ( $\mathcal{I}^\uparrow\text{-}\mathcal{S}^\downarrow$ ), the combinations of 3 objectives ( $\mathcal{I}^\uparrow\text{-}\mathcal{S}^\downarrow$  plus either  $\mathcal{C}^\uparrow$ ,  $\mathcal{F}^\uparrow$ ,  $\mathcal{B}^\downarrow$  or  $\mathcal{S}^\downarrow$ ), and all objectives ( $\mathcal{I}^\uparrow\text{-}\mathcal{S}^\downarrow\text{-}\mathcal{C}^\uparrow\text{-}\mathcal{F}^\uparrow\text{-}\mathcal{B}^\downarrow\text{-}\mathcal{T}^\downarrow$ ) are considered during the evolutionary process. The colored (underlined) cells indicate the highest (second highest) value per column, separately per each dataset, propagation model, and combination of objectives considered to calculate the  $HV$ .

| Algorithm         | Independent Cascade (IC)   |   |   |   |   |   | Weighted Cascade (WC)                               |   |   |   |   |   |   |
|-------------------|--|---|---|---|---|---|---|---|---|---|---|---|---|
|                   | $HV_{\mathcal{I}^\uparrow, \mathcal{S}^\downarrow}$  | $HV_{\mathcal{I}^\uparrow, \mathcal{S}^\downarrow, \mathcal{C}^\uparrow}$ | $HV_{\mathcal{I}^\uparrow, \mathcal{S}^\downarrow, \mathcal{F}^\uparrow}$ | $HV_{\mathcal{I}^\uparrow, \mathcal{S}^\downarrow, \mathcal{B}^\downarrow}$ | $HV_{\mathcal{I}^\uparrow, \mathcal{S}^\downarrow, \mathcal{T}^\downarrow}$ | $HV_{\text{all}}$                       | $HV_{\mathcal{I}^\uparrow, \mathcal{S}^\downarrow}$ | $HV_{\mathcal{I}^\uparrow, \mathcal{S}^\downarrow, \mathcal{C}^\uparrow}$ | $HV_{\mathcal{I}^\uparrow, \mathcal{S}^\downarrow, \mathcal{F}^\uparrow}$ | $HV_{\mathcal{I}^\uparrow, \mathcal{S}^\downarrow, \mathcal{B}^\downarrow}$ | $HV_{\mathcal{I}^\uparrow, \mathcal{S}^\downarrow, \mathcal{T}^\downarrow}$ | $HV_{\text{all}}$                       |   |
| cellular-core     | GDD [6]  | 4.99e-1 $\pm$ 3.72e-5   | 4.62e-1 $\pm$ 2.29e-4   | 4.19e-1 $\pm$ 1.83e-6   | 4.57e-1 $\pm$ 8.45e-5   | 6.10e-2 $\pm$ 3.62e-3                   | 3.21e-2 $\pm$ 2.94e-3                               | 4.11e-1 $\pm$ 2.26e-4   | 3.78e-1 $\pm$ 7.52e-5   | 3.01e-1 $\pm$ 1.21e-4   | 3.10e-1 $\pm$ 4.47e-4   | 7.45e-2 $\pm$ 5.62e-3                   | 2.68e-2 $\pm$ 4.93e-3                   |
|                   | CELF [3]   | 4.87e-1 $\pm$ 8.97e-4   | 4.58e-1 $\pm$ 7.80e-4   | 4.00e-1 $\pm$ 1.35e-3   | 4.60e-1 $\pm$ 1.12e-3   | 2.52e-2 $\pm$ 4.43e-2                   | 1.43e-2 $\pm$ 1.05e-2                               | 3.80e-1 $\pm$ 1.40e-3   | 3.47e-1 $\pm$ 1.49e-3   | 3.17e-1 $\pm$ 6.99e-3   | 2.53e-1 $\pm$ 1.84e-3   | 7.01e-2 $\pm$ 1.62e-3                   | 3.59e-2 $\pm$ 3.62e-3                   |
|                   | MOEA [1]   | 4.96e-1 $\pm$ 1.84e-3   | 4.68e-1 $\pm$ 1.80e-3   | 4.17e-1 $\pm$ 1.19e-2   | 4.53e-1 $\pm$ 2.14e-3   | 1.97e-2 $\pm$ 2.79e-2                   | 3.00e-3 $\pm$ 4.94e-3                               | 3.59e-1 $\pm$ 6.40e-3   | 3.26e-1 $\pm$ 6.57e-3   | 3.15e-1 $\pm$ 7.68e-3   | 2.53e-1 $\pm$ 1.84e-3   | 4.97e-2 $\pm$ 5.52e-3                   | 2.11e-2 $\pm$ 5.25e-3                   |
