Ella, Chidera, Takuto

Worksheet #1
Introduction and Mathematical Background

1. In one sentence each, define **complexity theory**, **computability theory**, and **automata theory**.

complexity theory: Complexity theory classifies problems according to their difficulty, often by the order of the algorithm, or big O time.

computability theory: Some problems are solvable and some are unsolvable.

automata theory: Automata theory defines the properties and limitations of various computational models.

2. Define each of the following mathematical sets and give three examples of elements of each set: N, Z, Q, N x Q, and R.

	N	**Z**	**Q**	**N x Q**	**R**
Definition	Set of natural numbers	Set of integers {..., -3, -2, -1, 0, 1, 2, 3, ...}	Set of rational numbers. x/y where x and y are both ints and y is not zero. $Q=\{m/n : m \in Z, n \in Z, n \neq 0\}$	Set of ordered pairs of each combination of elements from both sets; Cartesian product. $N \times Q = \{(x, y) : x \in N \text{ and } y \in Q\}$	Set of real numbers
Example	1	-1	7	(1,7)	$\sqrt{3}$
Example	9	-9	11/3	(9, 11/3)	1.7
Example	25	-25	15/2	(25, 15/2)	π

3. Write down any subset (⊆) relationships between the mathematical sets described in Question #2.

$N \subseteq Z$

$Q \subseteq R$
$N \subseteq R$
$Z \subseteq R$

4. Suppose we have the set A = {2, 3, 5, 7}. Write down all the elements in $\mathscr{P}$(A), the power set of A.

$\mathscr{P}$(A)={{}, {2}, {3}, {5}, {7}, {2, 3}, {2, 5}, {2, 7}, {3, 5}, {3, 7}, {5, 7}, {2, 3, 5}, {2, 3, 7}, {2, 5, 7}, {3, 5, 7}, {2, 3, 5, 7}}
$|\mathscr{P}(A)|=2^{|A|}= 16$

5. Give an example string **w** over the alphabet Σ = {a, b, c, d}. Also, give |**w**|.

w = abc

|**w**| = 3

6. Prove that any subset of size (n+1) from {1, 2, 3, ..., 2n}, contains (at least) two numbers that are consecutive.

Proof by contradiction:

Suppose any subset of size (2n) from {1,2,3, …,2n}, doesn't contain any pairs of two numbers that are consecutive. Only half of the elements would be valid in a given subset, because for any element selected, the following element is invalid (1).

1. The maximum size of a subset containing no consecutive elements is $\frac{2n}{2}$.
2. Assume $\frac{2n}{2} \geq n + 1$.
3. Reduce to $n \geq n + 1$. This cannot be true because n will always be less than n+1.
4. Therefore, since there are no possible subsets of size n+1 containing no consecutive numbers, our assumption is

incorrect and any subset of size n+1 must contain at least two consecutive numbers.

□