Tarea Sedra 13 / del TP lab 1 [TSB] 1.

$$T(s) = \frac{15}{s^3 + 6s^2 + 15s + 15} = S_{21}$$

$$|S_{21}|^2 = \frac{15}{s^3 + 6s^2 + 15s + 15} \cdot \frac{15}{(-s)^3 + 6(-s)^2 + 15(-s) + 15}$$

$$-s^3 + 6s^2 - 15s + 15$$

$$= \frac{15^2}{-s^6 + 6s^5 - 15s^4 + 15s^3 + -6s^5 + 36s^4 - 90s^3 + 90s^2 - 15s^4}$$

$$+ 90s^3 - 225s^2 + 225s - 15s^3 + 90s^2 - 225s + 225$$

$$\frac{225}{-s^6 + 6s^4 - 45s^2 + 225}$$

Polos de P(s)
$(\pm 2,2032026 \pm 1,3465413j)$
$(0 \pm 0j)$

$$|S_{11}|^2 + |S_{21}|^2 = 1 \implies |S_{11}|^2 = 1 - |S_{21}|^2$$

Polos de Q(s)
$(\pm 2,41029567 + 0j)$
$(\pm 1,83309155 \pm 1,80692757j)$

$$|S_{11}|^2 = \frac{-s^6 + 6 \cdot s^4 - 45 \cdot s^2 + 225 - 225}{-s^6 + 6s^4 - 45 \cdot s^2 + 225} = \frac{-s^6 + 6 \cdot s^4 - 45 s^2}{-s^6 + 6 \cdot s^4 - 45 s^2 + 225}$$

Buscando las raíces nos quedamos con

$$S_{11} = \frac{s^3 + \frac{22}{5} \cdot s^2 + \frac{67}{10} s}{s^3 + 6s^2 + 15s + 15}$$

$$Z = \frac{S_{11} + 1}{1 - S_{11}}$$

$$S_{11}(s) = \frac{P(s)}{Q(s)} \quad ; \quad P(s) = S(s^2 + 2 \cdot 2,2 \cdot S + 2,2^2 + 1,36^2)$$

Redondeando a dos decimal

$$P(s) = S(s^2 + 4,4S + 6,25)$$

$$Q(s) = (\cancel{s^2 + 2 \cdot 2,4 \cdot s + 2,4^2})(s^2 + 2 \cdot 1,83S + 1,83^2 + 1,8^2)$$

$$Q(s) = (s^2 + 3,66 \cdot S + 6,59)(S + 2,41)$$

$$Q(s) = s^3 + 2,41s^2 + 3,66 \cdot s^2 + 6,07 \cdot S + 6,59 \cdot S + 15,88$$

$$Q(s) = s^3 + 6,07 s^2 + 12,66 \cdot S + 15,88$$

$$S_{11}(s) = \frac{S(s^2 + 4,4S + 6,25)}{s^3 + 6,07 s^2 + 12,66 \cdot S + 15,88}$$

$$S_{11} = \frac{Z - R_{01}}{Z + R_{01}} \quad \Rightarrow \quad S_{11}(Z + R_{01}) = Z - R_{01} \quad \Rightarrow \quad S_{11} \cdot Z + S_{11} \cdot R_{01} = Z - R_{01}$$

$$Z(S_{11} - 1) = -R_{01} - S_{11} \cdot R_{01} \quad \Rightarrow \quad Z = \frac{-R_{01} - S_{11} R_{01}}{S_{11} - 1}$$

con $R_{01} = 1\Omega$

$$Z = \frac{S_{11} + 1}{1 - S_{11}} \qquad \cancel{R = S}$$

$$Z = \frac{\dfrac{S(s^2 + 4,4S + 6,25) + s^3 + 6,07s^2 + 12,66s + 15,88}{s^3 + 6,07s^2 + 12,66s + 15,88}}{\dfrac{s^3 + 6,07s^2 + 12,66s + 15,88 - s^3 - 4,4s^2 - 6,25s}{s^3 + 6,07s^2 + 12,66s + 15,88}}$$

$$Z = \frac{2S^3 + 10,47.S^2 + 18,91S + 15,88}{1,62.S^2 + 6,41.S + 15,88}$$

Si redondeamos nos queda

$$Z = \frac{2S^3 + 10.S^2 + 19S + 16}{2S^2 + 6S + 16}$$

Aplicando Cauer tramos

$$
\begin{array}{r|l}
2S^3 + 10S^2 + 19S + 16 & \underline{2S^2 + 6S + 16} \\
-\ \underline{2S^3 + 6S^2 + 16S} & S \\
2S^2 + 6S + 16 &
\end{array}
$$

$$
\begin{array}{r|l}
2S^2 + 6S + 16 & \underline{4S^2 + 3S + 16} \\
\underline{2S^2 + 3/2 S + 8} & \frac{1}{2} \\
\end{array}
$$

$$
\begin{array}{r|l}
4S^2 + 3S + 16 & \underline{9/2 S + 8} \\
\underline{4S^2 + 64/9 S} & 8/9 \cdot S \\
-37/9 S + 16 & \\
\end{array}
$$

$$9/2 S + 8 \qquad \begin{array}{|l} -37/9 S + 16 \end{array}$$

No se puede por falta de decimales

$$
\begin{array}{r|l}
16 + 19S + 10S^2 + 2S^3 & \underline{16 + 6S + 2S^2} \\
-\ \underline{16 + 6S + 2S^2} & 1 \\
\end{array}
$$

$$
\begin{array}{r|l}
16 + 6.S + 2S^2 & \underline{13S + 8S^2 + 2S^3} \\
\underline{16 + \frac{128}{13}.S + \frac{32}{13}.S^2} & 16/13 \cdot \frac{1}{S} \\
\end{array}
$$