|                   | MOEIM ( $\mathcal{I}^\uparrow\text{-}\mathcal{S}^\downarrow$ )                               | <u>4.99e-1 <math>\pm</math> 3.59e-4</u>                                   | <u>4.70e-1 <math>\pm</math> 5.59e-4</u>                                   | 4.18e-1 $\pm$ 4.33e-3   | 4.54e-1 $\pm$ 1.38e-3   | 0.00e0 $\pm$ 0.00e0                     | 3.43e-1 $\pm$ 1.08e-3                               | <u>4.13e-1 <math>\pm</math> 1.37e-3</u>                                   | <u>3.82e-1 <math>\pm</math> 1.36e-3</u>                                   | 3.52e-1 $\pm$ 1.63e-2   | 2.50e-1 $\pm$ 3.49e-4   | 7.06e-2 $\pm$ 5.46e-3                   | 1.67e-2 $\pm$ 6.13e-3                   |
|                   | MOEIM ( $\mathcal{I}^\uparrow\text{-}\mathcal{S}^\downarrow\text{-}\mathcal{C}^\uparrow$ )   | 4.95e-1 $\pm$ 1.66e-3   | <u>4.72e-1 <math>\pm</math> 3.53e-3</u>                                   | 4.17e-1 $\pm$ 1.01e-3   | 4.54e-1 $\pm$ 1.17e-3   | 1.92e-2 $\pm$ 2.49e-2                   | 3.00e-3 $\pm$ 4.51e-3                               | 4.09e-1 $\pm$ 1.20e-3   | <u>3.80e-1 <math>\pm</math> 1.20e-3</u>                                   | 3.52e-1 $\pm$ 1.69e-3   | 2.57e-1 $\pm$ 8.29e-4   | 7.90e-2 $\pm$ 4.57e-3                   | 2.76e-2 $\pm$ 4.90e-3                   |
|                   | MOEIM ( $\mathcal{I}^\uparrow\text{-}\mathcal{S}^\downarrow\text{-}\mathcal{F}^\uparrow$ )   | 4.95e-1 $\pm$ 6.50e-4   | 4.67e-1 $\pm$ 8.70e-4   | <u>4.64e-1 <math>\pm</math> 1.48e-3</u>                                     | 4.55e-1 $\pm$ 2.05e-3   | 5.67e-3 $\pm$ 4.11e-3                   | 2.31e-3 $\pm$ 5.26e-3                               | 4.06e-1 $\pm$ 1.39e-3   | 2.11e-1 $\pm$ 1.98e-3   | <u>3.89e-1 <math>\pm</math> 1.28e-3</u>                                     | 2.60e-1 $\pm$ 1.79e-3   | 7.61e-2 $\pm$ 2.28e-3                   | 3.40e-2 $\pm$ 4.03e-3                   |
|                   | MOEIM ( $\mathcal{I}^\uparrow\text{-}\mathcal{S}^\downarrow\text{-}\mathcal{B}^\downarrow$ ) | 4.91e-1 $\pm$ 1.82e-3   | 4.63e-1 $\pm$ 1.83e-3   | 4.41e-1 $\pm$ 4.53e-3   | <u>4.74e-1 <math>\pm</math> 1.79e-3</u>                                     | 1.38e-1 $\pm$ 2.81e-3                   | 3.45e-2 $\pm$ 3.63e-3                               | 3.94e-1 $\pm$ 9.16e-4   | 3.63e-1 $\pm$ 6.83e-4   | 3.61e-1 $\pm$ 8.82e-3   | 2.66e-1 $\pm$ 1.16e-3   | 8.83e-2 $\pm$ 6.41e-4                   | 3.07e-2 $\pm$ 1.45e-3                   |
|                   | MOEIM ( $\mathcal{I}^\uparrow\text{-}\mathcal{S}^\downarrow\text{-}\mathcal{T}^\downarrow$ ) | <u>4.98e-1 <math>\pm</math> 6.38e-4</u>                                   | 4.69e-1 $\pm$ 5.90e-4   | 4.22e-1 $\pm$ 8.15e-4   | <u>4.64e-1 <math>\pm</math> 3.20e-4</u>                                     | <u>1.60e-1 <math>\pm</math> 5.66e-3</u> | 4.57e-2 $\pm$ 3.57e-3                               | 4.10e-1 $\pm$ 7.90e-4   | 3.79e-1 $\pm$ 7.93e-4   | 3.61e-1 $\pm$ 5.31e-3   | 2.59e-1 $\pm$ 1.23e-3   | <u>9.04e-2 <math>\pm</math> 9.77e-4</u> | 3.56e-2 $\pm$ 8.13e-4                   |
|                   | MOEIM (all)  | 4.96e-1 $\pm$ 3.54e-4   | 4.61e-1 $\pm$ 6.62e-4   | 4.16e-1 $\pm$ 6.62e-4   | 4.66e-1 $\pm$ 1.19e-3   | <u>1.60e-1 <math>\pm</math> 3.90e-3</u> | <u>7.29e-2 <math>\pm</math> 3.17e-3</u>             | 3.59e-1 $\pm$ 6.39e-3   | 3.26e-1 $\pm$ 6.39e-3   | 3.51e-1 $\pm$ 3.36e-3   | 2.55e-1 $\pm$ 1.36e-3   | <u>4.79e-2 <math>\pm</math> 7.85e-4</u> | <u>2.00e-2 <math>\pm</math> 4.84e-4</u> |
