Josh, Ella, Richard

Worksheet #6
Pushdown Automata

1. When does a PDA stop executing?
A PDA stops executing when the stack becomes empty ($ has been popped).

2. Define a PDA that accepts the language {ww^r where w^r is w reversed}. Make your instructions of the form:

QAB → Q'A'W

where Q is the current state, A is the current string symbol, B is the popped stack symbol, Q' is the resulting state, A' is the string movement, and W is the string to be pushed on the stack. Use a binary alphabet.

Σ = {0,1}
Γ = {$, X,Y} (X represents zero, Y represents one)
Q = {q, q'}
q = q

Instructions

q0$ -> qR$X stack is empty, but reject, because entire string has not been read
q0Y -> qRYX continue scanning
q0X -> qRXX continue scanning
q0X-> q'Rε move into reverse state, do not add to stack

q1$ -> qR$Y stack is empty, but reject, because entire string has not been read
q1X -> qRXY continue scanning
q1Y -> qRYY continue scanning
q1Y -> q'Rε move to reverse state, do not add to stack

q'1$ -> q'N$ stack is empty, but reject, because entire string has not been read
q'0$ -> q'N$ stack is empty, but reject, because entire string has not been read
q'1X -> q'NX infinite loop (reject)
q'0X -> q'Rε pop from stack, continue scanning
q'1Y -> q'Rε pop from stack, continue scanning
q'0Y -> q'NY infinite loop (reject)

q'▭$ -> q'Nε (accept)
q'▭Y -> q'NY infinite loop (reject)
q'▭X-> q'NY infinite loop (reject)

q▭Y -> qNY infinite loop (reject)
q▭X-> qNY infinite loop (reject)
q▭$->qN$ infinite loop (reject)

3. Using your PDA from Question #2, show a series of matched instructions that are executed when working on the two following strings:

00111100

Instructions	**Stack**
q0$ -> qR$X	$X
q0X -> qRXX	$XX
q1X -> qRXY	$XXY
q1Y -> qRYY	$XXYY
q1Y -> q’Rε	$XXY
q’1Y -> q’Rε	$XX
q’0X -> q’Rε	$X
q’0X -> q’Rε	$
q’▭$ -> q’Nε	Accept!

0101

Instructions	**Stack**
q0$ -> qR$X	$X
q1X -> qRXY	$XY
q0Y -> qRYX	$XYX
q1X -> qRXY	$XYXY
q▭Y-> qNY	infinite loop (reject)

4. Use the CFL pumping lemma to prove that the language L = {a^n : where n is a perfect square} is not context-free.

Assume L is a CFL, so there exists a pumping length p >= 1.

Consider w = $a^{p^\wedge 2}$
Clearly a E L

By the pumping lemma, $|vxy| \leq p$
Then every w E L with $|w| \geq p$ can be written as w = uvxyz
$1 \leq |vxy| \leq p \leq |w|$
Consider the case uv^2xy^2z:
We know $|uvxyz| = n = p^2$
$|uv^2xy^2z| = |v| + |y| + |uvxyz| = p^2 + |v| + |y|$
Since we know |vy| >= 1 and |vxy| <= p,
$1 \leq |vy| \leq p$.
So, $|uv^2xy^2z|$ is in the range $[p^2 + 1, p^2 + p]$

But we know that n, the length of w, has to be perfect square, so the next valid string can only be of length $(p+1)^2 = p^2 + 2p + 1$, which is out of range of $p^2 + 1$ and $p^2 + p$. Contradiction! Therefore, the language L = {a^n : where n is a perfect square} is not context-free.