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Worksheet #11

Enumerability and More Decidability

1. What does it mean for a language to be enumerable?

A language, L, is enumerable if there exists a TM, M, (called a recognizer) such that on input w:

if $w \in L$, M terminates in the accept state

if $w \notin L$, M either terminates in the reject state OR loops forever

2. Is every decidable language enumerable? Is every enumerable language decidable?

Every decidable language is enumerable but not every enumerable language is decidable.

3. Is it possible for languages to exist that are not enumerable? If so, describe such a language. If not, then give a proof idea for why not.

Yes!! The language $\bar{A}_{TM}$ is not enumerable. If it were, A_{TM} would be decidable, but we KNOW it isn't.

4. Suppose you know that a language and its complement are both enumerable. What does this tell you about the decidability of the language?

If a language and its complement are both enumerable then the language itself is decidable.

5. How many languages are there over the given alphabet $\Sigma = \{0, 1\}$?

An infinitely uncountable amount, since a language can have an infinite length! Each string within a language must have a finite length, but there are an infinitely uncountable number of these. We saw this in step 1 of proving that the set of all languages is infinitely uncountable.

6. How many TMs are there?

   Each TM can be described by a finite length string.

   Each string can be converted to binary and added to a set S.

   We can put the S in lexicographical order with shorter strings first.

   Thereafter, we can make a bijection with $\mathbb{N}$. Since S is in order, we can map each element of S to a natural number.

   Any set that is finite or has a bijection (one-to-one and onto) with $\mathbb{N}$ is said to be countable.

   Thus, there are a countable number of TMs.