| facebook-combined | GDD [6]  | 5.15e-1 $\pm$ 4.42e-5   | 3.65e-1 $\pm$ 4.30e-3   | 2.95e-1 $\pm$ 2.38e-5   | 4.62e-1 $\pm$ 2.92e-4   | 0.00e0 $\pm$ 0.00e0                     | 0.00e0 $\pm$ 0.00e0                                 | 2.07e-1 $\pm$ 9.42e-5   | 1.84e-1 $\pm$ 4.89e-5   | 8.76e-2 $\pm$ 3.38e-5   | 1.70e-1 $\pm$ 5.91e-5   | 4.66e-3 $\pm$ 3.30e-3                   | 1.81e-4 $\pm$ 1.91e-4                   |
|                   | CELF [3]   | 5.18e-1 $\pm$ 4.49e-4   | 3.56e-1 $\pm$ 1.00e-3   | 2.84e-1 $\pm$ 9.19e-3   | 4.73e-1 $\pm$ 1.66e-3   | 0.00e0 $\pm$ 0.00e0                     | 0.00e0 $\pm$ 0.00e0                                 | 2.46e-1 $\pm$ 1.10e-3   | 1.84e-1 $\pm$ 7.23e-4   | 1.17e-1 $\pm$ 6.10e-3   | 2.04e-1 $\pm$ 6.49e-4   | 2.33e-3 $\pm$ 3.20e-3                   | 1.18e-4 $\pm$ 2.05e-4                   |
|                   | MOEA [1]   | 5.02e-1 $\pm$ 4.35e-3   | 3.47e-1 $\pm$ 5.72e-3   | 2.08e-1 $\pm$ 3.57e-3   | 4.74e-1 $\pm$ 1.52e-3   | 0.00e0 $\pm$ 0.00e0                     | 0.00e0 $\pm$ 0.00e0                                 | 1.06e-1 $\pm$ 5.52e-3   | 1.52e-1 $\pm$ 7.79e-3   | 1.21e-1 $\pm$ 4.25e-3   | 1.71e-1 $\pm$ 3.28e-3   | 0.00e0 $\pm$ 0.00e0                     | 1.25e-4 $\pm$ 1.40e-4                   |
|                   | MOEIM ( $\mathcal{I}^\uparrow\text{-}\mathcal{S}^\downarrow$ )                               | <u>5.22e-1 <math>\pm</math> 8.35e-4</u>                                   | <u>3.78e-1 <math>\pm</math> 2.52e-3</u>                                   | 3.18e-1 $\pm$ 5.60e-3   | 4.80e-1 $\pm$ 4.46e-3   | 0.00e0 $\pm$ 0.00e0                     | 0.00e0 $\pm$ 0.00e0                                 | <u>2.50e-1 <math>\pm</math> 2.73e-4</u>                                   | <u>1.89e-1 <math>\pm</math> 9.27e-4</u>                                   | 1.49e-1 $\pm$ 9.40e-4   | 2.06e-1 $\pm$ 1.25e-3   | 8.34e-3 $\pm$ 3.15e-3                   | 2.56e-4 $\pm$ 2.42e-4                   |
|                   | MOEIM ( $\mathcal{I}^\uparrow\text{-}\mathcal{S}^\downarrow\text{-}\mathcal{C}^\uparrow$ )   | 5.19e-1 $\pm$ 1.07e-3   | <u>3.93e-1 <math>\pm</math> 1.88e-3</u>                                   | 3.10e-1 $\pm$ 9.67e-3   | 4.78e-1 $\pm$ 1.31e-3   | 0.00e0 $\pm$ 0.00e0                     | 1.58e-4 $\pm$ 5.01e-4                               | 2.46e-1 $\pm$ 1.13e-3   | <u>1.95e-1 <math>\pm</math> 8.76e-4</u>                                   | 1.45e-1 $\pm$ 3.07e-4   | 2.04e-1 $\pm$ 1.37e-3   | 4.09e-3 $\pm$ 2.98e-3                   | 2.42e-4 $\pm$ 2.69e-4                   |
|                   | MOEIM ( $\mathcal{I}^\uparrow\text{-}\mathcal{S}^\downarrow\text{-}\mathcal{F}^\uparrow$ )   | 5.14e-1 $\pm$ 1.13e-3   | 3.71e-1 $\pm$ 4.36e-4   | <u>3.73e-1 <math>\pm</math> 2.21e-3</u>                                     | <u>4.86e-1 <math>\pm</math> 3.54e-3</u>                                     | 3.18e-3 $\pm$ 4.49e-3                   | 1.13e-4 $\pm$ 4.39e-4                               | 2.41e-1 $\pm$ 1.75e-3   | 1.87e-1 $\pm$ 2.13e-3   | <u>1.79e-1 <math>\pm</math> 1.40e-3</u>                                     | 2.02e-1 $\pm$ 1.32e-3   | 8.66e-3 $\pm$ 2.61e-3                   | 6.99e-4 $\pm$ 6.83e-4                   |
