

## Worksheet #4

**1. Describe in informal terms why the language  $A = \{0^n 1^n \text{ for } n > 0\}$  is not regular.**

In order to match the number of 0s and 1s the machine would need an arbitrarily large amount of memory. Without that it couldn't count the 0s for every word in the language. There must be a finite amount of states and that does not allow for counting the number of 0s or to then reference that when counting the 1s.

**2. Is the language  $A = \{0^n 1^n \text{ for } 0 < n < 1024\}$  regular? Why or why not?**

Yes it is. Because  $n$  can't be arbitrarily large (only being able to range from 0 to 1024) we can create a machine with a finite number of states that accepts the entire language. As long as we set the number of states equal to  $n$ , so that it can count zeros and then count backwards the same number using 1s to the final state the machine should accept any string in that range of  $0 < n < 1024$ .

**3. For  $\Sigma = \{0, 1\}$ , use the pumping lemma to prove that the language  $A = \{ww^r : w^r \text{ is } w \text{ in reverse order}\}$  is not regular.**

Assume  $A$  is a regular language, so there exists a pumping length  $p$ .

Consider  $s = 0^p 1^p 1^p 0^p = ww^r$

Clearly  $s \in A$ .

$|s| = |xyz| = 4p$ , so  $|s| > p$

So, by the pumping lemma,  $s$  can be written as  $s = xyz$  where  $y \neq \epsilon$ ,  $|xy| \leq p$ , and  $xy^iz \in A$  for all  $i \geq 1$ .

So by pumping lemma,  $|xy| \leq p$  and  $y$  must contain only 0's

Since  $y \neq \epsilon$ ,  $y$  must contain at least one 0

So  $y = 0^k$ , for some  $k > 0$

If  $y$  is pumped up and  $k$  surpasses the value of  $p$ , there will be more 0's at the beginning of the string than at the end, so by contradiction this must not be a regular language.

**4. For  $\Sigma = \{a, b, c\}$ , use the pumping lemma to prove that the language  $A = \{a^n b^n c^n \text{ for } n > 0\}$  is not regular.**

Assume  $A$  is a regular language, so there exists a pumping length  $p$ .

Consider  $w = a^p b^p c^p$

Clearly  $w \in A$ .

$|w| = |xyz| = 3p$

$|w| > p$

So, by the pumping lemma,  $w$  can be written as  $w = xyz$  where  $y \neq \epsilon$ ,  $|xy| \leq p$ , and  $xy^iz \in A$  for all  $i \geq 1$ .

So by pumping lemma,  $|xy| \leq p$  and  $y$  must contain only a's

Since  $y \neq \epsilon$ ,  $y$  must contain at least one  $a$

So  $y = a^k$ , for some  $k > 0$

Oh no!! There will be more  $a$ 's than the other symbols which will make the resulting string not in the language, but by the pumping lemma the resulting string should be, so this is a contradiction.