|                   | MOEIM ( $\mathcal{I}^\uparrow\text{-}\mathcal{S}^\downarrow\text{-}\mathcal{B}^\downarrow$ ) | 5.09e-1 $\pm$ 1.40e-3   | 3.54e-1 $\pm$ 2.80e-3   | 3.28e-1 $\pm$ 6.31e-4   | <u>4.91e-1 <math>\pm</math> 1.28e-3</u>                                     | <u>7.66e-2 <math>\pm</math> 4.09e-3</u> | 6.22e-2 $\pm$ 1.52e-3                               | 2.41e-1 $\pm$ 2.13e-3   | 1.80e-1 $\pm$ 2.15e-3   | 1.48e-1 $\pm$ 2.27e-3   | <u>2.07e-1 <math>\pm</math> 1.47e-3</u>                                     | <u>1.70e-2 <math>\pm</math> 1.35e-3</u> | 2.12e-3 $\pm$ 6.82e-4                   |
|                   | MOEIM ( $\mathcal{I}^\uparrow\text{-}\mathcal{S}^\downarrow\text{-}\mathcal{T}^\downarrow$ ) | <u>5.21e-1 <math>\pm</math> 3.96e-4</u>                                   | 3.70e-1 $\pm$ 3.36e-3   | 3.11e-1 $\pm$ 1.25e-2   | 4.82e-1 $\pm$ 5.86e-4   | <u>7.53e-2 <math>\pm</math> 2.84e-3</u> | <u>1.14e-2 <math>\pm</math> 1.19e-4</u>             | <u>2.48e-1 <math>\pm</math> 7.09e-4</u>                                   | 1.88e-1 $\pm$ 1.06e-3   | 1.51e-1 $\pm$ 2.52e-3   | <u>2.06e-1 <math>\pm</math> 9.59e-4</u>                                     | <u>1.83e-2 <math>\pm</math> 1.35e-3</u> | <u>2.00e-3 <math>\pm</math> 3.63e-4</u> |
|                   | MOEIM (all)  | 5.19e-1 $\pm$ 1.66e-3   | 3.53e-1 $\pm$ 3.71e-3   | <u>3.50e-1 <math>\pm</math> 1.43e-3</u>                                     | 4.78e-1 $\pm$ 1.31e-3   | 6.40e-2 $\pm$ 2.02e-3                   | <u>9.22e-2 <math>\pm</math> 3.10e-3</u>             | 2.22e-1 $\pm$ 4.23e-3   | 1.66e-1 $\pm$ 3.35e-3   | 1.60e-1 $\pm$ 2.19e-3   | 1.83e-1 $\pm$ 3.30e-3   | 1.61e-2 $\pm$ 3.21e-4                   | <u>3.89e-3 <math>\pm</math> 3.53e-4</u> |
| amazon            | GDD [6]  | 5.33e-1 $\pm$ 1.11e-4   | 8.34e-3 $\pm$ 1.71e-4   | 7.01e-3 $\pm$ 4.94e-4   | 9.48e-3 $\pm$ 3.19e-5   | 8.50e-3 $\pm$ 1.65e-4                   | 1.33e-3 $\pm$ 1.56e-4                               | 1.64e-1 $\pm$ 1.64e-4   | 1.43e-1 $\pm$ 1.30e-4   | 1.04e-1 $\pm$ 1.07e-4   | 1.09e-1 $\pm$ 1.51e-4   | 6.95e-3 $\pm$ 2.25e-3                   | 1.23e-3 $\pm$ 6.63e-4                   |
|                   | CELF [3]   | 1.34e-2 $\pm$ 1.15e-4   | 7.84e-3 $\pm$ 2.36e-4   | 9.33e-3 $\pm$ 9.90e-5   | 8.58e-3 $\pm$ 6.39e-5   | 8.51e-3 $\pm$ 8.23e-5                   | 1.70e-3 $\pm$ 7.55e-5                               | 1.86e-1 $\pm$ 5.41e-3   | 1.64e-1 $\pm$ 5.68e-3   | 1.22e-1 $\pm$ 4.75e-3   | 1.57e-1 $\pm$ 5.12e-3   | 1.93e-3 $\pm$ 1.79e-3                   | 1.42e-4 $\pm$ 2.06e-4                   |
|                   | MOEA [1]   | <u>1.55e-2 <math>\pm</math> 1.22e-4</u>                                   | 9.54e-3 $\pm$ 1.43e-4   | 1.06e-2 $\pm$ 2.45e-4   | 8.34e-3 $\pm$ 3.02e-5   | 9.06e-3 $\pm$ 1.13e-4                   | 1.61e-3 $\pm$ 7.57e-5                               | <u>2.90e-1 <math>\pm</math> 1.20e-3</u>                                   | <u>2.66e-1 <math>\pm</math> 9.10e-4</u>                                   | 2.08e-1 $\pm$ 1.24e-3   | 2.06e-1 $\pm$ 2.21e-3   | 2.83e-3 $\pm$ 6.74e-4                   | 2.22e-4 $\pm$ 1.38e-4                   |
|                   | MOEIM ( $\mathcal{I}^\uparrow\text{-}\mathcal{S}^\downarrow$ )                               | 1.52e-2 $\pm$ 9.41e-5   | <u>9.73e-3 <math>\pm</math> 1.11e-4</u>                                   | <u>1.13e-2 <math>\pm</math> 2.22e-4</u>                                     | 8.28e-3 $\pm$ 6.27e-5   | 9.50e-3 $\pm$ 2.05e-4                   | 2.02e-3 $\pm$ 1.0e-4                                | 2.80e-1 $\pm$ 3.58e-3   | 2.61e-1 $\pm$ 3.37e-3   | 2.07e-1 $\pm$ 6.51e-3   | 2.00e-1 $\pm$ 3.09e-3   | 0.00e0 $\pm$ 0.00e0                     | 3.66e-4 $\pm$ 2.95e-4                   |
|                   | MOEIM ( $\mathcal{I}^\uparrow\text{-}\mathcal{S}^\downarrow\text{-}\mathcal{C}^\uparrow$ )   | 1.42e-2 $\pm$ 2.73e-5   | 8.75e-3 $\pm$ 8.07e-5   | <u>1.23e-2 <math>\pm</math> 1.55e-4</u>                                     | 8.57e-3 $\pm$ 3.50e-4   | 9.37e-3 $\pm$ 9.72e-5                   | 2.47e-3 $\pm$ 1.16e-4                               | 2.71e-1 $\pm$ 1.50e-3   | 2.49e-1 $\pm$ 1.53e-3   | <u>2.33e-1 <math>\pm</math> 1.83e-3</u>                                     | 1.90e-1 $\pm$ 1.83e-3   | 6.40e-3 $\pm$ 2.60e-3                   | 3.61e-4 $\pm$ 3.24e-4                   |
|                   | MOEIM ( $\mathcal{I}^\uparrow\text{-}\mathcal{S}^\downarrow\text{-}\mathcal{F}^\uparrow$ )   | 1.42e-2 $\pm$ 8.04e-5   | 8.52e-3 $\pm$ 1.50e-4   | 1.11e-2 $\pm$ 2.64e-4   | <u>1.05e-2 <math>\pm</math> 6.82e-5</u>                                     | 1.04e-2 $\pm$ 3.95e-4                   | 2.75e-3 $\pm$ 1.08e-4                               | 2.82e-1 $\pm$ 1.56e-3   | 2.59e-1 $\pm$ 8.24e-4   | <u>2.12e-1 <math>\pm</math> 1.37e-3</u>                                     | 2.19e-1 $\pm$ 2.68e-3   | 3.98e-3 $\pm$ 7.49e-4                   | 4.47e-4 $\pm$ 1.46e-4                   |
|                   | MOEIM ( $\mathcal{I}^\uparrow\text{-}\mathcal{S}^\downarrow\text{-}\mathcal{B}^\downarrow$ ) | 1.52e-2 $\pm$ 2.64e-4   | 9.27e-3 $\pm$ 3.50e-4   | 1.08e-2 $\pm$ 4.58e-4   | 9.27e-3 $\pm$ 9.40e-5   | <u>1.14e-2 <math>\pm</math> 1.94e-4</u> | 2.26e-3 $\pm$ 1.19e-4                               | 2.81e-1 $\pm$ 2.54e-3   | 2.57e-1 $\pm$ 2.44e-3   | 2.01e-1 $\pm$ 2.72e-3   | <u>1.98e-1 <math>\pm</math> 2.72e-3</u>                                     | <u>1.74e-2 <math>\pm</math> 1.26e-3</u> | 4.28e-3 $\pm$ 8.34e-4                   |
|                   | MOEIM ( $\mathcal{I}^\uparrow\text{-}\mathcal{S}^\downarrow\text{-}\mathcal{T}^\downarrow$ ) | 1.36e-2 $\pm$ 4.98e-4   | 7.86e-3 $\pm$ 8.30e-5   | <u>1.05e-2 <math>\pm</math> 3.63e-3</u>                                     | <u>1.01e-2 <math>\pm</math> 2.72e-3</u>                                     | <u>1.14e-2 <math>\pm</math> 1.94e-4</u> | <u>2.26e-3 <math>\pm</math> 1.19e-4</u>             | 2.46e-1 $\pm$ 9.91e-4   | 2.26e-1 $\pm$ 7.77e-4   | 1.90e-1 $\pm$ 7.77e-4   | 1.79e-1 $\pm$ 1.52e-3   | <u>1.57e-2 <math>\pm</math> 1.46e-3</u> | <u>9.28e-3 <math>\pm</math> 8.34e-4</u> |
|                   | MOEIM (all)  | 1.52e-2 $\pm$ 1.11e-4   | 8.34e-3 $\pm$ 1.71e-4   | 7.01e-3 $\pm$ 4.94e-4   | 9.48e-3 $\pm$ 3.19e-5   | 8.50e-3 $\pm$ 1.65e-4                   | 1.33e-3 $\pm$ 1.56e-4                               | 1.64e-1 $\pm$ 1.64e-4   | 1.43e-1 $\pm$ 1.30e-4   | 1.04e-1 $\pm$ 1.07e-4   | 1.09e-1 $\pm$ 1.51e-4   | 6.95e-3 $\pm$ 2.25e-3                   | 1.23e-3 $\pm$ 6.63e-4                   |
| wiki-vote         | GDD [6]  | 1.79e-1 $\pm$ 3.33e-5   | 1.70e-1 $\pm$ 4.82e-5   | 1.63e-1 $\pm$ 5.54e-5   | 1.59e-1 $\pm$ 7.96e-5   | 2.13e-2 $\pm$ 2.28e-3                   | 7.07e-3 $\pm$ 1.62e-3                               | 6.94e-2 $\pm$ 1.15e-3   | 6.26e-2 $\pm$ 2.61e-3   | 6.42e-2 $\pm$ 9.02e-3   | 5.10e-2 $\pm$ 2.64e-3   | 1.47e-2 $\pm$ 6.13e-4                   | 8.78e-3 $\pm$ 4.62e-5                   |
|                   | CELF [3]   | 1.76e-1 $\pm$ 1.01e-3   | 1.67e-1 $\pm$ 9.63e-4   | 1.35e-1 $\pm$ 9.49e-3   | 1.63e-1 $\pm$ 4.87e-4   | 5.96e-3 $\pm$ 8.43e-3                   | 8.40e-4 $\pm$ 4.47e-3                               | 8.06e-2 $\pm$ 8.86e-4   | 7.48e-2 $\pm$ 9.77e-4   | 6.88e-2 $\pm$ 2.10e-3   | <u>5.96e-2 <math>\pm</math> 4.37e-4</u>                                     | 1.66e-2 $\pm$ 7.10e-4                   | 9.01e-3 $\pm$ 2.32e-4                   |
|                   | MOEA [1]   | 1.67e-1 $\pm$ 1.62e-3   | 1.51e-1 $\pm$ 1.00e-3   | 1.55e-1 $\pm$ 1.55e-3   | 1.37e-1 $\pm$ 1.34e-3   | 0.00e0 $\pm$ 0.00e0                     | 0.00e0 $\pm$ 0.00e0                                 | 5.20e-2 $\pm$ 1.49e-3   | 4.69e-2 $\pm$ 1.75e-3   | 4.59e-2 $\pm$ 2.13e-3   | 3.13e-2 $\pm$ 9.34e-4   | 6.96e-3 $\pm$ 4.33e-4                   |   |
|                   | MOEIM ( $\mathcal{I}^\uparrow\text{-}\mathcal{S}^\downarrow$ )                               | <u>1.84e-1 <math>\pm</math> 1.24e-4</u>                                   | <u>1.75e-1 <math>\pm</math> 1.96e-4</u>                                   | 1.45e-1 $\pm$ 2.03e-3   | 1.68e-1 $\pm$ 6.91e-4   | 1.66e-2 $\pm$ 4.14e-3                   | 4.69e-3 $\pm$ 2.62e-3                               | <u>8.55e-2 <math>\pm</math> 6.97e-3</u>                                   | <u>8.02e-2 <math>\pm</math> 1.02e-4</u>                                   | 7.82e-2 $\pm$ 1.11e-3   | 5.60e-2 $\pm$ 7.76e-5   | 1.80e-2 $\pm$ 2.54e-4                   | 9.25e-3 $\pm$ 1.51e-4                   |
|                   | MOEIM ( $\mathcal{I}^\uparrow\text{-}\mathcal{S}^\downarrow\text{-}\mathcal{C}^\uparrow$ )   | 1.81e-1 $\pm$ 4.71e-4   | <u>1.74e-1 <math>\pm</math> 4.62e-4</u>                                   | 1.58e-1 $\pm$ 5.29e-3   | 1.67e-1 $\pm$ 6.26e-4   | 1.70e-2 $\pm$ 8.50e-4                   | 5.10e-3 $\pm$ 1.16e-3                               | 8.28e-2 $\pm$ 1.97e-4   | 7.92e-2 $\pm$ 2.76e-4   | <u>8.08e-2 <math>\pm</math> 2.49e-4</u>                                     | 5.57e-2 $\pm$ 2.95e-4   | 1.77e-2 $\pm$ 1.23e-4                   | 1.02e-2 $\pm$ 2.30e-4                   |
|                   | MOEIM ( $\mathcal{I}^\uparrow\text{-}\mathcal{S}^\downarrow\text{-}\mathcal{F}^\uparrow$ )   | 1.81e-1 $\pm$ 4.93e-4   | 1.72e-1 $\pm$ 4.26e-4   | <u>1.78e-1 <math>\pm</math> 4.12e-4</u>                                     | 1.67e-1 $\pm$ 6.52e-4   | 1.11e-2 $\pm$ 3.46e-3                   | 4.77e-3 $\pm$ 1.29e-3                               | 8.21e-2 $\pm$ 2.64e-4   | 7.89e-2 $\pm$ 2.71e-4   | <u>8.02e-2 <math>\pm</math> 9.01e-4</u>                                     | 5.64e-2 $\pm$ 2.81  |   |   |

Table 2. Results of the statistical tests, performed by using the Holm-Bonferroni procedure [2], using “MOEIM (all)” as reference (Rank=8.82). Note that, for simplicity of notation, we omit “MOEIM” from the other settings of our algorithm. “Rank” refers to the ranking of the algorithm (the higher, the better). “z” is the statistic computed on the rank, “p” is the  $p$ -value, computed as the value of the cumulative normal distribution related to z.  $\alpha/j$  is the significance threshold adjusted for each row of the table. “Reject” indicates if the sequential null hypothesis (of equivalence between the reference and the algorithm on the row) can be rejected or not.

| Method                                 | Rank     | z         | p        | $\alpha/j$ | Rejected |
|--|----------|-----------|----------|------------|----------|
| $I^\uparrow.S^\downarrow.B^\downarrow$ | 6.56e+00 | -4.71e+00 | 1.21e-06 | 5.00e-02   | True     |
| $I^\uparrow.S^\downarrow.T^\downarrow$ | 6.52e+00 | -4.83e+00 | 6.76e-07 | 2.50e-02   | True     |
| $I^\uparrow.S^\downarrow.F^\uparrow$   | 4.97e+00 | -9.22e+00 | 1.54e-20 | 1.67e-02   | True     |
| $I^\uparrow.S^\downarrow.C^\uparrow$   | 3.55e+00 | -1.32e+01 | 3.24e-40 | 1.25e-02   | True     |
| GGD                                    | 3.46e+00 | -1.35e+01 | 9.96e-42 | 1.00e-02   | True     |
| CELF                                   | 3.36e+00 | -1.38e+01 | 2.31e-43 | 8.33e-03   | True     |
| MOEA                                   | 2.27e+00 | -1.69e+01 | 5.01e-64 | 7.14e-03   | True     |
| $I^\uparrow.S^\downarrow$              | 2.15e+00 | -1.72e+01 | 1.79e-66 | 6.25e-03   | True     |

Table 3. Results of the statistical tests using the Tukey HSD procedure [5] for pairwise comparisons. Note that, for simplicity of notation, we omit “MOEIM” from the settings of our algorithm. “Mean difference” refers to the mean of the difference on the hypervolume achieved by the methods in each pairwise comparison, “p-adj” refers to the adjusted  $p$ -value, “Lower CI” and “Upper CI” refer to the lower and upper bound for the 95% CI (Confidence Interval). “Reject” indicates if the null hypothesis (of statistical equivalence between Methods 1 and 2) can be rejected or not. We highlight the cases for which the hypothesis can be rejected.

| Method 1                               | Method 2                               | Mean difference | p-adj   | Lower CI | Upper CI | Rejected |
|--|--|-----------------|---------|----------|----------|----------|
| $I^\uparrow.S^\downarrow$              | $I^\uparrow.S^\downarrow.B^\downarrow$ | 6.90e-3         | 3.00e-4 | 2.10e-3  | 1.16e-2  | True     |
| $I^\uparrow.S^\downarrow$              | $I^\uparrow.S^\downarrow.C^\uparrow$   | 1.40e-3         | 9.93e-1 | -3.40e-3 | 6.20e-3  | False    |
| $I^\uparrow.S^\downarrow$              | $I^\uparrow.S^\downarrow.F^\uparrow$   | 2.10e-3         | 9.06e-1 | -2.70e-3 | 6.90e-3  | False    |
| $I^\uparrow.S^\downarrow$              | $I^\uparrow.S^\downarrow.T^\downarrow$ | 7.70e-3         | 0.00e+0 | 3.00e-3  | 1.25e-2  | True     |
| $I^\uparrow.S^\downarrow$              | all                                    | 1.20e-2         | 0.00e+0 | 7.30e-3  | 1.68e-2  | True     |
| $I^\uparrow.S^\downarrow$              | CELF                                   | 2.40e-3         | 9.80e-1 | -4.60e-3 | 9.40e-3  | False    |
| $I^\uparrow.S^\downarrow$              | GGD                                    | 3.80e-3         | 2.62e-1 | -1.00e-3 | 8.50e-3  | False    |
| $I^\uparrow.S^\downarrow$              | MOEA                                   | 0.00e+0         | 1.00e+0 | -4.70e-3 | 4.80e-3  | False    |
| $I^\uparrow.S^\downarrow.B^\downarrow$ | $I^\uparrow.S^\downarrow.C^\uparrow$   | -5.50e-3        | 1.09e-2 | -1.03e-2 | -7.00e-4 | True     |
| $I^\uparrow.S^\downarrow.B^\downarrow$ | $I^\uparrow.S^\downarrow.F^\uparrow$   | -4.80e-3        | 5.11e-2 | -9.50e-3 | 0.00e+0  | False    |
| $I^\uparrow.S^\downarrow.B^\downarrow$ | $I^\uparrow.S^\downarrow.T^\downarrow$ | 9.00e-4         | 1.00e+0 | -3.90e-3 | 5.60e-3  | False    |
| $I^\uparrow.S^\downarrow.B^\downarrow$ | all                                    | 5.20e-3         | 2.28e-2 | 4.00e-4  | 9.90e-3  | True     |
| $I^\uparrow.S^\downarrow.B^\downarrow$ | CELF                                   | -4.50e-3        | 5.53e-1 | -1.15e-2 | 2.50e-3  | False    |
| $I^\uparrow.S^\downarrow.B^\downarrow$ | GGD                                    | -3.10e-3        | 5.19e-1 | -7.90e-3 | 1.60e-3  | False    |
| $I^\uparrow.S^\downarrow.B^\downarrow$ | MOEA                                   | -6.90e-3        | 3.00e-4 | -1.16e-2 | -2.10e-3 | True     |
| $I^\uparrow.S^\downarrow.C^\uparrow$   | $I^\uparrow.S^\downarrow.F^\uparrow$   | 7.00e-4         | 1.00e+0 | -4.00e-3 | 5.50e-3  | False    |
| $I^\uparrow.S^\downarrow.C^\uparrow$   | $I^\uparrow.S^\downarrow.T^\downarrow$ | 6.40e-3         | 1.30e-3 | 1.60e-3  | 1.11e-2  | True     |
| $I^\uparrow.S^\downarrow.C^\uparrow$   | all                                    | 1.07e-2         | 0.00e+0 | 5.90e-3  | 1.54e-2  | True     |
| $I^\uparrow.S^\downarrow.C^\uparrow$   | CELF                                   | 1.00e-3         | 1.00e+0 | -6.00e-3 | 8.00e-3  | False    |
| $I^\uparrow.S^\downarrow.C^\uparrow$   | GGD                                    | 2.40e-3         | 8.36e-1 | -2.40e-3 | 7.10e-3  | False    |
| $I^\uparrow.S^\downarrow.C^\uparrow$   | MOEA                                   | -1.40e-3        | 9.94e-1 | -6.10e-3 | 3.40e-3  | False    |
| $I^\uparrow.S^\downarrow.F^\uparrow$   | $I^\uparrow.S^\downarrow.T^\downarrow$ | 5.60e-3         | 8.10e-3 | 8.00e-4  | 1.04e-2  | True     |
| $I^\uparrow.S^\downarrow.F^\uparrow$   | all                                    | 9.90e-3         | 0.00e+0 | 5.10e-3  | 1.47e-2  | True     |
| $I^\uparrow.S^\downarrow.F^\uparrow$   | CELF                                   | 3.00e-4         | 1.00e+0 | -6.80e-3 | 7.30e-3  | False    |
| $I^\uparrow.S^\downarrow.F^\uparrow$   | GGD                                    | 1.60e-3         | 9.79e-1 | -3.10e-3 | 6.40e-3  | False    |
| $I^\uparrow.S^\downarrow.F^\uparrow$   | MOEA                                   | -2.10e-3        | 9.12e-1 | -6.90e-3 | 2.70e-3  | False    |
| $I^\uparrow.S^\downarrow.T^\downarrow$ | all                                    | 4.30e-3         | 1.17e-1 | -5.00e-4 | 9.10e-3  | False    |
| $I^\uparrow.S^\downarrow.T^\downarrow$ | CELF                                   | -5.40e-3        | 3.03e-1 | -1.24e-2 | 1.70e-3  | False    |
| $I^\uparrow.S^\downarrow.T^\downarrow$ | GGD                                    | -4.00e-3        | 1.89e-1 | -8.80e-3 | 8.00e-4  | False    |
| $I^\uparrow.S^\downarrow.T^\downarrow$ | MOEA                                   | -7.70e-3        | 0.00e+0 | -1.25e-2 | -2.90e-3 | True     |
| all                                    | CELF                                   | -9.70e-3        | 7.00e-4 | -1.67e-2 | -2.60e-3 | True     |
| all                                    | GGD                                    | -8.30e-3        | 0.00e+0 | -1.31e-2 | -3.50e-3 | True     |
| all                                    | MOEA                                   | -1.20e-2        | 0.00e+0 | -1.68e-2 | -7.20e-3 | True     |
| CELF                                   | GGD                                    | 1.40e-3         | 1.00e+0 | -5.70e-3 | 8.40e-3  | False    |
| CELF                                   | MOEA                                   | -2.40e-3        | 9.81e-1 | -9.40e-3 | 4.70e-3  | False    |
| GGD                                    | MOEA                                   | -3.70e-3        | 2.71e-1 | -8.50e-3 | 1.00e-3  | False    |

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